Box-Behnken vs. Central Composite Design

Cameron, Amanda, Torstein, Aria, Chinmay

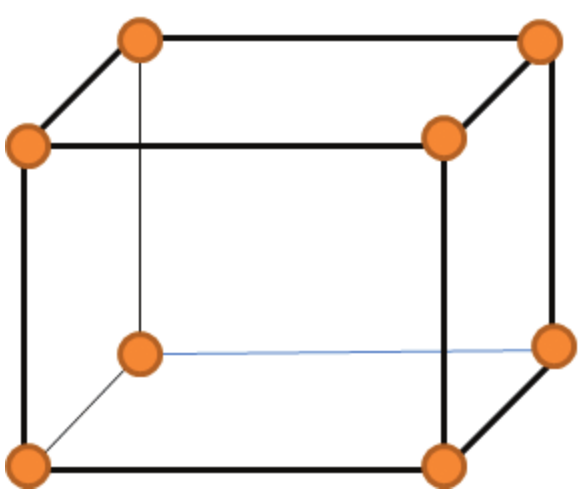
Summary

1. Factorial Design - Chinmay
2. Central Composite Design - Cameron
3. Box-Behnken Design - Aria
4. Method comparison – Amanda
5. Code comparison - Amanda
6. When to use – Aria / Torstein

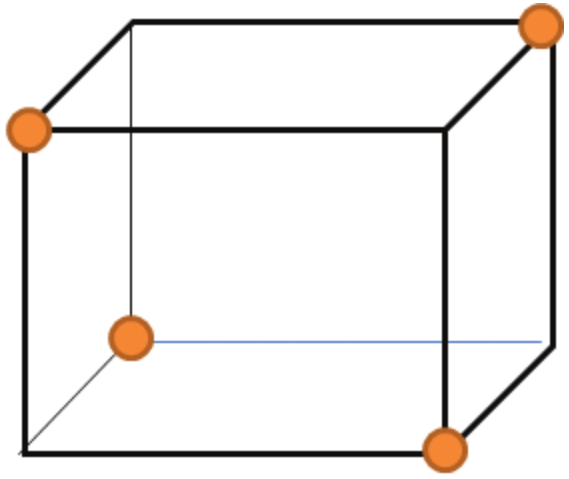
Two categories of DOE

- Classical Designs
  - Better as an introduction
  - Examples: Factorial Designs, Taguchi Array Designs, etc
- Modern Designs
  - More useful in real applications
  - Examples: Definitive Screening Designs, Custom Designs

Factorial Design - Recap



Full Factorial

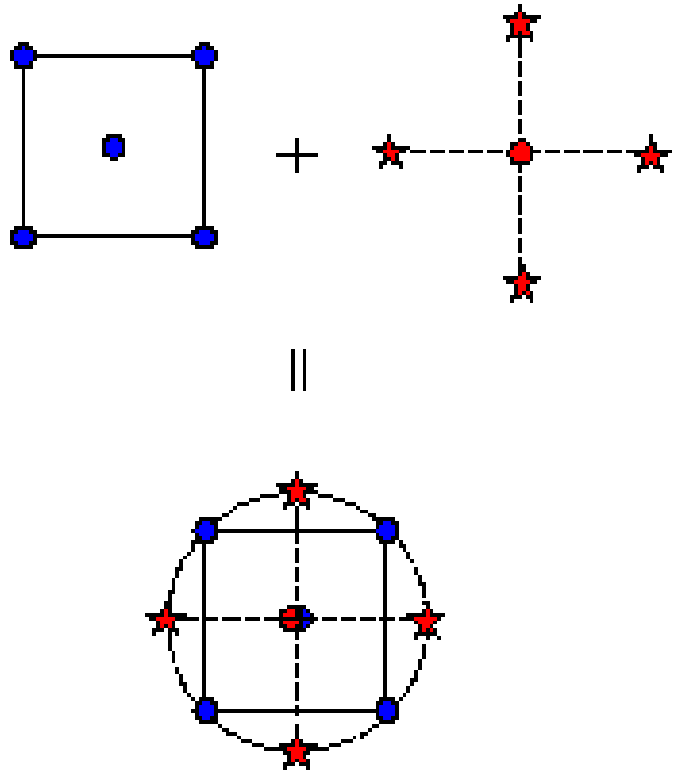


Fractional Factorial

Elazazy, M. S. (2017). Factorial Design and Machine Learning Strategies: Impacts on pharmaceutical analysis. In *InTech eBooks*. https://doi.org/10.5772/intechopen.69891

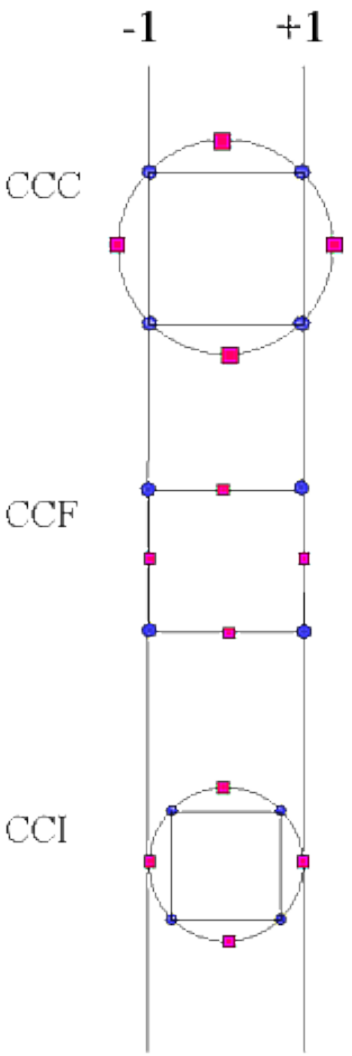
Central Composite Design (CCD)

