ELEMENTS BOOK 2

Fundamentals of Geometric Algebra

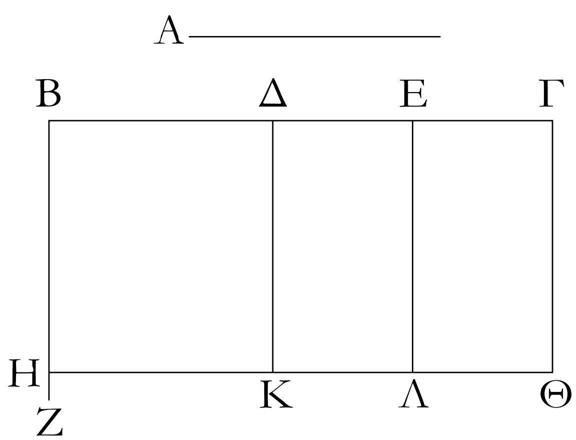
Ὅροι.

α′. Πᾶν παραλληλόγραμμον ὀρθογώνιον περιέχεσθαι λέγεται ὑπὸ δύο τῶν τὴν ὀρθὴν γωνίαν περιεχουσῶν εὐθειῶν.

β′. Παντὸς δὲ παραλληλογράμμου χωρίου τῶν περὶ τὴν διάμετρον αὐτοῦ παραλληλογράμμων ἓν ὁποιονοῦν σὺν τοῖς δυσὶ παραπληρώμασι γνώμων καλείσθω.

α′.

Ἐὰν ὦσι δύο εὐθεῖαι, τμηθῇ δὲ ἡ ἑτέρα αὐτῶν εἰς ὁσαδηποτοῦν τμήματα, τὸ περιεχόμενον ὀρθογώνιον ὑπὸ τῶν δύο εὐθειῶν ἴσον ἐστὶ τοῖς ὑπό τε τῆς ἀτμήτου καὶ ἑκάστου τῶν τμημάτων περιεχομένοις ὀρθογωνίοις.



Ἔστωσαν δύο εὐθεῖαι αἱ Α, ΒΓ, καὶ τετμήσθω ἡ ΒΓ, ὡς ἔτυχεν, κατὰ τὰ Δ, Ε σημεῖα· λέγω, ὅτι τὸ ὑπὸ τῶν Α, ΒΓ περιεχομένον ὀρθογώνιον ἴσον ἐστὶ τῷ τε ὑπὸ τῶν Α, ΒΔ περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ὑπὸ τῶν Α, ΔΕ καὶ ἔτι τῷ ὑπὸ τῶν Α, ΕΓ.

Ἤχθω γὰρ ἀπὸ τοῦ Β τῇ ΒΓ πρὸς ὀρθὰς ἡ ΒΖ, καὶ κείσθω τῇ Α ἴση ἡ ΒΗ, καὶ διὰ μὲν τοῦ Η τῇ ΒΓ παράλληλος ἤχθω ἡ ΗΘ, διὰ δὲ τῶν Δ, Ε, Γ τῇ ΒΗ παράλληλοι ἤχθωσαν αἱ ΔΚ, ΕΛ, ΓΘ.

Ἴσον δή ἐστι τὸ ΒΘ τοῖς ΒΚ, ΔΛ, ΕΘ. καί ἐστι τὸ μὲν ΒΘ τὸ ὑπὸ τῶν Α, ΒΓ· περιέχεται μὲν γὰρ ὑπὸ τῶν ΗΒ, ΒΓ, ἴση δὲ ἡ ΒΗ τῇ Α· τὸ δὲ ΒΚ τὸ ὑπὸ τῶν Α, ΒΔ· περιέχεται μὲν γὰρ ὑπὸ τῶν ΗΒ, ΒΔ, ἴση δὲ ἡ ΒΗ τῇ Α. τὸ δὲ ΔΛ τὸ ὑπὸ τῶν Α, ΔΕ· ἴση γὰρ ἡ ΔΚ, τουτέστιν ἡ ΒΗ, τῇ Α. καὶ ἔτι ὁμοίως τὸ ΕΘ τὸ ὑπὸ τῶν Α, ΕΓ· τὸ ἄρα ὑπὸ τῶν Α, ΒΓ ἴσον ἐστὶ τῷ τε ὑπὸ Α, ΒΔ καὶ τῷ ὑπὸ Α, ΔΕ καὶ ἔτι τῷ ὑπὸ Α, ΕΓ.

Ἐὰν ἄρα ὦσι δύο εὐθεῖαι, τμηθῇ δὲ ἡ ἑτέρα αὐτῶν εἰς ὁσαδηποτοῦν τμήματα, τὸ περιεχόμενον ὀρθογώνιον ὑπὸ τῶν δύο εὐθειῶν ἴσον ἐστὶ τοῖς ὑπό τε τῆς ἀτμήτου καὶ ἑκάστου τῶν τμημάτων περιεχομένοις ὀρθογωνίοις· ὅπερ

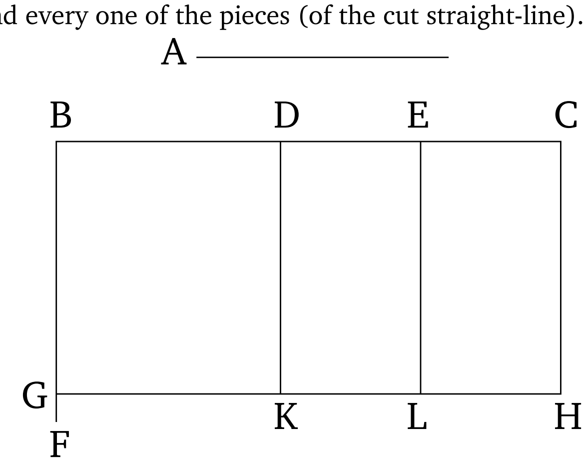
Definitions

1. Any rectangular parallelogram is said to be contained by the two straight-lines containing the right-angle.

2. And in any parallelogrammic figure, let any one whatsoever of the parallelograms about its diagonal, (taken) with its two complements, be called a gnomon.

Proposition 1†

If there are two straight-lines, and one of them is cut into any number of pieces whatsoever, then the rectangle contained by the two straight-lines is equal to the (sum of the) rectangles contained by the uncut (straight-line), and every one of the pieces (of the cut straight-line).



Let A and BC be the two straight-lines, and let BC be cut, at random, at points D and E. I say that the rectangle contained by A and BC is equal to the rectangle(s) contained by A and BD, by A and DE, and, finally, by A and EC.

For let BF have been drawn from point B, at right-angles to BC [Prop. 1.11], and let BG be made equal to A [Prop. 1.3], and let GH have been drawn through (point) G, parallel to BC [Prop. 1.31], and let DK, EL, and CH have been drawn through (points) D, E, and C (respectively), parallel to BG [Prop. 1.31].

So the (rectangle) BH is equal to the (rectangles) BK, DL, and EH. And BH is the (rectangle contained) by A and BC. For it is contained by GB and BC, and BG (is) equal to A. And BK (is) the (rectangle contained) by A and BD. For it is contained by GB and BD, and BG (is) equal to A. And DL (is) the (rectangle contained) by A and DE. For DK, that is to say BG [Prop. 1.34], (is) equal to A. Similarly, EH (is) also the (rectangle contained) by A and EC. Thus, the (rectangle contained) by A and BC is equal to the (rectangles contained) by A

ἔδει δεῖξαι.

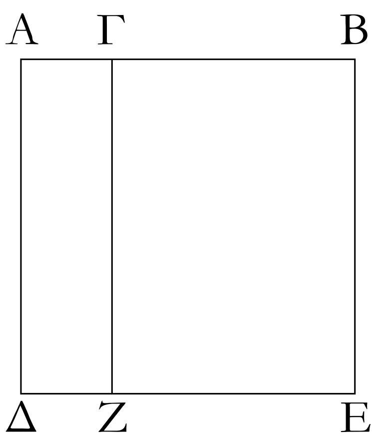
and BD, by A and DE, and, finally, by A and EC.

Thus, if there are two straight-lines, and one of them is cut into any number of pieces whatsoever, then the rectangle contained by the two straight-lines is equal to the (sum of the) rectangles contained by the uncut (straight-line), and every one of the pieces (of the cut straight-line). (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $a\,(b + c + d + \cdots) = a\,b + a\,c + a\,d + \cdots$.

β΄.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ὑπὸ τῆς ὅλης καὶ ἑκατέρου τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς ὅλης τετραγώνῳ.



Εὐθεῖα γὰρ ἡ ΑΒ τετμήσθω, ὡς ἔτυχεν, κατὰ τὸ Γ σημεῖον· λέγω, ὅτι τὸ ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ὑπὸ ΒΑ, ΑΓ περιεχομένου ὀρθογωνίου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ τετραγώνῳ.

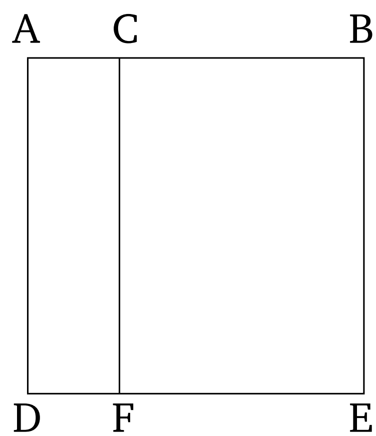
Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔΕΒ, καὶ ἤχθω διὰ τοῦ Γ ὁποτέρᾳ τῶν ΑΔ, ΒΕ παράλληλος ἡ ΓΖ.

Ἴσον δή ἐστι τὸ ΑΕ τοῖς ΑΖ, ΓΕ. καί ἐστι τὸ μὲν ΑΕ τὸ ἀπὸ τῆς ΑΒ τετράγωνον, τὸ δὲ ΑΖ τὸ ὑπὸ τῶν ΒΑ, ΑΓ περιεχόμενον ὀρθογώνιον· περιέχεται μὲν γὰρ ὑπὸ τῶν ΔΑ, ΑΓ, ἴση δὲ ἡ ΑΔ τῇ ΑΒ· τὸ δὲ ΓΕ τὸ ὑπὸ τῶν ΑΒ, ΒΓ· ἴση γὰρ ἡ ΒΕ τῇ ΑΒ. τὸ ἄρα ὑπὸ τῶν ΒΑ, ΑΓ μετὰ τοῦ ὑπὸ τῶν ΑΒ, ΒΓ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ τετραγώνῳ.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ὑπὸ τῆς ὅλης καὶ ἑκατέρου τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς ὅλης τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

Proposition 2†

If a straight-line is cut at random then the (sum of the) rectangle(s) contained by the whole (straight-line), and each of the pieces (of the straight-line), is equal to the square on the whole.



For let the straight-line AB have been cut, at random, at point C. I say that the rectangle contained by AB and BC, plus the rectangle contained by BA and AC, is equal to the square on AB.

For let the square $ADEB$ have been described on AB [Prop. 1.46], and let CF have been drawn through C, parallel to either of AD or BE [Prop. 1.31].

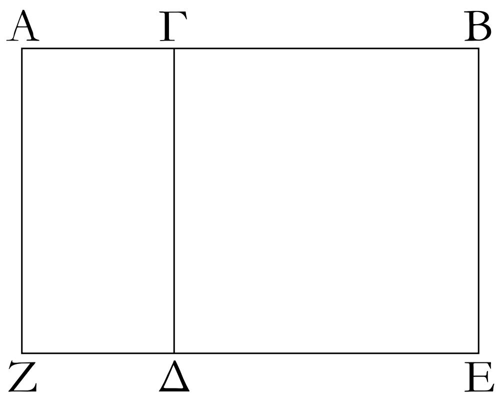
So the (square) AE is equal to the (rectangles) AF and CE. And AE is the square on AB. And AF (is) the rectangle contained by the (straight-lines) BA and AC. For it is contained by DA and AC, and AD (is) equal to AB. And CE (is) the (rectangle contained) by AB and BC. For BE (is) equal to AB. Thus, the (rectangle contained) by BA and AC, plus the (rectangle contained) by AB and BC, is equal to the square on AB.

Thus, if a straight-line is cut at random then the (sum of the) rectangle(s) contained by the whole (straight-line), and each of the pieces (of the straight-line), is equal to the square on the whole. (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $a\,b + a\,c = a^2$ if $a = b + c$.

γ′.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ὑπὸ τῆς ὅλης καὶ ἑνὸς τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ τε ὑπὸ τῶν τμημάτων περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τοῦ προειρημένου τμήματος τετραγώνῳ.



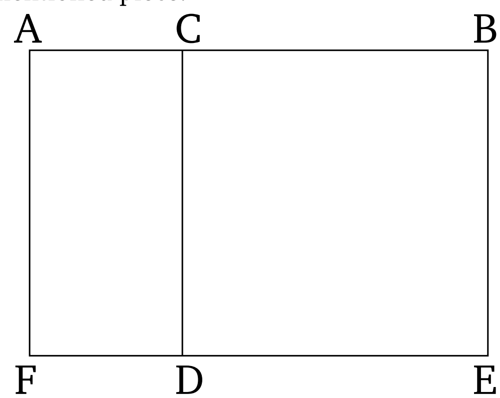
Εὐθεῖα γὰρ ἡ ΑΒ τετμήσθω, ὡς ἔτυχεν, κατὰ τὸ Γ· λέγω, ὅτι τὸ ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ τε ὑπὸ τῶν ΑΓ, ΓΒ περιεχομένῳ ὀρθογωνίῳ μετὰ τοῦ ἀπὸ τῆς ΒΓ τετραγώνου.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΓΒ τετράγωνον τὸ ΓΔΕΒ, καὶ διήχθω ἡ ΕΔ ἐπὶ τὸ Ζ, καὶ διὰ τοῦ Α ὁποτέρᾳ τῶν ΓΔ, ΒΕ παράλληλος ἤχθω ἡ ΑΖ. ἴσον δή ἐστι τὸ ΑΕ τοῖς ΑΔ, ΓΕ· καί ἐστι τὸ μὲν ΑΕ τὸ ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον· περιέχεται μὲν γὰρ ὑπὸ τῶν ΑΒ, ΒΕ, ἴση δὲ ἡ ΒΕ τῇ ΒΓ· τὸ δὲ ΑΔ τὸ ὑπὸ τῶν ΑΓ, ΓΒ· ἴση γὰρ ἡ ΔΓ τῇ ΓΒ· τὸ δὲ ΔΒ τὸ ἀπὸ τῆς ΓΒ τετράγωνον· τὸ ἄρα ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΓ, ΓΒ περιεχομένῳ ὀρθογωνίῳ μετὰ τοῦ ἀπὸ τῆς ΒΓ τετραγώνου.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ὑπὸ τῆς ὅλης καὶ ἑνὸς τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ τε ὑπὸ τῶν τμημάτων περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τοῦ προειρημένου τμήματος τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

Proposition 3†

If a straight-line is cut at random then the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), is equal to the rectangle contained by (both of) the pieces, and the square on the aforementioned piece.



