How to calculate p-values

Statistics and Big Data

Niccolò Salvini, PhD

UCSC

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Course: Statistics and Big Data

Overview

What Are P-Values?

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This question leads us to the concept of p-values, a critical tool in statistical hypothesis testing.

Example of Drug Testing

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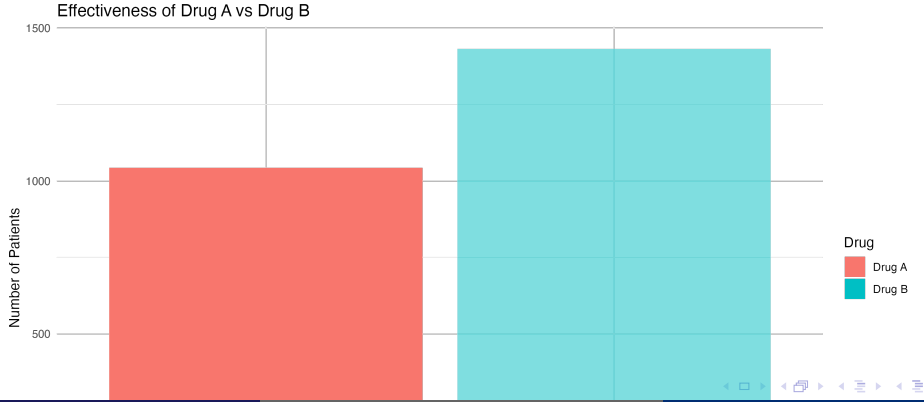

Understanding the Results

Now that we have observed the results, we can analyze the effectiveness:

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- Drug B: 0.1% cure rate (2 cured out of 1,434)

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Mathematical formulation: The observed proportions can be expressed as:

$$p_A = \frac{1043}{1046} \quad \text{and} \quad p_B = \frac{2}{1434}$$

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How confident can we be that Drug A is superior? This is where p-values come into play.

Defining P-Values

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Interpretation

A smaller p-value indicates stronger evidence against the null hypothesis.

The Threshold for Decision-Making

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In practical terms, this implies that if we repeated the experiment many times, we would expect to see a false positive in 5% of cases.

Exploring False Positives

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Conversely, a p-value of 0.01 suggests a significant difference, even if both groups received the same treatment.

Understanding False Positives

Terminology Alert

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This highlights the importance of selecting an appropriate threshold based on the context of the study.

The Importance of Effect Size

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For example, a p-value of 0.04 may arise from a small difference of 1 unit, while a p-value of 0.24 could correspond to a larger difference of 6 units.
Key insight: Statistical significance does not equate to practical significance.

Summary of Key Concepts

1. P-values help us assess the evidence against the null hypothesis.
2. A common threshold for significance is 0.05.
3. Small p-values indicate statistical significance but do not reflect effect size.

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Understanding these concepts is crucial for interpreting experimental results accurately.

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Exercise 4

Analyze the difference between statistical significance and practical significance in the context of drug effectiveness.