# DR_methods_comparison

April 29, 2025

# 1 Dimensionality Reduction Methods Comparison Task

## 1.1 Part 1: Generate Roulade Visualizations

### 1.1.1 1. Use the provided roulade generator functions to create 50-100 different visualizations by varying:

- Spiral density (spiral_density parameter)
- Number of points (spiral_steps parameter)
- Layer distance (width parameter)
- Angle between layers (angle parameter)
- Roll density (roll_density parameter)

### 1.1.2 2. For each parameter combination:

- Generate a 3D roulade structure
- Project it to 2D using PCA method
- Save the resulting 2D visualization as an image

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
import random
from mpl_toolkits.mplot3d import Axes3D
from sklearn.decomposition import PCA, KernelPCA
from sklearn.manifold import MDS, TSNE
import umap


class Roulade:
    def __init__(self):
        pass

    # Roulade point generator functions
    def uniform_spiral(self, density=3, steps=100):
        x, y = [], []
        for i in range(steps):
            x.append(
```

```
                (i / steps) ** 0.5 * math.cos((i / steps) ** 0.5 * density * np.
↪pi * 2)
            )
            y.append(
                (i / steps) ** 0.5 * math.sin((i / steps) ** 0.5 * density * np.
↪pi * 2)
            )
        return x, y

    def uniform_roll(self, xli, yli, width=7, angle=0.25, density=5,␣
↪noise_factor=0.05):
        nx, ny, nz, d = [], [], [], []
        d1 = [((x**2 + y**2) ** 0.5) for x, y in zip(xli, yli)]
        for i in range(density):
            nx.extend(
                [
                    (width + x) * math.cos(angle * i / density * np.pi * 2)
                    + (random.random() - 0.5) * noise_factor
                    for x in xli
                ]
            )
            ny.extend(
                [
                    (width + x) * math.sin(angle * i / density * np.pi * 2)
                    + (random.random() - 0.5) * noise_factor
                    for x in xli
                ]
            )
            nz.extend(yli)
            d.extend(d1)
        return nx, ny, nz, d

    def roll_generator(
        self,
        to_array=True,
        spiral_density=3,
        spiral_steps=20,
        width=7,
        angle=0.3,
        density=5,
    ):
        x, y = self.uniform_spiral(density=spiral_density, steps=spiral_steps)
        x, y, z, d = self.uniform_roll(x, y, width=width, angle=angle,␣
↪density=density)