- Goal: second order model for the response
- Components:
  - Factorial or fractional factorial design points
  - Center design point
  - Axial or star design points



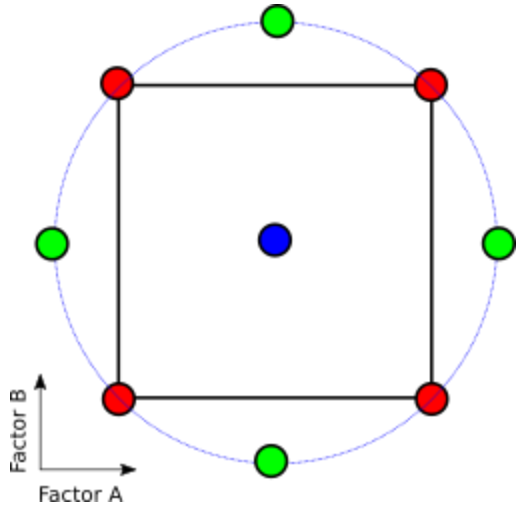
Central Composite Design (CCD)

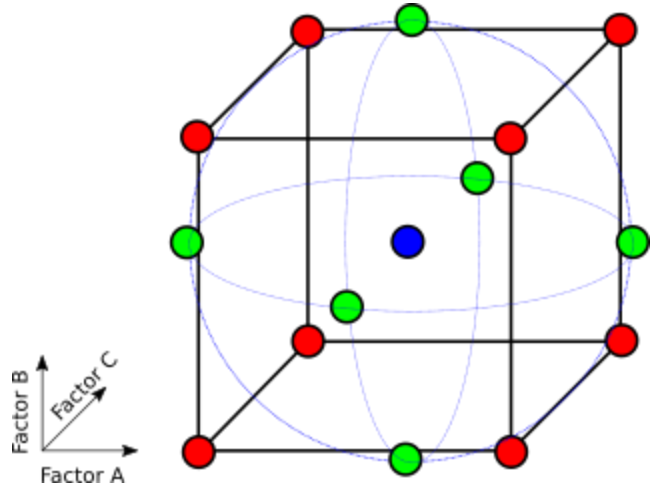
- Axial design points
  - Circumscribed (CCC)
    - Star points lie outside of the factorial design space and represent the new extreme values
  - Face Centered (CCF)
    - Star points at the centers of the factorial design faces
  - Inscribed (CCI)
    - Star points become the factorial design points



Central Composite Design (CCD)

- Rotatability
  - Design can be rotated around center point without impacting the reponse's variance
  - All points equal distance from center point
- Determining value of α:
  - α = (2^k)^0.25

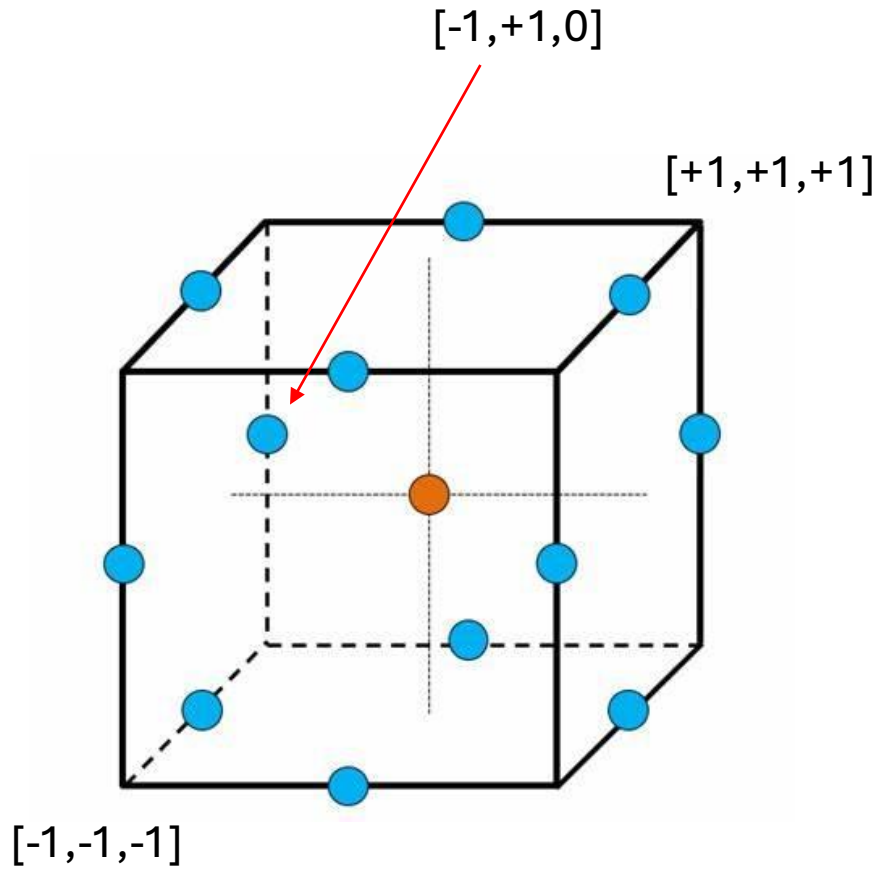


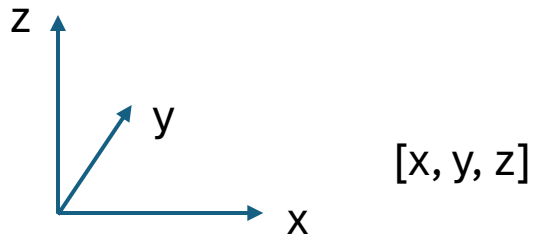


Box-Behnken Design (BBD)

