Kinematic Evaluation of Virtual Walking Trajectories

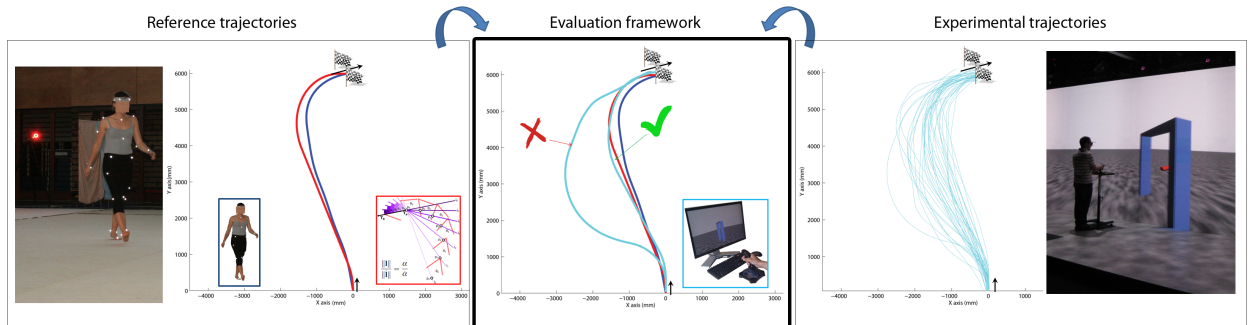Gabriel Cirio, Anne-Hélène Olivier, Maud Marchal, and Julien Pettré, *Member, IEEE*

Fig. 1. The objective of this paper is to evaluate various virtual locomotion conditions by comparing reference with virtual trajectories formed during goal-directed locomotion tasks. Reference trajectories (left) can be recorded through motion capture or be generated through a numerical model of human locomotion. The paper demonstrates the framework (center) over a set of experimental trajectories (right). For the purpose of demonstration, this paper compares frequently used virtual locomotion conditions.

Abstract—Virtual walking, a fundamental task in Virtual Reality (VR), is greatly influenced by the locomotion interface being used, by the specificities of input and output devices, and by the way the virtual environment is represented. No matter how virtual walking is controlled, the generation of realistic virtual trajectories is absolutely required for some applications, especially those dedicated to the study of walking behaviors in VR, navigation through virtual places for architecture, rehabilitation and training. Previous studies focused on evaluating the realism of locomotion trajectories have mostly considered the result of the locomotion task (efficiency, accuracy) and its subjective perception (presence, cybersickness). Few focused on the locomotion trajectory itself, but in situation of geometrically constrained task. In this paper, we study the realism of unconstrained trajectories produced during virtual walking by addressing the following question: did the user reach his destination by virtually walking along a trajectory he would have followed in similar real conditions? To this end, we propose a comprehensive evaluation framework consisting on a set of trajectographical criteria and a locomotion model to generate reference trajectories. We consider a simple locomotion task where users walk between two oriented points in space. The travel path is analyzed both geometrically and temporally in comparison to simulated reference trajectories. In addition, we demonstrate the framework over a user study which considered an initial set of common and frequent virtual walking conditions, namely different input devices, output display devices, control laws, and visualization modalities. The study provides insight into the relative contributions of each condition to the overall realism of the resulting virtual trajectories.

Index Terms—Locomotion, evaluation, motor control, vision, perception-action

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1 INTRODUCTION

Virtually walking in virtual environments is a fundamental requirement in numerous virtual reality (VR) applications. A wide range of devices and techniques have been proposed to achieve virtual locomotion, while a wide range of visualization modalities influence the way virtual locomotion is performed. In this paper, we propose to evaluate this set of virtual locomotion conditions that influence the *interactive virtual locomotion control loop*: does it allow users to steer their locomotion toward their goal following the same trajectory they would have followed in similar real conditions?

Recent work in Neuroscience has shown that humans perform basic locomotion tasks in a stereotyped manner [14]. In this context, stereo-typy means that, among the infinite possible trajectories *joining* A to B (two oriented points), all humans follow similar trajectories when *walking* from A to B. It is relevant to propose virtual locomotion conditions that preserve this specific steering, i.e. that naturally induce users to generate trajectories during goal-directed locomotion which conform to real ones. We particularly believe that this would open the VR field to the study of locomotion behaviors. This would allow to address challenging questions, such as studying how users would really navigate a new building at the stage of digital mockup, studying complex locomotion behaviors by leveraging the use of fully controlled virtual environments, and further motivate the use of VR for learning and training tasks.

Humans control their locomotion from a combination of sensory inputs from visual, proprioceptive and vestibular systems. In VR, the only way to preserve congruent and complete sensory input while virtually walking through a virtual environment is to enable users to perform real natural walking. Unfortunately, the physical limits of VR devices and VR displays, and the limited availability of large immersive displays, prevent using natural walking to walk more than a few steps. This issue received *a lot* of attention and numerous metaphors and devices were proposed. Considering, e.g., the Cyberwalk [35] on one hand and a simple keyboard-based technique on the other hand, one understands the *very large* variety of existing solutions. In addition, since virtual locomotion relies mostly on visual feedback, the

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many factors that influence the perception of the virtual environment also influence virtual locomotion. Among these factors are visualization modalities (viewpoint, field of view, stereoscopic vision, contrast, etc) and output display devices (desktop screen, head-mounted displays, CAVE-like environments, etc). Given different sets of locomotion conditions (i.e. different combinations of locomotion techniques, input and output devices, and visualization modalities), how do the generated trajectories compare? Do users steer their motion as in reality?

We argue that evaluating and comparing different locomotion conditions on this specific aspect is important. Beyond some basic and easily accessible properties (cost, required device, constraints, etc) and subjective questionnaires (cybersickness, presence), several objective quantitative criteria were proposed to compare locomotion conditions. In the evaluation of many VR interaction techniques, some performance and accuracy indicators were proposed to rank various different approaches accordingly. More specifically to navigation and locomotion behaviors, spatial judgment tasks were designed to evaluate subjective user appreciation [25]. Kinematic analysis of walking trajectories were also proposed [41]. However, geometrically constrained locomotion tasks were considered (walking in corridors): the control of speed with respect to time-to-collision could be analyzed. Our paper considers *unconstrained* goal-directed locomotion tasks: does the trajectory for virtually walking from A to B conform to reality?

Our contributions are the following:

1. a trajectography-based approach for the evaluation of different locomotion conditions in terms of conformity of the generated trajectories to their real counterpart. At the opposite of previous approaches, this work considers how users continuously steer their motion.

2. a complete framework, open and available to the community, for anyone to perform new evaluations. The framework incorporates a set of metrics to compare experimental virtual trajectories with reference ones, a set of real trajectories of goal-directed locomotion for reference, and a model to generate new reference trajectories. The model enables considering new situations which are not captured by the set of real motions. The suggested metrics analyze some global properties of trajectories (e.g., trajectory duration, length), some local characteristics (e.g., presence of stops, collisions with the target, etc.), as well as continuous variables (e.g., overall shape, speed profile, path curvature, etc.).

