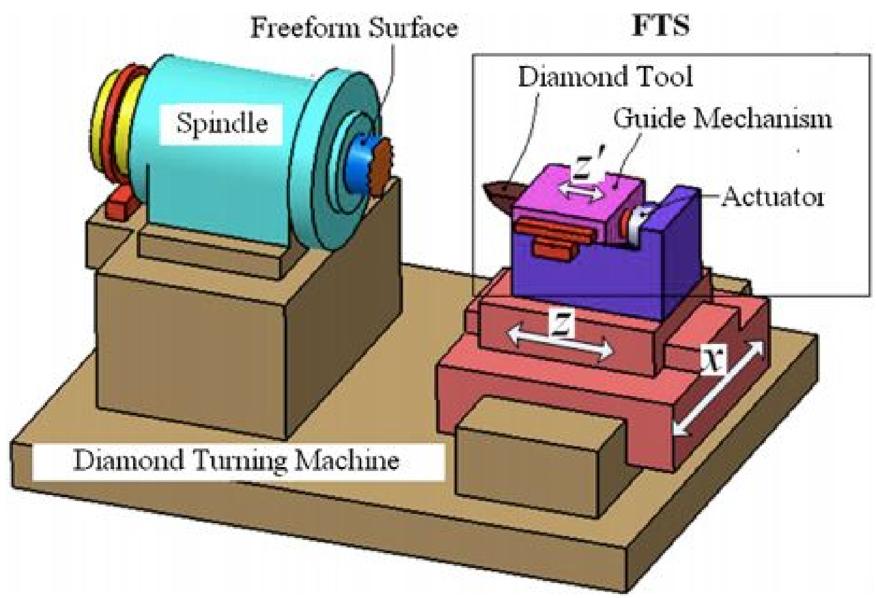
EE 141 Project



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Current Control Loop Design

$$L_c = 2 \times 10^{-4} H,\ R_c = 4\Omega,\ R_s = 0.2\Omega$$

$$\frac{V_a}{V_c} = 1 + \frac{20K}{4K} = 5 \qquad \frac{V_s}{V_x} = 5$$

$$V_a - V_{Back-EMF} = L_c I_c s + R_c I_c + R_s I_c$$

$$I_c = 5V_x$$

From the mechanical model, we know that

$$F = m_1 \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b_1 \frac{\mathrm{d}x}{\mathrm{d}t} + k_1 x$$

Rearranging, finds:

$$X(s) = \frac{K_f I_c}{m_1 s^2 + b_1 s + k_1}$$

From which we can find the voltage from the back EMF and plug it into the equation along with the relation between Ic and Vx.

$$V_a - \frac{2000 s V_x}{m_1 s^2 + b_1 s + k_1} = 10^{-3} s V_x + 21 V_x$$

$$V_a = V_x(10^{-3}s + 21 + \frac{2000s}{m_1 s^2 + b_1 s + k_1})$$

$$\frac{V_x}{V_a} = \frac{m_1 s^2 + b_1 s + k_1}{(10^{-3}s + 21)(m_1 s^2 + b_1 s + k_1) + 2000s}$$

$$\frac{V_s}{V_c} = \frac{V_a}{V_c} \times \frac{V_s}{V_x} \times \frac{V_x}{V_a}$$

$$\frac{V_s}{V_c} = \frac{25(m_1 s^2 + b_1 s + k_1)}{(10^{-3}s + 21)(m_1 s^2 + b_1 s + k_1) + 2000s}$$

By setting voltage from the back EMF to zero, we get the following equations.

$$V_a = 10^{-3} s V_x + 21 V_x$$

$$\frac{V_s}{V_c} = \frac{25}{10^{-3} s + 21}$$

We can see that our two answers are equivalent at high frequencies by dividing $m_1 s^2 + b_1 s + k_1$ from the numerator and denominator of the solution involving the back EMF. This yields:

$$\frac{25}{(10^{-3} s + 21) + \frac{2000s}{m_1 s^2 + b_1 s + k_1}}$$

Which at high frequencies, the 2000s tends towards zero so the solutions are the same. The reason that the back emf voltage is considered zero at higher frequencies is because the inductor in the motor resist sudden changes in current when it is first starting up. At higher frequencies, this inductor acts as a short circuit.

