ELSEVIER

Bone Vol. 24, No. 4
April 1999:291–295

REVIEW

Biot Theory: A Review of Its Application to Ultrasound Propagation Through Cancellous Bone

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To facilitate an understanding of the dependence of ultrasound velocity and attenuation upon the material and structural properties of cancellous bone, several theoretical concepts for ultrasound propagation have been adapted or developed, including the Biot theory and several scattering theories. Biot theory considers wave propagation through an elastic porous solid interspersed with fluid, considering the separate motion of the trabecular framework and morrow, respectively. The success achieved with the Biot theory has, to date, tended to be greater for the prediction of velocity than for attenuation. This article provides a review of the relevant literature, describing the physical parameters required for the Biot theory and their experimental determination. It is suggested that future developments should consider additional attenuation mechanisms, in particular, those due to scattering, local flow in microcracks, and surface roughness of the trabeculae. (Bone 24:291–295; 1999) © 1999 by Elsevier Science Inc. All rights reserved.

Key Words: Ultrasound; Cancellous; Bone; Theoretical models; Biot theory.

Introduction

Although a consensus for the clinical role of ultrasound for the assessment of osteoporosis is now emerging,[19,32,40] there is still a lack of clear understanding of the fundamental principles of ultrasound wave propagation through cancellous bone, and hence, the dependence of ultrasound velocity and attenuation upon material and structural parameters.[20,22,28,39] In vitro studies have confirmed the original hypotheses that ultrasound velocity and attenuation are dependent upon the elasticity and structure of cancellous bone, respectively, in addition to apparent bone density.[20,21,33,34,39]

Several theoretical concepts have been adapted or developed to model ultrasound propagation through cancellous bone, evaluated by complementary experimental studies.[6,31,37,38,39,48–50] There are two categories of attenuation mechanism related to the propagation of an ultrasound wave through a porous media: those related to bulk composition (absorption and relaxation), and those related to the spatial composition. Spatial inhomogeneities within a propagation medium will introduce a scattering effect, the magnitude depending on the relative size of the obstacles to the wavelength of the incident ultrasound wave.[37] Accurate measurement of true attenuation also requires acoustic field effects, such as diffraction and refraction, to be accounted for.

The "single" scattering theory considers the interaction and spatial redistribution of an ultrasound wave incident upon an obstacle.[37–39] The total attenuation is the summation of this single result over the number of obstacles. The "multiple" scattering theory considers an ensemble of obstacles, and averages their effect.[37–39] Again, the total attenuation is the summation of the ensembles. Although single and multiple scattering theories are well established in many fields of science, their validity in cancellous bone has not yet been thoroughly investigated. This is clearly not the case for ultrasound propagation through cancellous bone, a porous solid consisting of a complex interconnected array of individual trabeculae, interspaced with bone marrow and surrounded by a cortical shell.

An alternative model to the traditional scattering theories for the prediction of ultrasound propagation through cancellous is the Biot theory, which may also be applied to the less porous cortical bone. The Biot theory considers absorption (the loss of energy due to frictional and viscous forces opposing the motion of the bone marrow relative to the trabecular framework) and has been modified to take account of relaxation effects (energy is lost at a molecular level[51]) for the two media components, bone tissue and marrow in the case of cancellous bone. Although scattering is not fundamentally considered in the Biot theory, it has had considerable success in the last half a century in predicting qualitatively, if not quantitatively, the attenuation and the velocity of acoustic waves in a diverse variety of saturated porous media. The Biot theory has been applied in sciences as diverse as geophysics (to model waves produced by magma intrusions[16]) and sound-proofing (where the Biot theory has been shown to be superior to the more traditional continuity equations[15]). Much of this work is highly specialized and requires a good fundamental understanding of the "classical" Biot theory.

