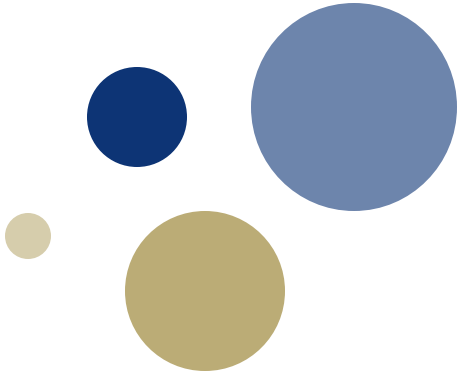

Bootstrapping and Jackknife

Week 04 - Topic 2

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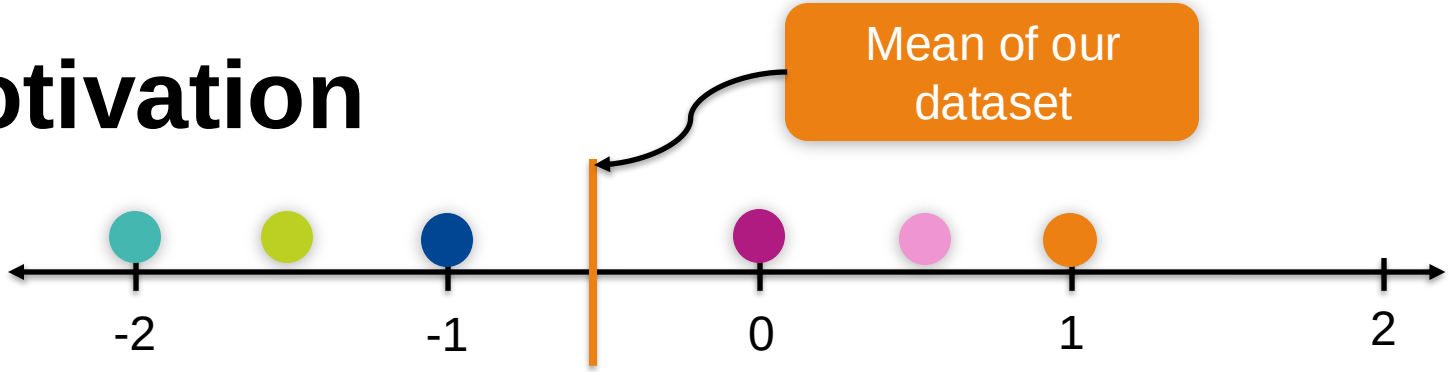
Motivation



Can we be confident now that this is our mean? How precise are we? 🤔

No! It would be nice to do the experiment again.

BUT this is very expensive. 🤑 Can we do it in another way?

Resampling methods

Create **n** subsets of our data

Calculate a desired statistic/estimator (e.g. mean, log variance, MSE)

Estimate the skill of our ML or estimate quantities of our dataset in a **non-parametric** way