From the opAmp circuit, we can find the equivalent resistance of R3, C1, and C2 to be:

$$Z = \frac{R_3 C_1 s + 1}{s C_1 + s C_2 + R_3 C_1 C_2 s^2}$$

Setting Vset to 0, we use KCL to find a relation between Vc and Vs.

$$V_c = \frac{-V_s}{R_2} Z$$

$$C_{elec} = \frac{Z}{R_2} = \frac{R_3 C_1 s + 1}{R_2 (C_1 s + C_2 s + R_3 C_1 C_2 s^2)}$$

This can be written in an easier format to analyze:

$$C_{elec} = \frac{1 + \frac{1}{sR_3C_1}}{\frac{R_2(C_1+C_2)}{R_3C_1}(\frac{sC_2R_3C_1}{C_1+C_2} + 1)}$$

Because the crossover frequency is 6e5, we know that the zero is one decade before (6e4) and the pole is one decade after (6e6) and the gain of the transfer function is 1 at the crossover frequency. In addition, we know from the conditions that:

$$\frac{-R_2}{R_1} = -0.5 \Rightarrow R_2 = 5000$$

Thus we have 3 equations and 3 unknowns and can solve for R3, C1, and C2.

$$K \times P_{elec}(jW) = 1$$

$$K(0.041641) = 1 \Rightarrow K = 24.015$$

$$K = \frac{R_3C_1}{R_2(C_1 + C_2)} \quad 24.015 = \frac{R_3C_1}{R_2(C_1 + C_2)}$$

$$\frac{1}{R_3C_1} = 6 \times 10^4$$

$$\frac{C_1 + C_2}{C_2R_3C_1} = 6 \times 10^6$$

Because we have solved for K and are able to decompose the variables that K is equal to into the other equations, we are able to solve for all of the variables and they are as such:

