NTNU

Norwegian University of Science and Technology

TK8117 Multivariate Data Analysis

Week 10 Advanced Topic 3:

Kalman Filters for Non-Gaussian Noises

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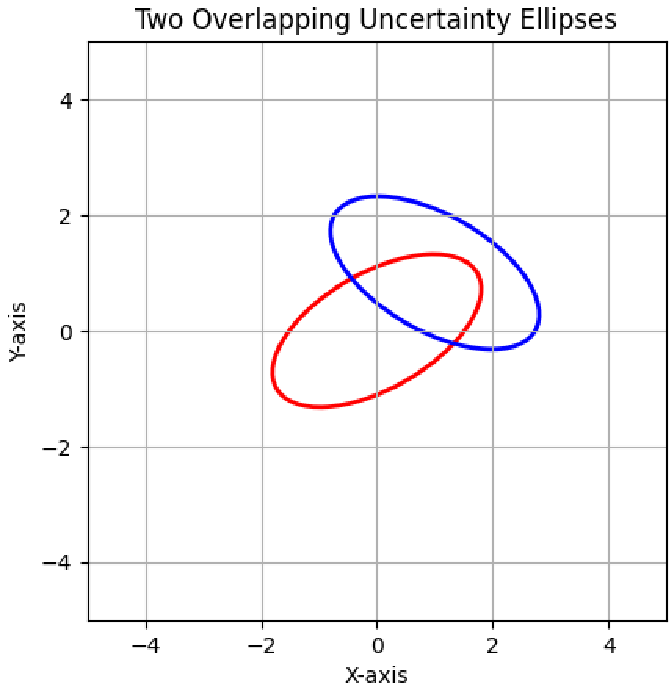
Outline

- Non-gaussian noise
- Using ‘’normal’’ filters
- H_∞ filter
- Estimates using IRLS method
- Sources

Non-gaussian noise

- Kalman filters propagate noise
- Assumes nice noise curves

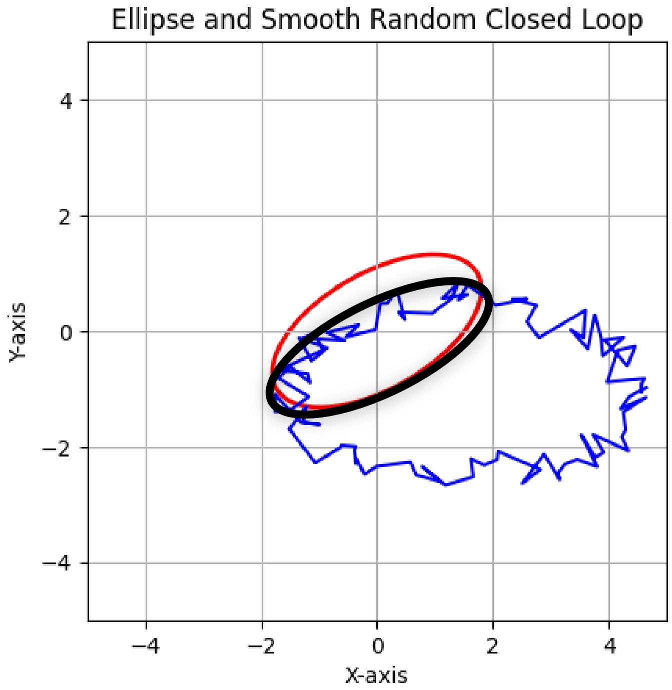


Non-gaussian noise

- Ugly noise is ugly
- Kalman filters requires:
  - Finite first and second moments

Using ‘’normal’’ filters

- Normal filters often work
- Ensemble and unscented filters are also good



H_∞ filter

- Also called **minimax** filter
- **Minimize** the maximum estimation error
- when we **maximize** the noise
- Makes no assumption about the noise

Daniel J. Simon, 2001 "From Here to Infinity"

H_∞ filter

$$min_{\hat{x}}\, max_{w,v}\, J$$

w, v – noise

J – measure of how good our estimator is

$$J = \frac{ave\|x_k - \hat{x}_k\|_Q}{ave\|w_k\|_W + ave\|v_k\|_V}$$

$min_{\hat{x}}\, max_{w,v}\, J$ – difficult to solve

Instead we solve:

$$J < \frac{1}{\gamma}$$

γ – constant chosen by us

H_∞ filter equations

The state estimate that forces $J < \frac{1}{\gamma}$ is given as:

$$L_k = (I - \gamma Q P_k + C^T V^{-1} C P_k)^{-1}$$
$$K_k = A P_k L_k C^T V^{-1}$$
$$\hat{x}_{k+1} = A\hat{x}_k + B u_k + K_k(y_k - C\hat{x}_k)$$
$$P_{k+1} = A P_k L_k A^T + W$$

