Introduction

Speaker Introduction

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- Technical Account Manager at Gurobi Optimization
  - Member of the Gurobi Technical Support team
  - Presales Engineer DACH/EMEA
- Ph.D. in Operations Research, University of Paderborn (Germany)
- Many years of experience in the development and design of decision support systems using mathematical optimization methods.

11 years ago, we were founded by the sharpest minds in computational mathematical programming

Gurobi by the numbers…

2,400	# of global customers
40	# of industries
39	% of employees with a PhD
59x	Gurobi speed increase since v1.0
50%	faster than closest competition

Machine Learning & Optimization

The two sides of AI

Two Ways to Draw a Conclusion

Inductive reasoning

- Derivation of general principles from specific observations

Deductive reasoning

- Deriving a conclusion from a set of multiple, factual premises

Two Forms of AI

Inductive AI – Machine Learning

- Discovering patterns in data

Deductive AI - Optimization

- Using known relationships to draw conclusions from data

Things you Know

- Manufacturing this part requires these raw materials
- Electricity generated here travels down these transmissions lines to get there
- A pilot must rest for at least 10 hours between shifts

Things you (Probably) Don't Know

- Is this a favorable position on a Go board?
- What steps to recognize a picture of a cat?
- How can I recognize that a machine is about to fail?

Things you (Perhaps) Didn't Notice

Correlations between certain actions and outcomes

A (Bad) Machine Learning Joke

"What's your greatest strength?

- *"I'm a fast learner"*

"What's 2+2?"

- *"6"*

"Not even close. It's 4."

- *"4"*

"Okay, what's 2+3?"

- *"4"*

"Always try MIP"

Optimization is used in many industries and across a broad range of business problems

- Gurobi is used in dozens of industries and by over 2,400 companies.
- The reason for such broad use is the ability to rapidly solve a wide range of problem types.

Planning usually involves a lot of "what if" analysis

- What if costs or prices change?
- What if customer demand changes?
- What is the impact of adding more capacity?

Modern MIP solvers contain a wealth of techniques, drawn from a broad range of domains

MIP is robust with respect to changes in the underlying model

- You can modify the **problem structure** (rules, conditions, logic, …)
- You can modify **data** (prices, costs, weights, …)
- You can modify **goals** (revenue, utilization, fairness, …)

Gurobi is a library

The **Gurobi Optimizer** is a high-performance solver that can be embedded into applications that solve decision problems.

The algorithmic details of solving optimization models are usually invisible for users

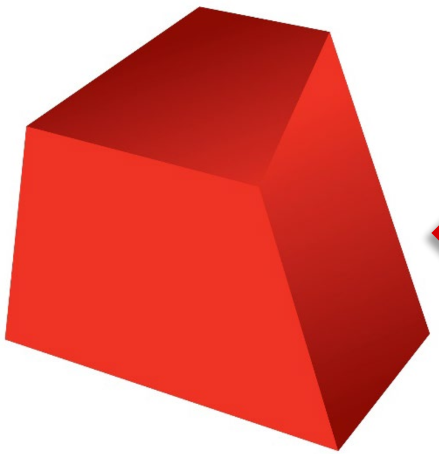
- In many cases: „Black box“ to transform selected input data into recommendations
- End-users are usually problem domain experts
- No need to expose internal details (but sometimes helpful)

Model development: An expert (modeller) designs the mathematical structure of the optimization model during development of the application.

Model generator: The application contains a configurable software component to generate optimization model instances based on data and preferences selected by the users.

Gurobi Integration

Gurobi Optimizer



APIs

Your Application

Linear programming

A *linear program* (LP) is an optimization problem of the form

$$\begin{array}{lll} \text{minimize} & c_1x_1 + c_2x_2 + \ldots + c_nx_n & \\ \text{subject to} & a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b_i, & i = 1, \ldots, m \\ & l_j \leq x_j \leq u_j, & j = 1, \ldots, n \end{array}$$

How does this help with business problems?!