3. a set of experiments conducted to serve two purposes: (1) illustrate our framework through a wide range of experimental data, and (2) provide an initial assessment of the conformity of virtual trajectories to their real counterpart under a set of commonly used conditions. Notably, it allows to answer an initial question: will virtual locomotion trajectories totally differ from real ones when steered under commonly used VR conditions?

The remaining of the paper is organized as follows. Section 2 describes related work. Section 3 describes the framework, with the different evaluation criteria and the model to generate reference trajectories. Section 4 describes the five different experiments conducted and their results. Finally, the results are discussed in Section 5 before concluding.

2 RELATED WORK

2.1 Human Locomotion

2.1.1 Goal-directed Locomotion

A human and his environment are an inseparable and complementary couple [10]. For example, walking can be described in a body-centered way, focusing on spatio-temporal features of gait, stability or segmental coordination. However, to navigate efficiently, the walker implements an adaptive locomotor behavior [28] which depends on the perception of the characteristics of the surrounding environment (goals, obstacles, etc). Different sensory systems convey the required information to the walker. Notably, the vestibular and the proprioceptive

systems provide absolute and relative information about the internal state of the walker. Conversely, the visual system provides information about the external environment, allowing the walker to estimate his position, his self motion, the movement of other elements in the environment as well as a precise information in advance about the nature of the environment [27].

The visual system plays a major role in our context, where virtual locomotion is used from a static real position. Visually guided locomotion has received considerable attention in the past [10, 27, 40, 22, 39, 6], and two main strategies to move towards a goal have been identified. The first one, proposed by Gibson [10], leverages the optical flow created by the apparent motion of each point composing the sequence of images perceived by the walker during his motion [17]. Steering toward a goal is achieved by superposing the focus of expansion of the flow with the target to reach. The second strategy, proposed by Rushton [33], aims at aligning the locomotor axis with the perceived egocentric direction of the target to reach. Studies have shown that both strategies are used during locomotion steering [40], with a different predominance depending on the amount of available visual information [12, 42, 39].

Concerning motor control, recent studies [1, 14] suggest that goal-directed locomotion planning is done globally rather than locally (step-by-step) [14]: human trajectories are stereotyped when measured at the center of mass of the walker, but not when measured at the feet. This stereotypy is found in the trajectory geometry as well as in the walkers velocity profile, with velocities decreasing before turning and constant velocities while turning, allowing the generation of smooth trajectories.

2.1.2 Numerical Models of Goal-directed Locomotion

Several models were proposed to generate global locomotion trajectories given a destination to reach. An initial approach was introduced by Reynolds [32] as a set of reactive controls. This approach is improved by Boulic [4] by considering oriented targets. Brogan and Johnson [5] introduce real human locomotion behavior in their model by computing the velocity according to acceleration and deceleration rules based on observations of real recordings.

Another set of goal-directed locomotion models further leverages real human locomotion observations by optimizing their parameters using recordings. A first approach, proposed by Fajen and Warren [8], uses a second order differential system and an attractor to cancel the angle between the heading direction and the position-target direction. The different parameters are optimized using data from trajectories generated by real humans in a VR environment. However, targets were not oriented, and velocities were constant. More recently, Arechavaleta et. al [1] generated trajectories by optimizing a unicycle model with recorded data for a given target. The underlying unicycle model was nonholonomic, i.e. with a coupling between the direction and the position of the body that prevents unnatural motions such as sidestepping.

A final set of goal-directed locomotion models relies on a cost function minimization approach to generate virtual trajectories. Pham et al. [29] compare the results of minimizing four different values (velocity, acceleration, jerk and snap), while knowing the initial and final velocity and acceleration, as well as the overall trajectory duration. Mombaur et al. [26] observes that trajectories are not always nonholonomic, and therefore combines nonholonomic and holonomic constraints as an optimal control problem with an objective function to minimize. The main drawback of these approaches is the requirement of the total trajectory time in advance, and the generation of the entire trajectory at once precluding the use of dynamic environments.

2.2 Locomotion in Virtual Reality

2.2.1 Locomotion techniques

Although natural walking is certainly the most ecological approach, the limited size of VR setups and input devices prevents from using it most of the time. Locomotion techniques are therefore required, which cannot preserve all the sensory channels involved in locomotion. These are techniques that unnoticeably play on the sensory chan-

nels (redirected walking [31], change blindness[37]), techniques that change the task by halting it [44], techniques that sacrifice equilibrioception (walk in place [34]), techniques that sacrifice proprioception [24], and techniques that sacrifice all kinesthetic channels (such as hand-held devices). A study evaluating the conformity of virtual trajectories generated through natural walking, walk-in-place and joystick flying to real trajectories showed that only natural walking correlates well with real data for different visual conditions [41].

2.2.2 Perception of self locomotion

Although virtual reality is a powerful way to conduct behavioral studies by fully controlling the experimental conditions, there are many variables that produce perceptual differences, thus potentially introducing a bias in the study [9]. Several studies using virtual environments and Head-Mounted Displays (HMD) revealed a distance compression effect [20, 23, 43], partially explained by the reduced field of view and the weight and the torques exerted by the HMD on the user's head. Similarly, the perceived velocity in a VR environment differs from the real locomotion velocity [2], introducing an additional bias. Other factors, such as the image contrast [13] and the point of view [25], also influence navigation efficiency in VR. In addition, gait studies have shown that navigation in VR is performed with increased gait instability compared to real walking conditions [16].

When studying human locomotor behavior in VR, the aforementioned factors potentially introduce deviations in the generated locomotor trajectories, such as earlier turns and larger paths. However, these deviations might not be necessarily significant, and locomotor behavior in real and virtual environments may be, in fact, quite similar [9]. It has been shown that distance compression is significantly reduced after five minutes of continuous visual feedback [25]. In addition, locomotor behavior might not always be solely based on perceptual cues, but also on task-specific controls, unchanged in a VR context [9].

2.3 Evaluation of Virtual Trajectories

Many VR studies on novel locomotion techniques or interfaces evaluate virtual trajectories using difference performance criteria [21, 7, 18, 35, 45] (such as task completion time, traveled distance, number of collisions, and path precision with respect to the ideal path), empirical observations of trajectory visualizations [45], as well as cognitive, presence and cybersickness questionnaires [45, 41, 36]. Although often sufficient in their context, these metrics cannot reliably evaluate the realism of a trajectory since they do not take into account its underlying shape and its kinematic aspects.

Several studies have used the mean point-by-point euclidean distance between trajectories as a distance metric [1, 29, 5]. By considering a sampling at constant timesteps, and not at constant curvilinear positions, this distance takes into account to some extent the temporal aspect of the trajectory. Fink et al. [9] used a different set of metrics, namely the mean radius of curvature along the full path, the maximum euclidean distance from a straight line between the origin and the target, and the minimum euclidean distance between the path and the obstacles of the virtual environment. In addition, they leverage their least squares trajectory optimization approach as Fajen and Warren [8] by using the mean fit values as a metric to account for the realism of the trajectories. Whitton et al. [41] used Principal Component Analysis to study a set of VR trajectories, and found that for their specific constrained task velocity profiles were mostly defined by the maximum velocity, the percent of time to reach the maximum velocity, and the maximum deceleration.

