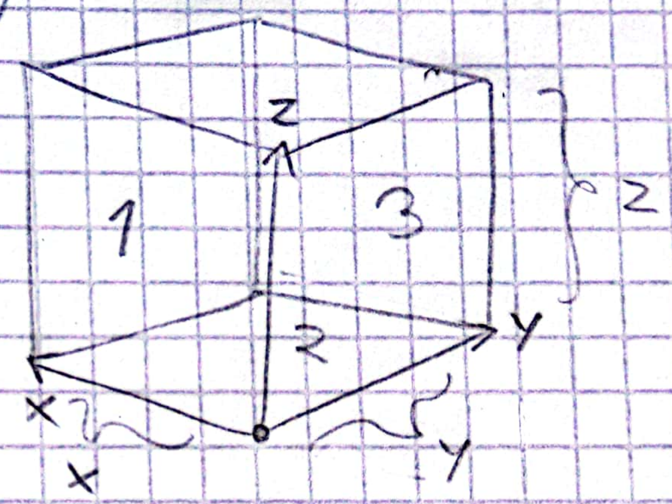
6.4.3)

$V(x,y,z) = xyz$

a)



b) Area 1: xz

Area 2: xy

Area 3: yz

En un prisma rectangular siempre hay 2 caras iguales.

$$A(x,y,z) = 12 = \underbrace{xy}_{} + 2yz + 2xz$$

Solo 1 debido a que el problema elimina una cara.

(1, 4.3)

e) $V(x,y,z) = xyz$; $A(x,y,z) = 12 = xy + 2yz + 2xz$
$= xy + 2yz + 2xz - 12$

$V_x = yz$ | $A_x = y + 2z$ $\qquad$ $G\ xy + 2yz + 2xz - 12 = 0$

$V_y = xz$ | $A_y = x + 2z$

$V_z = xy$ | $A_z = 2y + 2x = 2(y+x)$

$yz = \lambda(y+2z) \rightarrow yz = \lambda y + 2\lambda z \rightarrow y(z-\lambda) = 2\lambda z \rightarrow y = \dfrac{2\lambda z}{z-\lambda} \searrow$ $x = y$

$xz = \lambda(x+2z) \rightarrow xz = \lambda x + 2\lambda z \rightarrow x(z-\lambda) = 2\lambda z \rightarrow x = \dfrac{2\lambda z}{z-\lambda} \nearrow$

$x = y$; $xy = \lambda(2(y+x)) \rightarrow x^2 = \lambda(4x) \rightarrow \dfrac{x}{4} = \lambda$

$x = \dfrac{2\lambda z}{z-\lambda} \rightarrow \dfrac{2\left(\frac{x}{4}\right)z}{z-\frac{x}{4}} \rightarrow \left[\dfrac{\frac{x}{2}z}{\frac{4z-x}{4}}\right] \rightarrow \dfrac{4xz}{8z-2x} = x \rightarrow 4xz = 8zx - 2x^2$

$$x = \frac{2\lambda z}{z-\lambda} \rightarrow \frac{2\left(\frac{x}{4}\right)z}{z - \frac{x}{4}} \rightarrow \left[\frac{\frac{x}{2}z}{\frac{4z-x}{4}}\right] \rightarrow \frac{4xz}{8z-2x} = x \rightarrow 4xz = 8zx - 2x^2$$

$$\rightarrow \cancel{x} \rightarrow 4z = 8z - 2x \rightarrow -4z = -2x \rightarrow z = \frac{x}{2}$$

$$xy + 2yz + 2xz - 12 = 0 \rightarrow x^2 + 2x\left(\frac{x}{2}\right) + 2x\left(\frac{x}{2}\right) = 12$$

$$\rightarrow x^2 + 2x^2 = 12 \rightarrow 3x^2 = 12 \rightarrow x^2 = \frac{12}{3} \rightarrow x = \pm\sqrt{4} \rightarrow \pm 2 \text{ cm}$$

Tomamos $+2$, porque -2 es el mínimo.

