NTNU

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Instrumental variables for closed loop system identification

Week 09 Advanced Topic 5

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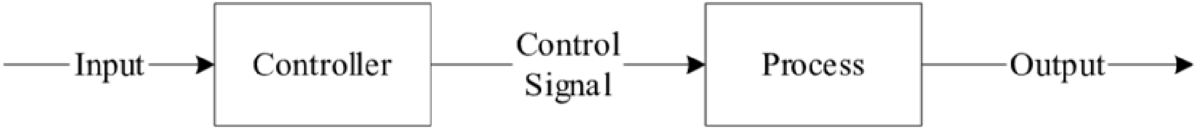
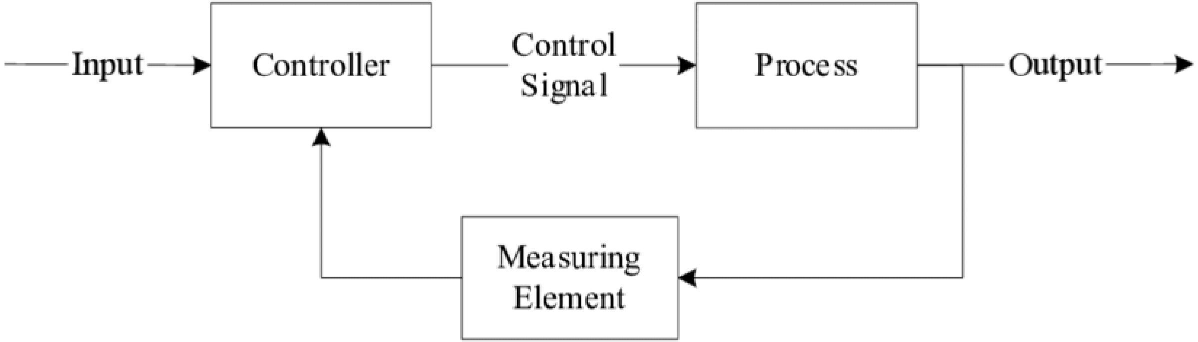
Outline

- Background
- Theory
- Coding example
- Conclusion

Closed loop



Open Loop System



Closed Loop System

System identification

- Collect input-output dataset
  - Sometimes not possible to do open-loop
- Feedback introduces bias
  - Identifying using u(y) not u
  - The input contains the output through feedback
  - Main difficulty is correlation between disturbance and control signal
- Consider transformed variables U and not the original ones u(y)

Instrumental variables motivation

- Estimation in closed loop - very complex topic
- For some systems – no input–output data in open loop
- Transforming into a consistent estimator
- Choose variables uncorrelated to noise

Simple Least Squares case

$$A(q)y(t) = B(q)u(t) + v(t)$$

$$\widehat{\Theta}_{LS} = \arg\min \frac{1}{N}\sum_{t=1}^{N}(y(t) - U(t)\Theta)^2$$

$$\widehat{\Theta}_{LS} = \left(\frac{1}{N}\sum_{t=1}^{N} U(t)^T U(t)\right)^{-1} \left(\frac{1}{N}\sum_{t=1}^{N} U(t)^T y(t)\right)$$

$$\widehat{\Theta}_{IV} = \left(\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T U(t)\right)^{-1} \left(\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T y(t)\right)$$

Let’s break it down

$$\widehat{\Theta}_{LS} \xrightarrow{N\to\infty} \Theta_0 + U^{-1}\mathbb{E}[U(t)^T v(t)]$$

$$\widehat{\Theta}_{IV} \xrightarrow{N\to\infty} \Theta_0 + (\tfrac{1}{N}\textstyle\sum_{t=1}^{N} \xi(t)^T U(t))^{-1} \tfrac{1}{N}\sum_{t=1}^{N} \xi(t)^T v(t)$$

$$(\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T U(t))^{-1} \ \exists$$

$$\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T v(t) = 0$$

$$\widehat{\Theta}_{IV} = (\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T U(t))^{-1} (\frac{1}{N}\sum_{t=1}^{N} \xi(t)^T y(t))$$

Instrumental variables requirements

- Correlated with the inputs

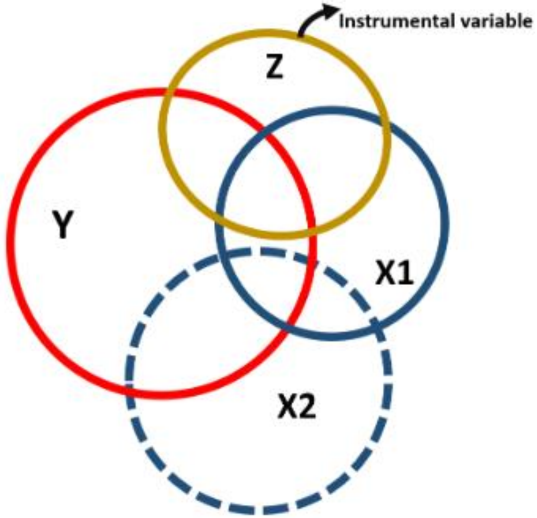
$$(\frac{1}{N}\sum_{t=1}^{N}\xi(t)^{T}U(t))^{-1}\ \exists$$