For let the straight-line AB have been cut, at random, at (point) C. I say that the rectangle contained by AB and BC is equal to the rectangle contained by AC and CB, plus the square on BC.

For let the square $CDEB$ have been described on CB [Prop. 1.46], and let ED have been drawn through to F, and let AF have been drawn through A, parallel to either of CD or BE [Prop. 1.31]. So the (rectangle) AE is equal to the (rectangle) AD and the (square) CE. And AE is the rectangle contained by AB and BC. For it is contained by AB and BE, and BE (is) equal to BC. And AD (is) the (rectangle contained) by AC and CB. For DC (is) equal to CB. And DB (is) the square on CB. Thus, the rectangle contained by AB and BC is equal to the rectangle contained by AC and CB, plus the square on BC.

Thus, if a straight-line is cut at random then the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), is equal to the rectangle contained by (both of) the pieces, and the square on the aforementioned piece. (Which is) the very thing it was required to show.

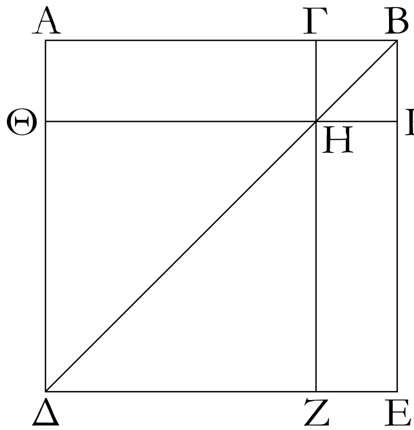
† This proposition is a geometric version of the algebraic identity: $(a + b)\,a = a\,b + a^2$.

δ′.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ἀπὸ τῆς ὅλης τετράγωνον ἴσον ἐστὶ τοῖς τε ἀπὸ τῶν τμημάτων τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν τμημάτων περιεχομένῳ ὀρθο-

Proposition 4†

If a straight-line is cut at random then the square on the whole (straight-line) is equal to the (sum of the) squares on the pieces (of the straight-line), and twice the γωνίῳ.

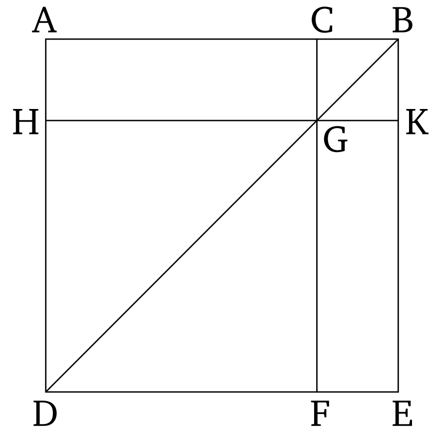


Εὐθεῖα γὰρ γραμμὴ ἡ ΑΒ τετμήσθω, ὡς ἔτυχεν, κατὰ τὸ Γ. λέγω, ὅτι τὸ ἀπὸ τῆς ΑΒ τετράγωνον ἴσον ἐστὶ τοῖς τε ἀπὸ τῶν ΑΓ, ΓΒ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ περιεχομένῳ ὀρθογωνίῳ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔΕΒ, καὶ ἐπεζεύχθω ἡ ΒΔ, καὶ διὰ μὲν τοῦ Γ ὁποτέρᾳ τῶν ΑΔ, ΕΒ παράλληλος ἤχθω ἡ ΓΖ, διὰ δὲ τοῦ Η ὁποτέρᾳ τῶν ΑΒ, ΔΕ παράλληλος ἤχθω ἡ ΘΚ. καὶ ἐπεὶ παράλληλός ἐστιν ἡ ΓΖ τῇ ΑΔ, καὶ εἰς αὐτὰς ἐμπέπτωκεν ἡ ΒΔ, ἡ ἐκτὸς γωνία ἡ ὑπὸ ΓΗΒ ἴση ἐστὶ τῇ ἐντὸς καὶ ἀπεναντίον τῇ ὑπὸ ΑΔΒ. ἀλλ' ἡ ὑπὸ ΑΔΒ τῇ ὑπὸ ΑΒΔ ἐστιν ἴση, ἐπεὶ καὶ πλευρὰ ἡ ΒΑ τῇ ΑΔ ἐστιν ἴση· καὶ ἡ ὑπὸ ΓΗΒ ἄρα γωνιά τῇ ὑπὸ ΗΒΓ ἐστιν ἴση· ὥστε καὶ πλευρὰ ἡ ΒΓ πλευρᾷ τῇ ΓΗ ἐστιν ἴση· ἀλλ' ἡ μὲν ΓΒ τῇ ΗΚ ἐστιν ἴση. ἡ δὲ ΓΗ τῇ ΚΒ· καὶ ἡ ΗΚ ἄρα τῇ ΚΒ ἐστιν ἴση· ἰσόπλευρον ἄρα ἐστὶ τὸ ΓΗΚΒ. λέγω δή, ὅτι καὶ ὀρθογώνιον. ἐπεὶ γὰρ παράλληλός ἐστιν ἡ ΓΗ τῇ ΒΚ [καὶ εἰς αὐτὰς ἐμπέπτωκεν εὐθεῖα ἡ ΓΒ], αἱ ἄρα ὑπὸ ΚΒΓ, ΗΓΒ γωνίαι δύο ὀρθαῖς εἰσιν ἴσαι. ὀρθὴ δὲ ἡ ὑπὸ ΚΒΓ· ὀρθὴ ἄρα καὶ ἡ ὑπὸ ΒΓΗ· ὥστε καὶ αἱ ἀπεναντίον αἱ ὑπὸ ΓΗΚ, ΗΚΒ ὀρθαί εἰσιν. ὀρθογώνιον ἄρα ἐστὶ τὸ ΓΗΚΒ· ἐδείχθη δὲ καὶ ἰσόπλευρον· τετράγωνον ἄρα ἐστίν· καί ἐστιν ἀπὸ τῆς ΓΒ. διὰ τὰ αὐτὰ δὴ καὶ τὸ ΘΖ τετράγωνόν ἐστιν· καί ἐστιν ἀπὸ τῆς ΘΗ, τουτέστιν [ἀπὸ] τῆς ΑΓ· τὰ ἄρα ΘΖ, ΚΓ τετράγωνα ἀπὸ τῶν ΑΓ, ΓΒ εἰσιν. καὶ ἐπεὶ ἴσον ἐστὶ τὸ ΑΗ τῷ ΗΕ, καί ἐστι τὸ ΑΗ τὸ ὑπὸ τῶν ΑΓ, ΓΒ· ἴση γὰρ ἡ ΗΓ τῇ ΓΒ· καὶ τὸ ΗΕ ἄρα ἴσον ἐστὶ τῷ ὑπὸ ΑΓ, ΓΒ· τὰ ἄρα ΑΗ, ΗΕ ἴσα ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ. ἔστι δὲ καὶ τὰ ΘΖ, ΓΚ τετράγωνα ἀπὸ τῶν ΑΓ, ΓΒ· τὰ ἄρα τέσσαρα τὰ ΘΖ, ΓΚ, ΑΗ, ΗΕ ἴσα ἐστὶ τοῖς τε ἀπὸ τῶν ΑΓ, ΓΒ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ περιεχομένῳ ὀρθογωνίῳ. ἀλλὰ τὰ ΘΖ, ΓΚ, ΑΗ, ΗΕ ὅλον ἐστὶ τὸ ΑΔΕΒ, ὅ ἐστιν ἀπὸ τῆς ΑΒ τετράγωνον· τὸ ἄρα ἀπὸ τῆς ΑΒ τετράγωνον ἴσον ἐστὶ τοῖς τε ἀπὸ τῶν ΑΓ, ΓΒ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ περιεχομένῳ ὀρθογωνίῳ.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ἀπὸ τῆς

rectangle contained by the pieces.



For let the straight-line AB have been cut, at random, at (point) C. I say that the square on AB is equal to the (sum of the) squares on AC and CB, and twice the rectangle contained by AC and CB.

For let the square $ADEB$ have been described on AB [Prop. 1.46], and let BD have been joined, and let CF have been drawn through C, parallel to either of AD or EB [Prop. 1.31], and let HK have been drawn through G, parallel to either of AB or DE [Prop. 1.31]. And since CF is parallel to AD, and BD has fallen across them, the external angle CGB is equal to the internal and opposite (angle) ADB [Prop. 1.29]. But, ADB is equal to ABD, since the side BA is also equal to AD [Prop. 1.5]. Thus, angle CGB is also equal to GBC. So the side BC is equal to the side CG [Prop. 1.6]. But, CB is equal to GK, and CG to KB [Prop. 1.34]. Thus, GK is also equal to KB. Thus, $CGKB$ is equilateral. So I say that (it is) also right-angled. For since CG is parallel to BK [and the straight-line CB has fallen across them], the angles KBC and GCB are thus equal to two right-angles [Prop. 1.29]. But KBC (is) a right-angle. Thus, BCG (is) also a right-angle. So the opposite (angles) CGK and GKB are also right-angles [Prop. 1.34]. Thus, $CGKB$ is right-angled. And it was also shown (to be) equilateral. Thus, it is a square. And it is on CB. So, for the same (reasons), HF is also a square. And it is on HG, that is to say [on] AC [Prop. 1.34]. Thus, the squares HF and KC are on AC and CB (respectively). And the (rectangle) AG is equal to the (rectangle) GE [Prop. 1.43]. And AG is the (rectangle contained) by AC and CB. For GC (is) equal to CB. Thus, GE is also equal to the (rectangle contained) by AC and CB. Thus, the (rectangles) AG and GE are equal to twice the (rectangle contained) by AC and CB. And HF and CK are the squares on AC and CB (respectively). Thus, the four (figures) HF, CK, AG, and GE are equal to the (sum of the) squares on ὅλης τετράγωνον ἴσον ἐστὶ τοῖς τε ἀπὸ τῶν τμημάτων τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν τμημάτων περιεχομένῳ ὀρθογωνίῳ· ὅπερ ἔδει δεῖξαι.

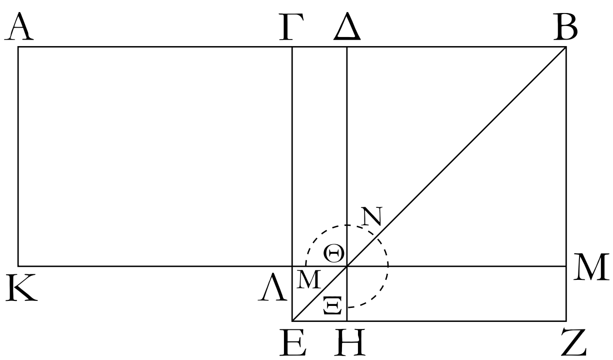
AC and BC, and twice the rectangle contained by AC and CB. But, the (figures) HF, CK, AG, and GE are (equivalent to) the whole of $ADEB$, which is the square on AB. Thus, the square on AB is equal to the (sum of the) squares on AC and CB, and twice the rectangle contained by AC and CB.

Thus, if a straight-line is cut at random then the square on the whole (straight-line) is equal to the (sum of the) squares on the pieces (of the straight-line), and twice the rectangle contained by the pieces. (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $(a+b)^2 = a^2 + b^2 + 2\,a\,b$.

ε′.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ εἰς ἴσα καὶ ἄνισα, τὸ ὑπὸ τῶν ἀνίσων τῆς ὅλης τμημάτων περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς μεταξὺ τῶν τομῶν τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ἡμισείας τετραγώνῳ.