$x = y \rightarrow y = +2\text{cm}$; $z = \frac{x}{2} \rightarrow \frac{2}{2} \rightarrow 1\text{ cm}$

$$V(x,y,z) = xyz = 2 \cdot 2 \cdot 1 = 4\text{ cm}^3$$

7.8.4)

Para $n=1$

$$\frac{\text{Casos Favorables}}{\text{Casos Posibles}} \Rightarrow \frac{\cancel{365}^{1}}{\cancel{365}} \cdot 100\% = 100\%$$

Para $n=2$

$$\left(\frac{365}{365} \cdot \frac{364}{365}\right) \cdot 100\% = 99{,}7\%$$

Para $n=3$

$$\left(\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365}\right) \cdot 100\% = 99{,}2\%$$

Así que:

$$\left(\prod_{n=0}^{364} \frac{|n-365|}{365}\right) \cdot 100\%$$] Formula General

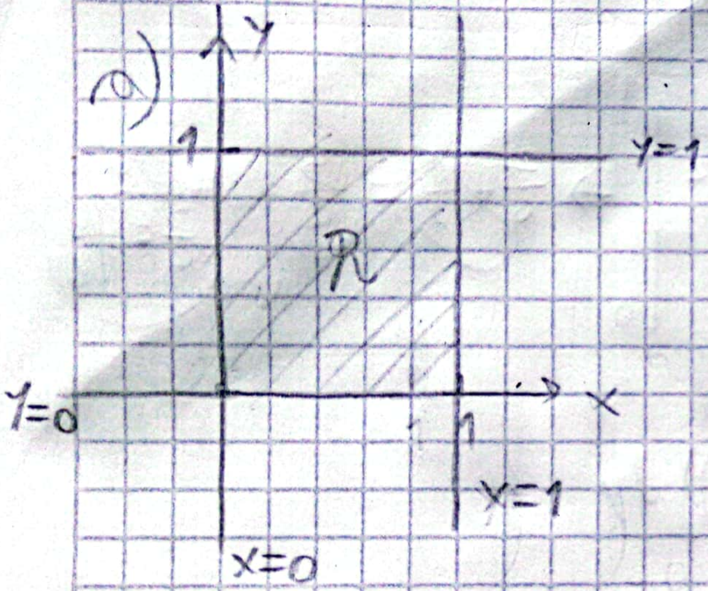
Llegamos hasta 364, porque $365-365=0$.

Ademas, cuando $n=0$; $\frac{365}{365}=1$, el cual es el caso $n=1$ en los ejemplos anteriores.

7.12.1)

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 \le x \le 1,\ 0 \le y \le 1. \\ 0 & \text{otro caso} \end{cases}$$

a)



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dA = 1$$

$$\int_0^1\int_0^1 \frac{2}{3}x + \frac{4}{3}y \; dx\, dy$$

$$\int_0^1 \left[\frac{1}{3}x^2 + \frac{4}{3}yx \Big|_0^1\right] dy \rightarrow \int_0^1 \frac{1}{3} + \frac{4}{3}y\; dy \rightarrow \frac{1}{3}x + \frac{2}{3}y^2 \Big|_0^1$$

$$\rightarrow \frac{1}{3} + \frac{2}{3} = \cancel{\frac{3}{3}} = 1 \checkmark$$

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$\rightarrow \frac{1}{3} + \frac{2}{3} = \cancel{\frac{3}{3}} = 1$ ✓

$$b)\ g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \rightarrow \int_0^1 \frac{2}{3}x + \frac{4}{3}y\,dy \rightarrow \frac{2}{3}xy + \frac{2}{3}y^2 \Big|_0^1$$

$$\rightarrow \frac{2}{3}x + \frac{2}{3} \rightarrow \frac{2}{3}(x+1) \quad Si\ 0 \leq x \leq 1$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \rightarrow \int_0^1 \frac{2}{3}x + \frac{4}{3}y\,dx \rightarrow \frac{1}{3}x^2 + \frac{4}{3}yx \Big|_0^1$$

$$\rightarrow \frac{1}{3} + \frac{4}{3}y \rightarrow \frac{1}{3}(1+4y) \quad Si:\ 0 \leq y \leq 1$$

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c) $E(x) = \int_{-\infty}^{\infty} x\, g(x)\, dx \rightarrow \int_0^1 x\left(\frac{2}{3}(x+1)\right) dx$