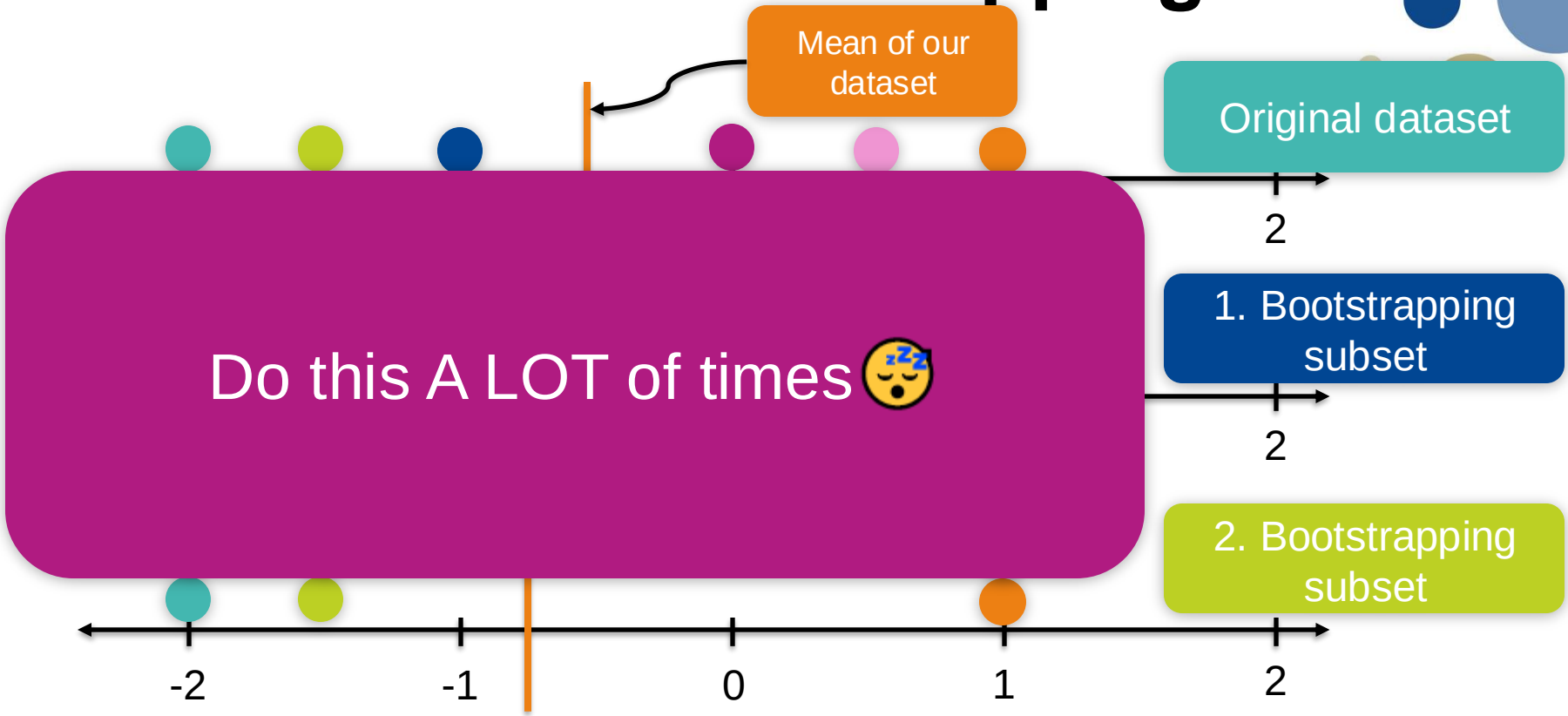
Bootstrapping & Jackknife

Two most popular methods for resampling

Bootstrapping 🥾 typically for **confidence interval/estimation**

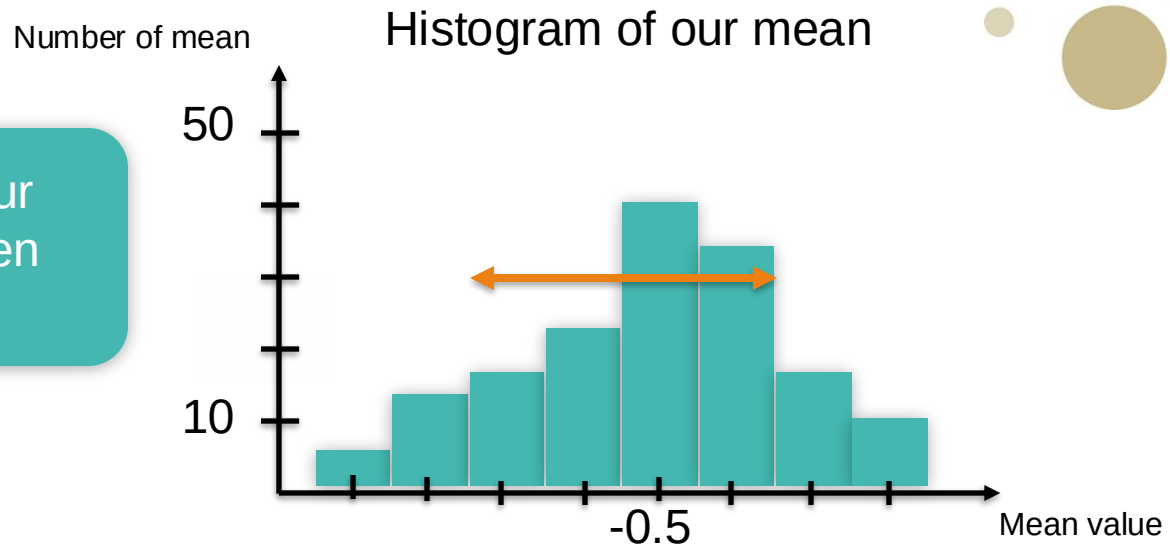
Jackknife 🔪 typically for **bias** and **variance**