- Uncorrelated with measurement noise

$$\frac{1}{N}\sum_{t=1}^{N}\xi(t)^{T}v(t) = 0$$

How to choose IV

- Delayed samples of the reference signal and/or output signal
- A nonsingular linear transformation of the IVs have no influence on the estimate
- Correlated with the input
- Uncorrelated with the noise



Different types of IV methods

- Closed-loop basic IV method
  - What I just showed
- Closed-loop extended IV method
  - Filter data by $L(q^{-1})$
  - $\bar{u}(t) = L(q^{-1})u(t), \quad \bar{y} = L(q^{-1})y(t)$
- Taylor-made IV identification
  - Provides unbiased estimate of linear regression models
- BELS method – Bias Eliminating Least Squares
  - Shown to be a particular form of Taylor-made IV estimator

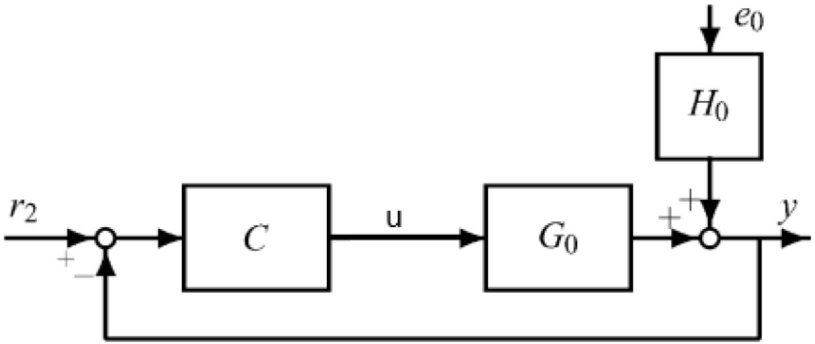
Example

- Closed-loop data sequences of length N=1000
- Monto Carlo simulation of 100 experiments

$$G_0(q) = \frac{0.5q^{-1}}{1 - 0.8q^{-1}}, \quad n = 1,$$

$$C(q) = \frac{0.0012 + 0.0002q^{-1} - 0.001q^{-2}}{0.5 - 0.9656q^{-1} + 0.4656q^{-2}}, m = 2,$$

$$H_0(q) = \frac{1 - 1.56q^{-1} + 1.045q^{-2} - 0.3338q^{-3}}{1 - 2.35q^{-1} + 2.09q^{-2} - 0.6675q^{-3}}.$$

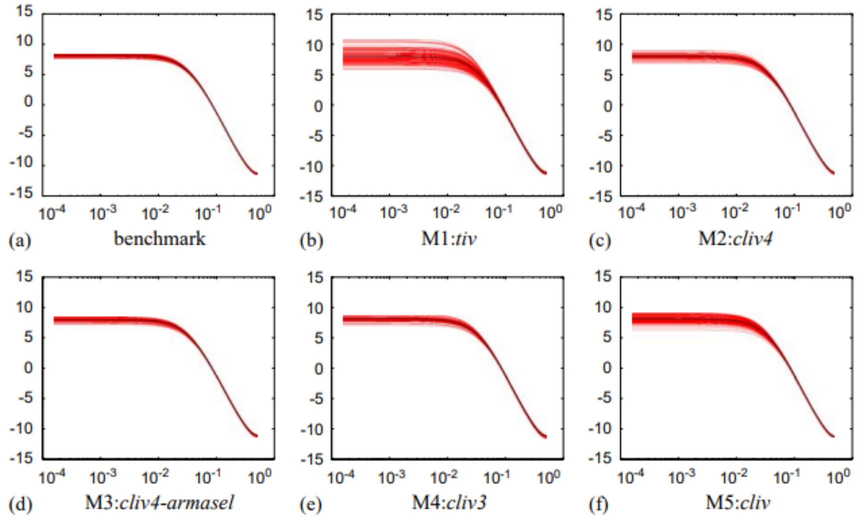


Gilson, M., & Van den Hof, P. (2005). Instrumental variable methods for closed-loop system identification. *Automatica*, 41(2), 241-249.
https://doi.org/10.1016/j.automatica.2004.09.016

Example

M1: tailor-made IV (*tiv*)/BELS with $m > n$, see Section 3.3;
M2: bootstrap IV (*cliv4*), see Section 5.1;
M3: bootstrap IV with automated noise model identification (*cliv4-armasel*), see Section 5.1;
M4: bootstrap IV with high-order least-squares (*cliv3*), see Section 5.2;
M5: basic closed-loop IV.