        if to_array:
            return np.array([x, y, z]).T, d
```

```
        else:
            return x, y, z, d
```

[2]:
```
def generate_roulades(num_roulades: int = 50):
    roulade_generator = Roulade()
    for i in range(num_roulades):
        pca = PCA(n_components=2)
        points_3D, d = roulade_generator.roll_generator(
            spiral_density=np.random.randint(1, 11),
            spiral_steps=np.random.randint(20, 200),
            width=np.random.randint(1, 20),
            angle=np.random.uniform(-1, 1),
            density=np.random.randint(1, 20),
        )
        points_2D = pca.fit_transform(points_3D)
        plt.scatter(points_2D[:, 0], points_2D[:, 1])
        plt.savefig(f"images/roulade{i}.png")
        plt.close()
```

[6]:
```
generate_roulades()
```

## 1.2 Part 2: Embed Visualization Images Using Dimensionality Reduction

Treat each saved image as a high-dimensional data point (where each pixel is a dimension) and reduce them to 2D using three methods:

### 1.2.1 1.PCA

- Implement basic PCA
- No additional hyperparameters required

### 1.2.2 2. t-SNE

- Implement with at least three different perplexity values (e.g., 5, 30, 50)

### 1.2.3 3. UMAP

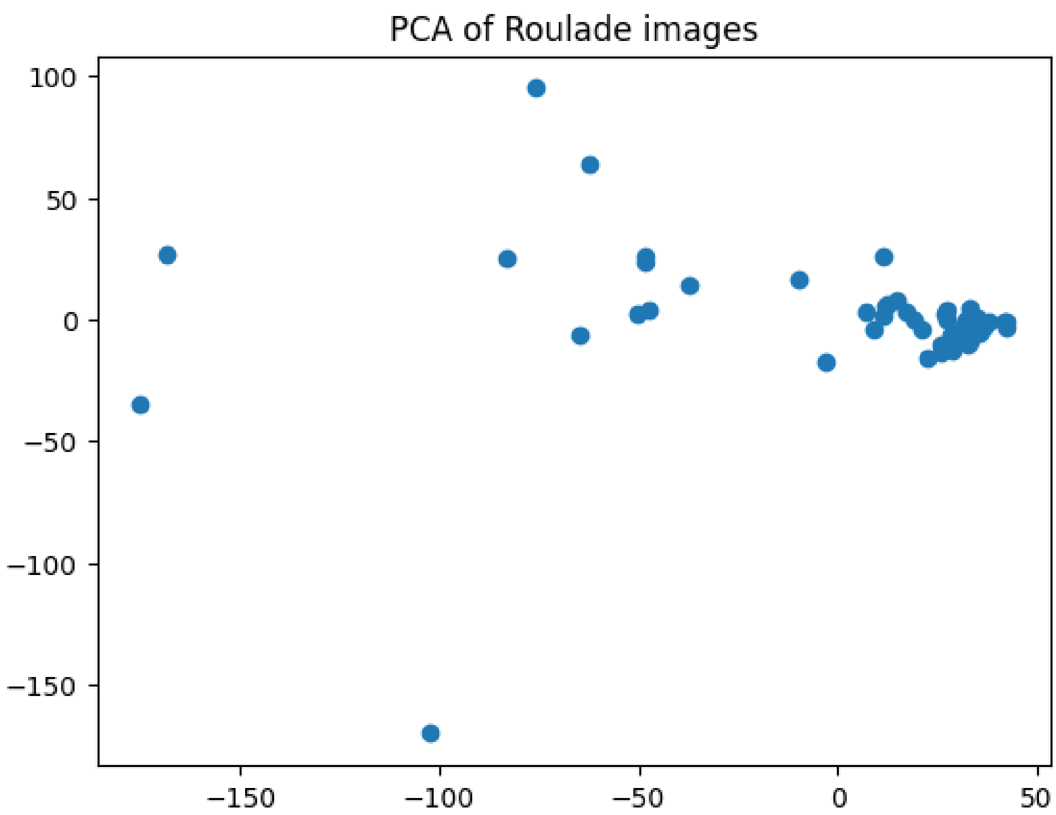
- Implement with at least three different combinations of n_neighbors and min_dist
- For example: (n_neighbors=5, min_dist=0.1), (n_neighbors=15, min_dist=0.5), etc.

[3]:
```
import pathlib

files = [f for f in pathlib.Path().glob("images/*.png")]
```

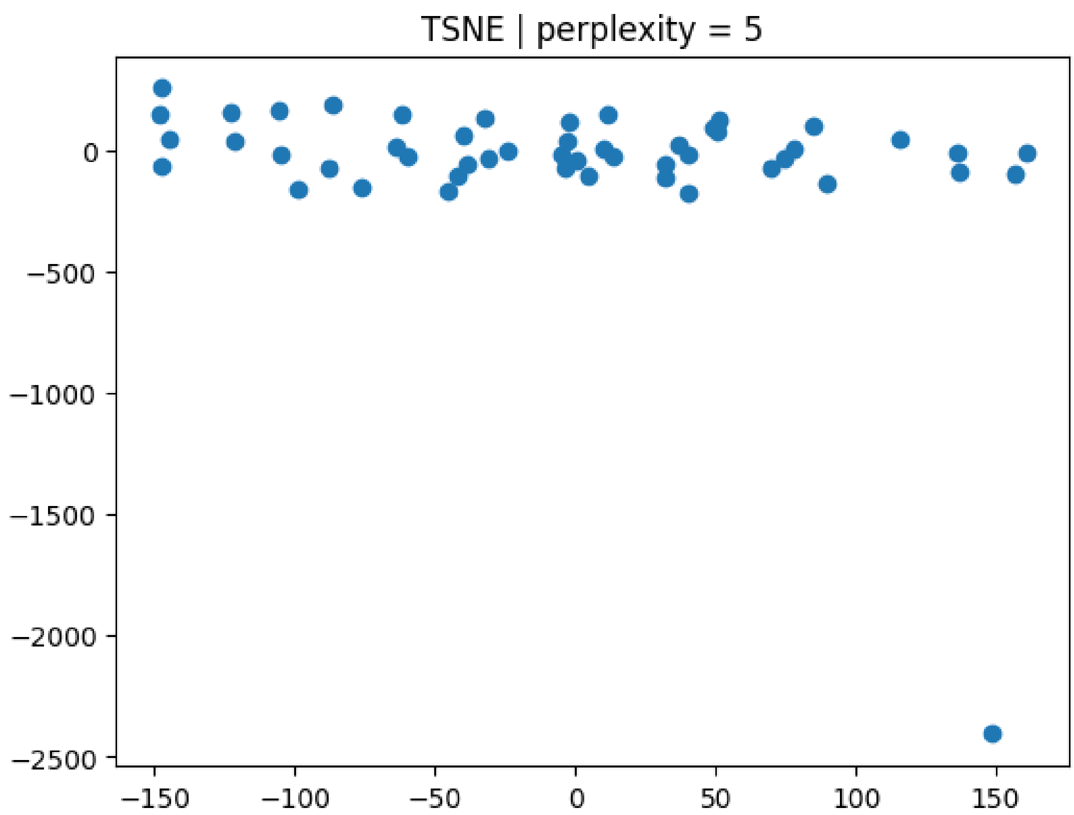
[4]:
```
image_vectors = []