- At least 3 levels per factor
  - E.g. [+1, -1, 0]

- Num points per input param = 2k(k-1) +1
  - 2*3(3-1) +1 = 13

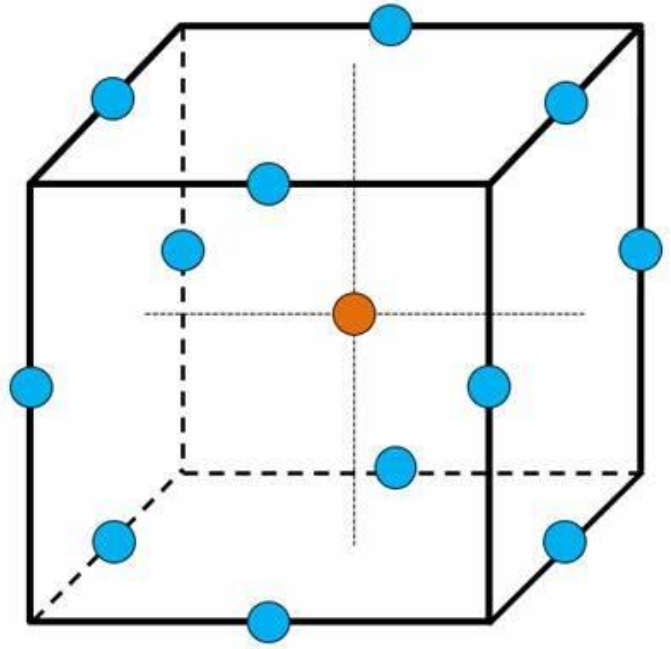




[x, y, z]

Box-Behnken Design (BBD)

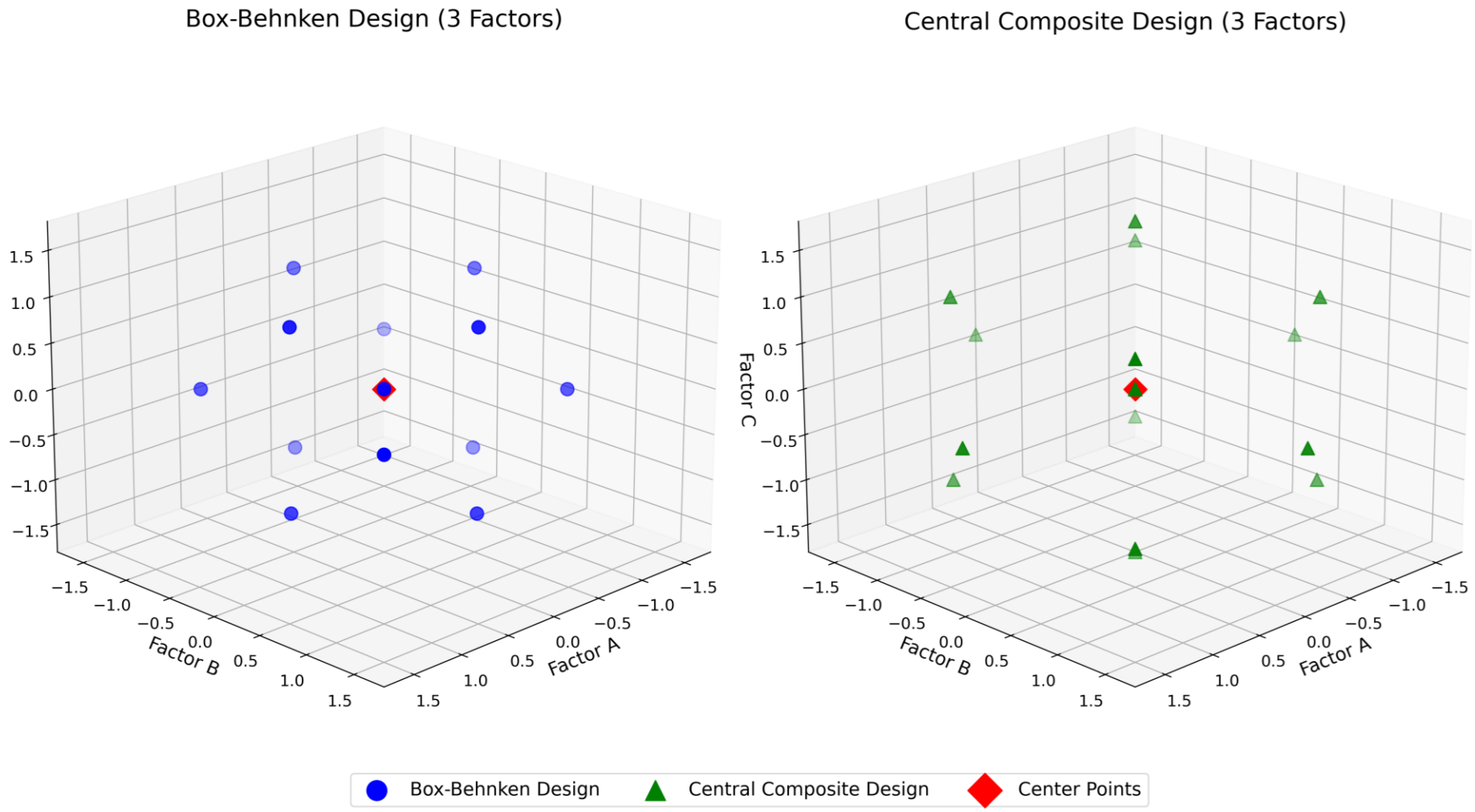
- No extreme points
- No axial points
  - No points that might be outside safe operating zone
- Assumes all features are significant
- No factorial or fractional factorial design embedded in it
- Efficiently estimates 1st and 2nd order effects
- Fewer design points