Continuous: LP / QP / QCP

Primal & dual simplex method

- Numerically stable (most challenging part)

Parallel barrier method with crossover

- Can effectively exploit multiple cores

Concurrent optimization

- Run both simplex and barrier simultaneously
- Solution is reported by first one to finish
- Great use of multiple CPU cores
- Best mix of speed and robustness

Karush-Kuhn-Tucker Conditions

Conditions for LP optimality:

- Primal feasibility: $Ax = b$ $(x \geq 0)$
- Dual feasibility: $A'y + z = c$ $(z \geq 0)$
- Complementarity: $x'z = 0$

	Primal feas	Dual feas	Complementarity
Primal simplex	Maintain	Goal	Maintain
Dual simplex	Goal	Maintain	Maintain
Barrier	Goal	Goal	Goal

Most real-life problems are not continuous

Business decisions need more:

- Integer numbers (1,2,3,….)
- Binary decisions (yes, no)

This can be done with an LP model but with additional **integrality constraints** on some variables.

Example: Binary choices. Instead of every value between the lower bound 0 and the upper bound 1 the solution value needs to be integer.

Result: Binary variables are only allowed to be 0 or 1 in any feasible solution.

This additional „feature" allows to model a huge number of functions, relationships, logic, etc.

MIP Problem Statement

A *mixed-integer program* (MIP) is an optimization problem of the form

$$\boxed{\begin{array}{ll} \textit{Minimize} & c^T x \\ \textit{Subject to} & Ax = b \\ & l \leq x \leq u \\ \multicolumn{2}{l}{\text{some or all } x_j \text{ integer}} \end{array}}$$

We give users tools that make them more efficient

Industries Transformed by MIP – Supply Chain

In the 1980's, software dominated by rules of thumb

- Example: theory of constraints (*The Goal*, Goldratt)

MIP widely adopted in the 1990's

Now the standard technology for supply-chain

- SAP, Oracle, JDA, Manhattan Associates, …

Industries Transformed by MIP – Electrical Power

Electrical power deregulated in the late 1990's

Need to create a market for electricity

Early solution techniques:

- Heuristics (Lagrangean relaxation)
- MIP (lots of models; no real usage)

EPRI report, June 1989:

- "Mixed-integer programming (MIP) is a powerful modeling tool. 'They are, however, theoretically complicated and computationally cumbersome'"

DIMACS meeting 1999:

- Bob Bixby demonstrated that MIP had improved to the point where practical power models could be solved

Within a few years, nearly every grid operator in the world was using MIP to solve these models

Industries Transformed by MIP – Sports Scheduling

Computing sports schedules quite complicated

- Stadium constraints, travel constraints, TV schedules, …

Done by hand for decades

- Example: Henry and Holly Stephenson scheduled Major League Baseball “by hand” from 1981-2004

Schedules now done using MIP:

- MLB since 2004
- NFL since 2007

Performance

Progress: LP (1988-2004)