for file in files:
    img = plt.imread(file).flatten()
    image_vectors.append(img)
```

```
X = np.stack(image_vectors)
```

[5]:
```
pca_embedding = PCA(n_components=2).fit_transform(X)
plt.scatter(pca_embedding[:, 0], pca_embedding[:, 1])
plt.title("PCA of Roulade images")
plt.show()
```

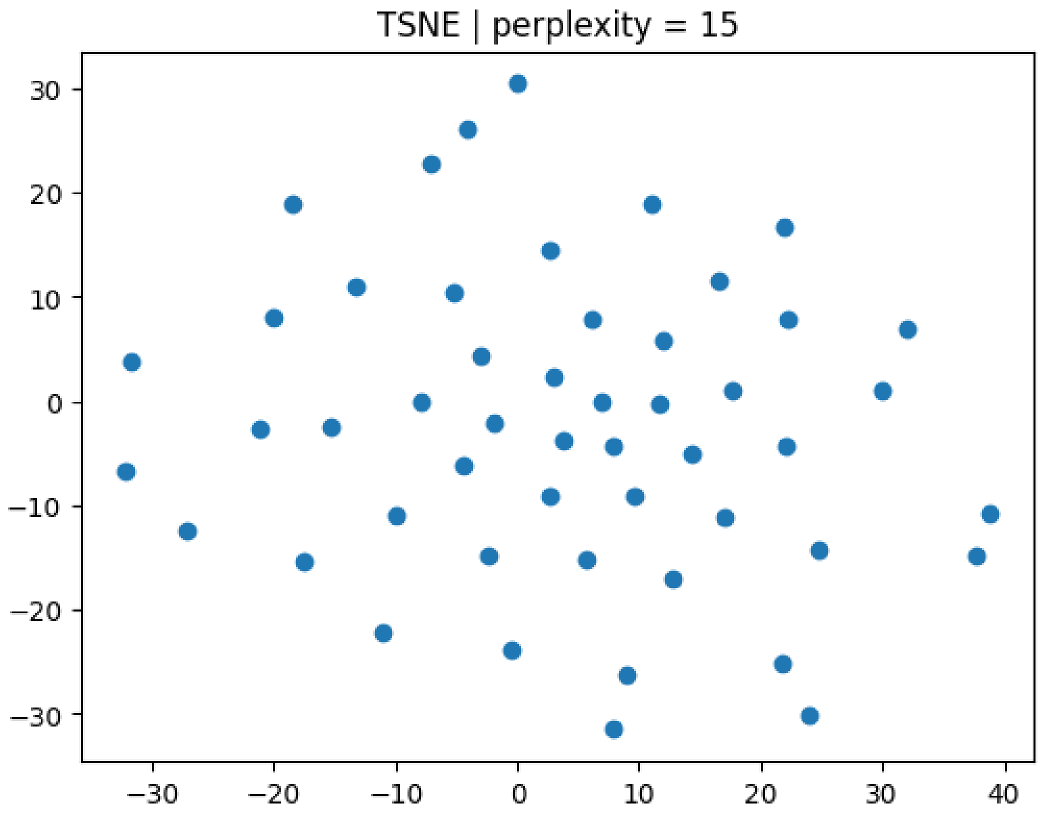



[6]:
```
tsne_embedding_5 = TSNE(n_components=2, perplexity=5).fit_transform(X)
plt.scatter(tsne_embedding_5[:, 0], tsne_embedding_5[:, 1])
plt.title("TSNE | perplexity = 5")
plt.show()
```

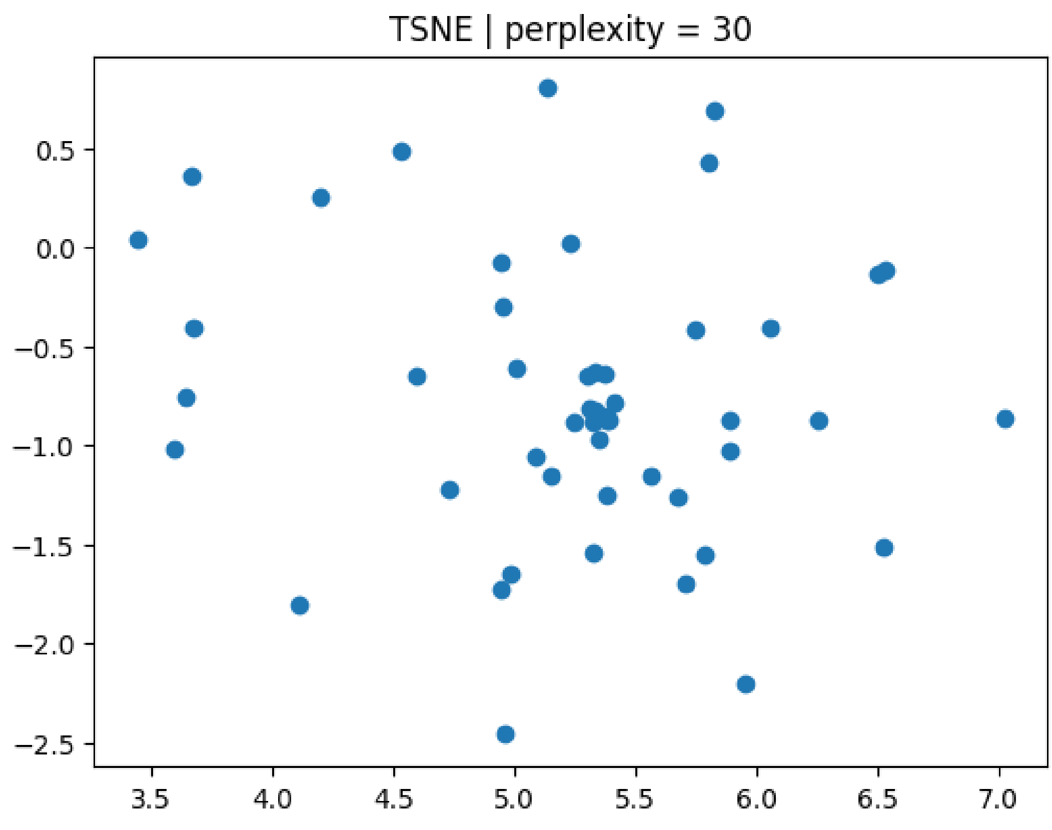



[7]:

```
tsne_embedding_15 = TSNE(n_components=2, perplexity=15).fit_transform(X)
plt.scatter(tsne_embedding_15[:, 0], tsne_embedding_15[:, 1])
plt.title("TSNE | perplexity = 15")
plt.show()
```




[8]:
```
tsne_embedding_30 = TSNE(n_components=2, perplexity=30).fit_transform(X)
plt.scatter(tsne_embedding_30[:, 0], tsne_embedding_30[:, 1])
plt.title("TSNE | perplexity = 30")
plt.show()
```




[9]:
```
umap_embedding_n25_d1 = umap.UMAP(n_neighbors=25, min_dist=1).fit_transform(X)
umap_embedding_n10_d05 = umap.UMAP(n_neighbors=10, min_dist=0.5).
  ↪fit_transform(X)
umnap_embedding_n5_d01 = umap.UMAP(n_neighbors=5, min_dist=0.1).fit_transform(X)
```