$$C_1 = 1.374 \times 10^{-10}$$

$$C_2 = 1.388 \times 10^{-12}$$

$$R_3 = 121287.86$$

By plugging in these values into the transfer function, we get:

```
C_elec =

        1.667e-05 s + 1
  --------------------------
  1.157e-13 s^2 + 6.94e-07 s
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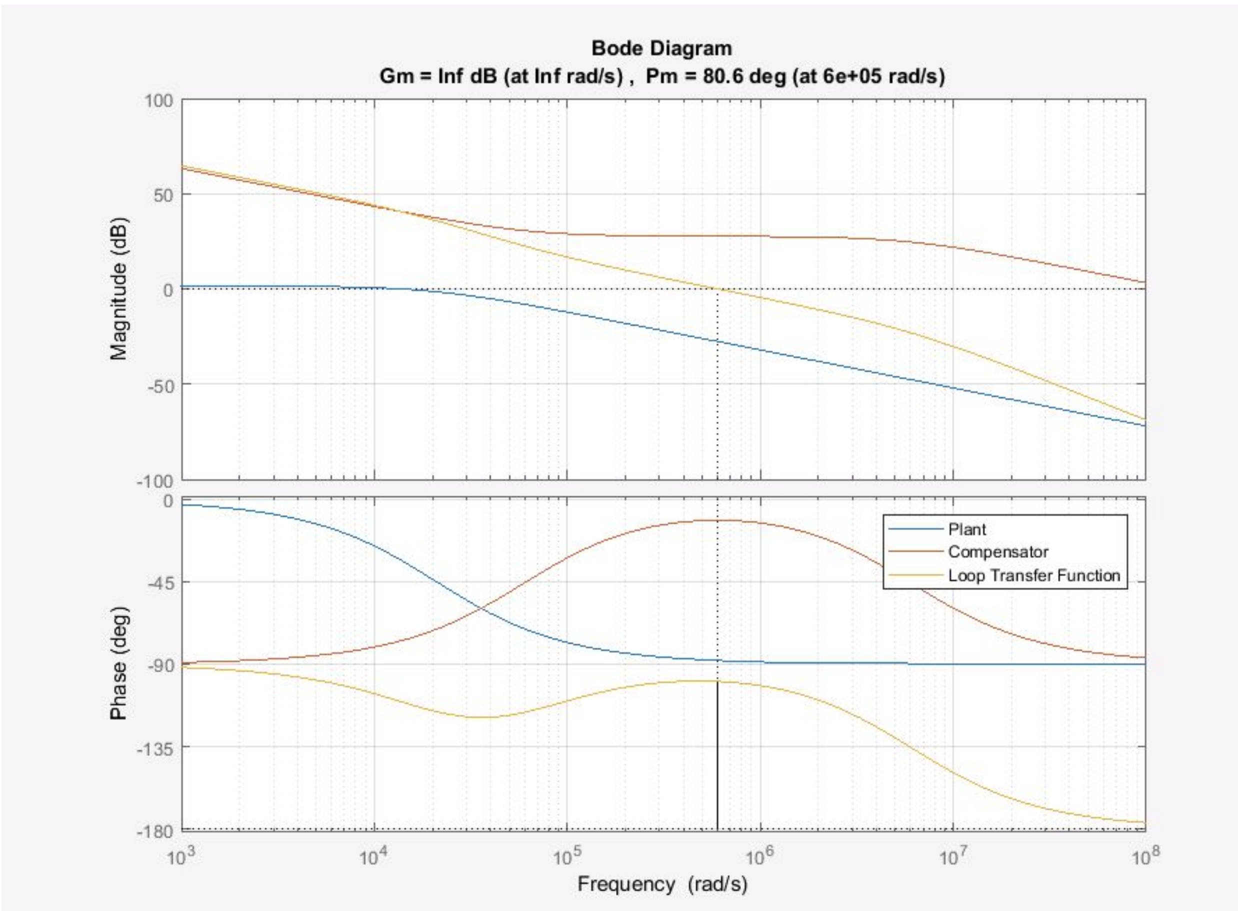


As you can see, the phase margin is greater than 60 degrees and the crossover frequency is indeed 6e5.

FTS Plant System Identification

The transfer function of the mechanical model that was solved earlier can be written as follows:

$$G_p = \frac{X}{I_c} = \frac{20}{m_1 s^2 + b_1 s + k_1} = \frac{\frac{20}{m_1}}{s^2 + \frac{b_1}{m_1}s + \frac{k_1}{m_1}}$$