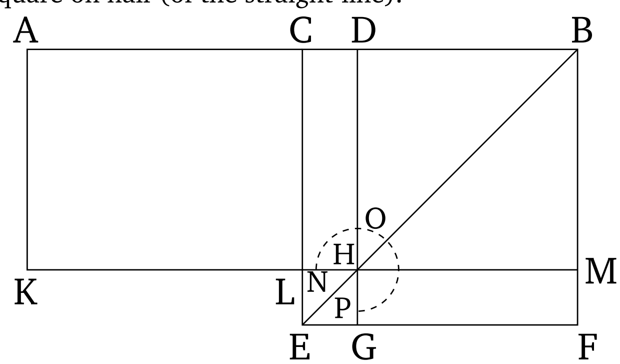


Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω εἰς μὲν ἴσα κατὰ τὸ Γ, εἰς δὲ ἄνισα κατὰ τὸ Δ· λέγω, ὅτι τὸ ὑπὸ τῶν ΑΔ, ΔΒ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΓΔ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΓΒ τετραγώνῳ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΓΒ τετράγωνον τὸ ΓΕΖΒ, καὶ ἐπεζεύχθω ἡ ΒΕ, καὶ διὰ μὲν τοῦ Δ ὁποτέρᾳ τῶν ΓΕ, ΒΖ παράλληλος ἤχθω ἡ ΔΗ, διὰ δὲ τοῦ Θ ὁποτέρᾳ τῶν ΑΒ, ΕΖ παράλληλος πάλιν ἤχθω ἡ ΚΜ, καὶ πάλιν διὰ τοῦ Α ὁποτέρᾳ τῶν ΓΛ, ΒΜ παράλληλος ἤχθω ἡ ΑΚ. καὶ ἐπεὶ ἴσον ἐστὶ τὸ ΓΘ παραπλήρωμα τῷ ΘΖ παραπληρώματι, κοινὸν προσκείσθω τὸ ΔΜ· ὅλον ἄρα τὸ ΓΜ ὅλῳ τῷ ΔΖ ἴσον ἐστίν. ἀλλὰ τὸ ΓΜ τῷ ΑΛ ἴσον ἐστίν, ἐπεὶ καὶ ἡ ΑΓ τῇ ΓΒ ἐστιν ἴση· καὶ τὸ ΑΛ ἄρα τῷ ΔΖ ἴσον ἐστίν. κοινὸν προσκείσθω τὸ ΓΘ· ὅλον ἄρα τὸ ΑΘ τῷ ΜΝΞ† γνώμονι ἴσον ἐστίν. ἀλλὰ τὸ ΑΘ τὸ ὑπὸ τῶν ΑΔ, ΔΒ ἐστιν· ἴση γὰρ ἡ ΔΘ τῇ ΔΒ· καὶ ὁ ΜΝΞ ἄρα γνώμων ἴσος ἐστὶ τῷ ὑπὸ ΑΔ, ΔΒ. κοινὸν προσκείσθω τὸ ΛΗ, ὅ ἐστιν ἴσον τῷ ἀπὸ τῆς ΓΔ· ὁ ἄρα ΜΝΞ γνώμων καὶ τὸ ΛΗ ἴσα ἐστὶ τῷ ὑπὸ τῶν ΑΔ, ΔΒ περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τῆς

Proposition 5‡

If a straight-line is cut into equal and unequal (pieces) then the rectangle contained by the unequal pieces of the whole (straight-line), plus the square on the (difference) between the (equal and unequal) pieces, is equal to the square on half (of the straight-line).



For let any straight-line AB have been cut—equally at C, and unequally at D. I say that the rectangle contained by AD and DB, plus the square on CD, is equal to the square on CB.

For let the square $CEFB$ have been described on CB [Prop. 1.46], and let BE have been joined, and let DG have been drawn through D, parallel to either of CE or BF [Prop. 1.31], and again let KM have been drawn through H, parallel to either of AB or EF [Prop. 1.31], and again let AK have been drawn through A, parallel to either of CL or BM [Prop. 1.31]. And since the complement CH is equal to the complement HF [Prop. 1.43], let the (square) DM have been added to both. Thus, the whole (rectangle) CM is equal to the whole (rectangle) DF. But, (rectangle) CM is equal to (rectangle) AL, since AC is also equal to CB [Prop. 1.36]. Thus, (rectangle) AL is also equal to (rectangle) DF. Let (rectangle) CH have been added to both. Thus, the whole (rectangle) AH is equal to the gnomon NOP. But, AH

ΓΔ τετραγώνῳ. ἀλλὰ ὁ ΜΝΞ γνώμων καὶ τὸ ΛΗ ὅλον ἐστὶ τὸ ΓΕΖΒ τετράγωνον, ὅ ἐστιν ἀπὸ τῆς ΓΒ· τὸ ἄρα ὑπὸ τῶν ΑΔ, ΔΒ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΓΔ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΓΒ τετραγώνῳ.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ εἰς ἴσα καὶ ἄνισα, τὸ ὑπὸ τῶν ἀνίσων τῆς ὅλης τμημάτων περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς μεταξὺ τῶν τομῶν τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ἡμισείας τετραγώνῳ. ὅπερ ἔδει δεῖξαι.

is the (rectangle contained) by AD and DB. For DH (is) equal to DB. Thus, the gnomon NOP is also equal to the (rectangle contained) by AD and DB. Let LG, which is equal to the (square) on CD, have been added to both. Thus, the gnomon NOP and the (square) LG are equal to the rectangle contained by AD and DB, and the square on CD. But, the gnomon NOP and the (square) LG is (equivalent to) the whole square $CEFB$, which is on CB. Thus, the rectangle contained by AD and DB, plus the square on CD, is equal to the square on CB.

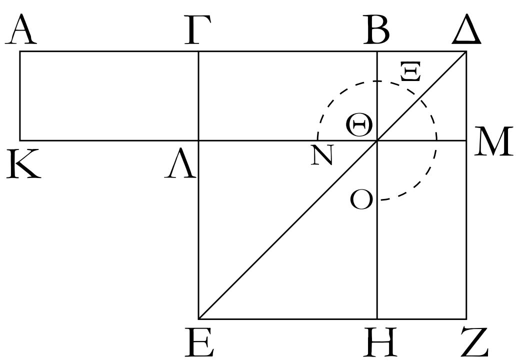
Thus, if a straight-line is cut into equal and unequal (pieces) then the rectangle contained by the unequal pieces of the whole (straight-line), plus the square on the (difference) between the (equal and unequal) pieces, is equal to the square on half (of the straight-line). (Which is) the very thing it was required to show.

† Note the (presumably mistaken) double use of the label M in the Greek text.

‡ This proposition is a geometric version of the algebraic identity: $a\,b + [(a+b)/2 - b]^2 = [(a+b)/2]^2$.

ϛ′.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ δίχα, προστεθῇ δέ τις αὐτῇ εὐθεῖα ἐπ᾽ εὐθείας, τὸ ὑπὸ τῆς ὅλης σὺν τῇ προσκειμένῃ καὶ τῆς προσκειμένης περιεχόμενον ὀρθόγώνιον μετὰ τοῦ ἀπὸ τῆς ἡμισείας τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς ἡμισείας καὶ τῆς προσκειμένης τετραγώνῳ.



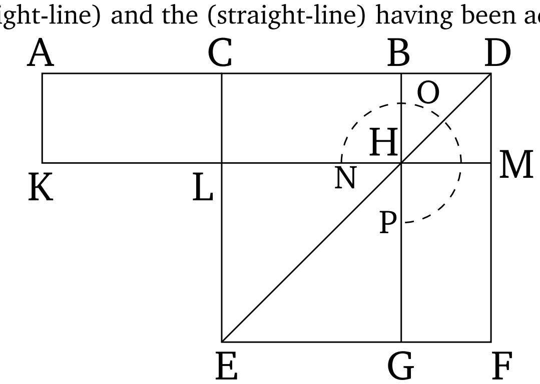
Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω δίχα κατὰ τὸ Γ σημεῖον, προσκείσθω δέ τις αὐτῇ εὐθεῖα ἐπ᾽ εὐθείας ἡ ΒΔ· λέγω, ὅτι τὸ ὑπὸ τῶν ΑΔ, ΔΒ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΓΒ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΓΔ τετραγώνῳ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΓΔ τετράγωνον τὸ ΓΕΖΔ, καὶ ἐπεζεύχθω ἡ ΔΕ, καὶ διὰ μὲν τοῦ Β σημείου ὁποτέρᾳ τῶν ΕΓ, ΔΖ παράλληλος ἤχθω ἡ ΒΗ, διὰ δὲ τοῦ Θ σημείου ὁποτέρᾳ τῶν ΑΒ, ΕΖ παράλληλος ἤχθω ἡ ΚΜ, καὶ ἔτι διὰ τοῦ Α ὁποτέρᾳ τῶν ΓΛ, ΔΜ παράλληλος ἤχθω ἡ ΑΚ.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΑΓ τῇ ΓΒ, ἴσον ἐστὶ καὶ τὸ ΑΛ

Proposition 6†

If a straight-line is cut in half, and any straight-line added to it straight-on, then the rectangle contained by the whole (straight-line) with the (straight-line) having being added, and the (straight-line) having being added, plus the square on half (of the original straight-line), is equal to the square on the sum of half (of the original straight-line) and the (straight-line) having been added.



For let any straight-line AB have been cut in half at point C, and let any straight-line BD have been added to it straight-on. I say that the rectangle contained by AD and DB, plus the square on CB, is equal to the square on CD.

For let the square $CEFD$ have been described on CD [Prop. 1.46], and let DE have been joined, and let BG have been drawn through point B, parallel to either of EC or DF [Prop. 1.31], and let KM have been drawn through point H, parallel to either of AB or EF [Prop. 1.31], and finally let AK have been drawn τῷ ΓΘ. ἀλλὰ τὸ ΓΘ τῷ ΘΖ ἴσον ἐστίν. καὶ τὸ ΑΛ ἄρα τῷ ΘΖ ἐστιν ἴσον. κοινὸν προσκείσθω τὸ ΓΜ· ὅλον ἄρα τὸ ΑΜ τῷ ΝΞΟ γνώμονί ἐστιν ἴσον. ἀλλὰ τὸ ΑΜ ἐστι τὸ ὑπὸ τῶν ΑΔ, ΔΒ· ἴση γάρ ἐστιν ἡ ΔΜ τῇ ΔΒ· καὶ ὁ ΝΞΟ ἄρα γνώμων ἴσος ἐστὶ τῷ ὑπὸ τῶν ΑΔ, ΔΒ [περιεχομένῳ ὀρθογωνίῳ]. κοινὸν προσκείσθω τὸ ΛΗ, ὅ ἐστιν ἴσον τῷ ἀπὸ τῆς ΒΓ τετραγώνῳ· τὸ ἄρα ὑπὸ τῶν ΑΔ, ΔΒ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΓΒ τετραγώνου ἴσον ἐστὶ τῷ ΝΞΟ γνώμονι καὶ τῷ ΛΗ. ἀλλὰ ὁ ΝΞΟ γνώμων καὶ τὸ ΛΗ ὅλον ἐστὶ τὸ ΓΕΖΔ τετράγωνον, ὅ ἐστιν ἀπὸ τῆς ΓΔ· τὸ ἄρα ὑπὸ τῶν ΑΔ, ΔΒ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΓΒ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΓΔ τετραγώνῳ.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ δίχα, προστεθῇ δέ τις αὐτῇ εὐθεῖα ἐπ᾿ εὐθείας, τὸ ὑπὸ τῆς ὅλης σὺν τῇ προσκειμένῃ καὶ τῆς προσκειμένης περιεχόμενον ὀρθόγώνιον μετὰ τοῦ ἀπὸ τῆς ἡμισείας τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς ἡμισείας καὶ τῆς προσκειμένης τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

through A, parallel to either of CL or DM [Prop. 1.31].

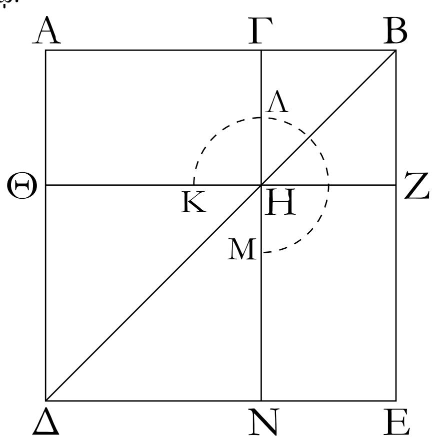
Therefore, since AC is equal to CB, (rectangle) AL is also equal to (rectangle) CH [Prop. 1.36]. But, (rectangle) CH is equal to (rectangle) HF [Prop. 1.43]. Thus, (rectangle) AL is also equal to (rectangle) HF. Let (rectangle) CM have been added to both. Thus, the whole (rectangle) AM is equal to the gnomon NOP. But, AM is the (rectangle contained) by AD and DB. For DM is equal to DB. Thus, gnomon NOP is also equal to the [rectangle contained] by AD and DB. Let LG, which is equal to the square on BC, have been added to both. Thus, the rectangle contained by AD and DB, plus the square on CB, is equal to the gnomon NOP and the (square) LG. But the gnomon NOP and the (square) LG is (equivalent to) the whole square $CEFD$, which is on CD. Thus, the rectangle contained by AD and DB, plus the square on CB, is equal to the square on CD.

Thus, if a straight-line is cut in half, and any straight-line added to it straight-on, then the rectangle contained by the whole (straight-line) with the (straight-line) having being added, and the (straight-line) having being added, plus the square on half (of the original straight-line), is equal to the square on the sum of half (of the original straight-line) and the (straight-line) having been added. (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $(2\,a+b)\,b+a^2=(a+b)^2$.

ζ΄.

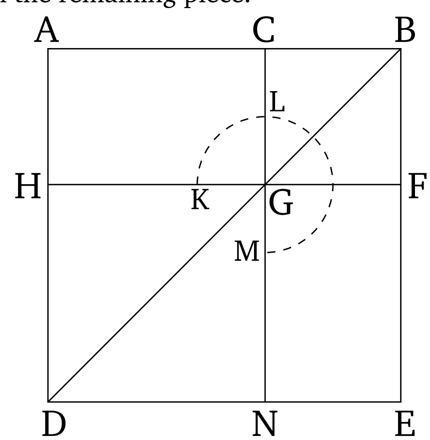
Ἐὰν εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ἀπὸ τῆς ὅλης καὶ τὸ ἀφ᾿ ἑνὸς τῶν τμημάτων τὰ συναμφότερα τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῆς ὅλης καὶ τοῦ εἰρημένου τμήματος περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνῳ.



Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω, ὡς ἔτυχεν, κατὰ τὸ Γ σημεῖον· λέγω, ὅτι τὰ ἀπὸ τῶν ΑΒ, ΒΓ τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῶν ΑΒ, ΒΓ περιεχομένῳ ὀρθογωνίῳ καὶ τῷ

Proposition 7†

If a straight-line is cut at random then the sum of the squares on the whole (straight-line), and one of the pieces (of the straight-line), is equal to twice the rectangle contained by the whole, and the said piece, and the square on the remaining piece.