Other studies focused on different gait parameters such as stride length, step width, variability in stride velocity and variability in step width in order to compare trajectories generated in virtual and real environments [16, 35]. Terziman et al. [38] also inspected the optimality of a trajectory performed when walking through a slalom without reference to real examples.

In this work, and as opposed to previous approaches, we base our evaluation on a comparison of reference and virtual trajectories

formed during goal directed locomotion tasks. Compared to previous kinematic-based studies, we benefit from recent results on locomotion stereotypy to avoid constraining locomotion paths (e.g., with walls): this enables us to introduce criteria about *the shape* of the path in combination with velocity profiles. We also introduce a model to eventually get rid of the need for trajectories performed in real conditions. Through the framework described in the next section, we expect a richer and more comprehensive set of trajectory evaluation criteria.

3 TRAJECTORY EVALUATION FRAMEWORK

In this section, we introduce our framework for the evaluation of virtual locomotion conditions. The framework considers virtual locomotion trajectories performed under a given set of studied conditions, as desired by experimenters. The framework checks their conformity with reference trajectories. In the framework, reference trajectories can be real recorded data when available, or can be generated through a numerical model of human locomotion.

As a result, the three main components of the framework are:

- a comprehensive set of criteria evaluating both geometrical and temporal aspects of the trajectories,

- a finite set of real trajectories, used as reference trajectories

- a locomotion model, used to generate reference trajectories when the corresponding real trajectory is not available

In this work, we evaluate trajectories produced during *goal-oriented* locomotion tasks in static obstacle-free environments. Therefore, the only objects composing the virtual environments are the initial and the destination positions with orientations which are materialized by gates. The geometry of gates are similar to [14], and we dispose of the trajectories captured in this study.

3.1 Evaluation Criteria

Nine criteria are used to evaluate the realism of a given set of virtual trajectories. Each criteria generates a value per trajectory. The data generated is analyzed using different tools, as presented in section 4.

errors : evaluates whether a trajectory contains errors making the trajectory fundamentally unrealistic, and therefore unacceptable. These errors detect behaviors that would never occur in a real context given the aforementioned task and conditions. The error criteria are unambiguous and without arbitrary thresholds. (1) A stop error is triggered when the subject has completely stopped during his trajectory. Detected when velocity equals 0 between the initial and destination gates. (2) A collision error is triggered when the subject has collided with the target gate. Detected when the position of the walker is inside the geometry of the gate. (3) An overshoot error is triggered when the subject overshot his path with respect to the gate. Detected when the position of the walker, projected onto the doorstep vector, goes past the outer side of the gate.

Only trajectories without errors are treated by the following criteria.

duration : evaluates duration, length, and average velocity of the trajectory relatively to the reference trajectory. Each subcriterion (duration, length, average velocity) is the ratio between the evaluated trajectory value and the reference trajectory value. A ratio of 1 means equal subcriterion values between trajectories.

tangential velocity profile : evaluates the similarity between the tangential velocity profile of the experimental trajectory and the tangential velocity profile of the reference trajectory. Since virtual environments can influence the perception of velocity, we aim at evaluating the differences in the variations of the velocity in time rather than the absolute differences, similarly to [29]. To this aim, (i.e., to remove the effect of absolute values) we subtract their mean values to each velocity profile and divide them by their standard deviation. Then, a cross correlation is computed between the reference and experimental standardized profiles . A value of 1 means an exact match.

angular velocity profile : evaluates the similarity between the angular velocity profile of the trajectory and the angular velocity profile of a reference trajectory. The approach is the same as for the *velocity profile* criterion.

smoothness : evaluates the smoothness of the trajectory. This criterion is inspired by studies which showed that human trajectories maximized smoothness [29], and that smoothness maximization could be achieved by jerk minimization. Smoothness is therefore evaluated by computing the mean jerk amplitude of the trajectory, as in [29]. Smaller values mean smoother trajectories.

shape : evaluates the mean euclidean distance between the trajectory and a reference trajectory. Both trajectories are resampled to an equal set of equidistant samples on the curvilinear axis, and distances are measured between corresponding samples. The evaluation is purely geometrical, since the uniform resampling removes the temporal component.

shape variability : evaluates stereotypy by inspecting the spatial spreading of various repetitions. This criteria is directly inspired by [14], who showed a stereotypic pattern of real human walking trajectories. It is computed through the sum of distances between sample points of each trajectory and sample points of the mean of all repetitions (i.e. all the experimental trajectories performed under identical conditions). This sum is divided by the length of the mean trajectories.

curvature : evaluates the relative mean curvature of the trajectory compared to a reference trajectory. The criterion is the ratio between the evaluated trajectory curvature and the reference trajectory curvature. The curvature at each sample is computed as in [9]. A ratio of 1 means equal mean curvature between trajectories.

final orientation : evaluates the angle between the final orientation of the trajectory and the orientation of the target.

3.2 Locomotion Model

Whenever a reference trajectory is required by the aforementioned criteria, a real trajectory can be used, provided it has the same target position and orientation as the virtual trajectory being evaluated. This is the purpose of the set of recorded trajectories available in the framework. However, the set of real trajectories encompasses a finite number of target positions and orientations. In addition, each real trajectory has a velocity component that depends on the subject who was recorded. If a reference trajectory is required with a target position and orientation not included in the real set, or with a different velocity component (e.g., a different comfort velocity), the reference trajectory can be generated using the locomotion model of the framework.

In principle, any locomotion model could be used as long as: (1) it is designed for goal-oriented tasks, with a target position and orientation, (2) it closely matches the geometrical and temporal components of real trajectories, (3) it does not require the availability of the real trajectory it tries to simulate. Among the models surveyed in Section 2, those leveraging parameter optimization [8, 1] and cost minimization [29, 26] produce simulated trajectories close to their real counterpart. However, Fajen and Warren model [8] does not consider oriented targets, and only matches the geometrical aspect of the real trajectory, since the simulated velocity is constant throughout the path. In addition, the model of Arechavaleta et. al [1] requires the real trajectory data to generate the simulated trajectory. Similarly, both cost minimization approaches [29, 26] require the real total trajectory time as input data.

