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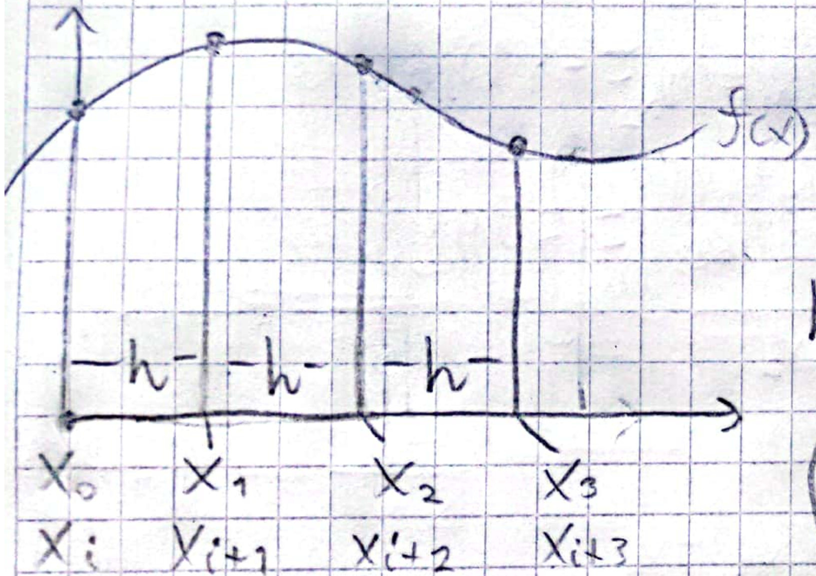
Punto 2.1 – Taller 2.



$f(x)$	$f(x_i)$	$f(x_{i+1})$	$f(x_{i+2})$	$f(x_{i+3})$
x	x_i	x_i+1	x_i+2	x_i+3

$$h = \frac{x_{i+3} - x_i}{3} \rightarrow x_3 = x_0 + 3h$$

$$x_2 = x_0 + 2h$$

$$x_1 = x_0 + h$$

$$x_0 = x_0$$

$$x = x_0 + ch$$

$$\int_{x_i}^{x_{i+3}} f(x)\,dx \approx \int_{x_0}^{x_3} P_3(x)\,dx$$

$$P_3(x) = \sum_{i=0}^{3} f(x_i) L_i(x)$$

$$\int_{x_0}^{x_3} P_3(x)\,dx = A$$

$$= f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x) + f(x_3)L_3(x)$$

$$A = f(x_0)\int_{x_0}^{x_3} L_0(x)\,dx + f(x_1)\int_{x_0}^{x_3} L_1(x)\,dx + f(x_2)\int_{x_0}^{x_3} L_2(x)\,dx + f(x_3)\int_{x_0}^{x_3} L_3(x)\,dx$$

$$L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(ch-h)(ch-2h)(ch-3h)}{(-h)(-2h)(-3h)} = \frac{\cancel{h}(c-1)\cancel{h}(c-2)\cancel{h}(c-3)}{-6\cancel{h^3}}$$

$$= -\frac{1}{6}\left((c-1)(c-2)(c-3)\right)$$

$$L_1 = \frac{1}{2}\left(c(c-2)(c-3)\right) \qquad L_3 = \frac{1}{6}\left(c(c-1)(c-2)\right)$$

$$L_2 = -\frac{1}{2}\left(c(c-1)(c-3)\right)$$

$$\frac{dx}{dc} = h \rightarrow dx = h\,dc$$

$$x = x_0 + ch \rightarrow \cancel{x_0} = \cancel{x_0} + ch \rightarrow c = 0$$

$$\hookrightarrow x_3 - x_0 = ch \rightarrow 3\cancel{h} = c\cancel{h} \rightarrow c = 3$$

$$\int_{x_0}^{x_3} \rightarrow \int_0^3$$

$$A = -\frac{f(x_0)h}{6}\int_0^3 \left((c-1)(c-2)(c-3)\right) dc + \frac{f(x_1)h}{2}\int_0^3 \left(c(c-2)(c-3)\right) dc$$

$$-\frac{f(x_2)h}{2}\int_0^3 \left(c(c-1)(c-3)\right) dc + \frac{f(x_3)h}{6}\int_0^3 \left(c(c-1)(c-2)\right) dc$$

$$(c-1)(c-2)(c-3) = (c^2-2c-c+2)(c-3) = c^3-2c^2-c^2+2c-3c^2+6c+3c-6$$
$$= c^3-6c^2+11c-6$$

$$\int_0^3 (c^3-6c^2+11c-6)\,dc = \frac{c^4}{4} - \frac{\cancel{6}c^3}{\cancel{3}} + \frac{11c^2}{2} - 6c \Big|_0^3$$

$$= \left(\frac{3^4}{4} - 2(3)^3 + \frac{11}{2}(3)^2 - 6(3)\right) - \cancel{\left(\frac{0^4}{4} - 2(0)^3 + \frac{11}{2}(0)^2 - 6(0)\right)}$$

$$= \frac{81}{4} - 2(27) + \frac{11}{2}(9) - 18 = \frac{81}{4} - 54 + \frac{99}{2} - 18 = -\frac{9}{4}$$

$$\int_0^3 (c(c-2)(c-3))\,dc = \frac{c^4}{4} - \frac{5c^3}{3} + \frac{\cancel{6}c^2}{\cancel{2}} \Big|_0^3 = \frac{9}{4}$$

$$\int_0^3 (c(c-1)(c-3))\,dc = \frac{c^4}{4} - \frac{4c^3}{3} + \frac{\cancel{3c^2}}{\cancel{2}} \Big|_0^3 = -\frac{9}{4}$$

$$\int_0^3 (c(c-1)(c-2))\,dc = \frac{c^4}{4} - \frac{\cancel{3}c^3}{\cancel{3}} + \frac{\cancel{2}c^2}{\cancel{2}} \Big|_0^3 = \frac{9}{4}$$

$$A = \frac{-h}{6} f(x_0)\left(-\frac{9}{4}\right) + \frac{h}{2} f(x_1)\left(\frac{9}{4}\right) - \frac{h}{2} f(x_2)\left(-\frac{9}{4}\right) + \frac{h}{6} f(x_3)\left(\frac{9}{4}\right)$$

$$\int_{x_0}^{x_3} f(x)\,dx \approx f(x_0)\left(\frac{3h}{8}\right) + f(x_1)\left(\frac{9h}{8}\right) + f(x_2)\left(\frac{9h}{8}\right) + f(x_3)\left(\frac{3h}{8}\right)$$

$$\int_{x_0}^{x_3} f(x)\,dx = \int_{x_i}^{x_{i+3}} f(x)\,dx \approx \frac{3h}{8}\left(f(x_i) + 3f(x_{i+1}) + 3f(x_{i+2}) + f(x_{i+3})\right)$$