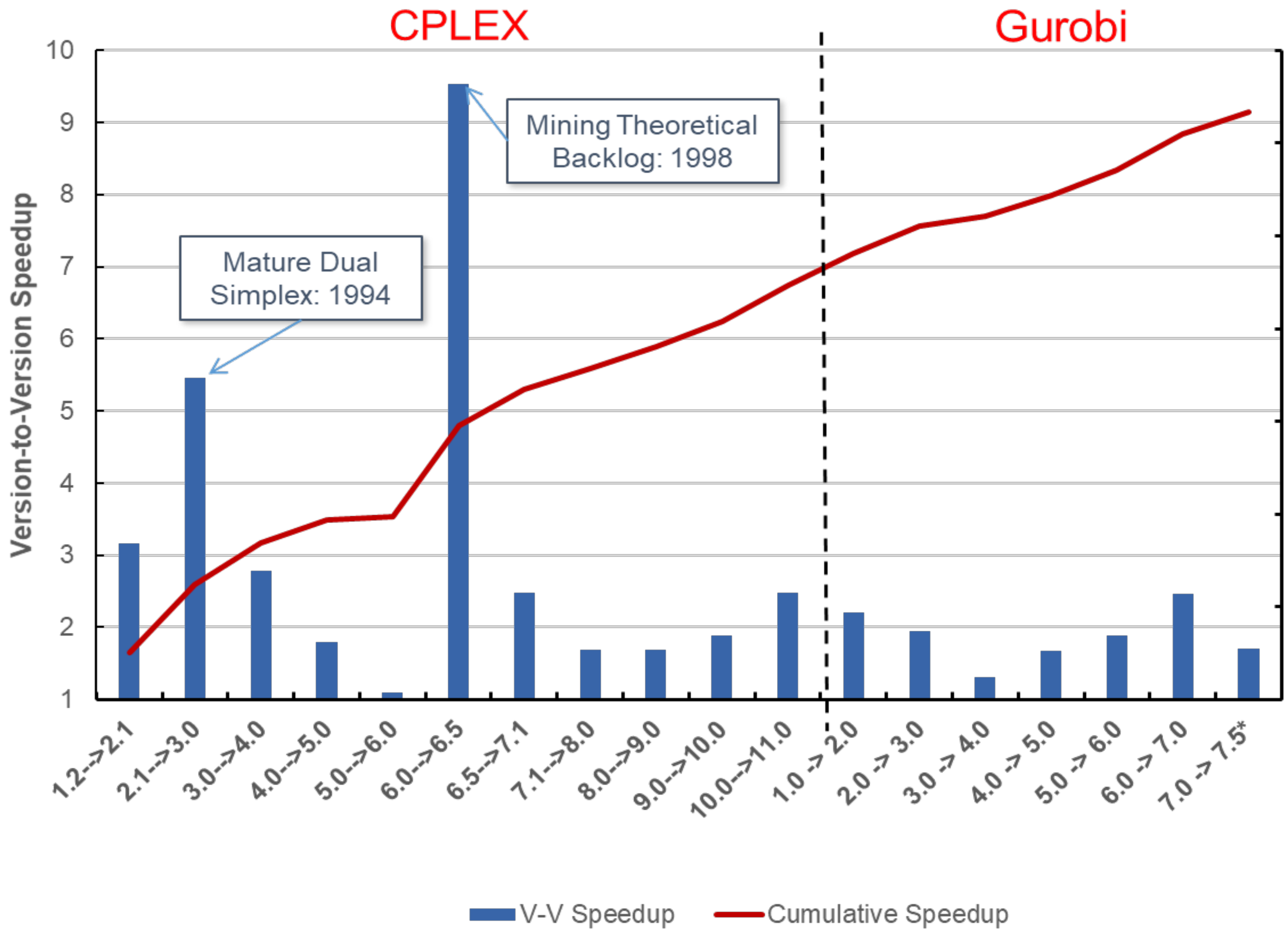
From 1988-2004:

- Improvement in algorithms:
  - Primal simplex in 1988 versus best of primal/dual/barrier: 3,300X
- Improvement in machines: 1,600X
- Net improvement: 5,300,000X
- Source: *Bixby*, Progress in Linear Programming, ORSA Journal on Computing, 1994