Given that none of the aforementioned models fulfills the three conditions to be adequate for our framework, we designed our own goal-directed locomotion model. The model is based on two simple observations. If $\mathbf{l}$ is the 2D vector from the walker position to the target position, α is the angle between $\mathbf{l}$ and the target orientation, $\mathbf{v}$ is the walker velocity and θ is the walker orientation, we observed that:

1. $\|\mathbf{l}\|$ decreases linearly with α as the walker moves along the trajectory path
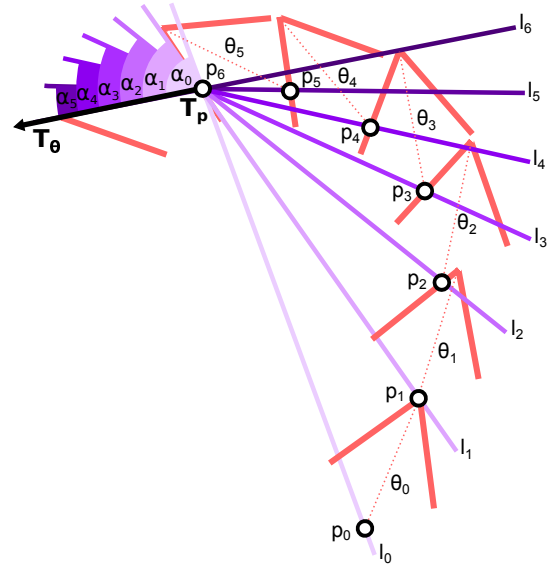

Fig. 2. Illustration of our locomotion model. T_p and T_θ are respectively the target position and orientation. At timestep 1, the walker has position p_1 and orientation θ_1. The angle α_2 is computed through Eq. 1 and explicit integration, thus defining the line l_2 to be reached by the walker at timestep 2. For this, the arrow shaped reachable space of the walker, oriented towards θ_1, is used to find the intersection points with l_2. The intersection point closest to T_p defines p_2, while the new orientation θ_2 is given by $p_2 - p_1$. This process is repeated until the walker's position matches T_p. At T_p, the walker's orientation matches T_θ.

2. $\mathbf{v}$ decreases with $\dot{\theta}$ when the walker turns, as already observed in previous models [5, 1, 26]

In the light of observation (1), we see $\|\mathbf{l}\|$ and α decrease at equal rates, i.e. humans move towards the target position and the target orientation at equal rates, thus assuming:

$$\frac{\|\mathbf{l}\|}{\|\dot{\mathbf{l}}\|} = \frac{\alpha}{\dot{\alpha}} \qquad (1)$$

at any trajectory instant. In addition, in order to define the relationship between $\mathbf{v}$ and $\dot{\theta}$, we plotted the tangential velocity values $\|\mathbf{v}\|$ and their corresponding angular velocity values $\dot{\theta}$ at each timestep, extracted from real trajectory data [14]. The plot, shown in Figure 3 (left), exhibits an arrow shaped contour. Leveraging this geometrical trait, we designed the relationship between $\mathbf{v}$ and $\dot{\theta}$ as an arrow-shaped contour defined by its height (maximum tangential velocity), its bottom (minimum tangential velocity) and its width (maximum angular velocity), thus allowing to compute the walker's arrow-shaped reachable space. As with existing parameter optimization models [8, 1], the aforementioned parameters are computed through optimization using a set of input trajectories.

The simulated walker is initialized at the starting position and orientation with maximum tangential velocity and zero angular velocity. At each timestep t, $\|\mathbf{l}\|$ and α are measured, and $\|\dot{\mathbf{l}}\|$ is computed. Then, $\dot{\alpha}$ can be obtained using Equation 1, which allows to compute α for timestep $t+1$ using explicit Euler integration. This new α defines a line that must be intersected by the walker arrow-shaped reachable space. If the line is intersected, the intersection defines the new position of the walker. Otherwise, the closest point to the line on the reachable space is selected as the new walker position. This algorithm is illustrated in Figure 2.

In order to validate the model, we calibrated it on a set of real trajectories [14]. Since the model parameters optimization problem is only tridimensional, and the parameter ranges are rather small (bounded by biomechanical values), we employed a brute force approach. The set of trajectories consisted in 40 gate positions, ranging from -1m to 1.1m
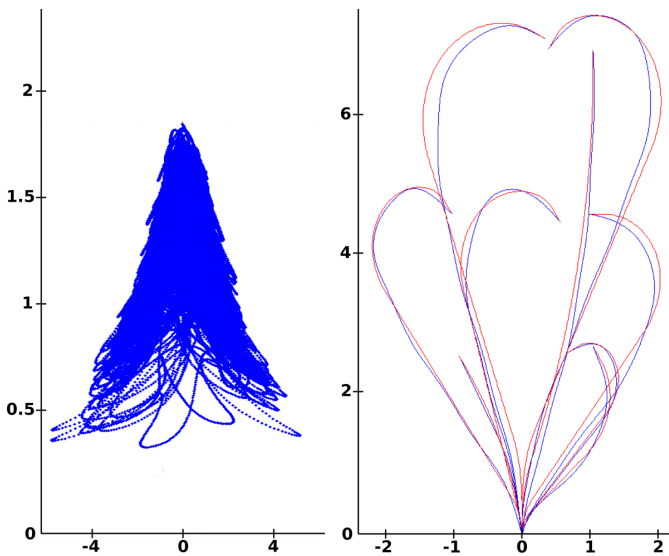
(games, virtual walkthroughs): the outputs of the input control device $(-1..+1)$ are linearly mapped to the tangential velocity of the user $-max\_tang\_speed..+max\_tang\_speed)$ and the angular velocity of the body $(-max\_ang\_speed..+max\_ang\_speed)$. Subjective (first person) cameras, $60°$ fields of view and desktop screens are arguably the most common conditions in consumer VR.

Experimental Apparatus Each experiment focuses in one of the aforementioned categories by evaluating the contributions of three common and frequently used conditions to the overall realism of virtual trajectories. The rest of the condition categories are set to their respective baseline conditions. Therefore, the experiments are as follows:

- *Experiment 1*: studies the influence of the input control device through the use of a *joystick* (baseline), a *keyboard* and a *gamepad*. The keyboard controls forward and backward motions using the up and down keys, while the gamepad uses the forward axis. The keyboard controls body rotation using the left and right keys, while the gamepad uses the lateral axis. Input control law, viewpoint, field of view and output display device categories are set to their respective baseline conditions.

- *Experiment 2*: studies the influence of the input control law through a *linear rate control law* (baseline), an *inertial rate control law*, and the *Joyman control law* [24]. The two non-reference control laws influence the tangential velocity in two different ways, both using human biomechanical observations. The inertial rate control law provides a common sensation of inertia when slowing down and heaviness when accelerating by limiting the maximum tangential acceleration of the linear control law to real values: 0.54 m.s^{-2}. The Joyman control law couples the angular velocity to the tangential velocity, thus making the user slow down progressively when turning (more details on [24]). Input control device, viewpoint, field of view, and output display device categories are set to their respective baseline conditions.

- *Experiment 3*: studies the influence of the viewpoint through the use of a *subjective camera* (baseline), a *third-person camera* and a *fixed camera*. The third-person camera is constantly located 4m behind the user avatar and 1m above it. The user avatar is a blue cylinder of 30cm radius and 1.6m high representing the body, topped by an extruded triangle with a base and a height of 60cm, and a thickness of 10cm, representing the head and pointing towards the line of sight. The fixed camera is a subjective camera that looks constantly at the target position. Input control device, input control law, field of view, and output display device categories are set to their respective baseline conditions, except when using the fixed camera condition, where the joystick lateral axis is used for side motion instead of angular velocity, with the same boundary velocities as for the tangential velocity.

- *Experiment 4*: studies the influence of the field of view through values of $60°$ (baseline), $45°$ and $90°$. Input control device, input control law, viewpoint, and output display device categories are set to their respective baseline conditions.