```
/Users/nicolas/studia/I_sem/wdzd/.venv/lib/python3.12/site-
packages/sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was
renamed to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/nicolas/studia/I_sem/wdzd/.venv/lib/python3.12/site-
packages/sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was
renamed to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/nicolas/studia/I_sem/wdzd/.venv/lib/python3.12/site-
packages/sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was
renamed to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

### 1.3 Part 3: Create Meta-Visualizations

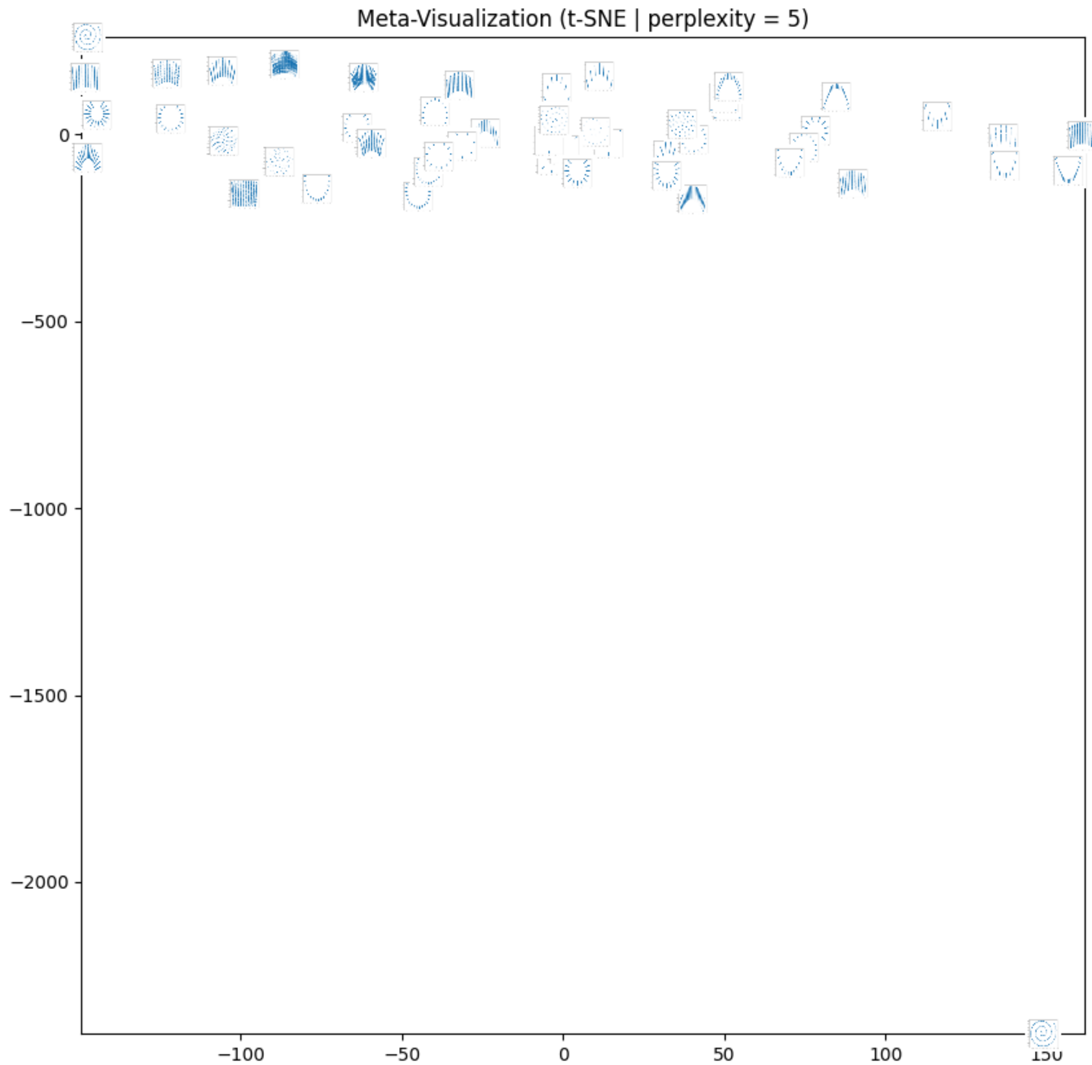
For each dimensionality reduction method:

1. Create a 2D scatter plot of the embedded images
2. Replace each point with a miniature of the original visualization image
3. Include clear labels indicating the parameter settings used for each method

[14]:
```
from matplotlib.offsetbox import OffsetImage, AnnotationBbox
from PIL import Image


def create_meta_visualizations(embeddings, files, method, params=None):
    embeddings = np.array([[embedding[0], embedding[1]] for embedding in␣
 ↪embeddings])
    image_paths = [file for file in files]

    fig, ax = plt.subplots(figsize=(10, 10))

    for (x, y), img_path in zip(embeddings, image_paths):
        img = Image.open(img_path)
        img = img.resize((100, 100))
        im = OffsetImage(img, zoom=0.2)
        ab = AnnotationBbox(im, (x, y), frameon=False)
        ax.add_artist(ab)

    ax.set_xlim(np.min(embeddings[:, 0]) - 1, np.max(embeddings[:, 0]) + 1)
    ax.set_ylim(np.min(embeddings[:, 1]) - 1, np.max(embeddings[:, 1]) + 1)
    if params is not None:
        ax.set_title(f"Meta-Visualization ({method} | {params})")
    else:
        ax.set_title(f"Meta-Visualization ({method})")
    plt.show()
```

[15]:
```
create_meta_visualizations(pca_embedding, files, "PCA")
```


```
[16]: create_meta_visualizations(tsne_embedding_5, files, "t-SNE", "perplexity = 5")
```

