Conformal Prediction Bands for Two-Dimensional Functional Time Series

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Abstract

Conformal Prediction (CP) is a versatile nonparametric technique used to quantify uncertainty in prediction problems. In this work, we propose an extension of such method to the case of time series of functions defined on a bivariate domain. Given the complex nature of data and the non-trivial dependence structure, we adapt the CP procedure, eventually deriving distribution-free prediction bands and providing performance bounds in terms of unconditional coverage and asymptotic exactness. The advantages of the CP method over the traditional Bootstrap approach are explored on synthetic data in a proper Appendix. Moreover, we extend the theory of autoregressive processes in Hilbert space in order to allow for functions with a bivariate domain. Given the novelty of the subject, we present estimation techniques for the Functional Autoregressive model (FAR) and for principal component analysis (PCA) for two-dimensional functional data. An *ad hoc* simulation study is implemented in order to investigate finite sample performances of estimators of the functional autoregressive model, comparing them with benchmark forecasting methods. Finally, we explore benefits and limits of the proposed approach on a real case study, employing a dataset from Copernicus Climate Change Service, collecting daily observations of Sea Level Anomalies of the Black Sea in the last twenty years.

Keywords: Conformal Prediction; Forecasting; Functional autoregressive process; Functional time series; Prediction band; Two-dimensional functional data.