- Points should be randomized
  - Except for center points
    - To check againts process instability
    - Center points are at start, end, and center of process

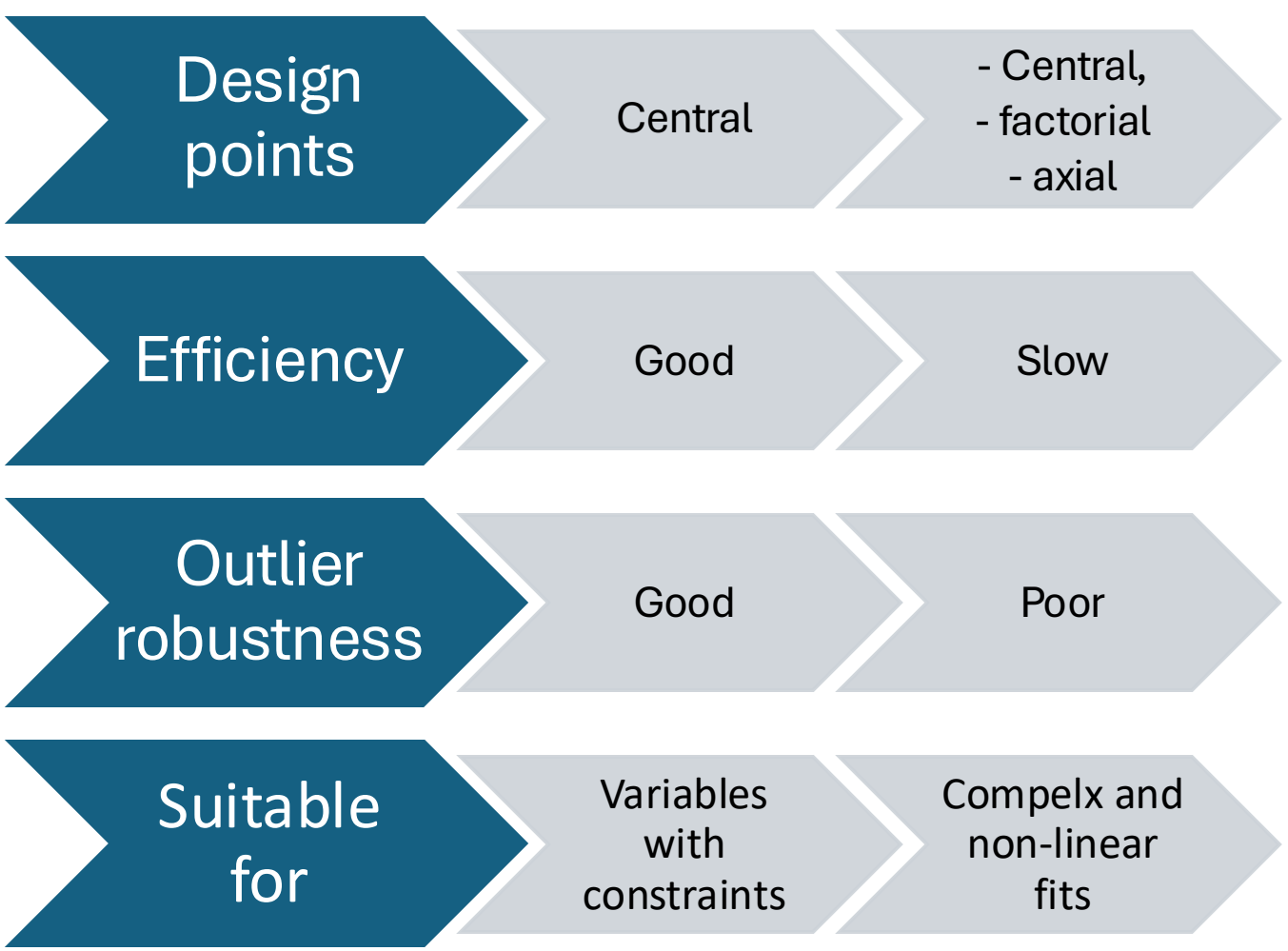
Comparison between BBD and CCD

Visual



Comparison between BBD and CCD

Theoretical



BBD

- 👍 Simple & requires few runs
- 👎 Limited boundary exploration

CCD