$\rightarrow \int_0^1 \frac{2}{3}x^2 + \frac{2}{3}x\, dx \rightarrow \frac{2}{9}x^3 + \frac{1}{3}x^2 \Big|_0^1 \rightarrow \frac{2}{9} + \frac{1}{3} = \frac{5}{9}$

$\frac{5}{9} = \frac{10}{18}$

d) $E(y) = \int_{-\infty}^{\infty} y\, h(y)\, dy \rightarrow \int_0^1 y\left(\frac{1}{3}(1+4y)\right) dy$

$\rightarrow \int_0^1 \frac{1}{3}y + \frac{4}{3}y^2\, dy \rightarrow \frac{1}{6}y^2 + \frac{4}{9}y^3 \Big|_0^1 \rightarrow \frac{1}{6} + \frac{4}{9} = \frac{11}{18}$

e) $$E(xy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dx\, dy \rightarrow \int_0^1\int_0^1 xy\left(\frac{2}{3}x + \frac{4}{3}y\right) dx\, dy$$

$$\rightarrow \int_0^1\int_0^1 \frac{2}{3}x^2 y + \frac{4}{3}y^2 x \, dx\, dy \rightarrow \int_0^1 \frac{2}{9}x^3 y + \frac{2}{3}y^2 x^2 \Big|_0^1 dy$$

$$\rightarrow \int_0^1 \frac{2}{9}y + \frac{2}{3}y^2\, dy \rightarrow \frac{1}{9}y^2 + \frac{2}{9}y^3 \Big|_0^1 \rightarrow \frac{1}{9} + \frac{2}{9} \rightarrow \frac{\cancel{3}}{\cancel{9}} \rightarrow \frac{1}{3}$$

$$\sigma_{xy} = E(xy) - E(x)E(y) = \frac{1}{3} - \left(\frac{10}{18}\right)\left(\frac{11}{18}\right) \rightarrow \frac{1}{3} - \frac{55}{162} = -\frac{1}{162}$$

$$= -0{,}00617$$

f) $\sigma_{xy} = E((x-\hat{\mu}_x)(y-\hat{\mu}_y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(x-\frac{10}{18}\right)\left(y-\frac{11}{18}\right)f(x,y)\,dA$

$$\rightarrow \int_0^1\int_0^1 \left(xy-\frac{11}{18}x-\frac{10}{18}y+\frac{55}{162}\right)\left(\frac{2}{3}x+\frac{4}{3}y\right)dx\,dy$$

$$\rightarrow \int_0^1\int_0^1 \frac{2}{3}x^2y+\frac{4}{3}xy^2-\frac{11}{27}x^2-\frac{20}{27}y^2-\frac{32}{27}xy+\frac{55}{243}x+\frac{110}{243}y\,dx\,dy$$

$$\rightarrow \int_0^1 \frac{2}{9}x^3y+\frac{2}{3}x^2y^2-\frac{11}{81}x^3-\frac{20}{27}y^2x-\frac{16}{27}x^2y+\frac{55}{486}x^2+\frac{110}{243}xy\Big|_0^1 dy$$

$$\to \int_0^1 \frac{2}{9}x^3y + \frac{2}{3}x^2y^2 - \frac{11}{81}x^3 - \frac{20}{27}y^2x - \frac{16}{27}x^2y + \frac{55}{486}x^2 + \frac{110}{243}xy \Big|_0^1 dy$$

$$\to \int_0^1 \frac{2}{9}y + \frac{2}{3}y^2 - \frac{11}{81} - \frac{20}{27}y^2 - \frac{16}{27}y + \frac{55}{486} + \frac{110}{243}y \; dy$$

$$\to \int_0^1 -\frac{2}{27}y^2 + \frac{20}{243}y - \frac{11}{486} \; dy \to -\frac{2}{81}y^3 + \frac{10}{243}y^2 - \frac{11}{486}y \Big|_0^1$$

$$\to -\frac{2}{81} + \frac{10}{243} - \frac{11}{486} = -\frac{1}{162} = -0{,}00617$$

