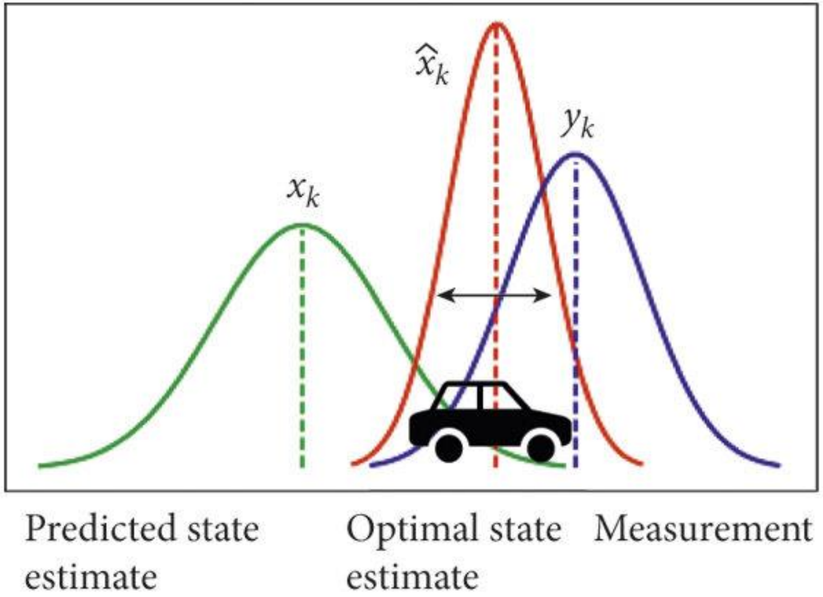
NTNU | Kunnskap for en bedre verden

System Identification with Kalman Filters

Advanced Topic 5

Kalman Filters



Kalman filter is an **algorithm** that helps us **make the best possible estimate** of a **changing value** by **combining information from measurements** that may be **noisy** or **uncertain**

Img source: *Journal of advanced transportation, 2021:1-16,* 10.1155/2021/6251399

Role of Kalman Filters in SysID

- In SysID, the goal is to create a mathematical model of a system **based on observed data**, particularly when the **system's internal structure is unknown or only partially known**

In this context:

1. Model Estimation: The Kalman filter *continuously updates a state-space model* of the system by using incoming data

2. Parameter Tuning: As the *filter processes more measurements*, it *updates the states (*like positions or speeds*) and system parameters.*

3. Handling Noise and Uncertainty: Since real-world data often has noise, *the Kalman filter's ability to separate the "true" signal from the noise is crucial in system identification*.

4. Application in Control Systems: Identifying the precise dynamics of a control system allows for more *accurate control strategies.*

Kalman filters enable us to understand how the system responds to different inputs, leading to better control and prediction of future behavior.

Mathematical foundation

State space model

$$x_{k+1} = Ax_k + Bu_k + w_k$$
$$y_k = Cx_k + Du_k + v_k$$

Predict

$$\bar{x} = Fx + Bu$$
$$\bar{P} = FPF^T + Q$$

Update

$$y = z - H\bar{x}$$
$$K = \bar{P}H^T(H\bar{P}H^T + R)^{-1}$$
$$x = \bar{x} + Ky$$
$$P = (I - KH)\bar{P}$$

Extensions for system identification

- Kalman Filters for nonlinear systems
  - Extended Kalman Filter (EKF)
  - Unscented Kalman Filter (UKF)
- EKF: Use Gaussian approx. to find an appropriate linear system representing the nonlinear dynamics
- Locally linearized state space model
  - Gradient estimation
- Estimation of time-varying systems
  - EKF with fading memory

Example

- Single-Degree-of-Freedom System:

$$\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = f(t)$$

- Employ EKF to update natural frequency ω_n and damping factor ζ
- The augmented state vector is defined as:
  - $\boldsymbol{y}(t) \equiv [\dot{x}, x, \omega_n, \zeta]^{\mathrm{T}}$
- The state-space representation of the ODE can be written as follows
  - $\dot{\boldsymbol{y}}(t) = g(\boldsymbol{y}(t), f(t); \omega_n, \zeta)$

Source: Huang, K., Yuen, KV. (2023). System Identification Using Kalman Filter and Extended Kalman Filter. In: Bayesian Real-Time System Identification.

Example

- Do Taylor expansion around the state $\boldsymbol{y}(t) = \boldsymbol{y}^*$:

$$\dot{\boldsymbol{y}}(t) = g|_{\boldsymbol{y}=\boldsymbol{y}^*} + \left.\frac{\partial g}{\partial \boldsymbol{y}}\right|_{\boldsymbol{y}=\boldsymbol{y}^*} (\boldsymbol{y} - \boldsymbol{y}^*) + \left.\frac{\partial g}{\partial \boldsymbol{f}}\right|_{\boldsymbol{y}=\boldsymbol{y}^*} \boldsymbol{f}$$

- Discretize the state-space representation:

$$\boldsymbol{y}_{i+1} = \mathbf{A}_i \boldsymbol{y}_i + \mathbf{B}_i \boldsymbol{f}_i + \boldsymbol{\delta}_i$$

- The matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ are given by:

$$\mathbf{A}_i = \exp\left(\mathbf{A}_y \Delta t\right)$$

$$\mathbf{B}_i = \mathbf{A}_y(\mathbf{A}_i - \mathbf{I}_4)\mathbf{B}_y$$

Source: Huang, K., Yuen, KV. (2023). System Identification Using Kalman Filter and Extended Kalman Filter. In: Bayesian Real-Time System Identification.