For let any straight-line AB have been cut, at random, at point C. I say that the (sum of the) squares on AB and BC is equal to twice the rectangle contained by AB and ἀπὸ τῆς ΓΑ τετραγώνῳ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔΕΒ· καὶ καταγεγράφθω τὸ σχῆμα.

Ἐπεὶ οὖν ἴσον ἐστὶ τὸ ΑΗ τῷ ΗΕ, κοινὸν προσκείσθω τὸ ΓΖ· ὅλον ἄρα τὸ ΑΖ ὅλῳ τῷ ΓΕ ἴσον ἐστίν· τὰ ἄρα ΑΖ, ΓΕ διπλάσιά ἐστι τοῦ ΑΖ. ἀλλὰ τὰ ΑΖ, ΓΕ ὁ ΚΛΜ ἐστι γνώμων καὶ τὸ ΓΖ τετράγωνον· ὁ ΚΛΜ ἄρα γνώμων καὶ τὸ ΓΖ διπλάσιά ἐστι τοῦ ΑΖ. ἔστι δὲ τοῦ ΑΖ διπλάσιον καὶ τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· ἴση γὰρ ἡ ΒΖ τῇ ΒΓ· ὁ ἄρα ΚΛΜ γνώμων καὶ τὸ ΓΖ τετράγωνον ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ. κοινὸν προσκείσθω τὸ ΔΗ, ὅ ἐστιν ἀπὸ τῆς ΑΓ τετράγωνον· ὁ ἄρα ΚΛΜ γνώμων καὶ τὰ ΒΗ, ΗΔ τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῶν ΑΒ, ΒΓ περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τῆς ΑΓ τετραγώνῳ. ἀλλὰ ὁ ΚΛΜ γνώμων καὶ τὰ ΒΗ, ΗΔ τετράγωνα ὅλον ἐστὶ τὸ ΑΔΕΒ καὶ τὸ ΓΖ, ἅ ἐστιν ἀπὸ τῶν ΑΒ, ΒΓ τετράγωνα· τὰ ἄρα ἀπὸ τῶν ΑΒ, ΒΓ τετράγωνα ἴσα ἐστὶ τῷ [τε] δὶς ὑπὸ τῶν ΑΒ, ΒΓ περιεχομένῳ ὀρθογωνίῳ μετὰ τοῦ ἀπὸ τῆς ΑΓ τετραγώνου.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ ἀπὸ τῆς ὅλης καὶ τὸ ἀφ᾽ ἑνὸς τῶν τμημάτων τὰ συναμφότερα τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῆς ὅλης καὶ τοῦ εἰρημένου τμήματος περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

BC, and the square on CA.

For let the square $ADEB$ have been described on AB [Prop. 1.46], and let the (rest of) the figure have been drawn.

Therefore, since (rectangle) AG is equal to (rectangle) GE [Prop. 1.43], let the (square) CF have been added to both. Thus, the whole (rectangle) AF is equal to the whole (rectangle) CE. Thus, (rectangle) AF plus (rectangle) CE is double (rectangle) AF. But, (rectangle) AF plus (rectangle) CE is the gnomon KLM, and the square CF. Thus, the gnomon KLM, and the square CF, is double the (rectangle) AF. But double the (rectangle) AF is also twice the (rectangle contained) by AB and BC. For BF (is) equal to BC. Thus, the gnomon KLM, and the square CF, are equal to twice the (rectangle contained) by AB and BC. Let DG, which is the square on AC, have been added to both. Thus, the gnomon KLM, and the squares BG and GD, are equal to twice the rectangle contained by AB and BC, and the square on AC. But, the gnomon KLM and the squares BG and GD is (equivalent to) the whole of $ADEB$ and CF, which are the squares on AB and BC (respectively). Thus, the (sum of the) squares on AB and BC is equal to twice the rectangle contained by AB and BC, and the square on AC.

Thus, if a straight-line is cut at random then the sum of the squares on the whole (straight-line), and one of the pieces (of the straight-line), is equal to twice the rectangle contained by the whole, and the said piece, and the square on the remaining piece. (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $(a+b)^2 + a^2 = 2\,(a+b)\,a + b^2$.

η'.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ τετράκις ὑπὸ τῆς ὅλης καὶ ἑνὸς τῶν τμημάτων περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνου ἴσον ἐστὶ τῷ ἀπό τε τῆς ὅλης καὶ τοῦ εἰρημένου τμήματος ὡς ἀπὸ μιᾶς ἀναγραφέντι τετραγώνῳ.

Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω, ὡς ἔτυχεν, κατὰ τὸ Γ σημεῖον· λέγω, ὅτι τὸ τετράκις ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΑΓ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ, ΒΓ ὡς ἀπὸ μιᾶς ἀναγραφέντι τετραγώνῳ.

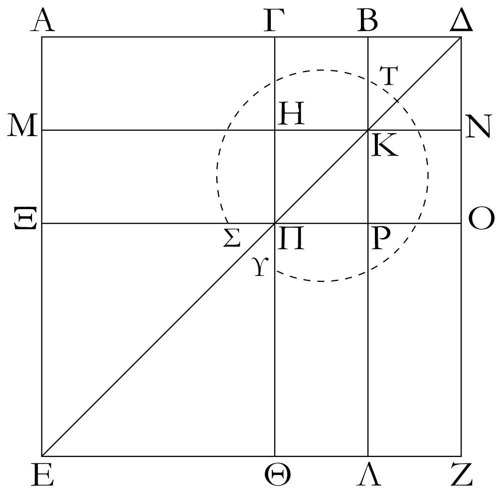
Ἐκβεβλήσθω γὰρ ἐπ᾽ εὐθείας [τῇ ΑΒ εὐθεῖα] ἡ ΒΔ, καὶ κείσθω τῇ ΓΒ ἴση ἡ ΒΔ, καὶ ἀναγεγράφθω ἀπὸ τῆς ΑΔ τετράγωνον τὸ ΑΕΖΔ, καὶ καταγεγράφθω διπλοῦν τὸ σχῆμα.

Proposition 8†

If a straight-line is cut at random then four times the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), plus the square on the remaining piece, is equal to the square described on the whole and the former piece, as on one (complete straight-line).

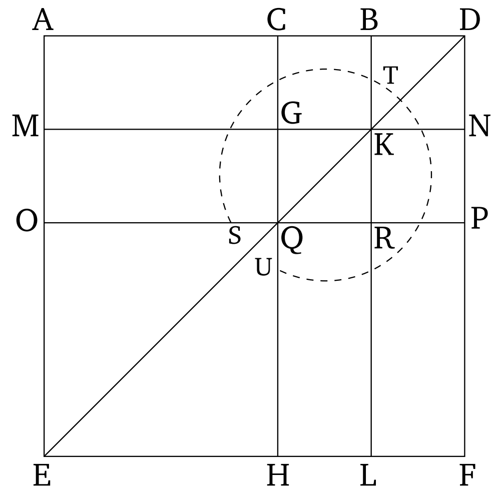
For let any straight-line AB have been cut, at random, at point C. I say that four times the rectangle contained by AB and BC, plus the square on AC, is equal to the square described on AB and BC, as on one (complete straight-line).

For let BD have been produced in a straight-line [with the straight-line AB], and let BD be made equal to CB [Prop. 1.3], and let the square $AEFD$ have been described on AD [Prop. 1.46], and let the (rest of the) figure have been drawn double.



Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΓΒ τῇ ΒΔ, ἀλλὰ ἡ μὲν ΓΒ τῇ ΗΚ ἐστιν ἴση, ἡ δὲ ΒΔ τῇ ΚΝ, καὶ ἡ ΗΚ ἄρα τῇ ΚΝ ἐστιν ἴση. διὰ τὰ αὐτὰ δὴ καὶ ἡ ΠΡ τῇ ΡΟ ἐστιν ἴση. καὶ ἐπεὶ ἴση ἐστὶν ἡ ΒΓ τῇ ΒΔ, ἡ δὲ ΗΚ τῇ ΚΝ, ἴσον ἄρα ἐστὶ καὶ τὸ μὲν ΓΚ τῷ ΚΔ, τὸ δὲ ΗΡ τῷ ΡΝ. ἀλλὰ τὸ ΓΚ τῷ ΡΝ ἐστιν ἴσον· παραπληρώματα γὰρ τοῦ ΓΟ παραλληλογράμμου· καὶ τὸ ΚΔ ἄρα τῷ ΗΡ ἴσον ἐστίν· τὰ τέσσαρα ἄρα τὰ ΔΚ, ΓΚ, ΗΡ, ΡΝ ἴσα ἀλλήλοις ἐστίν. τὰ τέσσαρα ἄρα τετραπλάσιά ἐστι τοῦ ΓΚ. πάλιν ἐπεὶ ἴση ἐστὶν ἡ ΓΒ τῇ ΒΔ, ἀλλὰ ἡ μὲν ΒΔ τῇ ΒΚ, τουτέστι τῇ ΓΗ ἴση, ἡ δὲ ΓΒ τῇ ΗΚ, τουτέστι τῇ ΗΠ, ἐστιν ἴση, καὶ ἡ ΓΗ ἄρα τῇ ΗΠ ἴση ἐστίν. καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν ΓΗ τῇ ΗΠ, ἡ δὲ ΠΡ τῇ ΡΟ, ἴσον ἐστὶ καὶ τὸ μὲν ΑΗ τῷ ΜΠ, τὸ δὲ ΠΛ τῷ ΡΖ. ἀλλὰ τὸ ΜΠ τῷ ΠΛ ἐστιν ἴσον· παραπληρώματα γὰρ τοῦ ΜΛ παραλληλογράμμου· καὶ τὸ ΑΗ ἄρα τῷ ΡΖ ἴσον ἐστίν· τὰ τέσσαρα ἄρα τὰ ΑΗ, ΜΠ, ΠΛ, ΡΖ ἴσα ἀλλήλοις ἐστίν· τὰ τέσσαρα ἄρα τοῦ ΑΗ ἐστι τετραπλάσια. ἐδείχθη δὲ καὶ τὰ τέσσαρα τὰ ΓΚ, ΚΔ, ΗΡ, ΡΝ τοῦ ΓΚ τετραπλάσια· τὰ ἄρα ὀκτώ, ἃ περιέχει τὸν ΣΤΥ γνώμονα, τετραπλάσιά ἐστι τοῦ ΑΚ. καὶ ἐπεὶ τὸ ΑΚ τὸ ὑπὸ τῶν ΑΒ, ΒΔ ἐστιν· ἴση γὰρ ἡ ΒΚ τῇ ΒΔ· τὸ ἄρα τετράκις ὑπὸ τῶν ΑΒ, ΒΔ τετραπλάσιόν ἐστι τοῦ ΑΚ. ἐδείχθη δὲ τοῦ ΑΚ τετραπλάσιος καὶ ὁ ΣΤΥ γνώμων· τὸ ἄρα τετράκις ὑπὸ τῶν ΑΒ, ΒΔ ἴσον ἐστὶ τῷ ΣΤΥ γνώμονι. κοινὸν προσκείσθω τὸ ΞΘ, ὅ ἐστιν ἴσον τῷ ἀπὸ τῆς ΑΓ τετραγώνῳ· τὸ ἄρα τετράκις ὑπὸ τῶν ΑΒ, ΒΔ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ ΑΓ τετραγώνου ἴσον ἐστὶ τῷ ΣΤΥ γνώμονι καὶ τῷ ΞΘ. ἀλλὰ ὁ ΣΤΥ γνώμων καὶ τὸ ΞΘ ὅλον ἐστὶ τὸ ΑΕΖΔ τετράγωνον, ὅ ἐστιν ἀπὸ τῆς ΑΔ· τὸ ἄρα τετράκις ὑπὸ τῶν ΑΒ, ΒΔ μετὰ τοῦ ἀπὸ ΑΓ ἴσον ἐστὶ τῷ ἀπὸ ΑΔ τετραγώνῳ· ἴση δὲ ἡ ΒΔ τῇ ΒΓ. τὸ ἄρα τετράκις ὑπὸ τῶν ΑΒ, ΒΓ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ ΑΓ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΔ, τουτέστι τῷ ἀπὸ τῆς ΑΒ καὶ ΒΓ ὡς ἀπὸ μιᾶς ἀναγραφέντι τετραγώνῳ.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ, ὡς ἔτυχεν, τὸ τετράκις ὑπὸ τῆς ὅλης καὶ ἑνὸς τῶν τμημάτων περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνου ἴσου



Therefore, since CB is equal to BD, but CB is equal to GK [Prop. 1.34], and BD to KN [Prop. 1.34], GK is thus also equal to KN. So, for the same (reasons), QR is equal to RP. And since BC is equal to BD, and GK to KN, (square) CK is thus also equal to (square) KD, and (square) GR to (square) RN [Prop. 1.36]. But, (square) CK is equal to (square) RN. For (they are) complements in the parallelogram CP [Prop. 1.43]. Thus, (square) KD is also equal to (square) GR. Thus, the four (squares) DK, CK, GR, and RN are equal to one another. Thus, the four (taken together) are quadruple (square) CK. Again, since CB is equal to BD, but BD (is) equal to BK—that is to say, CG—and CB is equal to GK—that is to say, GQ—CG is thus also equal to GQ. And since CG is equal to GQ, and QR to RP, (rectangle) AG is also equal to (rectangle) MQ, and (rectangle) QL to (rectangle) RF [Prop. 1.36]. But, (rectangle) MQ is equal to (rectangle) QL. For (they are) complements in the parallelogram ML [Prop. 1.43]. Thus, (rectangle) AG is also equal to (rectangle) RF. Thus, the four (rectangles) AG, MQ, QL, and RF are equal to one another. Thus, the four (taken together) are quadruple (rectangle) AG. And it was also shown that the four (squares) CK, KD, GR, and RN (taken together are) quadruple (square) CK. Thus, the eight (figures taken together), which comprise the gnomon STU, are quadruple (rectangle) AK. And since AK is the (rectangle contained) by AB and BD, for BK (is) equal to BD, four times the (rectangle contained) by AB and BD is quadruple (rectangle) AK. But the gnomon STU was also shown (to be equal to) quadruple (rectangle) AK. Thus, four times the (rectangle contained) by AB and BD is equal to the gnomon STU. Let OH, which is equal to the square on AC, have been added to both. Thus, four times the rectangle contained by AB and BD, plus the square on AC, is equal to the gnomon STU, and the (square) OH. But, ἐστὶ τῷ ἀπό τε τῆς ὅλης καὶ τοῦ εἰρημένου τμήματος ὡς ἀπὸ μιᾶς ἀναγραφέντι τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

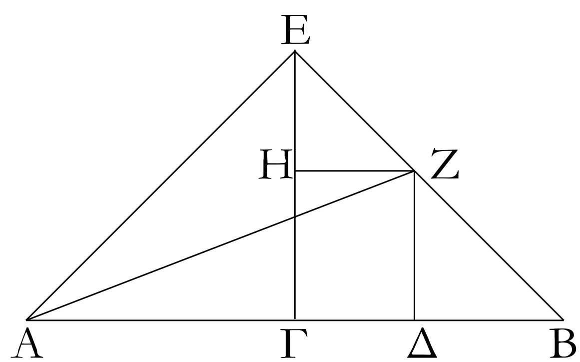
the gnomon STU and the (square) OH is (equivalent to) the whole square $AEFD$, which is on AD. Thus, four times the (rectangle contained) by AB and BD, plus the (square) on AC, is equal to the square on AD. And BD (is) equal to BC. Thus, four times the rectangle contained by AB and BC, plus the square on AC, is equal to the (square) on AD, that is to say the square described on AB and BC, as on one (complete straight-line).