This article is intended to review the relevant literature and allow those interested in ultrasound propagation through bone to use the Biot theory as the powerful theoretical tool and also to demonstrate to those engaged in histomorphometrical and mechanical studies the need to determine the values of the physical parameters used in the Biot theory. The Biot theory may be used to model the propagation of ultrasound through cancellous bone,[38,48] and to explain the existence of the second ultrasonic wave observed in cortical bone.[27]

The Classical Biot Theory

The theory of elastic wave propagation in porous media was developed, almost single-handedly, by M. A. Biot through several papers published over a period of 20 years. It was originally

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8756-3282/99/$20.00
PII S8756-3282(99)00011-3

applied to the analysis of ultrasound geophysical test data through porous rock samples.[7–12] The theory is derived by considering the separate motion of the solid elastic frame and the interspersed fluid, induced by the ultrasonic wave. The energy loss due to the friction existing between the frame and the fluid is then analyzed. The theory gives rise to three elastic parameters that are dependent on the structural and elastic properties of the porous media. The elastic parameters P, Q, and R, have been defined by Biot[7,8] and Biot and Willis.[12] Stoll,[46] using the assumption that porosity is constant for small strains, linked the elastic coefficients to measurable physical constants. Using notation similar to Biot,[7,8] Plona and Johnson,[42] and Williams,[48] the Biot parameters are defined as:

$$P = \frac{\beta\left(\frac{K_s}{K_f} - 1\right)K^* + \beta^2 K_s + (1 - 2\beta)(K_s - K^*)}{1 - \beta - \frac{K^*}{K_s} + \beta\,\frac{K_s}{K_f}} + 4\,\frac{\mu^*}{3} \tag{1}$$

$$Q = \frac{\left(1 - \beta - \frac{K^*}{K_s}\right)\beta K_s}{1 - \beta - \frac{K^*}{K_s} + \beta\,\frac{K_s}{K_f}} \tag{2}$$

$$R = \frac{K_s\beta^2}{1 - \beta - \frac{K^*}{K_s} + \beta\,\frac{K_s}{K_f}} \tag{3}$$

where K_s is the intrinsic bulk modulus of the solid material, K^* is the bulk modulus of the frame, K_f is the bulk modulus of the fluid, μ^* is the shear modulus of the frame, and β is the porosity (volume fraction of the fluid phase). These equations may be found in slightly different forms,[44,46,53] depending upon the precise derivation used, but they are equivalent.

The other three main parameters are the mass coefficients that describe the effects of viscous and inertial drag, taking into account the fact that the relative fluid flow through the pores can be nonuniform:

$$\rho_{11} + \rho_{12} = \rho_1 \tag{4}$$

$$\rho_{22} + \rho_{12} = \rho_2 \tag{5}$$

$$\rho_{12} = -(\alpha(\omega) - 1)\beta\rho_f \tag{6}$$

$$\rho_1 = (1 - \beta)\rho_s \tag{7}$$

$$\rho_2 = \beta\rho_f \tag{8}$$

where ρ_s and ρ_f are the densities of the solid and the fluid phase, respectively. The just noted parameters are similar to Biot,[7,8] but they are complex terms having been modified by Johnson et al.,[25] to take account the theory of dynamic tortuosity and permeability. ρ_{11} is the effective density of the solid moving through the liquid, ρ_{22} is the effective density of the fluid moving through the solid, ρ_{12} is the inertial drag that the solid exerts on the fluid, and $\alpha(\omega)$ is the Johnson–Koplik–Dashen (JKD) dynamic tortuosity.[1,25] The aforementioned parameters can be defined without using the JKD formulation of tortuosity and, in this case, the tortuosity is a purely geometric variable (sometimes referred to as the sinuosity).[46] The JKD tortuosity is the formulation most commonly used and is adequate for most situations. The underestimation of the imaginary part at low frequencies has been corrected for by Pride,[41] although this is not required when dealing with clinical ultrasound frequencies through cancellous bone, so we will not concern ourselves with it.