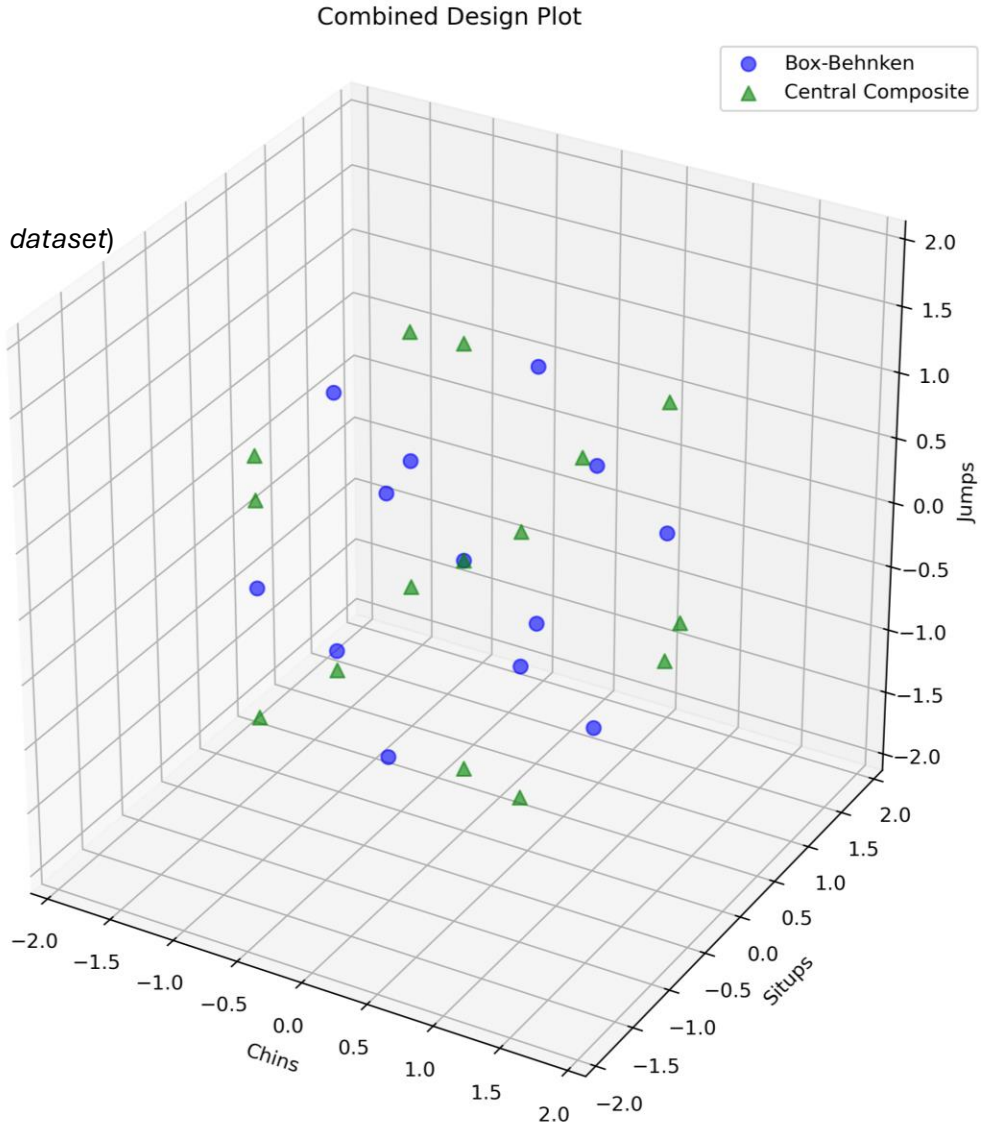
- 👍 Comprehensive design space
- 👎 Resource intensive, more *tuning* needed

Real Example

Data overview

- Physical Exercise dataset with 3 variables, 20 samples (*Linnerud dataset*)

	Chins	Situps	Jumps	Weight
0	5.0	162.0	60.0	191.0
1	2.0	110.0	60.0	189.0
2	12.0	101.0	101.0	193.0
3	12.0	105.0	37.0	162.0
4	13.0	155.0	58.0	189.0