Thus, if a straight-line is cut at random then four times the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), plus the square on the remaining piece, is equal to the square described on the whole and the former piece, as on one (complete straight-line). (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $4\,(a+b)\,a + b^2 = [(a+b)+a]^2$.

ϑ´.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ εἰς ἴσα καὶ ἄνισα, τὰ ἀπὸ τῶν ἀνίσων τῆς ὅλης τμημάτων τετράγωνα διπλάσιά ἐστι τοῦ τε ἀπὸ τῆς ἡμισείας καὶ τοῦ ἀπὸ τῆς μεταξὺ τῶν τομῶν τετραγώνου.

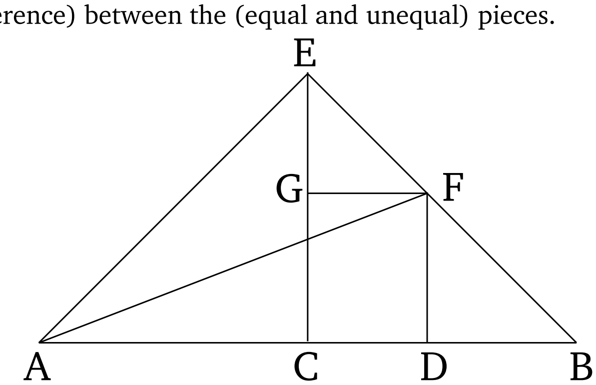


Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω εἰς μὲν ἴσα κατὰ τὸ Γ, εἰς δὲ ἄνισα κατὰ τὸ Δ· λέγω, ὅτι τὰ ἀπὸ τῶν ΑΔ, ΔΒ τετράγωνα διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων.

Ἤχθω γὰρ ἀπὸ τοῦ Γ τῇ ΑΒ πρὸς ὀρθὰς ἡ ΓΕ, καὶ κείσθω ἴση ἑκατέρᾳ τῶν ΑΓ, ΓΒ, καὶ ἐπεζεύχθωσαν αἱ ΕΑ, ΕΒ, καὶ διὰ μὲν τοῦ Δ τῇ ΕΓ παράλληλος ἤχθω ἡ ΔΖ, διὰ δὲ τοῦ Ζ τῇ ΑΒ ἡ ΖΗ, καὶ ἐπεζεύχθω ἡ ΑΖ. καὶ ἐπεὶ ἴση ἐστὶν ἡ ΑΓ τῇ ΓΕ, ἴση ἐστὶ καὶ ἡ ὑπὸ ΕΑΓ γωνία τῇ ὑπὸ ΑΕΓ. καὶ ἐπεὶ ὀρθή ἐστιν ἡ πρὸς τῷ Γ, λοιπαὶ ἄρα αἱ ὑπὸ ΕΑΓ, ΑΕΓ μιᾷ ὀρθῇ ἴσαι εἰσίν· καί εἰσιν ἴσαι· ἡμίσεια ἄρα ὀρθῆς ἐστιν ἑκατέρα τῶν ὑπὸ ΓΕΑ, ΓΑΕ. δὶα τὰ αὐτὰ δὴ καὶ ἑκατέρα τῶν ὑπὸ ΓΕΒ, ΕΒΓ ἡμίσειά ἐστιν ὀρθῆς· ὅλη ἄρα ἡ ὑπὸ ΑΕΒ ὀρθή ἐστιν. καὶ ἐπεὶ ἡ ὑπὸ ΗΕΖ ἡμίσειά ἐστιν ὀρθῆς, ὀρθὴ δὲ ἡ ὑπὸ ΕΗΖ· ἴση γάρ ἐστι τῇ ἐντὸς καὶ ἀπεναντίον τῇ ὑπὸ ΕΓΒ· λοιπὴ ἄρα ἡ ὑπὸ ΕΖΗ ἡμίσειά ἐστιν

Proposition 9†

If a straight-line is cut into equal and unequal (pieces) then the (sum of the) squares on the unequal pieces of the whole (straight-line) is double the (sum of the) square on half (the straight-line) and (the square) on the (difference) between the (equal and unequal) pieces.



For let any straight-line AB have been cut—equally at C, and unequally at D. I say that the (sum of the) squares on AD and DB is double the (sum of the squares) on AC and CD.

For let CE have been drawn from (point) C, at right-angles to AB [Prop. 1.11], and let it be made equal to each of AC and CB [Prop. 1.3], and let EA and EB have been joined. And let DF have been drawn through (point) D, parallel to EC [Prop. 1.31], and (let) FG (have been drawn) through (point) F, (parallel) to AB [Prop. 1.31]. And let AF have been joined. And since AC is equal to CE, the angle EAC is also equal to the (angle) AEC [Prop. 1.5]. And since the (angle) at C is a right-angle, the (sum of the) remaining angles (of triangle AEC), EAC and AEC, is thus equal to one rightὀρθῆς· ἴση ἄρα [ἐστὶν] ἡ ὑπὸ ΗΕΖ γωνία τῇ ὑπὸ ΕΖΗ· ὥστε καὶ πλευρὰ ἡ ΕΗ τῇ ΗΖ ἐστιν ἴση. πάλιν ἐπεὶ ἡ πρὸς τῷ Β γωνία ἡμίσειά ἐστιν ὀρθῆς, ὀρθὴ δὲ ἡ ὑπὸ ΖΔΒ· ἴση γὰρ πάλιν ἐστὶ τῇ ἐντὸς καὶ ἀπεναντίον τῇ ὑπὸ ΕΓΒ· λοιπὴ ἄρα ἡ ὑπὸ ΒΖΔ ἡμίσειά ἐστιν ὀρθῆς· ἴση ἄρα ἡ πρὸς τῷ Β γωνία τῇ ὑπὸ ΔΖΒ· ὥστε καὶ πλευρὰ ἡ ΖΔ πλευρᾷ τῇ ΔΒ ἐστιν ἴση. καὶ ἐπεὶ ἴση ἐστὶν ἡ ΑΓ τῇ ΓΕ, ἴσον ἐστὶ καὶ τὸ ἀπὸ ΑΓ τῷ ἀπὸ ΓΕ· τὰ ἄρα ἀπὸ τῶν ΑΓ, ΓΕ τετράγωνα διπλάσιά ἐστι τοῦ ἀπὸ ΑΓ. τοῖς δὲ ἀπὸ τῶν ΑΓ, ΓΕ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΕΑ τετράγωνον· ὀρθὴ γὰρ ἡ ὑπὸ ΑΓΕ γωνία· τὸ ἄρα ἀπὸ τῆς ΕΑ διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΑΓ. πάλιν, ἐπεὶ ἴση ἐστὶν ἡ ΕΗ τῇ ΗΖ, ἴσον καὶ τὸ ἀπὸ τῆς ΕΗ τῷ ἀπὸ τῆς ΗΖ· τὰ ἄρα ἀπὸ τῶν ΕΗ, ΗΖ τετράγωνα διπλάσιά ἐστι τοῦ ἀπὸ τῆς ΗΖ τετραγώνου. τοῖς δὲ ἀπὸ τῶν ΕΗ, ΗΖ τετραγώνοις ἴσον ἐστὶ τὸ ἀπὸ τῆς ΕΖ τετράγωνον· τὸ ἄρα ἀπὸ τῆς ΕΖ τετράγωνον διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΗΖ. ἴση δὲ ἡ ΗΖ τῇ ΓΔ· τὸ ἄρα ἀπὸ τῆς ΕΖ διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΓΔ. ἔστι δὲ καὶ τὸ ἀπὸ τῆς ΕΑ διπλάσιον τοῦ ἀπὸ τῆς ΑΓ· τὰ ἄρα ἀπὸ τῶν ΑΕ, ΕΖ τετράγωνα διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων. τοῖς δὲ ἀπὸ τῶν ΑΕ, ΕΖ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΑΖ τετράγωνον· ὀρθὴ γάρ ἐστιν ἡ ὑπὸ ΑΕΖ γωνία· τὸ ἄρα ἀπὸ τῆς ΑΖ τετράγωνον διπλάσιόν ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ. τῷ δὲ ἀπὸ τῆς ΑΖ ἴσα τὰ ἀπὸ τῶν ΑΔ, ΔΖ· ὀρθὴ γὰρ ἡ πρὸς τῷ Δ γωνία· τὰ ἄρα ἀπὸ τῶν ΑΔ, ΔΖ διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων. ἴση δὲ ἡ ΔΖ τῇ ΔΒ· τὰ ἄρα ἀπὸ τῶν ΑΔ, ΔΒ τετράγωνα διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ εἰς ἴσα καὶ ἄνισα, τὰ ἀπὸ τῶν ἀνίσων τῆς ὅλης τμημάτων τετράγωνα διπλάσιά ἐστι τοῦ τε ἀπὸ τῆς ἡμισείας καὶ τοῦ ἀπὸ τῆς μεταξὺ τῶν τομῶν τετραγώνου· ὅπερ ἔδει δεῖξαι.

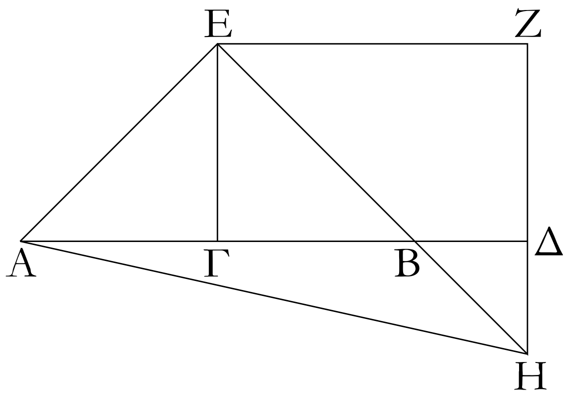
angle [Prop. 1.32]. And they are equal. Thus, (angles) CEA and CAE are each half a right-angle. So, for the same (reasons), (angles) CEB and EBC are also each half a right-angle. Thus, the whole (angle) AEB is a right-angle. And since GEF is half a right-angle, and EGF (is) a right-angle—for it is equal to the internal and opposite (angle) ECB [Prop. 1.29]—the remaining (angle) EFG is thus half a right-angle [Prop. 1.32]. Thus, angle GEF [is] equal to EFG. So the side EG is also equal to the (side) GF [Prop. 1.6]. Again, since the angle at B is half a right-angle, and (angle) FDB (is) a right-angle—for again it is equal to the internal and opposite (angle) ECB [Prop. 1.29]—the remaining (angle) BFD is half a right-angle [Prop. 1.32]. Thus, the angle at B (is) equal to DFB. So the side FD is also equal to the side DB [Prop. 1.6]. And since AC is equal to CE, the (square) on AC (is) also equal to the (square) on CE. Thus, the (sum of the) squares on AC and CE is double the (square) on AC. And the square on EA is equal to the (sum of the) squares on AC and CE. For angle ACE (is) a right-angle [Prop. 1.47]. Thus, the (square) on EA is double the (square) on AC. Again, since EG is equal to GF, the (square) on EG (is) also equal to the (square) on GF. Thus, the (sum of the squares) on EG and GF is double the square on GF. And the square on EF is equal to the (sum of the) squares on EG and GF [Prop. 1.47]. Thus, the square on EF is double the (square) on GF. And GF (is) equal to CD [Prop. 1.34]. Thus, the (square) on EF is double the (square) on CD. And the (square) on EA is also double the (square) on AC. Thus, the (sum of the) squares on AE and EF is double the (sum of the) squares on AC and CD. And the square on AF is equal to the (sum of the squares) on AE and EF. For the angle AEF is a right-angle [Prop. 1.47]. Thus, the square on AF is double the (sum of the squares) on AC and CD. And the (sum of the squares) on AD and DF (is) equal to the (square) on AF. For the angle at D is a right-angle [Prop. 1.47]. Thus, the (sum of the squares) on AD and DF is double the (sum of the) squares on AC and CD. And DF (is) equal to DB. Thus, the (sum of the) squares on AD and DB is double the (sum of the) squares on AC and CD.

Thus, if a straight-line is cut into equal and unequal (pieces) then the (sum of the) squares on the unequal pieces of the whole (straight-line) is double the (sum of the) square on half (the straight-line) and (the square) on the (difference) between the (equal and unequal) pieces. (Which is) the very thing it was required to show.

