Appendix

Description of the mathematical steps necessary for computing E and H inside a multilayer stack

Common Assumptions

- We consider a forward propagating plane wave in the form

$$\vec{A} = \vec{A}_m \cdot exp(-i\vec{k} \cdot \vec{r} + i\omega t).$$

  In such a way that

$$\partial_t \vec{A} = i\omega \vec{A},$$
$$\partial_j \vec{A} = -ik_j \vec{A},$$

  where $j = x, y, z$.

- We consider isotropic media with $\mu = \mu_0$.

- The optical axis is z.

- We consider waves with impinging at angle theta with $k_x = 0$. Therefore we can write $k_y = k \sin\theta$ and $k_z = k \cos\theta$.

- Inside a medium with refractive index n we write $k = n \cdot k_0$.

- We have to consider that $n = f(z)$ where $f(z)$ is a piecewise function, constant in each layer of the multilayer. We therefore refer to the refractive index with n_z in order to remind that its value depends on the layer in which we are located, but its derivative is null everywhere.
  Therefore $k_z = k_0 n_z \cos\theta$ and $k_y = k_0 n_z \sin\theta$.

- Since k_y has to be preserved, we write

$$k_y = k_0 \beta,$$

  where $\beta = n_{(z=0)} \cdot \sin\theta$.

- For reference, consider that the curl of a vector takes the form:

$$\nabla \wedge \vec{A} = \hat{x}(\partial_y A_z - \partial_z A_y) + \hat{y}(\partial_z A_x - \partial_x A_z) + \hat{z}(\partial_x A_y - \partial_y A_x),$$

  where $\hat{j}$ is a versor in the direction j.

s-polarization (E is parallel to the multilayer)

Maxwell's curl equation for E takes the form:

$$\nabla \wedge \vec{E} = -i\omega\mu_0\vec{H}$$

Since $E_y = E_z = 0$, for a forward propagating wave we can write:

$$\nabla \times \vec{E}^{+} = \hat{y} \cdot \partial_z E_x^{+} - \hat{z}\partial_y E_x^{+} = -i\omega\mu_0\vec{H}^{+}$$

$$H_y^{+} = \frac{1}{-i\omega\mu_0} \cdot (-ik_z)E_x^{+} = \frac{n_z k_0 \cos\theta}{\omega\mu_0}E_x^{+};$$

$$H_z^{+} = -\frac{1}{-i\omega\mu_0} \cdot (-ik_y)E_x^{+} = \frac{k_0\beta}{\omega\mu_0}E_x^{+}.$$

Therefore

$$H_y^{+} = n_z/Z_0 \;\cos\theta\; E_x^{+};$$

$$H_z^{+} = 1/Z_0 \;\beta\; E_x^{+}.$$

For the backward propagating wave the derivative along z changes sign, therefore

$$H_y^{-} = -n_z/Z_0 \;\cos\theta\; E_x^{-},$$

while for H_z we get the same result

$$H_z^{-} = 1/Z_0 \;\beta\; E_x^{-}.$$

s-polarization (H is parallel to the multilayer)

$$\nabla \times \vec{H} = +i\omega\varepsilon E$$

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