- *Experiment 5*: studies the influence of the output display device through a desktop screen (baseline), a head-monted display (HMD), and an immersive projection setup (IPS). The HMD was an eMagin Z800. The IPS consisted in a 9.6mx3.1m floor and front wall, with stereoscopic display. An optical tracking system tracked the position of the user's head for both the HMD and the immersive projection setup, thus allowing to dissociate the viewing direction from the body orientation. Since the user naturally stands when using the immersive projection setup, all 3 conditions where conducted in a standing position to avoid any experimental bias. Input control device, input control law, viewpoint, and field of view categories are set to their respective baseline conditions.

Fig. 3. Left: plot of the relationship between $\|\mathbf{v}\|$ (vertical axis, in m.s^{-1}) and $\dot{\theta}$ (horizontal axis, in rad.s^{-1}) from a set of real trajectories, forming an arrow-shaped contour. Right: plot of a set of real trajectories (in blue) and their corresponding simulated trajectories (in red) using our locomotion model. Units in meters.

in lateral position and from 2m to 7m in forward position, and 12 gate orientations evenly distributed in the unit circle. Figure 3 (right) shows several real trajectories and their corresponding simulated trajectories, highlighting a close fit between each couple of paths. Using the *shape* criterion, we computed the average distance between real and simulated trajectories, resulting in a mean value of 12.8cm. Results are similar to [9] (15.6cm average distance), and under the experimental variability as in [29] (15.1cm) for the same set of trajectories. With a similar performance to previous models, our model fulfills the three aforementioned requirements. In addition, it enjoys the benefits of approaches based on optimization, namely the capacity to be automatically configured for different locomotion behaviors.

4 EXPERIMENTS

In this section, we describe five different experiments that were conducted with two objectives in mind: (1) illustrate our framework through a wide range of experimental data, and (2) provide an initial assessment of the conformity of virtual trajectories to their real counterpart under a set of commonly used conditions.

All five experiments consisted on the same task: navigate from a starting position and orientation A to a target position and orientation B represented by an oriented gate (i.e. a gate whose sides are deep enough to clearly indicate orientation). The instructions were to pass under the target gate while matching its orientation.

We focus on five different categories of conditions that usually vary between different VR applications and domains (visualization, walk-throughs, games, training, etc): input control device, input control law, viewpoint (camera), field of view, and output display device.

Baseline set of conditions In order to facilitate comparisons between experiments, a baseline (control) condition is chosen for each category, resulting in a baseline set of conditions always repeated across all experiments. This baseline set consists on a joystick as input control device, a linear rate control law, a subjective camera, a $60°$ field of view and a desktop screen as output display device. The choice of using a joystick as input device is justified by the wider degrees of freedom when using a large stick compared to smaller gamepad sticks or the binary input of a keyboard. Forward and backward motion is controlled by the forward axis, while body rotation is controlled by the lateral axis. The linear rate control law is a classic control law found in many VR applications involving frequent locomotion

The virtual scene, shown in Figure 1 in desktop and IPS conditions, had minimal visual cues and consisted in two gates, one at the origin and oriented forward and the other at the target position with the target orientation. Both gates were 1m wide and 1.8m tall. A 80cm red arrow located at the center of the target gate and pointing outwards indicated the orientation of the target gate. The floor was a 30mx30m plane, centered at the origin, with an isotropic noisy grey texture. The sky was of a uniform light gray.

The possible gate positions were (in floor 2D coordinates and mm): (1) (-1000, 2000), (2) (-300, 2000), (3) (1100, 2000), (4) (-1000, 6000), (5) (-300, 6000), (6) (1100, 6000), thus covering short and long types of trajectories to both sides. The possible gate orientations were: (a) 0°, (b) 150°, (c) 270°, with 0° being a right side orientation and turning counter-clockwise, thus creating three distinct trajectory headings. Figure 4a) illustrates all possible gate positions and orientations as well as corresponding reference trajectories.

Population Each experiment had twelve participants: experiment 1 had 1 female and 11 males, aged from 22 to 31 ($M = 26.18, SD = 2.18$); experiment 2 had 2 females and 10 males, aged from 24 to 30 ($M = 26.05, SD = 2.06$); experiment 3 and 4 had 3 females and 9 males, aged from 24 to 28 ($M = 26.20, SD = 1.2$); experiment 5 had 2 females and 10 males, aged from 22 to 33 ($M = 27.5, SD = 2.8$). Some subjects participated in several experiments. None had any known vision or perception disorders that could alter the perception of the visual environment. They were all unpaid volunteers and naïve to the purpose of the experiments.

Procedure Before the beginning of an experiment, the three variable conditions were explained orally to the subject. He was given the instruction to pass under the first gate at the origin, and then navigate to the target gate and match the orientation of the target gate (and the arrow) when passing under it. Before each trial, the position and orientation was reset to 1m before the origin (behind the first gate), and looking at the origin. Before using each condition, six different training trials using the condition were given to the subject with the explicit instruction to practice the task.

In the experiment, participants completed all three conditions (the conditions depended on the experiment number) and the order of the conditions was counterbalanced across participants. In each condition, the participants were exposed in random order to all 18 gate combinations (positions 1 to 6 and orientations a to c) with three repetitions for each combination. In experiment 2 to 5, we reduced the number of combinations to remove gates that did not provide useful results, i.e. gates that produced small trajectory variabilities. The resulting combination was a set of 10 gates: (1,a),(1,c),(2,a),(2,c),(3,b),(3,c),(4,a),(4,c),(5,c),(6,c). In experiment 1 participants completed a total of 162 trials (18x3x3) with an average duration of 25 minutes, while in experiments 2 to 5 participants completed a total of 90 trials (10x3x3) with an average duration of 15 minutes.

Collected data For each trial, we recorded the virtual position and orientation of the subject at each timestep. For HMD and IPS conditions, we also recorded the head tracker readings (i.e. position and orientation of the head). This data is fed to the framework to analyze the virtual trajectories.

Results For all data, we performed Shapiro tests that rejected the normality hypothesis on the data distribution (thus excluding the use of an ANOVA). Thus, we used a non-parametric Friedman test for differences among the conditions according to the criteria defined in section 3. Post-hoc comparisons were performed using Wilcoxon signed-rank tests with a threshold of 0.05 for significance. Reported p-values are adjusted for multiple comparisons. Data were first filtered with the different error criteria defined in section 3. Figure 4 provides these statistical results for the 5 experiments as well as, for each condition, experimental trajectories and median values for all the criteria defined in Section 3.1. Most criteria are unitless (duration, tangential and angular velocity profiles, shape variability, curvature). Smoothness unit is m.s^{-2}. Shape unit is m. Final orientation unit is radian.

5 DISCUSSION

Experiment 1 We could expect, as a first major finding, that these experimental trajectories, generated while seated and through a manual task, would be completely different from real trajectories generated through natural walking . Indeed, a manual and a bipedal task use entirely different members, with different internal mechanics, different kinematic constraints, different inertias, etc. However, a quick glance at the resulting trajectories illustrated in Figure 4 already shows striking similarities with reference trajectories, as show in Figure 5: smooth and long curves, progressive reorientations, and the peculiar property of real trajectories of making an offset to one side (e.g. the right) even if the target is on the other side (e.g. the left) when large reorientations are required. When looking more closely using the evaluation criteria, the *shape* criterion shows an average distance of 183.45mm, with average *durations*, *lengths* and *velocities* close to the reference trajectories. Since the visual guidance of locomotion relies heavily on optical flow [40], a possible explanation for this overall similarity could be that subjects aim at obtaining an optical flow similar to what they would have obtained in a real trajectory. This would generate similar trajectories, independently of the input device being used and the manual motion required for its manipulation. Nevertheless, with an average maximum distance of 312.21mm, and an average *velocity profile* match of 0.53, virtual trajectories exhibit a significant difference with their real counterpart.