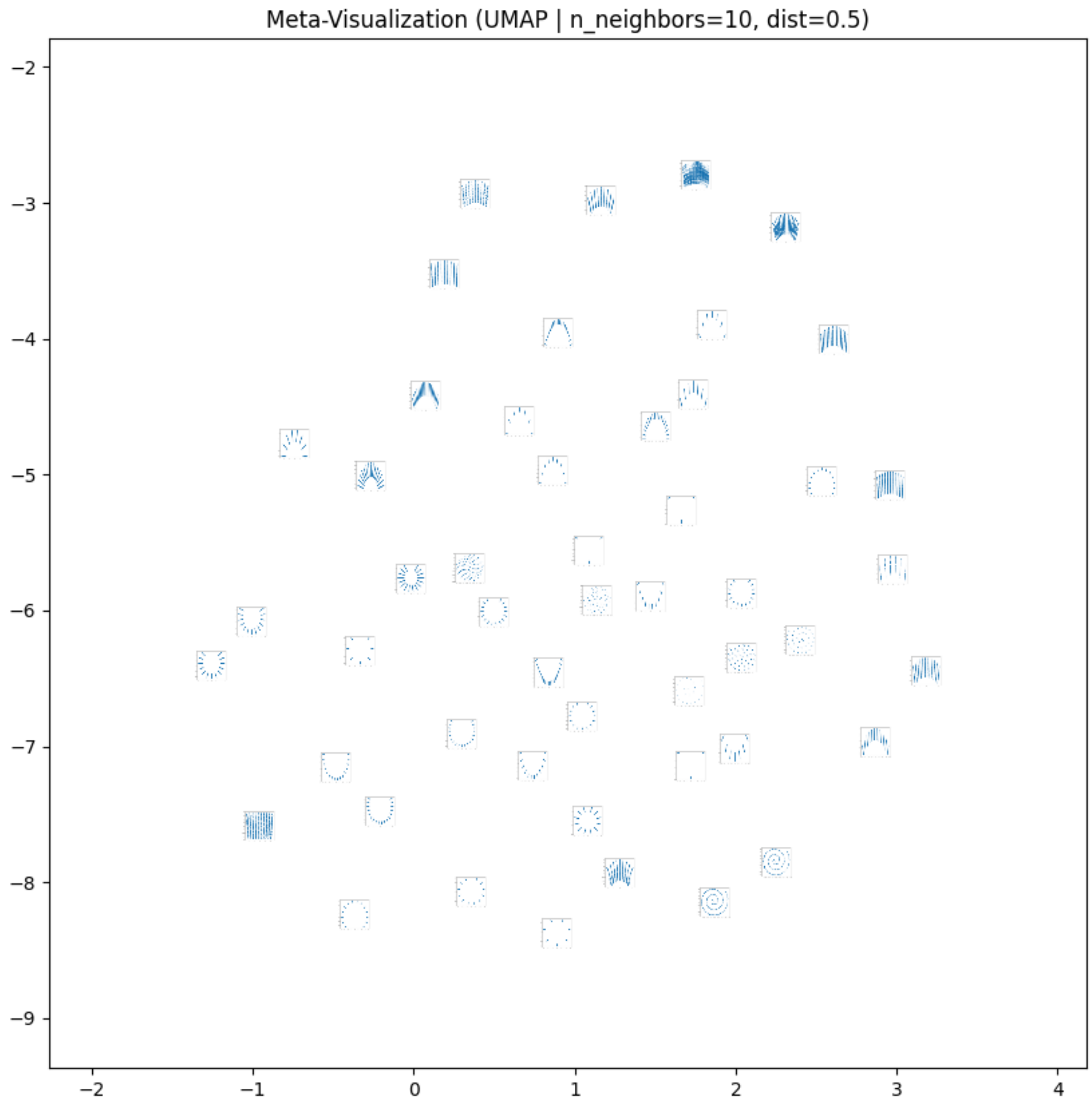



[17]:
```
create_meta_visualizations(tsne_embedding_15, files, "t-SNE", "perplexity=15")
```