Biot theory predicts three modes of propagation for an ultrasonic wave in a porous media. Two dilatational waves (longitudinal waves), termed waves of the first kind and waves of the second kind, alternatively named fast and slow waves; and one rotational (shear wave).[7–11] The usual explanation given for the existence of the separate fast and slow waves is that the fast wave represents the fluid and solid vibrating in phase and the slow corresponds to vibration in antiphase. The Biot theory also predicts two regimes of wave dispersion, separated by a characteristic frequency, given by $f_c = \beta\eta/k_0$, where η is the fluid kinematic viscosity and k_0 is the permeability of the sample. Below this critical frequency, the slow wave is diffuse, but above it the slow wave propagates.[35] The wave equations for these waves are:

$$V^2_{(\text{fast/slow})} = \frac{\Delta \pm \sqrt{\Delta^2 - 4(\rho_{11}\rho_{22} - \rho_{12}^2(PR - Q^2)}}{2(\rho_{11}\rho_{22} - \rho_{12}^2)} \tag{9}$$

where:

$$\Delta = P\rho_{22} - 2Q\rho_{12} + R\rho_{11}$$

and:

$$V^2_{(\text{shear})} = \mu\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right) \tag{10}$$

Equation 9 has two complex roots, corresponding to the fast and the slow waves and equation 10 has one; the real part of the root (q_r), provides the wave speed as ω/q_r (m s^{-1}), where ω is the angular frequency of the wave. The imaginary part of the root (q_i), provides the attenuation (Np m^{-1}).[49]

The Biot Theory Applied to Cancellous Bone

The Biot theory has been applied to cancellous bone with varying degrees of success. McKelvie[37] demonstrated that predictions by the Biot theory agreed better with experimental results obtained from the calcaneus than predictions made by the two scattering theories. McKelvie[37,38] predicted qualitatively the dependence of attenuation upon ultrasound frequency in cancellous bone; the attenuation values were of the right order of magnitude, but did not reproduce the full range of experimental values observed in natural tissue. However, McKelvie[37,38] was unable to predict correctly the trends in ultrasound velocity. Conversely, Williams[48] used a limited formulation of the Biot theory to calculate velocities alone and found good agreement for the fast wave velocity to predict experimental values obtained from tibial and femoral bovine cancellous bone samples.[3] Williams et al.[49] then expanded his formulation of the Biot theory to consider attenuation using the JKD formulation of dynamic tortuosity. Good agreement for fast wave velocity was again found. For attenuation, although the predicted trends were similar to those observed experimentally in cancellous bone, the experimental values were considerably higher than those predicted by the Biot theory. It should be pointed out, however, that because of experimental difficulties many parameters were either estimated or were tunable. Williams et al.[49] and Wu et al.[52] suggested that there may be additional losses due to reflections at the flat surfaces of the samples, or by other factors such as diffraction, scattering, or phase cancellation. For the experimental validation of the Biot theory applied to cancellous bone, most studies have failed to find, or have neglected to look for, the slow wave. Although it has been observed in some studies[23,42] it has been suggested that experimental conditions favoring out-of-phase motion of the solid and fluid must exist to enable the slow wave to be measured.[42,49,50] A short numerical simulation of osteoporosis using Biot theory has been carried out by Leclaire et al.,[35] although a comparison and validation with experimental data were not performed.

An important feature of the Biot theory is that it contains several directionally dependent parameters, noting that ultrasound velocity and attenuation are also highly dependent upon the direction of propagation in anisotropic media such as cancellous bone. To date, experimental correlations of ultrasound attenuation in cancellous bone have been with density, which is not directionally dependent.[33] The Biot theory could therefore be used to help explain the directional component of ultrasound attenuation in cancellous bone.[45]

Physical Parameters and Experimental Determination

The greatest difficulty in the application of the Biot theory to cancellous bone is the large number of physical parameters that must be measured or estimated. A large amount of information exists in the literature about the experimental determination of the parameters necessary for the Biot theory, applying both general experimental methods and those more specifically applicable to cancellous bone. In what follows specific values for these parameters are not provided, because they are so dependent on the particular tissue samples being studied; however, for typical values, see Currey.[14]

For cancellous bone, many of the parameters required by the Biot theory are unknown and may only be estimated. The intrinsic ultrasonic (velocity and attenuation) and physical parameters (density [ρ_s], Young's modulus [E_s], bulk modulus [K_s] and Poisson's ratio [υ_s]) for cancellous bone tissue are assumed to be those for solid bone material. Even with this assumption, some difficulty can arise, however, in the experimental measurement of these parameters. For example, it is usual to calculate the intrinsic bulk modulus from the Young's modulus,[27,30] using the following relationship:

$$K_s = \frac{E_s}{3(1 - 2\upsilon_s)} \tag{11}$$

Once the intrinsic material properties have been measured or calculated it is possible to calculate the parameter values for the trabecular frame[2,4,17] (μ^*, E^*, and K^*) using the relationships:

$$\mu^* = \frac{E^*}{2(1 + \upsilon^*)} \qquad \text{(shear modulus)} \tag{12}$$

$$E^* = E_s(1 - \beta)^n \qquad \text{(Young's modulus)} \tag{13}$$

where β is porosity and n varies between 1 and 3:

$$K^* = \frac{E^*}{3(1 - 2\upsilon^*)} \qquad \text{(bulk modulus)} \tag{14}$$

Poisson's ratio is either assumed to have a specific value (typically 0.5), or is experimentally measured, with inherent difficulty. The Young's modulus for the cancellous bone frame can be determined using three methods—by calculation, using Equation 13; by testing the bone in a compressive testing machine at a low strain rate (this is not ideal because the Young's modulus of cancellous bone is strain-sensitive,[27,36] the dynamic modulus is a factor of two larger than that which is measured for a low strain rate); or by ultrasound measurement (where a conversion factor is not needed, the Young's modulus calculated from the velocity of a propagating ultrasonic bar wave and the density of the medium of propagation), using:

$$V_{\text{bar}}^2 = \frac{E}{\rho} \tag{15}$$

There is some doubt, however, regarding the validity of assuming bar wave propagation.[48]

Because bone marrow is composed mainly of fat with very little blood and tissue fluid, the physical parameters for fat are normally used for the pore fluid.[38] Water may be substituted as the pore fluid, particularly if the theoretical results are to be compared with experiments performed in vitro where the marrow is often completely removed and replaced with water.

Permeability relates the rate of fluid or gas flow through a material to the sample thickness, the cross-sectional area, and the pressure causing the flow, defined by the relationship known as Darcy's law. A simple method for measuring the permeability has been outlined by Njeh,[39] which involves measuring the velocity of fluid flow through a cancellous bone specimen, using a vertical column of fluid kept at a constant height.

One of the most elusive parameters in the Biot theory is the tortuosity (α_∞), or the sinuosity. This is defined as the ratio of the length of true path of flow for a fluid to the shortest distance between the inflow and the outflow.[43] It is important to realize that this definition is kinematic, not geometric. In the JKD theory of dynamic tortuosity, α_∞ becomes wholly dependent on the geometry of the sample. Although there has been some success in calculating the tortuosity for a given structure, it has been limited to simple structures that do not even begin to approximate the complex structure of cancellous bone, and therefore it must be measured. The most common method for measurement of tortuosity is by measuring the electrical resistivity formation factor (F),[13] the ratio of conductivity of saline to the conductivity of the sample saturated with saline. If F is then multiplied by the porosity of the sample, then it is found to be a good measure of the tortuosity. This method is based on the assumption that electric current will flow along the same paths as a fluid and, although there are still a few doubts as to the validity of this assumption, this is probably the most widely used measure of tortuosity. Other methods to measure the tortuosity do exist, most notably the use of ultrasound; that is, the tortuosity calculated from the limiting value of the slow wave velocity, at the high frequency limit, as outlined by Leclaire et al.[35]

The final parameter to be measured is the pore size parameter (Λ) introduced by Johnson et al.[25,26] Λ is a measure of the intrinsic, dynamically interconnected pore sizes. The pore size parameter can be estimated by measuring the mean trabecular plate separation (MTPS), using standard histomorphometric techniques. For cancellous bone, the pore size parameter is assumed to be half of the mean trabecular plate separation.[49]

It may not always be necessary to measure all of the last three parameters. It may be enough to know two of them and the porosity of the samples, because an approximate relationship (Equation 16) between permeability, tortuosity, pore size, and the porosity was suggested by Johnson et al.[26] and Banavar and Johnson[5] for a variety of smooth pore shapes. Other attempts have also been made to relate the mass transport parameters (e.g., permeability, tortuosity, and pore size) to the pore microgeometry, either by calculation or simulation[24,29]:

$$M = \frac{8\alpha_\infty k_0}{(\beta\Lambda^2)} \tag{16}$$

where k_0 is the permeability and M is of the order of 1. Williams found M to have a value of approximately 4 for bovine samples.[49] Although some work has been done to validate this relationship for cancellous bone, further work is still required.