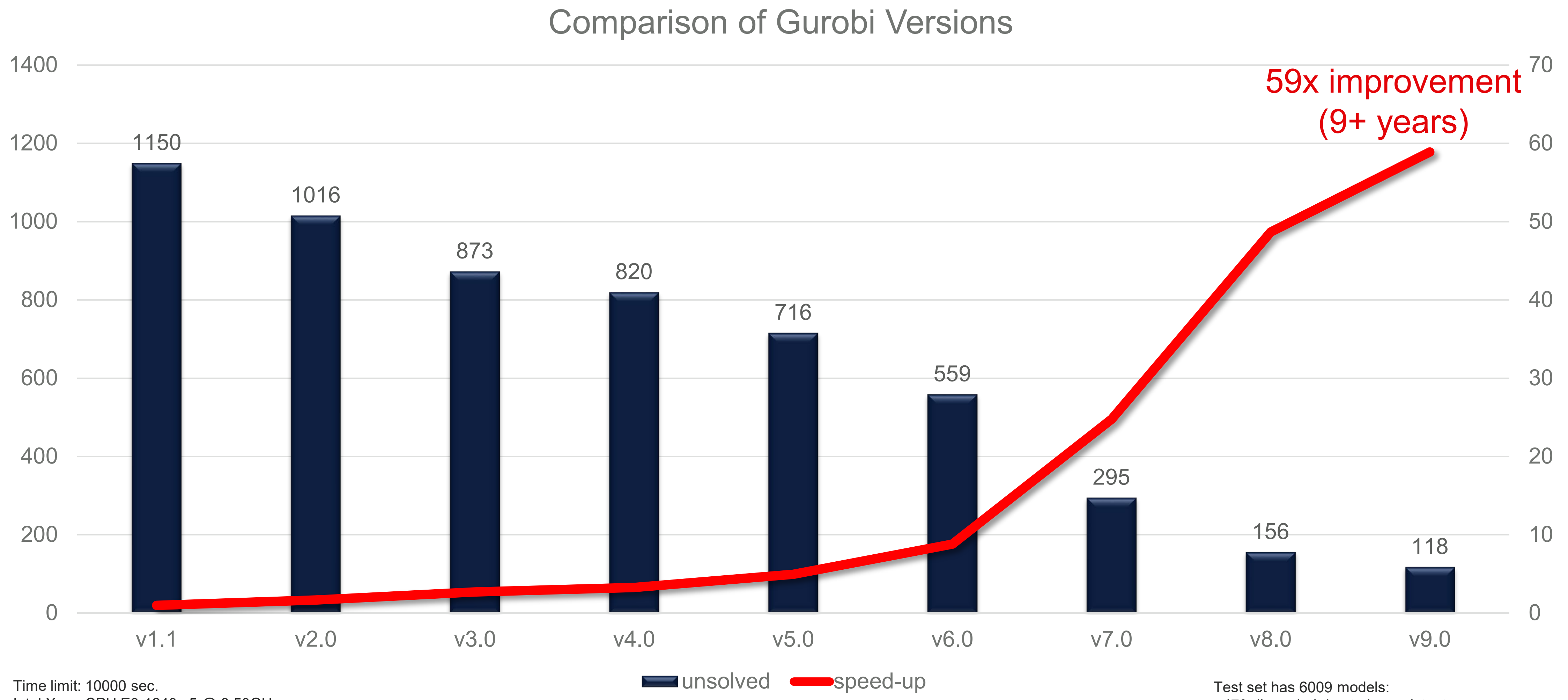
Impact:

- What would take 2 months before now takes 1 second
- LP is now (mostly) considered a solved problem
  - Regularly solve models with millions of variables and constraints

Progress: MIP (1990-2017)