Example

- Where:

$$\mathbf{A}_y = \left.\frac{\partial g}{\partial \boldsymbol{y}}\right|_{\boldsymbol{y}=\boldsymbol{y}_i} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_n^2 & -2\zeta\omega_n & -2\omega_n x(t) - 2\zeta\dot{x}(t) & -2\omega_n\dot{x}(t) \\ 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & -\eta \end{bmatrix}$$

$$\mathbf{B}_y = \left.\frac{\partial g}{\partial \boldsymbol{f}}\right|_{\boldsymbol{y}=\boldsymbol{y}_i} = [0, 1, 0, 0]^{\mathrm{T}}$$

- The remainder term $\boldsymbol{\delta}_i$ is defined as

$$\boldsymbol{\delta}_i = \mathbf{A}_y(\mathbf{A}_i - \mathbf{I}_4)\left(g|_{\boldsymbol{y}=\boldsymbol{y}_i} - \mathbf{A}_y\boldsymbol{y}_i\right)$$

Source: Huang, K., Yuen, KV. (2023). System Identification Using Kalman Filter and Extended Kalman Filter. In: Bayesian Real-Time System Identification.

Example

- The observation equation is given by:

$$z_i = \mathbf{H}\boldsymbol{y}_i + \boldsymbol{n}_i$$

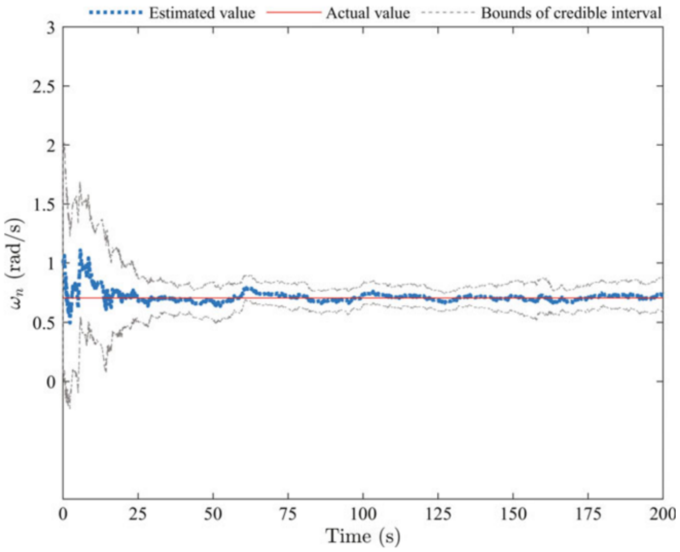
- Where the observation matrix is defined as:

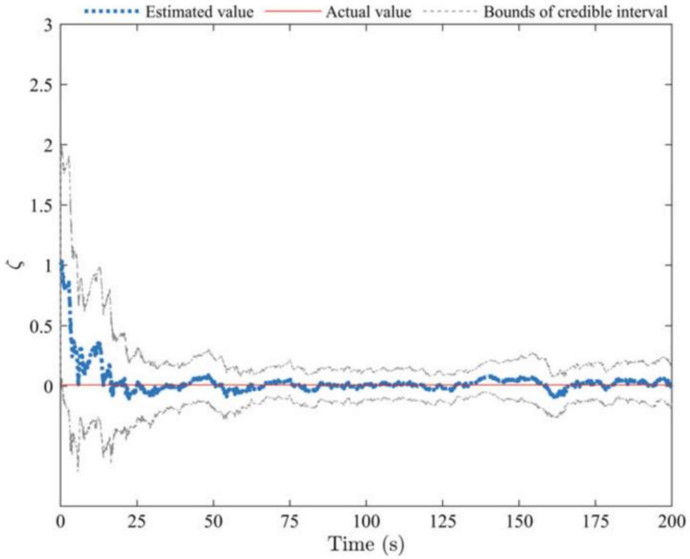
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Source: Huang, K., Yuen, KV. (2023). System Identification Using Kalman Filter and Extended Kalman Filter. In: Bayesian Real-Time System Identification.

Example

- Results:





Source: Huang, K., Yuen, KV. (2023). System Identification Using Kalman Filter and Extended Kalman Filter. In: Bayesian Real-Time System Identification.

Advantages and Challenges

- Advantages
  - Recursive estimation
  - Noise reduction
  - Adaptability to nonlinear models
  - Simultaneous state and parameter estimation
- Challenges
  - Sensitivity to model assumptions
  - Computational complex
  - Needs parameter initialization
  - Unstable in highly nonlinear systems

Thank you!