When comparing the three different conditions (keyboard, joystick, gamepad), one could expect a significant difference in the velocity profiles. The keyboard input is binary, while the gamepad has a limited range of motion compared to the joystick. The higher number of *stop errors* for keyboard and gamepad suggests that a possible strategy for both devices was to move to a given location, stop, reorient, and start moving again, bearing no similarity whatsoever with real trajectories. With the joystick, on the other side, there were much lesser stops (37 against 223 and 237 respectively), and although the *velocity profile* criterion did not yield a significant effect of the condition, it did show a significantly better velocity profile match for the joystick when considering only longer trajectories with large reorientations. This suggests that, when given enough locomotion time, a more continuous control of the velocity can yield a higher conformity to real trajectories.

Experiment 2 In the light of the results of Experiment 1, different sets of conditions could be improved to generate virtual trajectories with a higher conformity to their real counterpart. While a real walker directly controls his motion, a virtual walker has a double control loop: he controls the input device, which in turn controls the motion in the VE. Experiment 1 showed that subjects try to reproduce real trajectories, but significant differences remain. By playing on the way the device controls the motion, there are biomechanical behaviors present in any real locomotion task and lost by the use of a manual interface that could be reincorporated through the use of a particular control law.

Results suggest that there was an opposite effect of the inertial and the Joyman control law regarding the geometric trajectory: the inertial law is significantly worse in the *shape* criterion, as well as the *curvature*. This can be explained by the simple fact that the inertial law is harder to control, as the inertial motion has to be anticipated in a virtual context. It was therefore not well used or understood, and the inertial motions resulted in higher deviations and more exaggerated trajectories. However, we can also observe that both control laws behave similarly compared to the baseline rate control law regarding duration/velocity (significantly slower motions due to lower velocities) and smoothness (significantly lower smoothness value, meaning a smoother trajectory). We hypothesize that since both laws are a novelty to users used to simple yet effective rate control laws of games and simulations, they required a higher cognitive load and therefore a higher concentration, leading to slower and more continuously focused manipulations, and thus smoother trajectories. The Joyman by itself exhibits an excellent *curvature*, and his results are good overall. With more training and perhaps a different set of parameters, the Joyman control law could overcome its slowness and prove to be a positive influence in the overall conformity to real trajectories.

a)

Criterion	Joystick	Keyboard	Gamepad
Stop error	37	237	223
Collision error	5	17	27
Overshoot error	9	5	4
Duration $\chi^2=3.27$ $p=0.003$	1.07	1.03	1.01
Length $\chi^2=3.67$ $p=0.0007$	1	0.98	0.98
Average velocity $\chi^2=1.63$ $p=0.23$	0.97	0.97	0.98
Tangential velocity profile $\chi^2=1.84$ $p=0.16$	0.59	0.49	0.50
Angular velocity profile $\chi^2=1.83$ $p=0.16$	0.80	0.80	0.78
Smoothness $\chi^2=0.61$ $p=0.81$	2.66	2.83	2.91
Shape $\chi^2=3.88$ $p=0.004$	221.01	162.97	166.36
Shape variability $\chi^2=2.17$ $p=0.34$	4.87	4.39	4.65
Curvature $\chi^2=2.65$ $p=0.02$	0.90	0.91	0.87
Final orientation $\chi^2=0.41$ $p=0.91$	0.12	0.11	0.13

b) (above table)

c)

Criterion	Linear	Inertial	Joyman
Stop error	18	5	6
Collision error	33	30	38
Overshoot error	5	13	1
Duration $\chi^2=3.27$ $p=0.003$	1.11	1.23	1.46
Length $\chi^2=1.43$ $p=0.33$	1.01	1.01	1.01
Average velocity $\chi^2=3.47$ $p=0.01$	0.92	0.87	0.71
Tangential velocity profile $\chi^2=2.04$ $p=0.1$	0.62	0.61	0.72
Angular velocity profile $\chi^2=1.13$ $p=0.33$	0.82	0.80	0.82
Smoothness $\chi^2=3.47$ $p=0.001$	3.15	2.60	2.52
Shape $\chi^2=3.47$ $p=0.002$	322.76	364.69	294.88
Shape variability $\chi^2=5.16$ $p=0.07$	5.81	5.92	5.31
Curvature $\chi^2=3.67$ $p=0.0008$	0.90	0.91	0.87
Final orientation $\chi^2=0.41$ $p=0.91$	0.16	0.15	0.16

d)

Criterion	Subjective camera	Third-person camera	Fixed camera
Stop error	53	37	44
Collision error	24	5	4
Overshoot error	18	11	4
Duration $\chi^2=3.67$ $p=0.0007$	1.31	1.47	1.11
Length $\chi^2=3.27$ $p=0.03$	1.09	1.21	1.08
Average velocity $\chi^2=2.86$ $p=0.01$	0.87	0.86	1
Tangential velocity profile $\chi^2=2.45$ $p=0.04$	0.67	0.64	0.56
Angular velocity profile $\chi^2=2.45$ $p=0.04$	0.81	0.76	0.72
Smoothness $\chi^2=3.06$ $p=0.006$	2.66	2.54	3.69
Shape $\chi^2=2.24$ $p=0.06$	306.14	413.61	318.44
Shape variability $\chi^2=4.17$ $p=0.12$	6.27	5.68	5.61
Curvature $\chi^2=3.88$ $p=0.0003$	0.89	0.93	1.10
Final orientation $\chi^2=1.84$ $p=0.16$	0.12	0.11	0.11

e)

Criterion	45°	60°	90°
Stop error	65	26	7
Collision error	41	17	4
Overshoot error	19	5	3
Duration $\chi^2=0.41$ $p=0.91$	1.21	1.26	1.26
Length $\chi^2=1.63$ $p=0.23$	1.03	1.06	1.06
Average velocity $\chi^2=1.22$ $p=0.44$	0.89	0.90	0.87
Tangential velocity profile $\chi^2=1.84$ $p=0.16$	0.72	0.71	0.64
Angular velocity profile $\chi^2=1.02$ $p=0.56$	0.79	0.81	0.80
Smoothness $\chi^2=0.41$ $p=0.91$	2.96	2.91	2.70
Shape $\chi^2=3.06$ $p=0.007$	252.07	270.51	223.08
Shape variability $\chi^2=18.5$ $p=0.0001$	7.75	5.59	4.83
Curvature $\chi^2=2.24$ $p=0.06$	0.89	0.90	0.95
Final orientation $\chi^2=1.02$ $p=0.56$	0.15	0.13	0.16

f)