† This proposition is a geometric version of the algebraic identity: $a^2 + b^2 = 2[([a+b]/2)^2 + ([a+b]/2 - b)^2]$.

ι΄.

Ἐὰν εὐθεῖα γραμμὴ τμηθῇ δίχα, προστεθῇ δέ τις αὐτῇ εὐθεῖα ἐπ᾽ εὐθείας, τὸ ἀπὸ τῆς ὅλης σὺν τῇ προσκειμένῃ καὶ τὸ ἀπὸ τῆς προσκειμένης τὰ συναμφότερα τετράγωνα διπλάσιά ἐστι τοῦ τε ἀπὸ τῆς ἡμισείας καὶ τοῦ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς ἡμισείας καὶ τῆς προσκειμένης ὡς ἀπὸ μιᾶς ἀναγραφέντος τετραγώνου.

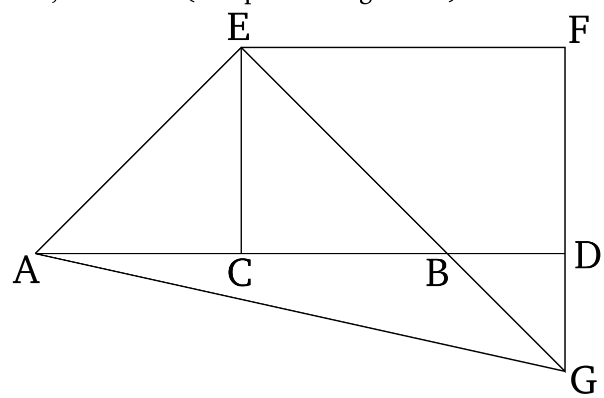


Εὐθεῖα γάρ τις ἡ ΑΒ τετμήσθω δίχα κατὰ τὸ Γ, προσκείσθω δέ τις αὐτῇ εὐθεῖα ἐπ᾽ εὐθείας ἡ ΒΔ· λέγω, ὅτι τὰ ἀπὸ τῶν ΑΔ, ΔΒ τετράγωνα διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων.

Ἤχθω γὰρ ἀπὸ τοῦ Γ σημείου τῇ ΑΒ πρὸς ὀρθὰς ἡ ΓΕ, καὶ κείσθω ἴση ἑκατέρᾳ τῶν ΑΓ, ΓΒ, καὶ ἐπεζεύχθωσαν αἱ ΕΑ, ΕΒ· καὶ διὰ μὲν τοῦ Ε τῇ ΑΔ παράλληλος ἤχθω ἡ ΕΖ, διὰ δὲ τοῦ Δ τῇ ΓΕ παράλληλος ἤχθω ἡ ΖΔ. καὶ ἐπεὶ εἰς παραλλήλους εὐθείας τὰς ΕΓ, ΖΔ εὐθεῖά τις ἐνέπεσεν ἡ ΕΖ, αἱ ὑπὸ ΓΕΖ, ΕΖΔ ἄρα δυσὶν ὀρθαῖς ἴσαι εἰσίν· αἱ ἄρα ὑπὸ ΖΕΒ, ΕΖΔ δύο ὀρθῶν ἐλάσσονές εἰσιν· αἱ δὲ ἀπ᾽ ἐλασσόνων ἢ δύο ὀρθῶν ἐκβαλλόμεναι συμπίπτουσιν· αἱ ἄρα ΕΒ, ΖΔ ἐκβαλλόμεναι ἐπὶ τὰ Β, Δ μέρη συμπεσοῦνται. ἐκβεβλήσθωσαν καὶ συμπιπτέτωσαν κατὰ τὸ Η, καὶ ἐπεζεύχθω ἡ ΑΗ. καὶ ἐπεὶ ἴση ἐστὶν ἡ ΑΓ τῇ ΓΕ, ἴση ἐστὶ καὶ γωνία ἡ ὑπὸ ΕΑΓ τῇ ὑπὸ ΑΕΓ· καὶ ὀρθὴ ἡ πρὸς τῷ Γ· ἡμίσεια ἄρα ὀρθῆς [ἐστιν] ἑκατέρα τῶν ὑπὸ ΕΑΓ, ΑΕΓ. διὰ τὰ αὐτὰ δὴ καὶ ἑκατέρα τῶν ὑπὸ ΓΕΒ, ΕΒΓ ἡμίσειά ἐστιν ὀρθῆς· ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΑΕΒ. καὶ ἐπεὶ ἡμίσεια ὀρθῆς ἐστιν ἡ ὑπὸ ΕΒΓ, ἡμίσεια ἄρα ὀρθῆς καὶ ἡ ὑπὸ ΔΒΗ. ἔστι δὲ καὶ ἡ ὑπὸ ΒΔΗ ὀρθή· ἴση γάρ ἐστι τῇ ὑπὸ ΔΓΕ· ἐναλλὰξ γάρ· λοιπὴ ἄρα ἡ ὑπὸ ΔΗΒ ἡμίσειά ἐστιν ὀρθῆς· ἡ ἄρα ὑπὸ ΔΗΒ τῇ ὑπὸ ΔΒΗ ἐστιν ἴση· ὥστε καὶ πλευρὰ ἡ ΒΔ πλευρᾷ τῇ ΗΔ ἐστιν ἴση. πάλιν, ἐπεὶ ἡ ὑπὸ ΕΗΖ ἡμίσειά ἐστιν ὀρθῆς, ὀρθὴ δὲ ἡ πρὸς τῷ Ζ· ἴση γάρ ἐστι τῇ ἀπεναντίον τῇ πρὸς τῷ Γ· λοιπὴ ἄρα ἡ ὑπὸ ΖΕΗ ἡμίσειά ἐστιν ὀρθῆς· ἴση ἄρα ἡ ὑπὸ ΕΗΖ γωνία τῇ ὑπὸ ΖΕΗ· ὥστε καὶ πλευρὰ ἡ ΗΖ πλευρᾷ τῇ ΕΖ ἐστιν ἴση. καὶ ἐπεὶ [ἴση ἐστὶν ἡ ΕΓ τῇ ΓΑ], ἴσον ἐστὶ [καὶ] τὸ ἀπὸ τῆς ΕΓ τετράγωνον τῷ ἀπὸ τῆς ΓΑ

Proposition 10†

If a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line).



For let any straight-line AB have been cut in half at (point) C, and let any straight-line BD have been added to it straight-on. I say that the (sum of the) squares on AD and DB is double the (sum of the) squares on AC and CD.

For let CE have been drawn from point C, at right-angles to AB [Prop. 1.11], and let it be made equal to each of AC and CB [Prop. 1.3], and let EA and EB have been joined. And let EF have been drawn through E, parallel to AD [Prop. 1.31], and let FD have been drawn through D, parallel to CE [Prop. 1.31]. And since some straight-line EF falls across the parallel straight-lines EC and FD, the (internal angles) CEF and EFD are thus equal to two right-angles [Prop. 1.29]. Thus, FEB and EFD are less than two right-angles. And (straight-lines) produced from (internal angles whose sum is) less than two right-angles meet together [Post. 5]. Thus, being produced in the direction of B and D, the (straight-lines) EB and FD will meet. Let them have been produced, and let them meet together at G, and let AG have been joined. And since AC is equal to CE, angle EAC is also equal to (angle) AEC [Prop. 1.5]. And the (angle) at C (is) a right-angle. Thus, EAC and AEC [are] each half a right-angle [Prop. 1.32]. So, for the same (reasons), CEB and EBC are also each half a right-angle. Thus, (angle) AEB is a right-angle. And since EBC is half a right-angle, DBG (is) thus also half a right-angle [Prop. 1.15]. And BDG is also a right-angle. For it is equal to DCE. For (they are) alternate (angles) τετραγώνῳ· τὰ ἄρα ἀπὸ τῶν ΕΓ, ΓΑ τετράγωνα διπλάσιά ἐστι τοῦ ἀπὸ τῆς ΓΑ τετραγώνου. τοῖς δὲ ἀπὸ τῶν ΕΓ, ΓΑ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΕΑ· τὸ ἄρα ἀπὸ τῆς ΕΑ τετράγωνον διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΑΓ τετραγώνου. πάλιν, ἐπεὶ ἴση ἐστὶν ἡ ΖΗ τῇ ΕΖ, ἴσον ἐστὶ καὶ τὸ ἀπὸ τῆς ΖΗ τῷ ἀπὸ τῆς ΖΕ· τὰ ἄρα ἀπὸ τῶν ΗΖ, ΖΕ διπλάσιά ἐστι τοῦ ἀπὸ τῆς ΕΖ. τοῖς δὲ ἀπὸ τῶν ΗΖ, ΖΕ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΕΗ· τὸ ἄρα ἀπὸ τῆς ΕΗ διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΕΖ. ἴση δὲ ἡ ΕΖ τῇ ΓΔ· τὸ ἄρα ἀπὸ τῆς ΕΗ τετράγωνον διπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΓΔ. ἐδείχθη δὲ καὶ τὸ ἀπὸ τῆς ΕΑ διπλάσιον τοῦ ἀπὸ τῆς ΑΓ· τὰ ἄρα ἀπὸ τῶν ΑΕ, ΕΗ τετράγωνα διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων. τοῖς δὲ ἀπὸ τῶν ΑΕ, ΕΗ τετραγώνοις ἴσον ἐστὶ τὸ ἀπὸ τῆς ΑΗ τετράγωνον· τὸ ἄρα ἀπὸ τῆς ΑΗ διπλάσιόν ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ. τῷ δὲ ἀπὸ τῆς ΑΗ ἴσα ἐστὶ τὰ ἀπὸ τῶν ΑΔ, ΔΗ· τὰ ἄρα ἀπὸ τῶν ΑΔ, ΔΗ [τετράγωνα] διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ [τετραγώνων]. ἴση δὲ ἡ ΔΗ τῇ ΔΒ· τὰ ἄρα ἀπὸ τῶν ΑΔ, ΔΒ [τετράγωνα] διπλάσιά ἐστι τῶν ἀπὸ τῶν ΑΓ, ΓΔ τετραγώνων.

Ἐὰν ἄρα εὐθεῖα γραμμὴ τμηθῇ δίχα, προστεθῇ δέ τις αὐτῇ εὐθεῖα ἐπ' εὐθείας, τὸ ἀπὸ τῆς ὅλης σὺν τῇ προσκειμένῃ καὶ τὸ ἀπὸ τῆς προσκειμένης τὰ συναμφότερα τετράγωνα διπλάσιά ἐστι τοῦ τε ἀπὸ τῆς ἡμισείας καὶ τοῦ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς ἡμισείας καὶ τῆς προσκειμένης ὡς ἀπὸ μιᾶς ἀναγραφέντος τετραγώνου· ὅπερ ἔδει δεῖξαι.

[Prop. 1.29]. Thus, the remaining (angle) DGB is half a right-angle. Thus, DGB is equal to DBG. So side BD is also equal to side GD [Prop. 1.6]. Again, since EGF is half a right-angle, and the (angle) at F (is) a right-angle, for it is equal to the opposite (angle) at C [Prop. 1.34], the remaining (angle) FEG is thus half a right-angle. Thus, angle EGF (is) equal to FEG. So the side GF is also equal to the side EF [Prop. 1.6]. And since [EC is equal to CA] the square on EC is [also] equal to the square on CA. Thus, the (sum of the) squares on EC and CA is double the square on CA. And the (square) on EA is equal to the (sum of the squares) on EC and CA [Prop. 1.47]. Thus, the square on EA is double the square on AC. Again, since FG is equal to EF, the (square) on FG is also equal to the (square) on FE. Thus, the (sum of the squares) on GF and FE is double the (square) on EF. And the (square) on EG is equal to the (sum of the squares) on GF and FE [Prop. 1.47]. Thus, the (square) on EG is double the (square) on EF. And EF (is) equal to CD [Prop. 1.34]. Thus, the square on EG is double the (square) on CD. But it was also shown that the (square) on EA (is) double the (square) on AC. Thus, the (sum of the) squares on AE and EG is double the (sum of the) squares on AC and CD. And the square on AG is equal to the (sum of the) squares on AE and EG [Prop. 1.47]. Thus, the (square) on AG is double the (sum of the squares) on AC and CD. And the (sum of the squares) on AD and DG is equal to the (square) on AG [Prop. 1.47]. Thus, the (sum of the) [squares] on AD and DG is double the (sum of the) [squares] on AC and CD. And DG (is) equal to DB. Thus, the (sum of the) [squares] on AD and DB is double the (sum of the) squares on AC and CD.

Thus, if a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line). (Which is) the very thing it was required to show.

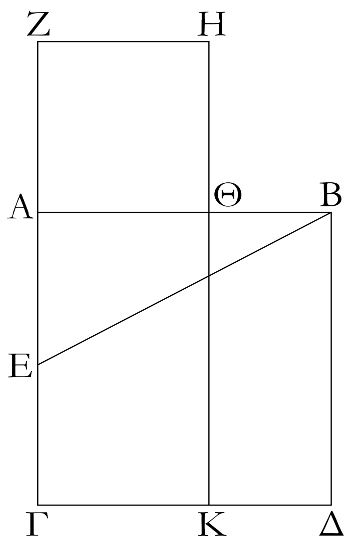
† This proposition is a geometric version of the algebraic identity: $(2\,a+b)^2+b^2=2\,[a^2+(a+b)^2]$.

ια'.

Τὴν δοθεῖσαν εὐθεῖαν τεμεῖν ὥστε τὸ ὑπὸ τῆς ὅλης καὶ τοῦ ἑτέρου τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον εἶναι τῷ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνῳ.

Proposition 11†

To cut a given straight-line such that the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), is equal to the square on the remaining piece.