In every release, we crank up the speed



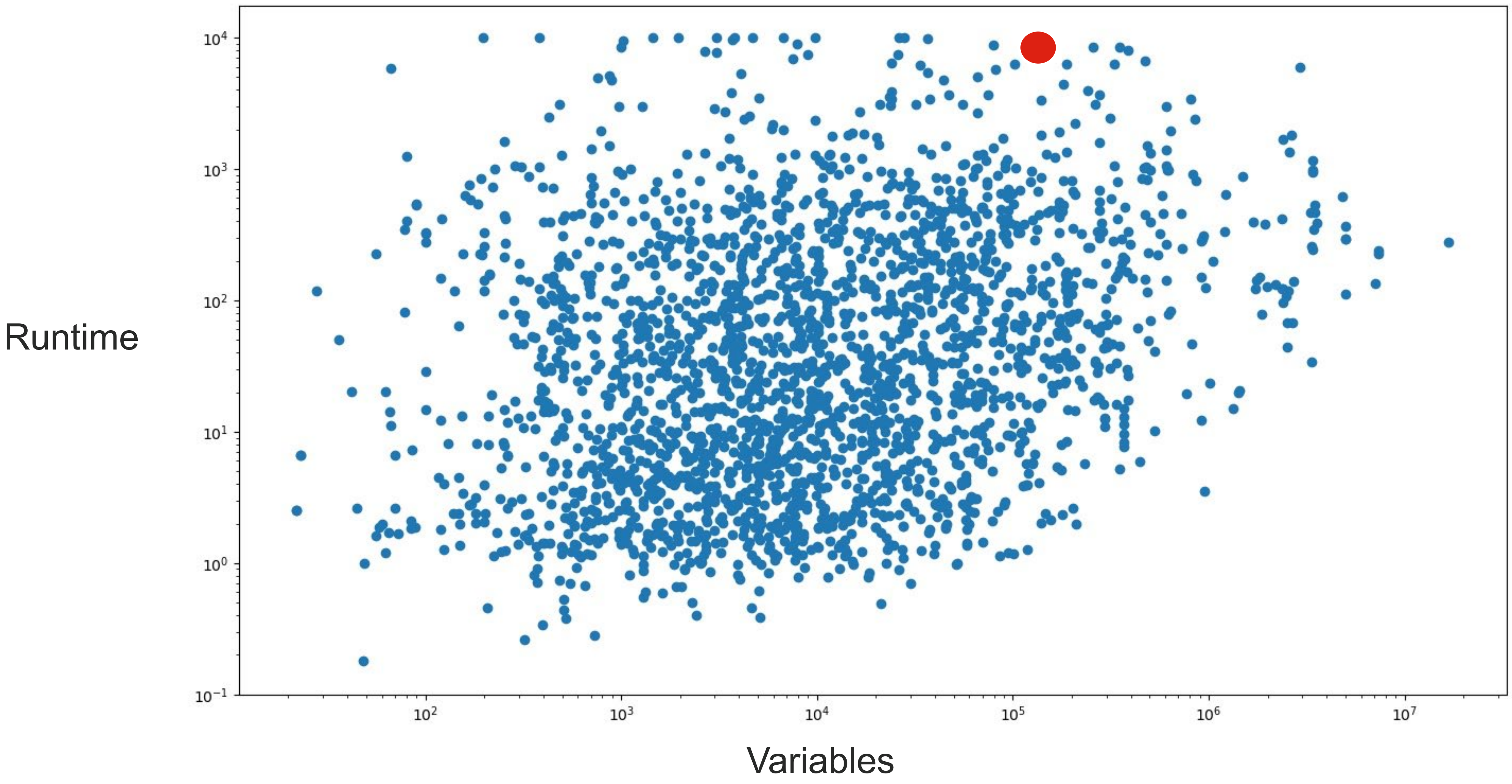
Time limit: 10000 sec.
Intel Xeon CPU E3-1240 v5 @ 3.50GHz
4 cores, 8 hyper-threads
32 GB RAM

Test set has 6009 models:
- 473 discarded due to inconsistent answers
- 1580 discarded that none of the versions can solve
- speed-up measured on >100s bracket: 2197 models

Gurobi MIP Library

(5927 models)

Runtime Versus # of Variables



Next Steps

Interested?

- Try Gurobi!
  - AMLD 2020 Workshop participants can get a free product evaluation
  - Academic and research licenses are free!

- Contact me (siefen@gurobi.com) directly or our sales team at info@gurobi.de