K_k - Gain matrix, will converge after a few steps and can be used as a constant matrix for Steady-state H_∞

γ – must be chosen such that the eigenvalues of P_k has a magnitude less than 1. A too large γ will result in no solution for H_∞

γ, Q, P_0, V & W are the tuning parameters

H_∞ filter Key points

- Minimax filter

- Assumes Murphy’s Law – the noise we have is the worst possible noise

- No statistical assumption about the noise

A Modified Kalman Filter for Non-gaussian Measurement Noise

- Kalman Filter with Robust Sequential Estimator (RSE)
- Using RSE to handle non-Gaussian data as outliers

Mirza J. Muhammed, 2011 "A Modified Kalman Filter for Non-gaussian Measurement Noise"

How it works..

measurements with noise

modeling non-gaussian part as outliers

how modeling works..

one sample

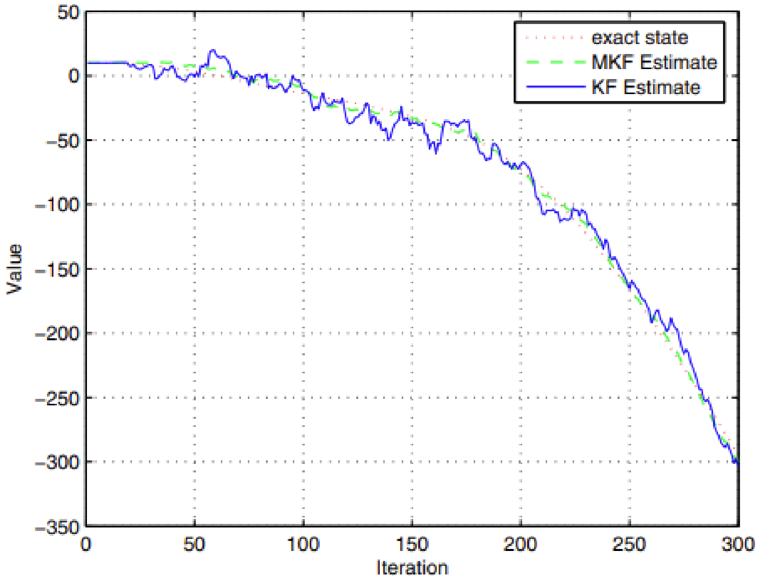
maximum likelihood estimates for t-distribution error model

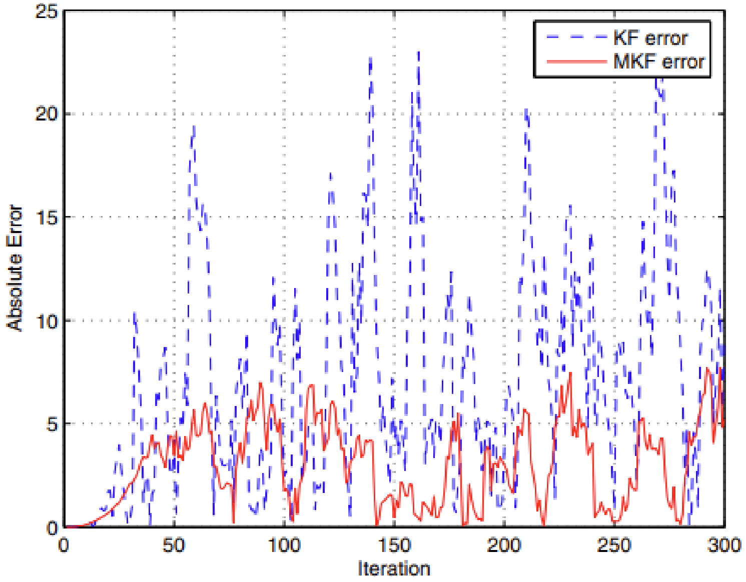
update the sample via Iteratively Reweighted Least Squares (IRLS)

Outlier classification with weights done by RSE

updates the state only with valid data

Results





standart vs. modified Kalman Filter estimates

standart vs. modified Kalman Filter mean square errors

Sources

- Mirza J. Muhammed, 2011 "A Modified Kalman Filter for Non-gaussian Measurement Noise"
- Dan Simon, 2001, "From Here to Infinity"