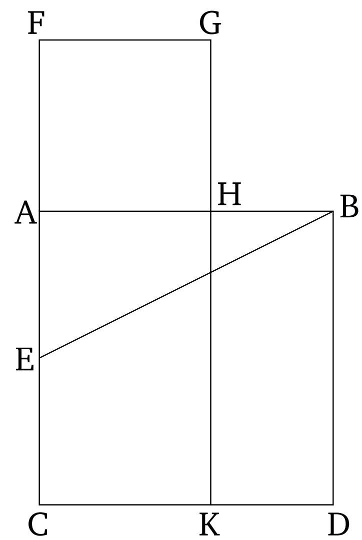


Ἔστω ἡ δοθεῖσα εὐθεῖα ἡ ΑΒ· δεῖ δὴ τὴν ΑΒ τεμεῖν ὥστε τὸ ὑπὸ τῆς ὅλης καὶ τοῦ ἑτέρου τῶν τμημάτων περιεχόμενον ὀρθογώνιον ἴσον εἶναι τῷ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνῳ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΒΔΓ, καὶ τετμήσθω ἡ ΑΓ δίχα κατὰ τὸ Ε σημεῖον, καὶ ἐπεζεύχθω ἡ ΒΕ, καὶ διήχθω ἡ ΓΑ ἐπὶ τὸ Ζ, καὶ κείσθω τῇ ΒΕ ἴση ἡ ΕΖ, καὶ ἀναγεγράφθω ἀπὸ τῆς ΑΖ τετράγωνον τὸ ΖΘ, καὶ διήχθω ἡ ΗΘ ἐπὶ τὸ Κ· λέγω, ὅτι ἡ ΑΒ τέτμηται κατὰ τὸ Θ, ὥστε τὸ ὑπὸ τῶν ΑΒ, ΒΘ περιεχόμενον ὀρθογώνιον ἴσον ποιεῖν τῷ ἀπὸ τῆς ΑΘ τετραγώνῳ.

Ἐπεὶ γὰρ εὐθεῖα ἡ ΑΓ τέτμηται δίχα κατὰ τὸ Ε, πρόσκειται δὲ αὐτῇ ἡ ΖΑ, τὸ ἄρα ὑπὸ τῶν ΓΖ, ΖΑ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΑΕ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΕΖ τετραγώνῳ. ἴση δὲ ἡ ΕΖ τῇ ΕΒ· τὸ ἄρα ὑπὸ τῶν ΓΖ, ΖΑ μετὰ τοῦ ἀπὸ τῆς ΑΕ ἴσον ἐστὶ τῷ ἀπὸ ΕΒ. ἀλλὰ τῷ ἀπὸ ΕΒ ἴσα ἐστὶ τὰ ἀπὸ τῶν ΒΑ, ΑΕ· ὀρθὴ γὰρ ἡ πρὸς τῷ Α γωνία· τὸ ἄρα ὑπὸ τῶν ΓΖ, ΖΑ μετὰ τοῦ ἀπὸ τῆς ΑΕ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΒΑ, ΑΕ. κοινὸν ἀφῃρήσθω τὸ ἀπὸ τῆς ΑΕ· λοιπὸν ἄρα τὸ ὑπὸ τῶν ΓΖ, ΖΑ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ τετραγώνῳ. καί ἐστι τὸ μὲν ὑπὸ τῶν ΓΖ, ΖΑ τὸ ΖΚ· ἴση γὰρ ἡ ΑΖ τῇ ΖΗ· τὸ δὲ ἀπὸ τῆς ΑΒ τὸ ΑΔ· τὸ ἄρα ΖΚ ἴσον ἐστὶ τῷ ΑΔ. κοινὸν ἀφῃρήσθω τὸ ΑΚ· λοιπὸν ἄρα τὸ ΖΘ τῷ ΘΔ ἴσον ἐστίν. καί ἐστι τὸ μὲν ΘΔ τὸ ὑπὸ τῶν ΑΒ, ΒΘ· ἴση γὰρ ἡ ΑΒ τῇ ΒΔ· τὸ δὲ ΖΘ τὸ ἀπὸ τῆς ΑΘ· τὸ ἄρα ὑπὸ τῶν ΑΒ, ΒΘ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ ΘΑ τετραγώνῳ.

Ἡ ἄρα δοθεῖσα εὐθεῖα ἡ ΑΒ τέτμηται κατὰ τὸ Θ ὥστε τὸ ὑπὸ τῶν ΑΒ, ΒΘ περιεχόμενον ὀρθογώνιον ἴσον ποιεῖν τῷ ἀπὸ τῆς ΘΑ τετραγώνῳ· ὅπερ ἔδει ποιῆσαι.



Let AB be the given straight-line. So it is required to cut AB such that the rectangle contained by the whole (straight-line), and one of the pieces (of the straight-line), is equal to the square on the remaining piece.

For let the square $ABDC$ have been described on AB [Prop. 1.46], and let AC have been cut in half at point E [Prop. 1.10], and let BE have been joined. And let CA have been drawn through to (point) F, and let EF be made equal to BE [Prop. 1.3]. And let the square FH have been described on AF [Prop. 1.46], and let GH have been drawn through to (point) K. I say that AB has been cut at H such as to make the rectangle contained by AB and BH equal to the square on AH.

For since the straight-line AC has been cut in half at E, and FA has been added to it, the rectangle contained by CF and FA, plus the square on AE, is thus equal to the square on EF [Prop. 2.6]. And EF (is) equal to EB. Thus, the (rectangle contained) by CF and FA, plus the (square) on AE, is equal to the (square) on EB. But, the (sum of the squares) on BA and AE is equal to the (square) on EB. For the angle at A (is) a right-angle [Prop. 1.47]. Thus, the (rectangle contained) by CF and FA, plus the (square) on AE, is equal to the (sum of the squares) on BA and AE. Let the square on AE have been subtracted from both. Thus, the remaining rectangle contained by CF and FA is equal to the square on AB. And FK is the (rectangle contained) by CF and FA. For AF (is) equal to FG. And AD (is) the (square) on AB. Thus, the (rectangle) FK is equal to the (square) AD. Let (rectangle) AK have been subtracted from both. Thus, the remaining (square) FH is equal to the (rectangle) HD. And HD is the (rectangle contained) by AB and BH. For AB (is) equal to BD. And FH (is) the (square) on AH. Thus, the rectangle contained by AB

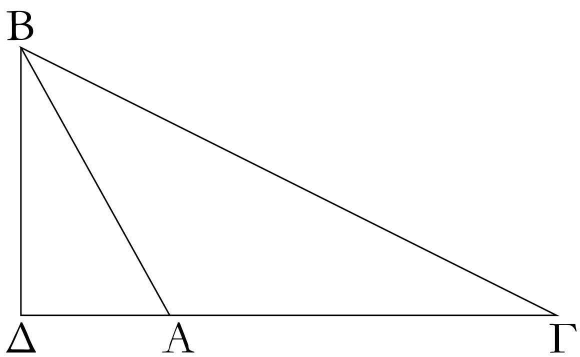
and BH is equal to the square on HA.

Thus, the given straight-line AB has been cut at (point) H such as to make the rectangle contained by AB and BH equal to the square on HA. (Which is) the very thing it was required to do.

† This manner of cutting a straight-line—so that the ratio of the whole to the larger piece is equal to the ratio of the larger to the smaller piece—is sometimes called the "Golden Section".

ιβ′.

Ἐν τοῖς ἀμβλυγωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ἀμβλεῖαν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον μεῖζόν ἐστι τῶν ἀπὸ τῶν τὴν ἀμβλεῖαν γωνίαν περιεχουσῶν πλευρῶν τετραγώνων τῷ περιεχομένῳ δὶς ὑπό τε μιᾶς τῶν περὶ τὴν ἀμβλεῖαν γωνίαν, ἐφ᾽ ἣν ἡ κάθετος πίπτει, καὶ τῆς ἀπολαμβανομένης ἐκτὸς ὑπὸ τῆς καθέτου πρὸς τῇ ἀμβλείᾳ γωνίᾳ.



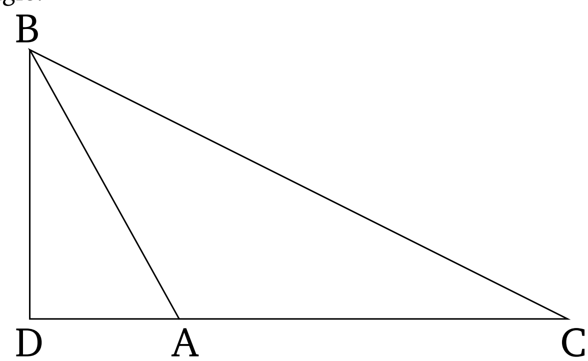
Ἔστω ἀμβλυγώνιον τρίγωνον τὸ ΑΒΓ ἀμβλεῖαν ἔχον τὴν ὑπὸ ΒΑΓ, καὶ ἤχθω ἀπὸ τοῦ Β σημείου ἐπὶ τὴν ΓΑ ἐκβληθεῖσαν κάθετος ἡ ΒΔ. λέγω, ὅτι τὸ ἀπὸ τῆς ΒΓ τετράγωνον μεῖζόν ἐστι τῶν ἀπὸ τῶν ΒΑ, ΑΓ τετραγώνων τῷ δὶς ὑπὸ τῶν ΓΑ, ΑΔ περιεχομένῳ ὀρθογωνίῳ.

Ἐπεὶ γὰρ εὐθεῖα ἡ ΓΔ τέτμηται, ὡς ἔτυχεν, κατὰ τὸ Α σημεῖον, τὸ ἄρα ἀπὸ τῆς ΔΓ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΓΑ, ΑΔ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΓΑ, ΑΔ περιεχομένῳ ὀρθογωνίῳ. κοινὸν προσκείσθω τὸ ἀπὸ τῆς ΔΒ· τὰ ἄρα ἀπὸ τῶν ΓΔ, ΔΒ ἴσα ἐστὶ τοῖς τε ἀπὸ τῶν ΓΑ, ΑΔ, ΔΒ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΓΑ, ΑΔ [περιεχομένῳ ὀρθογωνίῳ]. ἀλλὰ τοῖς μὲν ἀπὸ τῶν ΓΔ, ΔΒ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΓΒ· ὀρθὴ γὰρ ἡ προς τῷ Δ γωνία· τοῖς δὲ ἀπὸ τῶν ΑΔ, ΔΒ ἴσον τὸ ἀπὸ τῆς ΑΒ· τὸ ἄρα ἀπὸ τῆς ΓΒ τετράγωνον ἴσον ἐστὶ τοῖς τε ἀπὸ τῶν ΓΑ, ΑΒ τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν ΓΑ, ΑΔ περιεχομένῳ ὀρθογωνίῳ· ὥστε τὸ ἀπὸ τῆς ΓΒ τετράγωνον τῶν ἀπὸ τῶν ΓΑ, ΑΒ τετραγώνων μεῖζόν ἐστι τῷ δὶς ὑπὸ τῶν ΓΑ, ΑΔ περιεχομένῳ ὀρθογωνίῳ.

Ἐν ἄρα τοῖς ἀμβλυγωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ἀμβλεῖαν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον μεῖζόν ἐστι τῶν ἀπὸ τῶν τὴν ἀμβλεῖαν γωνίαν περιεχουσῶν

Proposition 12†

In obtuse-angled triangles, the square on the side subtending the obtuse angle is greater than the (sum of the) squares on the sides containing the obtuse angle by twice the (rectangle) contained by one of the sides around the obtuse angle, to which a perpendicular (straight-line) falls, and the (straight-line) cut off outside (the triangle) by the perpendicular (straight-line) towards the obtuse angle.



Let ABC be an obtuse-angled triangle, having the angle BAC obtuse. And let BD be drawn from point B, perpendicular to CA produced [Prop. 1.12]. I say that the square on BC is greater than the (sum of the) squares on BA and AC, by twice the rectangle contained by CA and AD.

For since the straight-line CD has been cut, at random, at point A, the (square) on DC is thus equal to the (sum of the) squares on CA and AD, and twice the rectangle contained by CA and AD [Prop. 2.4]. Let the (square) on DB have been added to both. Thus, the (sum of the squares) on CD and DB is equal to the (sum of the) squares on CA, AD, and DB, and twice the [rectangle contained] by CA and AD. But, the (square) on CB is equal to the (sum of the squares) on CD and DB. For the angle at D (is) a right-angle [Prop. 1.47]. And the (square) on AB (is) equal to the (sum of the squares) on AD and DB [Prop. 1.47]. Thus, the square on CB is equal to the (sum of the) squares on CA and AB, and twice the rectangle contained by CA and AD. So the square on CB is greater than the (sum of the) squares on πλευρῶν τετραγώνων τῷ περιχομένῳ δὶς ὑπό τε μιᾶς τῶν περὶ τὴν ἀμβλεῖαν γωνίαν, ἐφ᾽ ἣν ἡ κάθετος πίπτει, καὶ τῆς ἀπολαμβανομένης ἐκτὸς ὑπὸ τῆς καθέτου πρὸς τῇ ἀμβλείᾳ γωνίᾳ· ὅπερ ἔδει δεῖξαι.

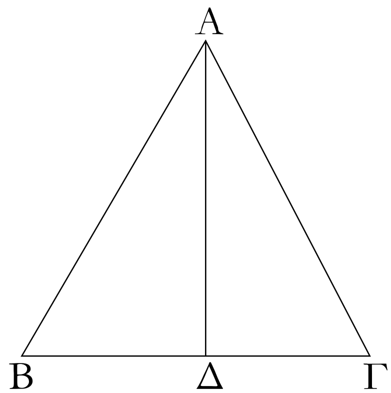
CA and AB by twice the rectangle contained by CA and AD.

Thus, in obtuse-angled triangles, the square on the side subtending the obtuse angle is greater than the (sum of the) squares on the sides containing the obtuse angle by twice the (rectangle) contained by one of the sides around the obtuse angle, to which a perpendicular (straight-line) falls, and the (straight-line) cut off outside (the triangle) by the perpendicular (straight-line) towards the obtuse angle. (Which is) the very thing it was required to show.