Let us start with Bootstrapping…



Result of our Bootstrapping🥾

Save the statistics of our subsets & calculate even better stuff😲



Confidence😎 interval of our dataset

Recipe for the Bootstrapping

1. Choose a size for your bootstrap subset (same as dataset but with lager dataset can be also just 80%)

2. Choose a number of repetition for your bootstrapping (some references say around 200, some say <1000)

3. Calculate your desired statistic for each bootstrap subset (e.g. mean, MSE)

4. Evaluate your statistics (e.g. confidence interval, standard error)

Be careful when using it for ML…

Reduce bootstrap subset size to **lower than** dataset size (if your dataset is relatively small)

Use the bootstrap subset for training and **out-of-bag** subset for testing

Use out-of-bag subset because we do not want any **data leakage** between training and test set

Pros & Cons of Bootstrapping

Non-parametric method	Computationally expensive
Mimics samping from a larger dataset	Not more information than in the original dataset
Simple confidence interval estimation	Limited in its capabilities for small datasets

🤨 Okay, but for what can I use it?

Check the confidence of my model's predictions

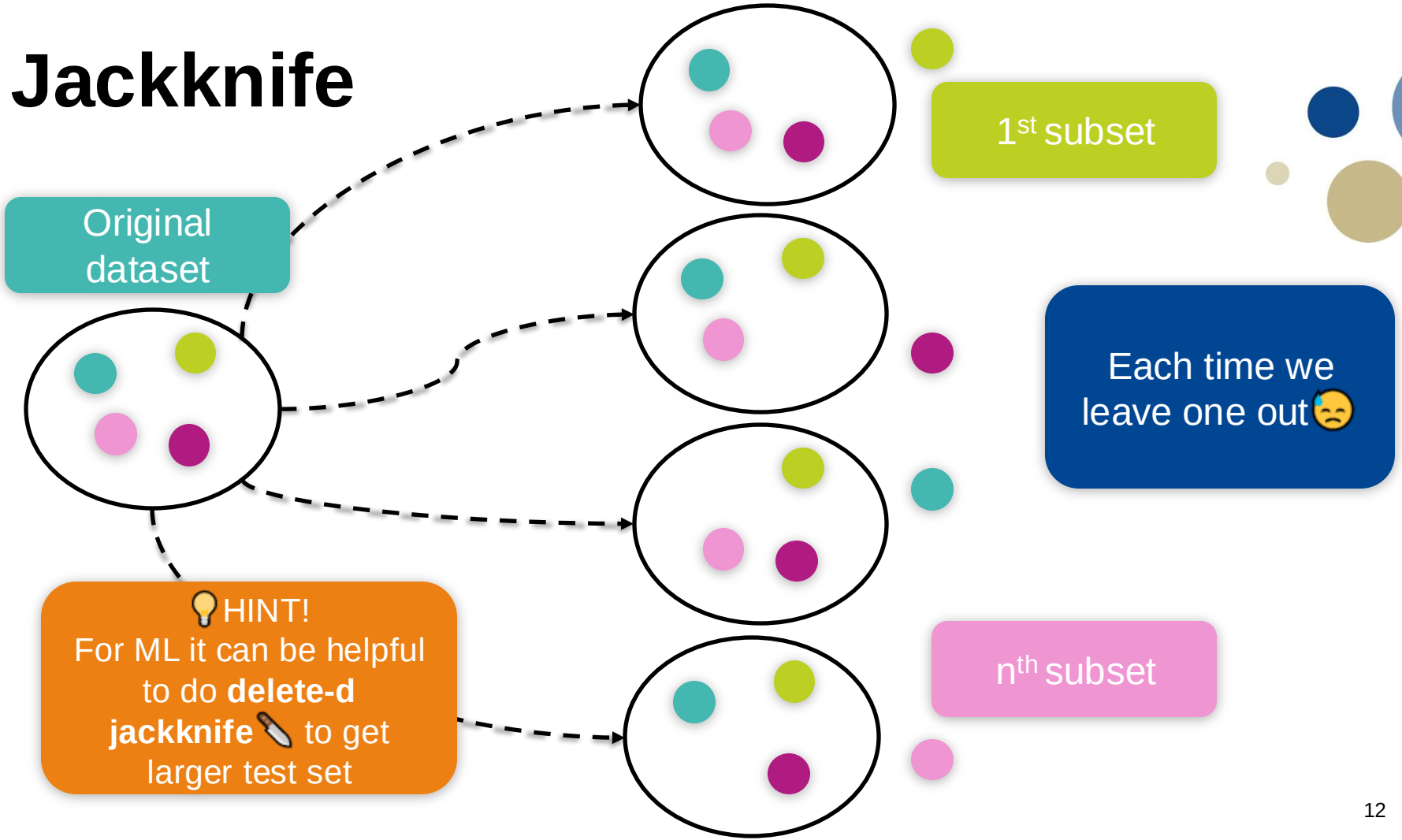
Validate several models in terms of different statistics