Example 1

Let both designs have <u>one center point</u>

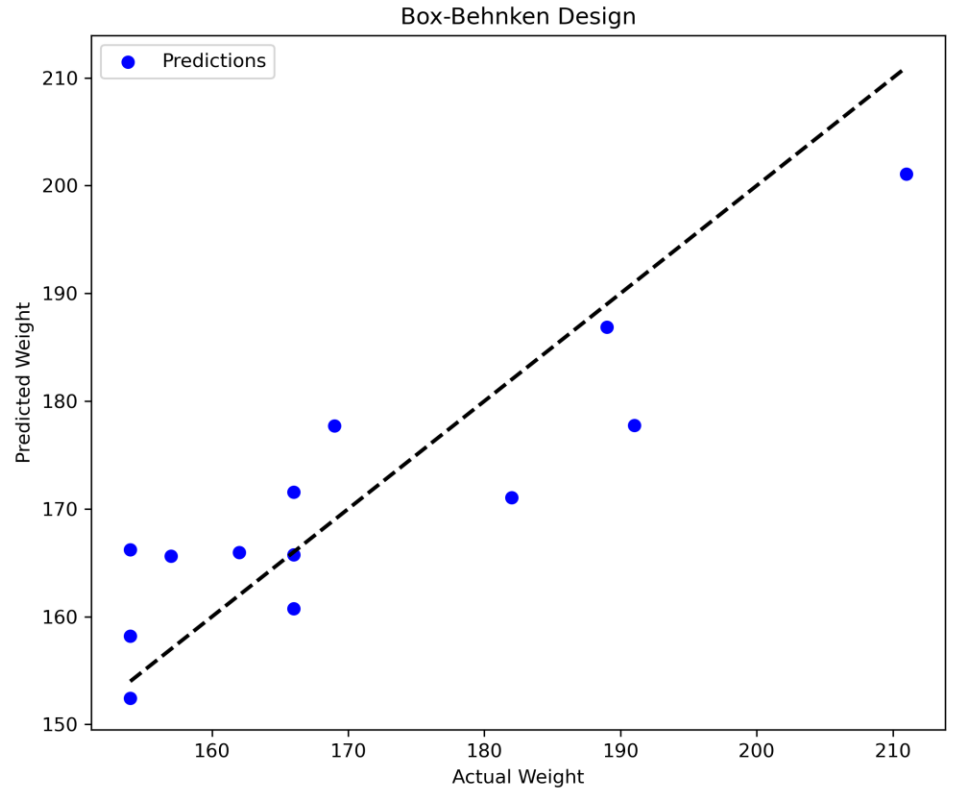
→ BBD: Run 13 experiments

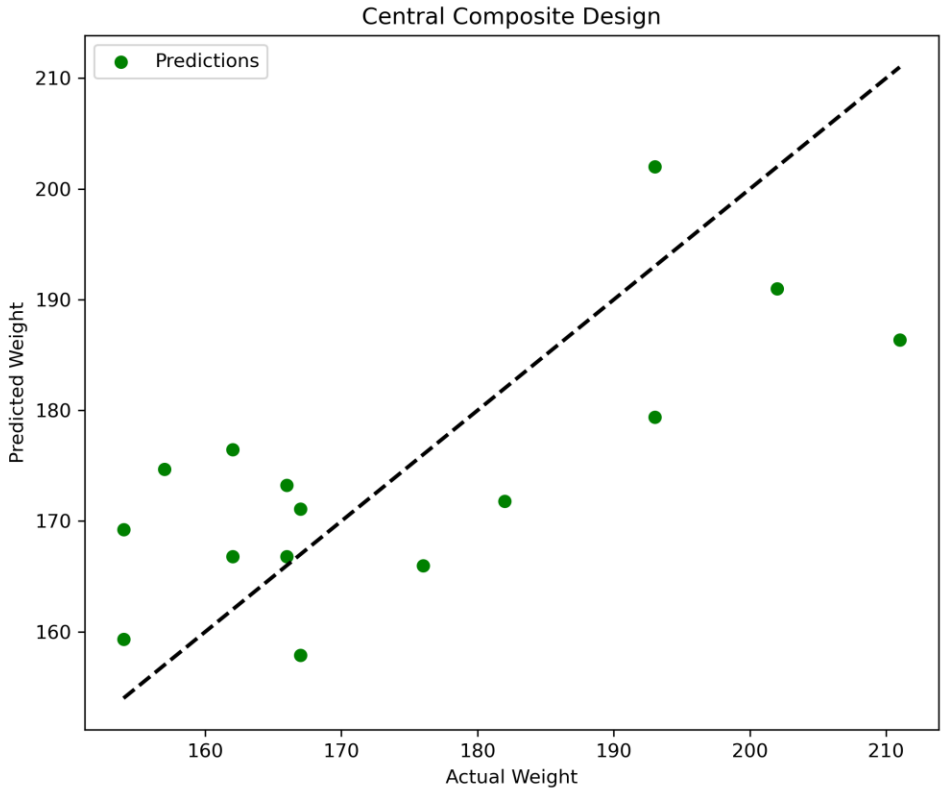
→ CCD: Run 15 experiments

Real Example

Prediction results

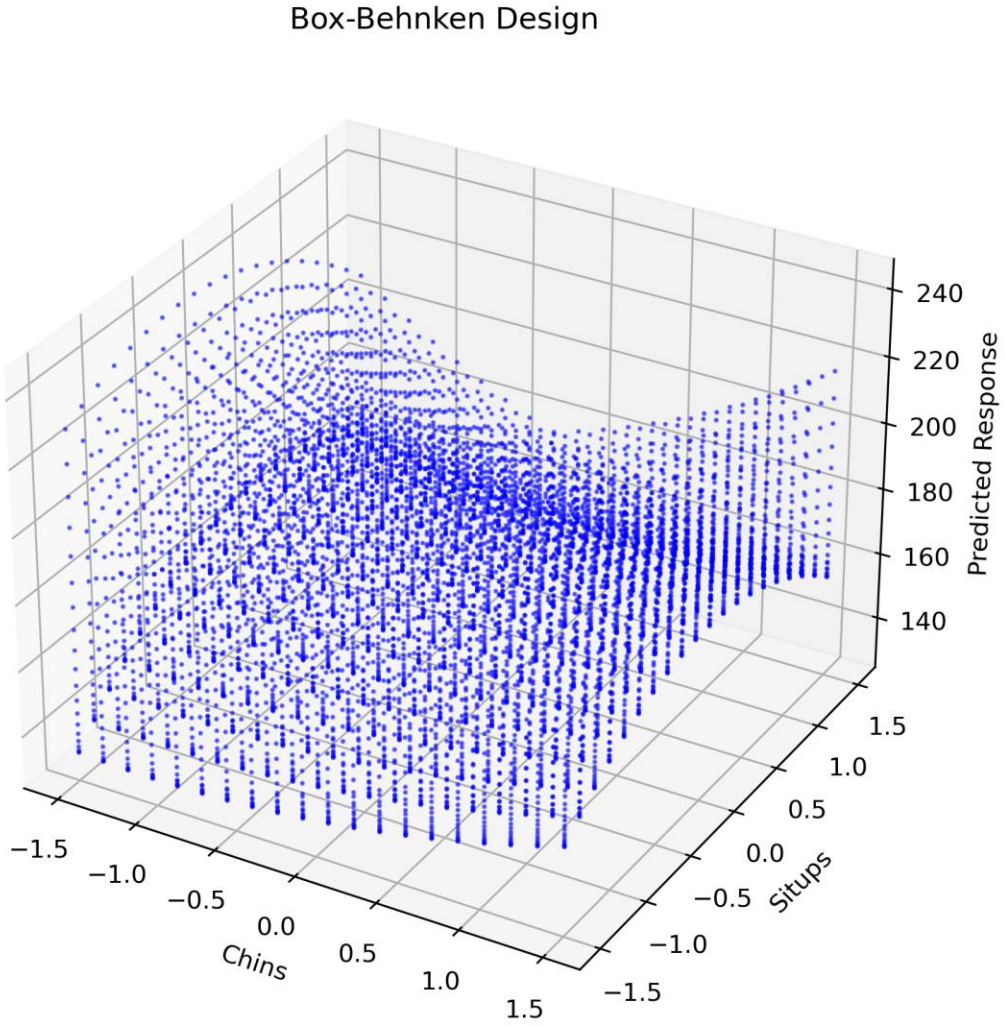
	BBD	CCD
MSE	60.9	144
R^2	0.784	0.526

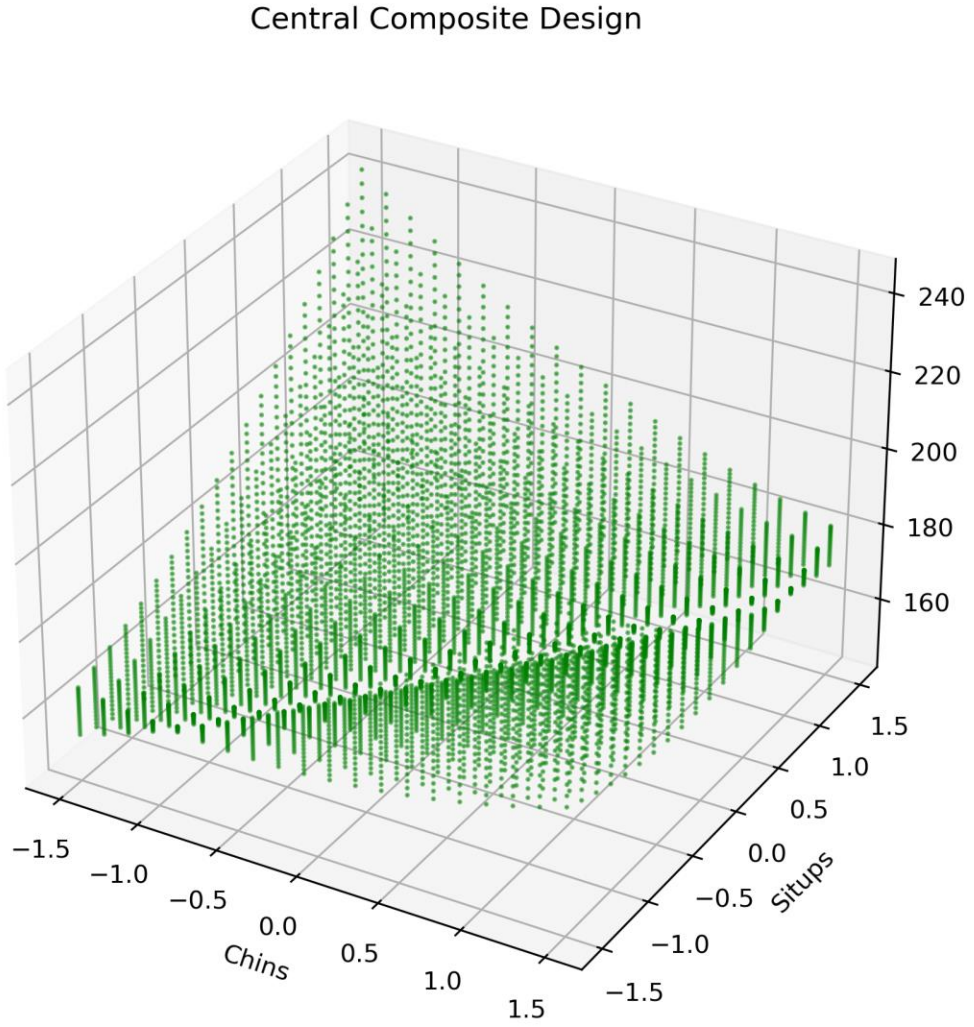




Real Example

Prediction results





Real Example

Adjust the number experiments

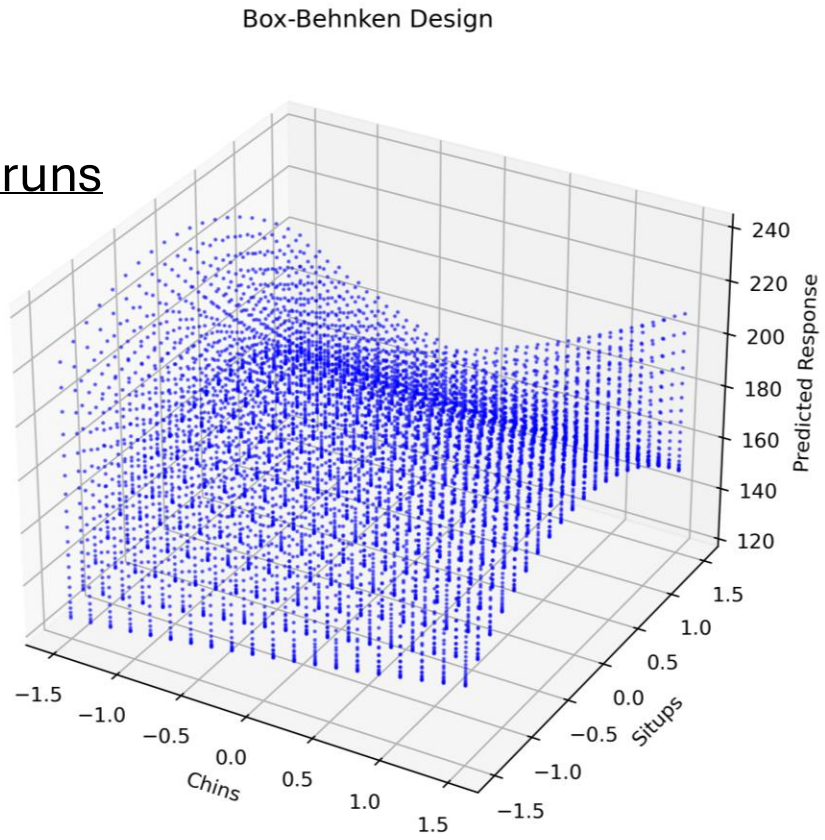
Example 2

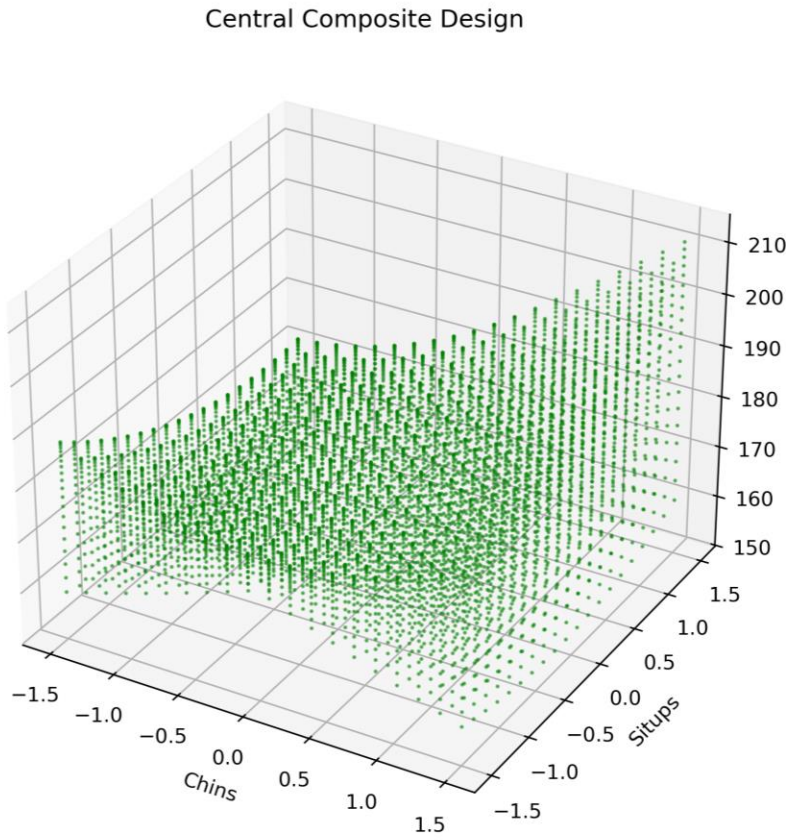
Let us do both designs with 14 runs

→ BBD: Two center points

→ CCD: Zero center points

	BBD	CCD
MSE	62.4	238
R^2	0.763	0.266





Summarized

Box-Behnken

- ✅ Requires fewer experiments
- ✅ No experiments outside operating conditions
- ❌ Requires more precise control of experiment variables

Central Composite

- ✅ Easy to create from exploratory factorial experiments
- ✅ Robust to missing/failing experiments
- ✅ Better predictions for edge of parameter space

When to use

Box-Behnken

- When experiment parameters have strict boundaries
- When experiments are expensive

Central Composite

- When partially factorial designed experiment results exists
- When the edges of the design space are relevant

Sources

- https://www.itl.nist.gov/div898/handbook/pri/section3/pri3361.htm
- https://www.itl.nist.gov/div898/handbook/pri/section3/pri3362.htm
- Lecture71 (Data2Decision) Response Surface Modeling – YouTube
- What are response surface designs, central composite designs, and Box-Behnken designs? - Minitab
- https://develve.net/Rotatable.html