We can use the second order equivalence equations in order to solve for the unknown variables.

$$20log_{10}(M_{pw}) = 20 \Rightarrow M_{pw} = 10$$

$$\frac{1}{M_{pw}} = 2\zeta\sqrt{1-\zeta^2} \Rightarrow \zeta = 0.05$$

$$W_n = \frac{10^3}{\sqrt{1-2\zeta^2}} \Rightarrow W_n = 10002.509$$

We know that second order derivatives come in the form:

$$\frac{W_n^2}{s^2 + 2\zeta W_n + W_n^2}$$

So we can plug in the variables to create:

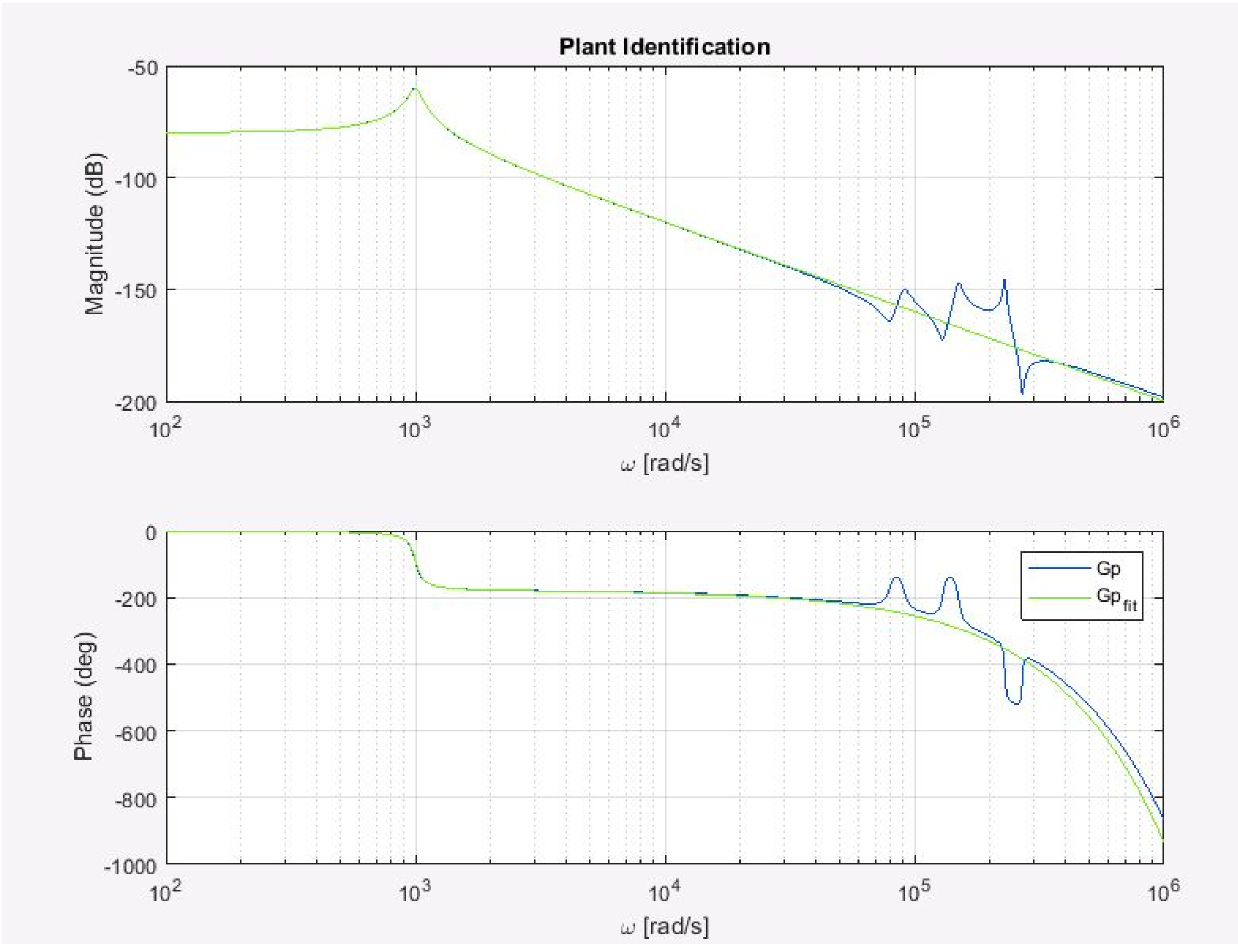
$$\frac{1002.509^2}{s^2 + 100.2509s + 1002.509^2}$$

In order to solve for b1, k1, and m1, we look back at our previous equation for Gp at the top of the page. We find:

$$m_1 = 1.99 \times 10^{-5}, b_1 = 0.001995, k_1 = 19.99$$