g) $f(x,y) = g(x)\,h(y)$

$$\frac{2}{3}(x+2y) = \left(\frac{2}{3}(x+1)\right)\left(\frac{1}{3}(1+4y)\right) \to \left(\frac{2}{3}x+\frac{2}{3}\right)\left(\frac{1}{3}+\frac{4}{3}y\right)$$

$$= \frac{2}{9}x + \frac{8}{9}xy + \frac{2}{9} + \frac{8}{9}y$$

$$\frac{2}{3}(x+2y) \neq \frac{2}{9}(x+4xy+4y+1)$$

X e Y <u>NO</u> son independientes

9.7.1) $A(x_1, x_2, \dots, x_n;\quad A \sim N(\mu, \sigma)$

$$\mathcal{L}(x; \mu, \sigma^2) = \frac{1}{2\pi\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L}(x; \mu, \sigma^2) = \frac{1}{2\pi\sqrt{\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L}(\mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{2\pi\sqrt{\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L}(\mu, \sigma^2) = \frac{1}{(2\pi\sqrt{\sigma^2})^n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\ln(\mathcal{L}(\mu, \sigma^2)) = \ln\left(\frac{1}{(2\pi\sqrt{\sigma^2})^n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\right)$$

$$\ln(\mathcal{L}(\mu, \sigma^2)) = \ln\left(\frac{1}{(2\pi\sqrt{\sigma^2})^n}\right) + \cancel{\ln}\left(\cancel{e}^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\right)$$

$$\ln(\mathcal{L}(\mu,\sigma^2)) = \ln\left(\frac{1}{(2\pi\sqrt{\sigma^2})^n}\right) \ldots$$

$$\ln(\mathcal{L}(\mu,\sigma^2)) = \ln\left(\frac{1}{(2\pi\sqrt{\sigma^2})^n}\right) + \cancel{\ln}\left(\cancel{e}^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\right)$$

$$\ln(\mathcal{L}(\mu,\sigma^2)) = \cancel{\ln(1)}^{\,0} - \ln\left((2\pi\sqrt{\sigma^2})^n\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\ln(\mathcal{L}(\mu,\sigma^2)) = -n\ln(2\pi\sqrt{\sigma^2}) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\mu}\ln(\mathcal{L}(\mu,\sigma^2)) = -\frac{1}{2\sigma^2}\left(2\sum_{i=1}^{n}(x_i-\mu)(-1)\right)$$

$$\frac{\partial}{\partial\mu}\ln(\mathcal{L}(\mu,\sigma^2)) = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$; Igualamos $\frac{\partial}{\partial\mu}\ln(\mathcal{L}(\mu,\sigma^2)) = 0$

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0 \rightarrow \cdot\sigma^2 \rightarrow \sum_{i=1}^{n}x_i - \sum_{i=1}^{n}\mu = 0 \rightarrow \sum_{i=1}^{n}x_i - n\mu = 0$$

$$n\mu = \sum_{i=1}^{n}x_i \rightarrow \boxed{\mu = \frac{1}{n}\sum_{i=1}^{n}x_i = \bar{X}} \qquad \hat{\mu} = \bar{x}$$

$$\ln(L(\mu,\sigma^2)) = -n\ln(2\pi\sqrt{\sigma^2}) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\sigma^2}\ln(L(\mu,\sigma^2)) = -n\frac{\cancel{2\pi}}{\cancel{2\pi}\sqrt{\sigma^2}}\left(\frac{1}{2\sqrt{\sigma^2}}\right) - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2\left(-(\sigma^2)^{-2}\right)$$

$$\frac{\partial}{\partial\sigma^2}\ln(L(\mu,\sigma^2)) = -\frac{n}{2\sigma^2} + \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2\frac{1}{(\sigma^2)^2}$$

$$\frac{\partial}{\partial\sigma^2}\ln(L(\mu,\sigma^2)) = 0;\ -\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\frac{1}{2\sigma^2}\left(-n + \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right) = 0 \rightarrow \cdot 2\sigma^2 \rightarrow -n + \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\left(\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right)^{-1} = (n)^{-1} \rightarrow \frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\mu)^2} = \frac{1}{n} \rightarrow \boxed{\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$