† This proposition is equivalent to the well-known cosine formula: $BC^2 = AB^2 + AC^2 - 2\,AB\,AC\,\cos BAC$, since $\cos BAC = -AD/AB$.

ιγ΄.

Ἐν τοῖς ὀξυγωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀξεῖαν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον ἔλαττόν ἐστι τῶν ἀπὸ τῶν τὴν ὀξεῖαν γωνίαν περιεχουσῶν πλευρῶν τετραγώνων τῷ περιεχομένῳ δὶς ὑπό τε μιᾶς τῶν περὶ τὴν ὀξεῖαν γωνίαν, ἐφ᾽ ἣν ἡ κάθετος πίπτει, καὶ τῆς ἀπολαμβανομένης ἐντὸς ὑπὸ τῆς καθέτου πρὸς τῇ ὀξείᾳ γωνίᾳ.

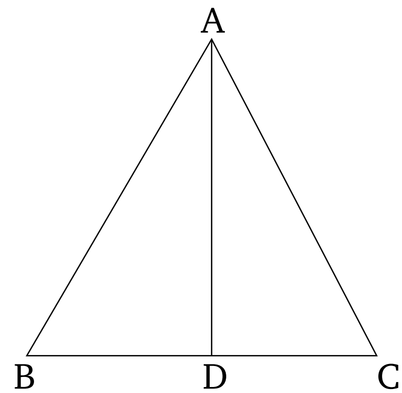


Ἔστω ὀξυγώνιον τρίγωνον τὸ ΑΒΓ ὀξεῖαν ἔχον τὴν πρὸς τῷ Β γωνίαν, καὶ ἤχθω ἀπὸ τοῦ Α σημείου ἐπὶ τὴν ΒΓ κάθετος ἡ ΑΔ· λέγω, ὅτι τὸ ἀπὸ τῆς ΑΓ τετράγωνον ἔλαττόν ἐστι τῶν ἀπὸ τῶν ΓΒ, ΒΑ τετραγώνων τῷ δὶς ὑπὸ τῶν ΓΒ, ΒΔ περιεχομένῳ ὀρθογωνίῳ.

Ἐπεὶ γὰρ εὐθεῖα ἡ ΓΒ τέτμηται, ὡς ἔτυχεν, κατὰ τὸ Δ, τὰ ἄρα ἀπὸ τῶν ΓΒ, ΒΔ τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῶν ΓΒ, ΒΔ περιεχομένῳ ὀρθογωνίῳ καὶ τῷ ἀπὸ τῆς ΔΓ τετραγώνῳ. κοινὸν προσκείσθω τὸ ἀπὸ τῆς ΔΑ τετράγωνον· τὰ ἄρα ἀπὸ τῶν ΓΒ, ΒΔ, ΔΑ τετράγωνα ἴσα ἐστὶ τῷ τε δὶς ὑπὸ τῶν ΓΒ, ΒΔ περιεχομένῳ ὀρθογωνίῳ καὶ τοῖς ἀπὸ τῶν ΑΔ, ΔΓ τετραγώνιος. ἀλλὰ τοῖς μὲν ἀπὸ τῶν ΒΔ, ΔΑ ἴσον τὸ ἀπὸ τῆς ΑΒ· ὀρθὴ γὰρ ἡ πρὸς τῷ Δ γωνίᾳ· τοῖς δὲ ἀπὸ τῶν ΑΔ, ΔΓ ἴσον τὸ ἀπὸ τῆς ΑΓ· τὰ ἄρα ἀπὸ τῶν ΓΒ, ΒΑ ἴσα ἐστὶ τῷ τε ἀπὸ τῆς ΑΓ καὶ τῷ δὶς ὑπὸ τῶν ΓΒ, ΒΔ· ὥστε μόνον τὸ ἀπὸ τῆς ΑΓ ἔλαττόν ἐστι

Proposition 13†

In acute-angled triangles, the square on the side subtending the acute angle is less than the (sum of the) squares on the sides containing the acute angle by twice the (rectangle) contained by one of the sides around the acute angle, to which a perpendicular (straight-line) falls, and the (straight-line) cut off inside (the triangle) by the perpendicular (straight-line) towards the acute angle.



Let ABC be an acute-angled triangle, having the angle at (point) B acute. And let AD have been drawn from point A, perpendicular to BC [Prop. 1.12]. I say that the square on AC is less than the (sum of the) squares on CB and BA, by twice the rectangle contained by CB and BD.

For since the straight-line CB has been cut, at random, at (point) D, the (sum of the) squares on CB and BD is thus equal to twice the rectangle contained by CB and BD, and the square on DC [Prop. 2.7]. Let the square on DA have been added to both. Thus, the (sum of the) squares on CB, BD, and DA is equal to twice the rectangle contained by CB and BD, and the (sum of the) squares on AD and DC. But, the (square) on AB (is) equal to the (sum of the squares) on BD and DA. For the angle at (point) D is a right-angle [Prop. 1.47].

τῶν ἀπὸ τῶν ΓΒ, ΒΑ τετραγώνων τῷ δὶς ὑπὸ τῶν ΓΒ, ΒΔ περιεχομένῳ ὀρθογωνίῳ.

Ἐν ἄρα τοῖς ὀξυγωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀξεῖαν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον ἔλαττόν ἐστι τῶν ἀπὸ τῶν τὴν ὀξεῖαν γωνίαν περιεχουσῶν πλευρῶν τετραγώνων τῷ περιεχομένῳ δὶς ὑπό τε μιᾶς τῶν περὶ τὴν ὀξεῖαν γωνίαν, ἐφ᾽ ἣν ἡ κάθετος πίπτει, καὶ τῆς ἀπολαμβανομένης ἐντὸς ὑπὸ τῆς καθέτου πρὸς τῇ ὀξείᾳ γωνίᾳ· ὅπερ ἔδει δεῖξαι.

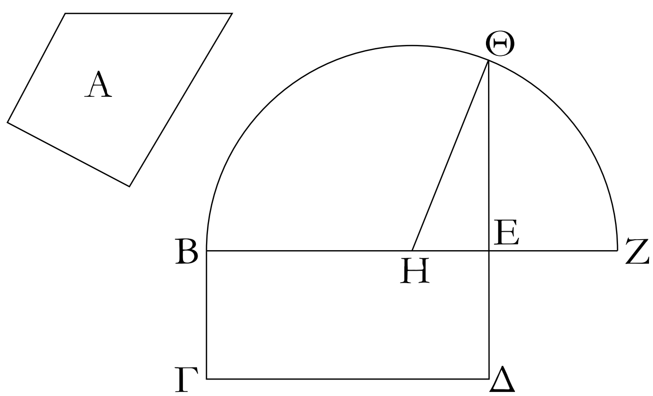
And the (square) on AC (is) equal to the (sum of the squares) on AD and DC [Prop. 1.47]. Thus, the (sum of the squares) on CB and BA is equal to the (square) on AC, and twice the (rectangle contained) by CB and BD. So the (square) on AC alone is less than the (sum of the) squares on CB and BA by twice the rectangle contained by CB and BD.

Thus, in acute-angled triangles, the square on the side subtending the acute angle is less than the (sum of the) squares on the sides containing the acute angle by twice the (rectangle) contained by one of the sides around the acute angle, to which a perpendicular (straight-line) falls, and the (straight-line) cut off inside (the triangle) by the perpendicular (straight-line) towards the acute angle. (Which is) the very thing it was required to show.

† This proposition is equivalent to the well-known cosine formula: $AC^2 = AB^2 + BC^2 - 2\,AB\,BC\,\cos ABC$, since $\cos ABC = BD/AB$.

ιδ′.

Τῷ δοθέντι εὐθυγράμμῳ ἴσον τετράγωνον συστήσασθαι.



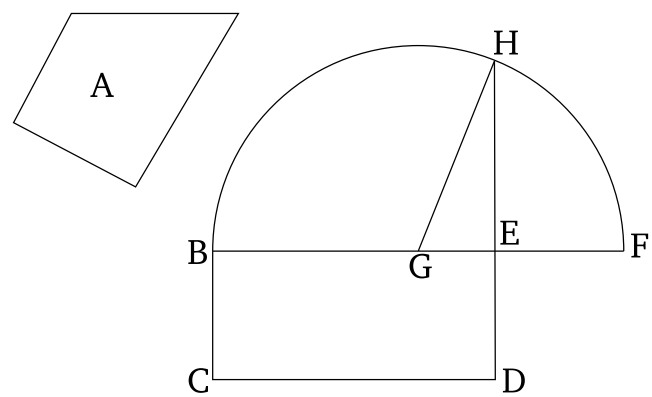
Ἔστω τὸ δοθὲν εὐθύγραμμον τὸ Α· δεῖ δὴ τῷ Α εὐθυγράμμῳ ἴσον τετράγωνον συστήσασθαι.

Συνεστάτω γὰρ τῷ Α ἐυθυγράμμῳ ἴσον παραλληλόγραμμον ὀρθογώνιον τὸ ΒΔ· εἰ μὲν οὖν ἴση ἐστὶν ἡ ΒΕ τῇ ΕΔ, γεγονὸς ἂν εἴη τὸ ἐπιταχθέν. συνέσταται γὰρ τῷ Α εὐθυγράμμῳ ἴσον τετράγωνον τὸ ΒΔ· εἰ δὲ οὔ, μία τῶν ΒΕ, ΕΔ μείζων ἐστίν. ἔστω μείζων ἡ ΒΕ, καὶ ἐκβεβλήσθω ἐπὶ τὸ Ζ, καὶ κείσθω τῇ ΕΔ ἴση ἡ ΕΖ, καὶ τετμήσθω ἡ ΒΖ δίχα κατὰ τὸ Η, καὶ κέντρῳ τῷ Η, διαστήματι δὲ ἑνὶ τῶν ΗΒ, ΗΖ ἡμικύκλιον γεγράφθω τὸ ΒΘΖ, καὶ ἐκβεβλήσθω ἡ ΔΕ ἐπὶ τὸ Θ, καὶ ἐπεζεύχθω ἡ ΗΘ.

Ἐπεὶ οὖν εὐθεῖα ἡ ΒΖ τέτμηται εἰς μὲν ἴσα κατὰ τὸ Η, εἰς δὲ ἄνισα κατὰ τὸ Ε, τὸ ἄρα ὑπὸ τῶν ΒΕ, ΕΖ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΕΗ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΗΖ τετραγώνῳ. ἴση δὲ ἡ ΗΖ τῇ ΗΘ· τὸ ἄρα ὑπὸ τῶν ΒΕ, ΕΖ μετὰ τοῦ ἀπὸ τῆς ΗΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΗΘ. τῷ δὲ ἀπὸ τῆς ΗΘ ἴσα ἐστὶ τὰ ἀπὸ τῶν ΘΕ, ΕΗ

Proposition 14

To construct a square equal to a given rectilinear figure.



Let A be the given rectilinear figure. So it is required to construct a square equal to the rectilinear figure A.

For let the right-angled parallelogram BD, equal to the rectilinear figure A, have been constructed [Prop. 1.45]. Therefore, if BE is equal to ED then that (which) was prescribed has taken place. For the square BD, equal to the rectilinear figure A, has been constructed. And if not, then one of the (straight-lines) BE or ED is greater (than the other). Let BE be greater, and let it have been produced to F, and let EF be made equal to ED [Prop. 1.3]. And let BF have been cut in half at (point) G [Prop. 1.10]. And, with center G, and radius one of the (straight-lines) GB or GF, let the semi-circle BHF have been drawn. And let DE have been produced to H, and let GH have been joined.

Therefore, since the straight-line BF has been cut—equally at G, and unequally at E—the rectangle conτετράγωνα· τὸ ἄρα ὑπὸ τῶν ΒΕ, ΕΖ μετὰ τοῦ ἀπὸ ΗΕ ἴσα ἐστὶ τοῖς ἀπὸ τῶν ΘΕ, ΕΗ. κοινὸν ἀφῃρήσθω τὸ ἀπὸ τῆς ΗΕ τετράγωνον· λοιπὸν ἄρα τὸ ὑπὸ τῶν ΒΕ, ΕΖ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς ΕΘ τετραγώνῳ. ἀλλὰ τὸ ὑπὸ τῶν ΒΕ, ΕΖ τὸ ΒΔ ἐστιν· ἴση γὰρ ἡ ΕΖ τῇ ΕΔ· τὸ ἄρα ΒΔ παραλληλόγραμμον ἴσον ἐστὶ τῷ ἀπὸ τῆς ΘΕ τετραγώνῳ. ἴσον δὲ τὸ ΒΔ τῷ Α εὐθυγράμμῳ. καὶ τὸ Α ἄρα εὐθύγραμμον ἴσον ἐστὶ τῷ ἀπὸ τῆς ΕΘ ἀναγραφησομένῳ τετραγώνῳ.

Τῷ ἄρα δοθέντι εὐθυγράμμῳ τῷ Α ἴσον τετράγωνον συνέσταται τὸ ἀπὸ τῆς ΕΘ ἀναγραφησόμενον· ὅπερ ἔδει ποιῆσαι.

tained by BE and EF, plus the square on EG, is thus equal to the square on GF [Prop. 2.5]. And GF (is) equal to GH. Thus, the (rectangle contained) by BE and EF, plus the (square) on GE, is equal to the (square) on GH. And the (sum of the) squares on HE and EG is equal to the (square) on GH [Prop. 1.47]. Thus, the (rectangle contained) by BE and EF, plus the (square) on GE, is equal to the (sum of the squares) on HE and EG. Let the square on GE have been taken from both. Thus, the remaining rectangle contained by BE and EF is equal to the square on EH. But, BD is the (rectangle contained) by BE and EF. For EF (is) equal to ED. Thus, the parallelogram BD is equal to the square on HE. And BD (is) equal to the rectilinear figure A. Thus, the rectilinear figure A is also equal to the square (which) can be described on EH.

Thus, a square—(namely), that (which) can be described on EH—has been constructed, equal to the given rectilinear figure A. (Which is) the very thing it was required to do.