The recently developed technique of microcomputed tomography, having a spatial resolution in the region of 20 μm, shows great promise for the nondestructive determination of structural parameters including tortuosity, pore size, and geometry. For example, Uchiyama et al.[47] demonstrated good correlation with conventional histomorphometry of the human ilium. Biot theory essentially assumes isotropic behavior, whereas most elastic and structural parameters of cancellous bone are anisotropic. Hence, for a true representation of ultrasound propagation through cancellous bone, consideration of sample orientation should be given.

The Future

Even though the Biot theory is a mature theory and has had great successes in many of its applications, it has been found in some circumstances to be only an approximation, providing only qualitative information. Although it works very well for many artificial fluid-saturated porous media, when applied to natural materials, particularly rocks and cancellous bone, it consistently underestimates the experimentally observed attenuation by several orders of magnitude. These deviations are indicative of non-Biot dissipative mechanisms. It has been suggested, for sandstones, that the discrepancies between Biot theory and experimental ultrasonic measurements are due to the complex pore shapes. Two non-Biot attenuation mechanisms have been proposed: (1) local flow in microcracks found in between grain contacts; and (2) attenuation due to the roughness of the pores.[18] Calculations performed including these dissipative mechanisms seem to correct for the deviations from the traditional Biot theory.

We suggest that similar non-Biot mechanisms might explain some of the deviations from Biot theory that are seen in experiments with cancellous bone. Although bone will not have noticeable grain boundaries, the loading of bone causes microcracks to appear and it is possible that local flow mechanisms in these microcracks are having a similar effect to flow in the grain boundaries of sandstones. Also, cancellous bone is not as smooth as might have first been thought because there are numerous reabsorption pits on the surface of the bone that arise as part of the remodeling cycle. Theoretical calculations are required to demonstrate that these new dissipative mechanisms provide the right order of magnitude corrections to the traditional Biot theory, alongside experimental work as performed by Gist,[18] where small microcracks in the bone would be filled, and the pore surfaces smoothed.

Conclusions

The Biot theory has shown some success in predicting ultrasound propagation in cancellous bone, in particular, fast wave velocity. It has not yet, however, accurately predicted attenuation and there is a general absence of a measurable slow wave expected from the theory. This is partially due to incomplete knowledge of the parameters, resulting in highly "tunable" implementation with inherent inconclusive validation. Cancellous bone lies at the limits of the Biot theory's applicability and, as a result, insights gained in modeling ultrasonic propagation through bone may find application to other areas of research. The Biot theory certainly shows promise and needs to be explored in greater depth both theoretically, where more work is required to compare its predictions with other modeling approaches, in particular scattering, and experimentally, to determine more exact values for the parameters used within the theory.

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Date Received: September 30, 1997
Date Revised: December 3, 1998
Date Accepted: December 3, 1998

Appendix: Notation

f_c	characteristic frequency
k_0	permeability of sample
n	exponent value
E	Young's modulus of sample
E_s	Young's modulus of cancellous bone tissue
E^*	Young's modulus of frame
K_s	bulk modulus of cancellous bone tissue
K^*	bulk modulus of frame
K_f	bulk modulus of fluid
M	predicted permeability ratio, introduced by Johnson et al.[26]
P, Q, R	Biot parameters
V	ultrasound velocity
$\alpha(\omega)$	Johnson–Koplik–Dashen dynamic tortuosity[25]
α_∞	tortuosity
β	porosity (volume fraction of fluid phase)
η	kinematic viscosity of fluid
μ^*	shear modulus of frame
υ_s	Poisson's ratio of cancellous bone tissue
υ^*	Poisson's ratio of frame
ρ_s	density of solid phase
ρ_f	density of fluid phase
ρ_{11}	effective density of the solid moving through the liquid
ρ_{22}	effective density of the fluid moving through the solid
ρ_{12}	inertial drag that the solid exerts on the fluid
Δ	combined Biot parameter $(P\rho_{22} - 2Q\rho_{12} + R\rho_{11})$
Λ	interconnected pore size, introduced by Johnson et al.[26]