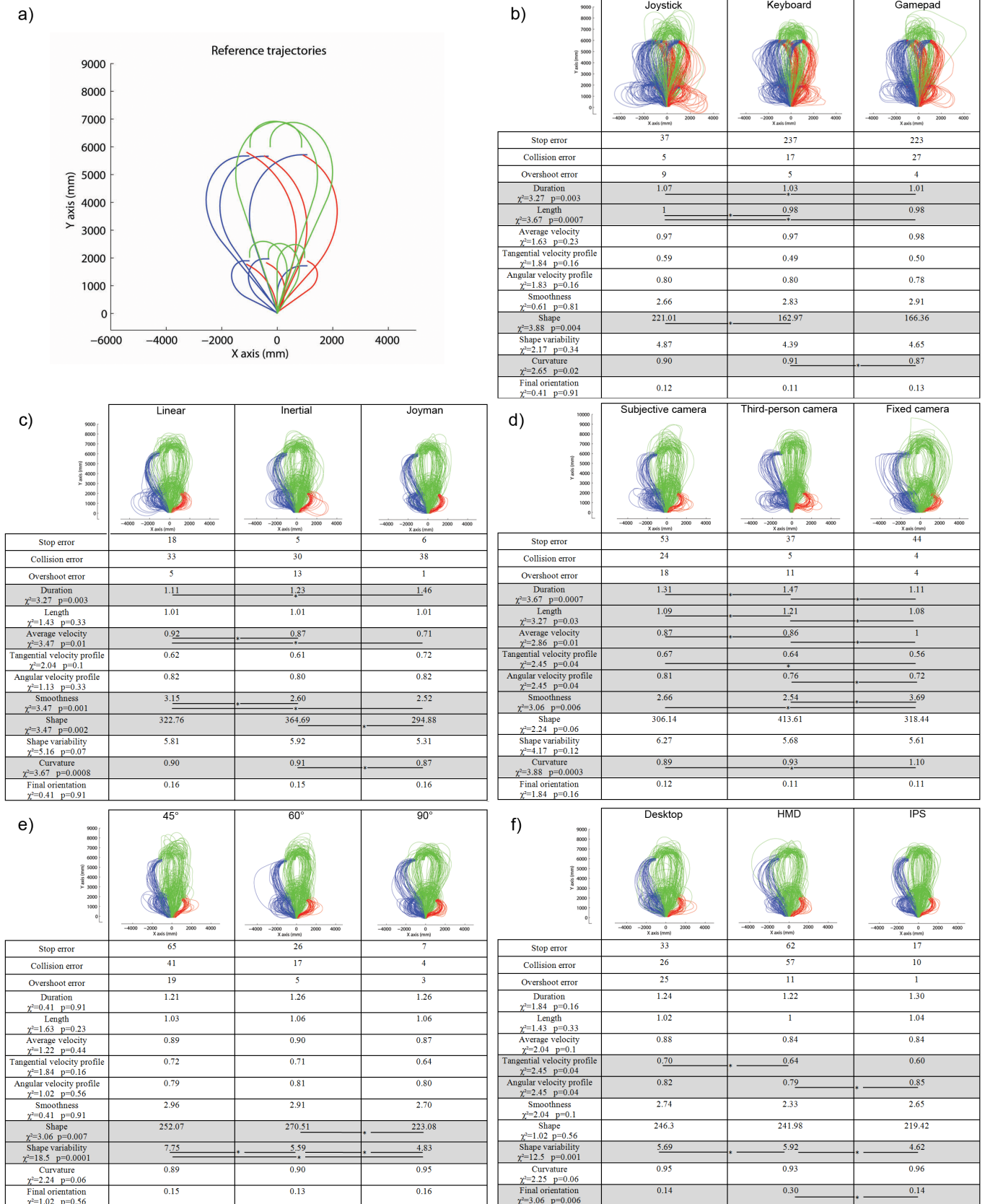| Criterion	Desktop	HMD	IPS
Stop error	33	62	17
Collision error	26	57	10
Overshoot error	25	11	1
Duration $\chi^2=1.84$ $p=0.16$	1.24	1.22	1.30
Length $\chi^2=1.43$ $p=0.33$	1.02	1	1.04
Average velocity $\chi^2=2.04$ $p=0.1$	0.88	0.84	0.84
Tangential velocity profile $\chi^2=2.45$ $p=0.04$	0.70	0.64	0.60
Angular velocity profile $\chi^2=2.45$ $p=0.04$	0.82	0.79	0.85
Smoothness $\chi^2=2.04$ $p=0.1$	2.74	2.33	2.65
Shape $\chi^2=1.02$ $p=0.56$	246.3	241.98	219.42
Shape variability $\chi^2=12.5$ $p=0.001$	5.69	5.92	4.62
Curvature $\chi^2=2.25$ $p=0.06$	0.95	0.93	0.96
Final orientation $\chi^2=3.06$ $p=0.006$	0.14	0.30	0.14

Fig. 4. Experimental Results. a) Reference trajectories for all possible gate positions (gate angles are color-coded: 0°, 150°, 270°, are respectively illustrated in blue, red and green). b) to f) Respectively for Experience 1 to 5: each table illustrates experimental trajectories for each studied condition, and reports median values for all the criteria defined in Section 3.1. Significant differences between conditions are represented through a line with a star.
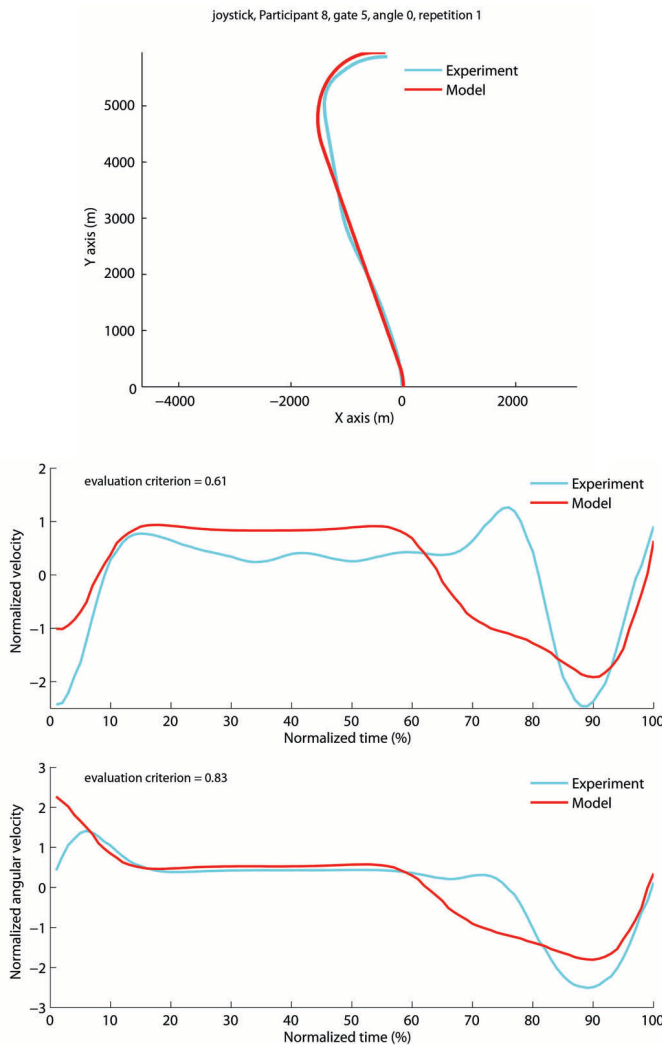
Fig. 5. Plot of the trajectory (top), velocity profile (medium) and angular velocity profile (bottom) of a given trial of the Joystick condition (in blue) and the corresponding reference trajectory computed through our model (in red). The similarities are striking, considering the experimental condition was generated through a manual and not a bipedal task.