However, when plotting this transfer function, we find that the gain is incorrect and the transfer function also needs a time delay. After playing around with values in order to match the given plots, we find that:

$$G_p = \frac{10^{-4}e^{-1.319\times10^{-5}s}(1002.509^2)}{s^2 + 100.2509s + 1002.509^2}$$



Above is the Gpfit that I created overlayed on the target Gp. I used the second order approximation in order to get the two plots to coincide at lower frequencies.

<u>Position Control Loop Design</u>

We know that Cmech takes the form of a PI + lead compensator in the form:

$$C_{mech} = K_p(1 + \frac{K_i}{s})(\frac{s+z}{s+p}) = K_p(\frac{s+K_i}{s})(\frac{s+z}{s+p})$$

The lead compensator can also be written in the form:

$$LeadCompensator = \frac{1 + (\alpha)(\tau)s}{1 + (\tau)s}$$

Since we want to factor out the disturbance at 10^5 and include that at 10^3, we want Wg as 10^4. We know that alpha greater than 15 is not good design, so starting at a 50 degree phase margin, we increase by increments of 5 until we find an acceptable alpha. We choose the phase margin as 55 and are able to find alpha and tau:

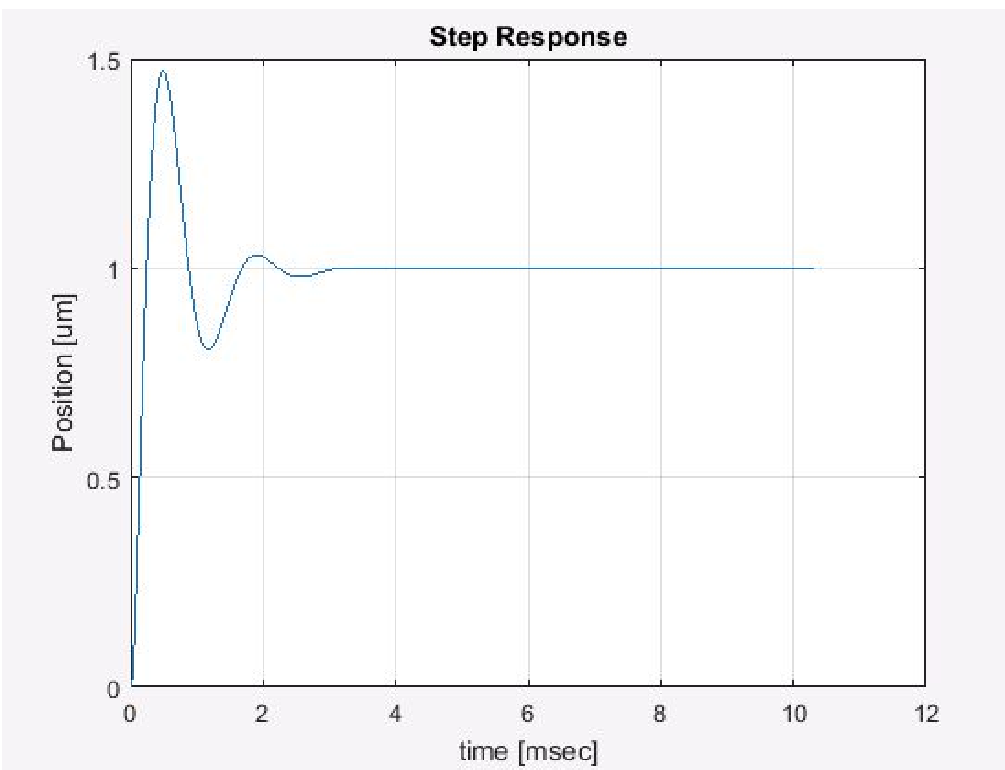
$$sin(55) = \frac{\alpha - 1}{\alpha + 1} \Rightarrow \alpha = 10.06$$

$$\tau = \frac{1}{W_g\sqrt{\alpha}} \Rightarrow \tau = 3.1528 \times 10^{-5}$$

Therefore, the lead compensator in Cmech is:

$$LeadCompensator = \frac{1 + (10.06)(3.1528 \times 10^{-5})s}{1 + (3.1528 \times 10^{-5})s}$$

The plot of the step response is:

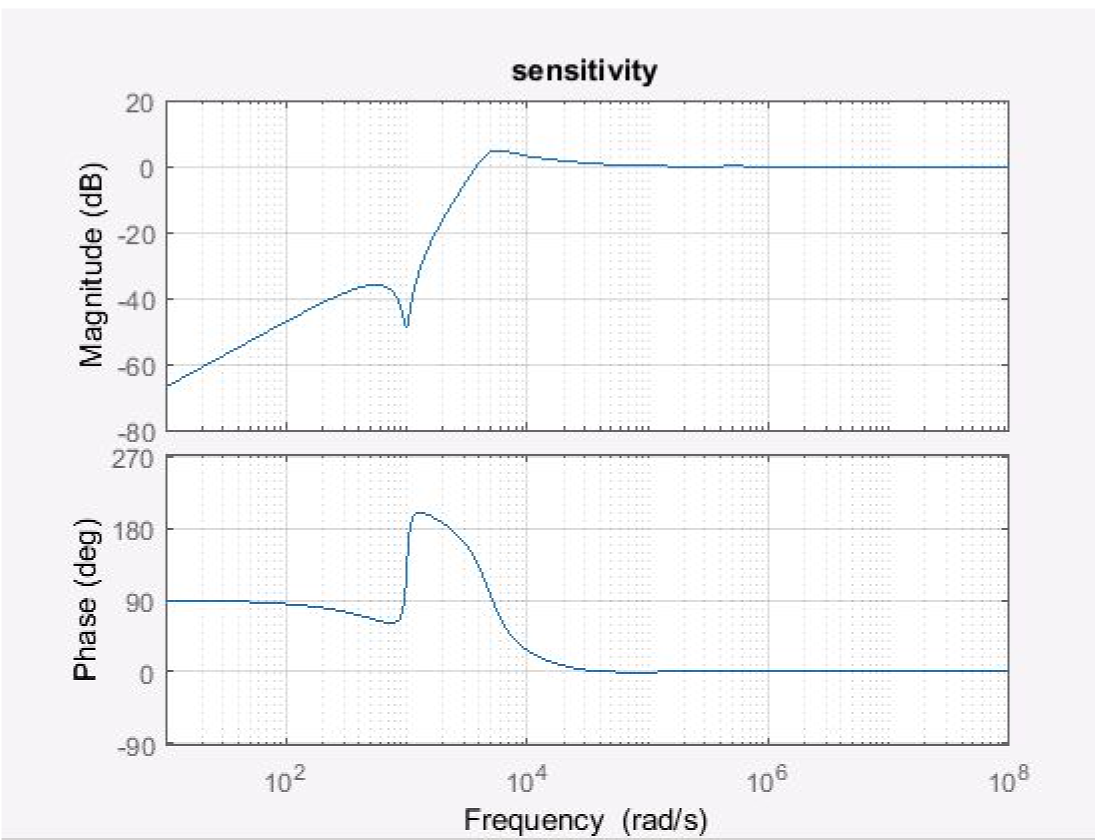


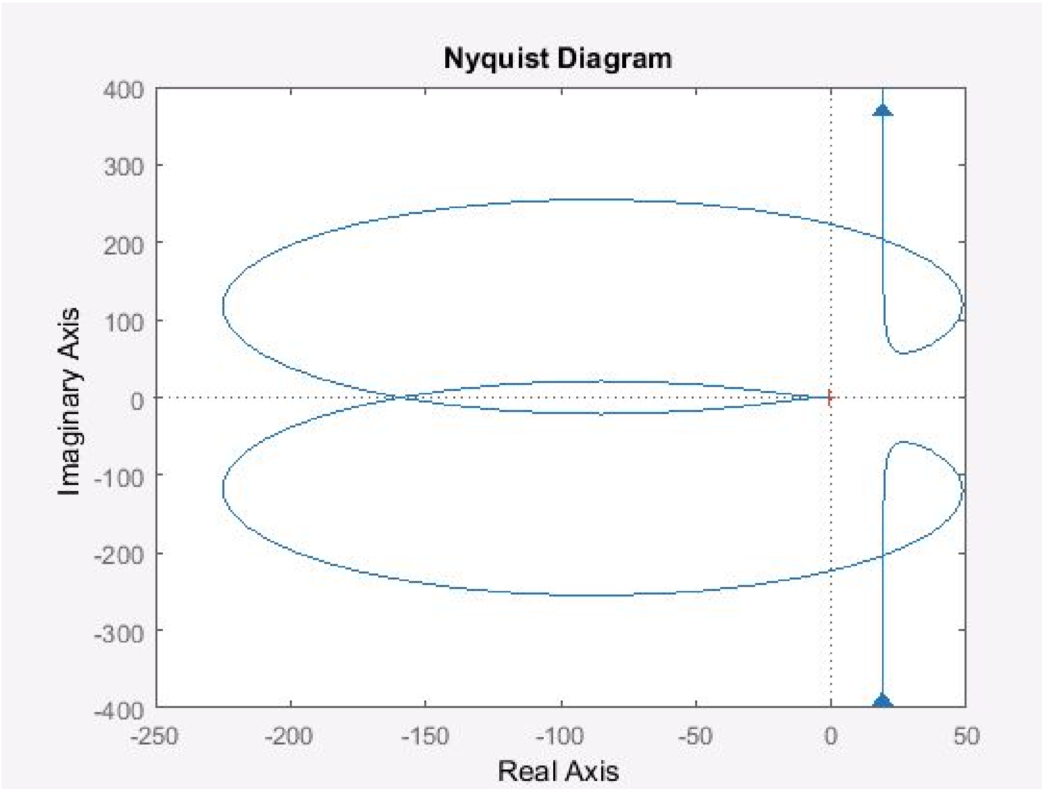
And the step info:

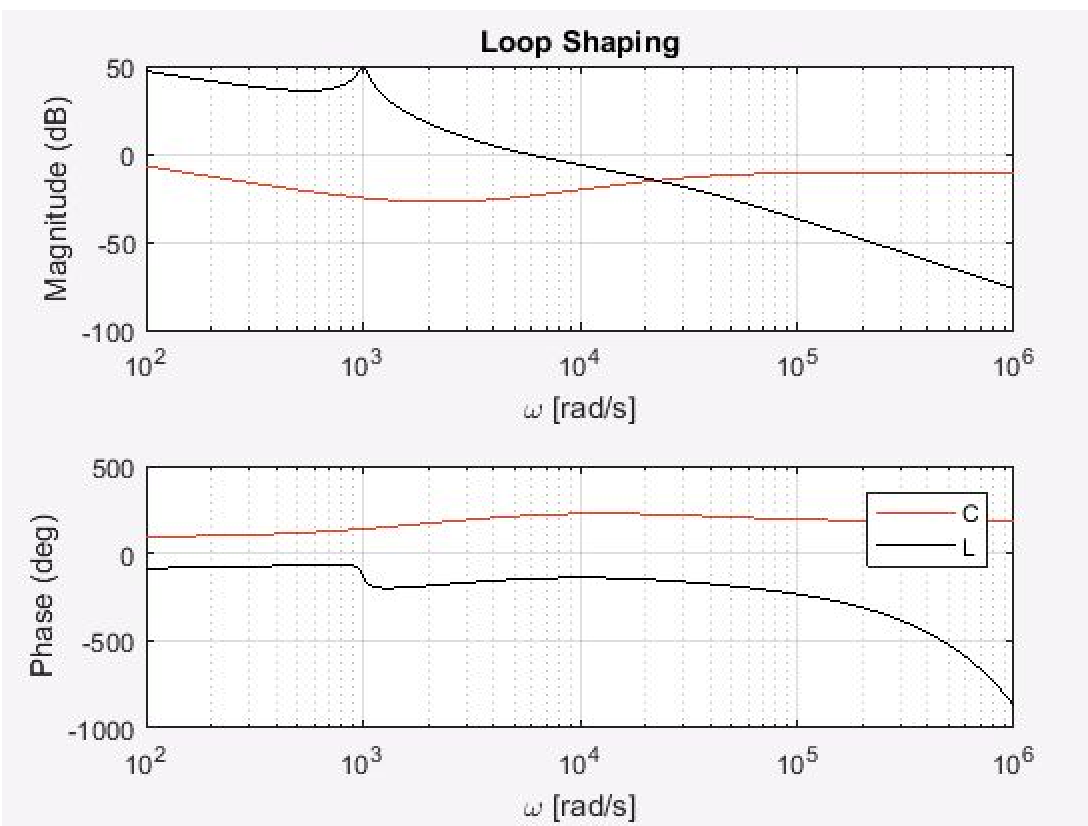
```
     RiseTime: 1.6506e-04
 SettlingTime: 0.0026
  SettlingMin: 0.8053
  SettlingMax: 1.4712
    Overshoot: 47.1179
   Undershoot: 0
         Peak: 1.4712
     PeakTime: 4.8921e-04
```

The step info showed an overshoot of 47 which is not ideal. By decreasing Ki and increasing gain K, the overshoot is decreased at the expense of increasing rise time, peak time, and settling time. Increasing gain also increases stability

Plots requested:




Nyquist Diagram
400
300
200
100
0
-100
-200
-300
-400
Imaginary Axis
-250
-200
-150
-100
-50
0
50
Real Axis


Loop Shaping
50
0
-50
-100
Magnitude (dB)
10^2
10^3
10^4
10^5
10^6
ω [rad/s]
500
0
-500
-1000
Phase (deg)
C
L
10^2
10^3
10^4
10^5
10^6
ω [rad/s]