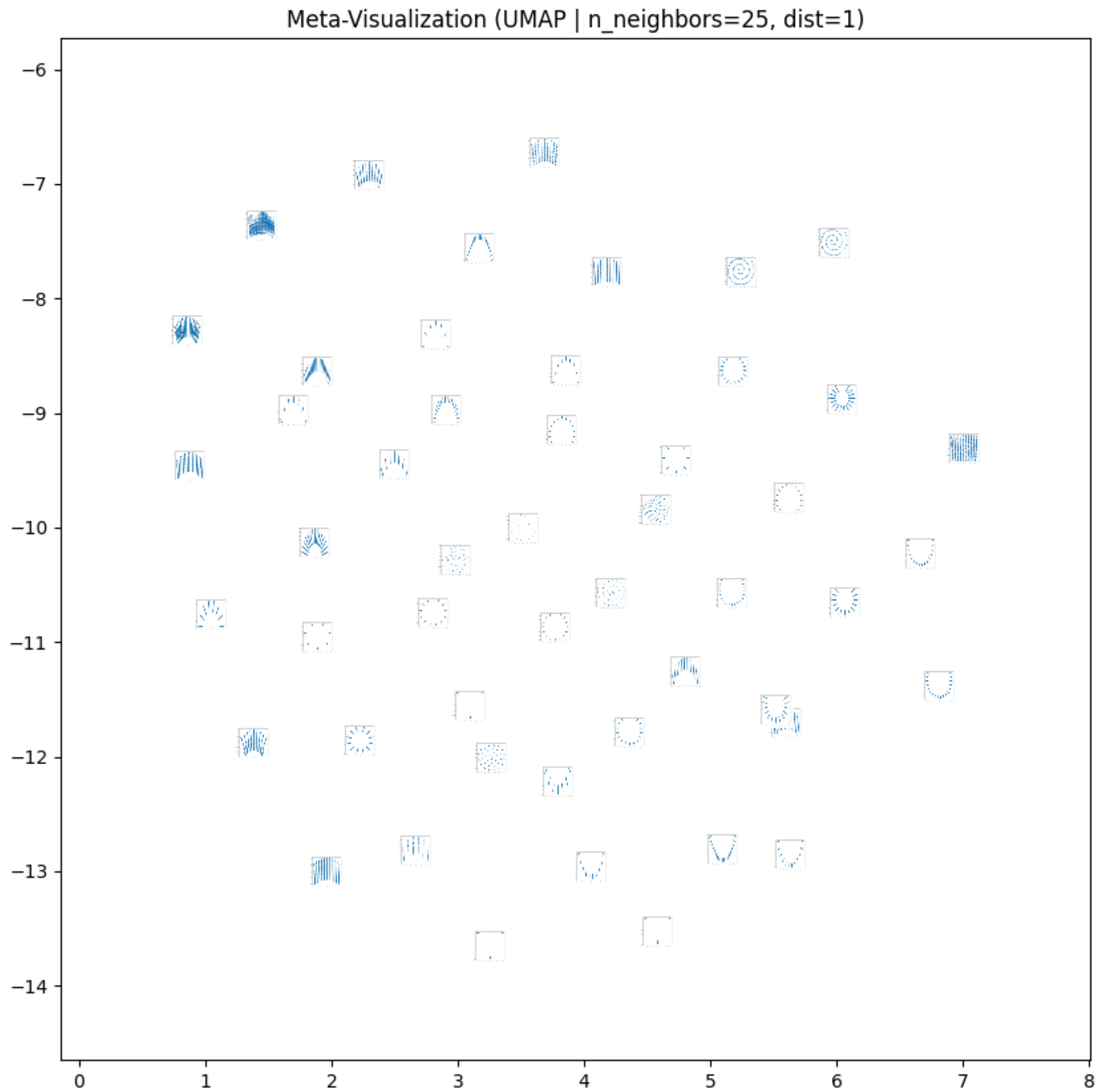

[18]:
```
create_meta_visualizations(tsne_embedding_30, files, "t-SNE", "perplexity=30")
```


[19]:
```
create_meta_visualizations(
    umnap_embedding_n5_d01, files, "UMAP", "n_neighbors=5, dist=0.1"
)
```


[20]:
```
create_meta_visualizations(
    umap_embedding_n10_d05, files, "UMAP", "n_neighbors=10, dist=0.5"
)
```




[21]:
```
create_meta_visualizations(
    umap_embedding_n25_d1, files, "UMAP", "n_neighbors=25, dist=1"
)
```




## 1.4 Part 4: Analysis

Write a 300-500 word analysis that addresses:

- Which method best preserves the spiral structure of the original roulade?
- How do different hyperparameter settings affect the results?
- What patterns can you observe in how similar visualizations are grouped?
- How do the quantitative results from the cf metric align with visual assessments?

Among the dimensionality reduction methods evaluated—PCA, t-SNE, and UMAP—**t-SNE with a perplexity of 15** appears to best preserve the original data structure based on both visual assessments and the quantitative **class fidelity (cf)** metric. The cf metric, which measures how well local neighborhoods are preserved, shows that t-SNE consistently achieves the highest scores, especially for smaller numbers of neighbors. This aligns well with the visual plots, where t-SNE clearly maintains distinct and compact clusters that closely resemble the original class distributions.

Hyperparameters play a significant role in the outcome of both t-SNE and UMAP. For t-SNE, **perplexity** controls the balance between local and global structure. A **perplexity of 5** tends to overemphasize local detail, often resulting in fragmented and overly tight clusters. On the other hand, a **perplexity of 30** smooths out the embedding, possibly blending class boundaries. **Perplexity 15** offers a balanced representation, effectively preserving both neighborhood integrity and broader class layout.

For UMAP, the **n_neighbors** parameter determines the local versus global emphasis, while **min_dist** controls how tightly points are packed in the low-dimensional space. The setting **(n_neighbors=5, min_dist=0.1)** leads to very tight, distinct clusters, but sometimes at the expense of inter-class relationships. The setting **(n_neighbors=15, min_dist=0.5)** results in a more globally cohesive layout but with slightly fuzzier class boundaries. The setting **(n_neighbors=25, min_dist=1)** results in a globally cohesive layout with clear class boundaries. Overall, it's a good choice of method and a set of parameters too.

When comparing visual groupings with cf metric results, there's a clear alignment: methods that visually show tight, well-separated clusters (like t-SNE with optimal perplexity or UMAP with high number of n_neigbors) tend to score higher on class fidelity. PCA, lacking nonlinear modeling capacity, shows overlapping class distributions and scores lowest on the cf metric, highlighting its limitation in preserving neighborhood structure for complex manifolds like MNIST digits.