Experiment 3 Previous studies have shown that the viewing direction and the head orientation anticipate (predate) the walking direction, and that this anticipation plays an important role in locomotion [11, 30, 15, 14]. Experiments 1 and 2 use viewpoints (cameras) that are fixed to the body of the virtual walker. For these experiments, we computed the percentage of total time where the *shape* distance increased (48.2%), the percentage of total time where the walker was not looking at the gate (28.3%), and both (24.2%). These results suggest an increase in trajectory deviation when the walker does not look at his target. Thus, having the walker see his target might improve the conformity of virtual trajectories.

Giving the user full control of his head and his body independently with a subjective camera turned out to be extremely complex for the user under our experimental conditions, preventing any plausible motion. Therefore, the third-person camera and the fixed camera are two ways to give the user the full view of the target without complicating his control. In addition, the third-person camera changes the perception of the VE using an allocentric point of view, i.e. position and orientation are expressed in a global reference frame [19]. This should provide a better perception of the relative position and orientation of the walker and the target since this allocentric representation facilitates

mental manipulations and simulation of the relation between objects [3].

The third-person camera shows significantly slower and longer trajectories than the other two conditions, showing an influence of the viewpoint in the way the task is executed. However, results are rather disappointing since no conclusion can be drawn beyond this. We can only observe a lower number of *collisions* with the gate than with the subjective camera (5 times less), which is not surprising given that the subject can see the full gate and body when reaching the target. On the other hand, the fixed camera clearly exhibits significant worse values for the *velocity profile*, the *angular velocity profile*, and the *smoothness*. This shows that not only the fixed camera does not improve the conformity of virtual trajectories to real ones, but the sidestepping it uses has a negative influence. This is an interesting result in the way that the introduction of a clearly non realistic locomotion behavior (sidestepping) that is successfully used in many games and simulations generates non realistic behaviors, which are successfully detected by our framework.

Experiment 4 Instead of changing the viewpoint condition to have the target in sight, a different approach is to modify the subject's field of view to encompass a wider viewing area, and therefore have the target in sight during a longer period of time. Naturally, a wider field of view is expected to reduce the average *shape* distance to reference trajectories, while a smaller field of view is expected to do the opposite.

Results conform to what was expected. Indeed, a wider field of view allowed to significantly reduce the *shape* distance compared to the baseline field of view. This emphasizes the need to have locomotion conditions that allow the user to see his target, but without influencing other factors as in Experiment 3. In addition, the *shape variability* was significantly reduced, suggesting that subjects often have to guess, with variable luck, where the target gate is located when it is out of sight. This is confirmed by the significant increase of *shape variability* for a small field of view. The large field of view also allowed to drastically reduce the total number of errors (14) compared to the baseline (49) and the small (102) fields of view.

Experiment 5 Exploring more immersive setups allows us to get closer to real walking conditions. Through the IPS and the HMD, the subject controls his head independently and naturally, while controlling his body through a manual task as in previous experiments. The stereoscopic vision provides a better perception of depth cues and of the relative positions and distances, while the IPS provides a field of view close to reality. The HMD, however, requires the user to choose between seeing the target, or seeing the focus of expansion of the optical flow, both important in a locomotion task.

A first surprising result lies in the performance of the IPS condition. Given the similarity in visual conditions between the IPS and reality (depth, stereo, field of view, scale, independent head, etc), we expected a significant improvement of most criteria. However, it is not the case: the *shape* distance is reduced, but not in a significant way, while the *shape variability* is also reduced, but it is only significant compared to the HMD condition. The total number of errors is lower with the IPS (17) than with the HMD (62) and with the desktop screen (33). A second surprising result is the particularly low performance of the HMD. The *velocity profile* and *angular velocity profile* are significantly worse than the baseline and the IPS conditions respectively. The HMD has the higher number of total errors, twice than the baseline, and a significantly higher variability than the other two conditions. After analyzing the head orientation in the HMD condition, we found that most users did not use their head as in real life, but rather stayed locked with the torso, looking forward, as if the viewpoint was not independent. This behavior could be explained by the lack of focus of expansion during head turns, or by the complexity of efficiently navigating with the joystick when the view was not aligned with the motion direction. This latter hypothesis is supported by the *final orientation* criterion, where the angle is significantly worse than in the other two conditions, showing a problem in the head/navigation coupling. In the end, the HMD condition is reduced to a baseline condition, but with a small physical

field of view, and a less comfortable apparatus, which could explain its low performance.

6 CONCLUSION

In this paper, we proposed to study different VR locomotion conditions by comparing reference trajectories to virtual trajectories generated by users during a simple oriented locomotion task (i.e. move from an oriented position to another). We differ from the use of traditional naturalness, effectiveness and precision criteria by proposing a set of trajectographical criteria. Through this approach, we evaluate the influence of a given set of virtual locomotion conditions in the generation of virtual trajectories that conform to real human locomotion. This is particularly important in the context of VR applications based on realistic trajectories within virtual environments. Therefore, we propose a comprehensive evaluation framework taking a set of experimental virtual trajectories as input and evaluating them through a comparison to a set of reference trajectories using different trajectographical criteria. These reference trajectories are either real trajectories generated through a similar real task, or simulated trajectories generated through a goal-oriented locomotion model, thus allowing to evaluate any sort of trajectory independently of the target (position and orientation) and the execution velocity.

In addition, we conducted a study to provide an initial assessment of the conformity of virtual trajectories to their real counterpart under a set of commonly used conditions: input devices, output display devices, control laws, and visualization modalities. Our major finding suggests that, no matter the condition, subjects seem to try to generate trajectories that conform to real ones. This is achieved with more or less success, depending on the conditions, but virtual trajectories always exhibit some fundamental characteristics of real locomotion. A different outcome could have been expected, with virtual and real trajectories differing completely. It is difficult to determine what pushes subjects to exhibit this realistic behavior, yet it is a result that comforts our work: we have to give the user the means to strengthen this behavior, and this is exactly the aim of our framework.

Future work will focus on making the framework available to the community, in order to improve it through additional criteria and new models. The overwhelming number of input devices, output devices and visualization conditions prevents a centralized study, thus making the availability of the framework an important step of our work. Existing trajectorgraphical evaluation criteria [41] could be incorporated, as well as existing and new locomotion models, which would extend the applications of our framework by allowing to compare locomotion models between them. Other locomotion tasks could be studied, in particular those involving the interaction with static or dynamic objects of the virtual environment. In the context of our research group, we are very interested in allowing the realistic execution of locomotion tasks while interacting with real or virtual users (involving realistic collision avoidance and target following) in a fully controlled VR environment.

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