Problema 5 _ Derivación:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) + O(h^5)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) - O(h^5)$$

$$f(x+2h) = f(x) + 2hf'(x) + \frac{(2h)^2}{2}f''(x) + \frac{(2h)^3}{3!}f'''(x) + \frac{(2h)^4}{4!}f^{(4)}(x) + O(h^5)$$

$$f(x-2h) = f(x) - 2hf'(x) + \frac{(2h)^2 f''(x)}{2} - \frac{(2h)^3}{3!}f'''(x) + \frac{(2h)^4}{4!}f^{(4)}(x) - O(h^5)$$

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(1)

$$-f(x+h) - f(x-h) - 2f(x) - h^2 f''(x) - \frac{h^4}{12}f^{(4)}(x) - O(h^6)$$

(2)

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2 \cancel{f''(x)} + \frac{4h^4}{3!}f^{(4)}(x) + O(h^6)$$

$$4 \cdot (1) \quad -4f(\alpha+h) - 4f(\alpha-h) = -8f(\alpha) - \cancel{4h^2 f''(\alpha)} - \frac{h^4}{3} f^{(4)}(\alpha) - 4O(h^6)$$

$$4 \cdot (1) + (2) = -4f(\alpha+h) - 4f(\alpha-h) + f(\alpha+2h) + f(\alpha-2h)$$

$$= -6f(\alpha) + h^4 f^{(4)}(\alpha) + 3O(h^6)$$

Finalmente:

$$f^{(4)}(x) = \frac{f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h)}{h^4} + \mathcal{O}(h^6)$$

$$f^{(4)}(x_i) \simeq \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$$

El Orden de error es de $\mathcal{O}(h^6)$