Get more statistics on my dataset (i.e. bias, standard error, confidence interval)

Short break

Did we understand everything so far?

Jackknife


Original dataset
1st subset
Each time we leave one out 😓
HINT!
For ML it can be helpful to do **delete-d jackknife** 🔪 to get larger test set
nth subset

Now, we have to do some maths

$\phi_n(x)$ – our estimator (e.g. mean)

$ps_i(x)$ – our i[th] pseudovalue

x - is the original subset

$x_{[i]}$ - is the subset without one sample

$$ps_i(x) = n\phi_n(x) - (n - 1)(\phi_n(x) - \phi_{n-1}(x_{[i]}))$$

Result of our Jackknife 

Bias corrected version of $\phi(x)$ / Jackknife mean

$$ps(x) = \frac{1}{n}\sum_{i=1}^{n} ps_i(x)$$

Jackknife variance

$$V_{ps}(x) = \frac{1}{n-1}\sum_{i=1}^{n}\left(ps_i(x) - ps(x)\right)^2$$

We can also calculate the Jackknife p-values, Jackknife confidence intervall, etc.

Pros & Cons of Jackknife

Estimating and compensating the bias in the estimator

Observation assumed to be i.i.d.

Less computationally expensive than **Bootstrapping** 🥾

Performs poorly on non-linear or continuous statistics, i.e. median

🤨 Okay, but for what can I use it?

Estimate AND correct the bias of my estimator

Validate several models in terms of their estimator

Get more statistics on my dataset (i.e. bias, standard error, confidence interval)

Bootstrapping 🥾 vs. Jackknife 🔪

Bootstrapping will give different results when repeated, whereas **Jackknife** gives exactly the same

Jackknife is computationally less expensive than **Bootstrapping**

Delete-1 Jackknife only works with smooth, differentiable statistics (e.g. means, ratios, proportions) not with medians or quantiles

Bootstrapping typically for confidence intervall estimation and **Jackknife** for bias estimation

References

- Jackknifing:
  - https://www.math.wustl.edu/~sawyer/handouts/Jackknife.pdf
  - https://www.sciencedirect.com/topics/mathematics/jackknife-resampling
  - https://si.biostat.washington.edu/sites/default/files/modules/2017_sisg_1_9_v3.pdf
  - https://www.stat.berkeley.edu/~hhuang/STAT152/Jackknife-Bootstrap.pdf
  - https://myweb.uiowa.edu/pbreheny/uk/teaching/621/notes/9-6.pdf

- Bootstrapping:
  - https://www.youtube.com/watch?v=Xz0x-8-cgaQ
  - https://machinelearningmastery.com/a-gentle-introduction-to-the-bootstrap-method/
  - https://www.lancaster.ac.uk/stor-i-student-sites/jack-trainer/bootstrapping-in-statistics/
  - https://online.stat.psu.edu/stat500/lesson/11/11.2/11.2.1

- General:
  - https://stats.stackexchange.com/questions/249333/comparison-of-the-jacknife-vs-the-bootstrap