M. Gilson, P. Van den Hof / Automatica 41 (2005) 241–249



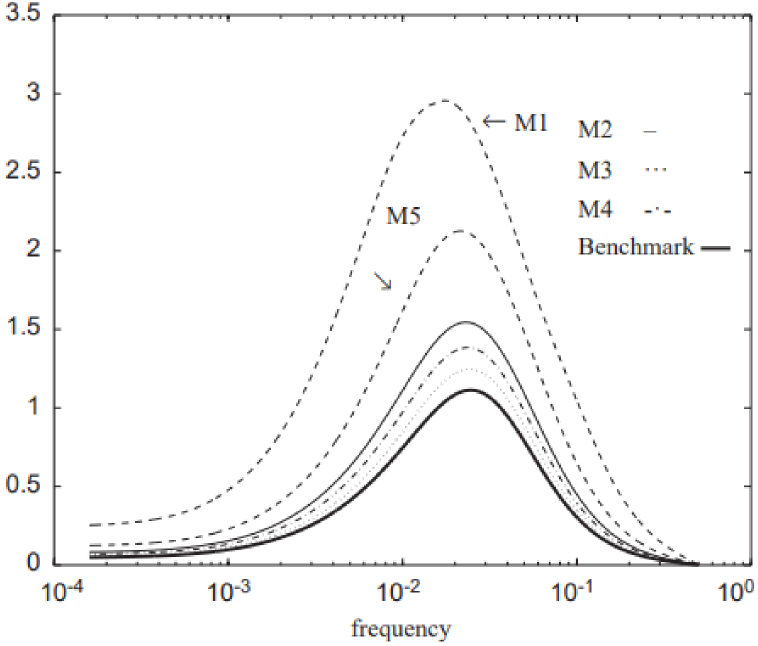


Fig. 3. Average frequency response error $g(\omega)$ for several IV methods; results are averaged over 100 Monte Carlo experiments.

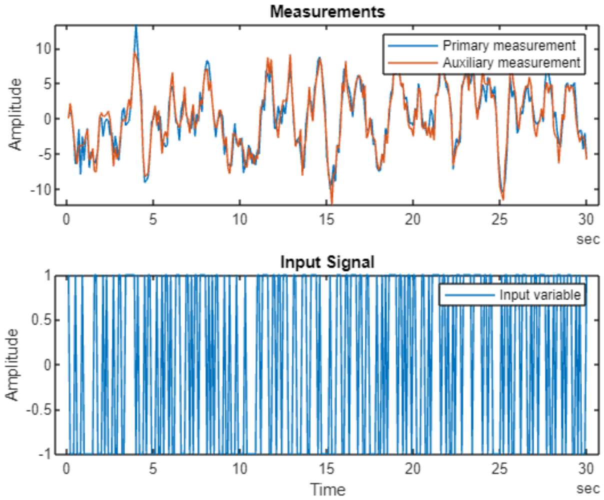
Code example

```
% Load input/output data from a SISO system
load IODataForIVExample IOData
idplot(IOData)
```

```
% Estimate an ARX Model of order na = 2, nb = 2, nk = 1.
% This amounts to using the regressors
% y(t-1), y(t-2), u(t-1), u(t-2)
na = 2;
nb = 2;
nk = 1;
sysARX = arx(IOData,[na nb nk])
```

```
sysARX =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 - 0.8282 z^-1 + 0.09171 z^-2

  B(z) = 0.8512 z^-1 + 1.233 z^-2
```




```
sysIV = ivx(IOData, [na nb nk], X)
```

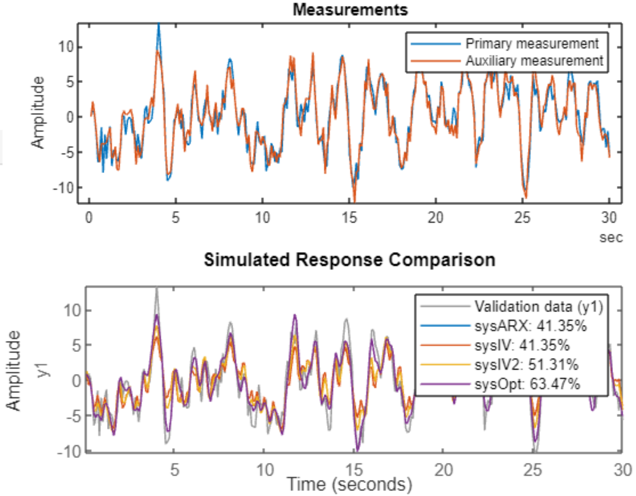
```
sysIV =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 - 0.8282 z^-1 + 0.09171 z^-2

  B(z) = 0.8512 z^-1 + 1.233 z^-2
```

```
sysOpt = iv4(IOData, [na nb nk])
```

```
sysOpt =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 - 1.504 z^-1 + 0.7069 z^-2

  B(z) = 0.9289 z^-1 + 0.6264 z^-2
```



Conclusion

- When to use instrumental variables?
  - Input is correlated with the output – closed loop
- How to choose instrumental variables?
  - Correlated with the input
  - Uncorrelated with the noise
- Pros
  - Unbiased estimators
  - Handles input/output correlation
- Cons
  - Finding valid instruments
  - Weak instruments problem