Linear Regression Final

Statistics and Big Data

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UCSC

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Course: Statistics and Big Data

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Overview

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In this lecture, we will explore the foundational concepts of linear regression, including fitting a line to data, calculating R^2, and determining statistical significance through p-values.

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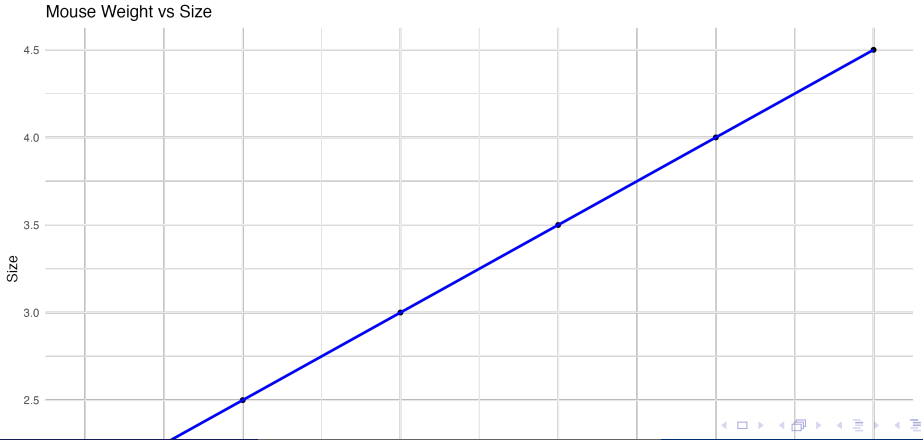

Fitting a Line to Data

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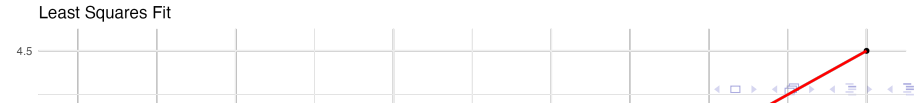
The objective is to minimize the sum of squared residuals.

Mathematical Formulation

$$\text{Sum of Squared Residuals} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$



Understanding Residuals

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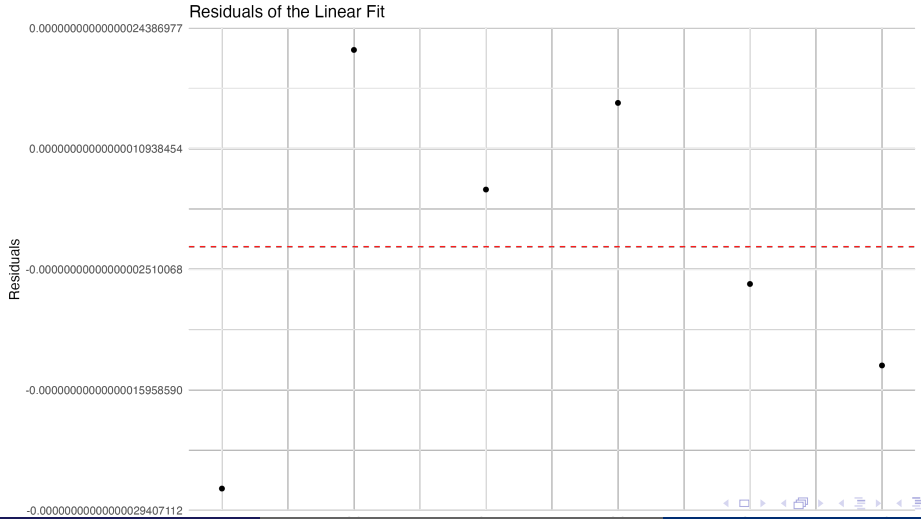
Visualizing the Fitting Process

Here we visualize the process of fitting a line to our dataset.



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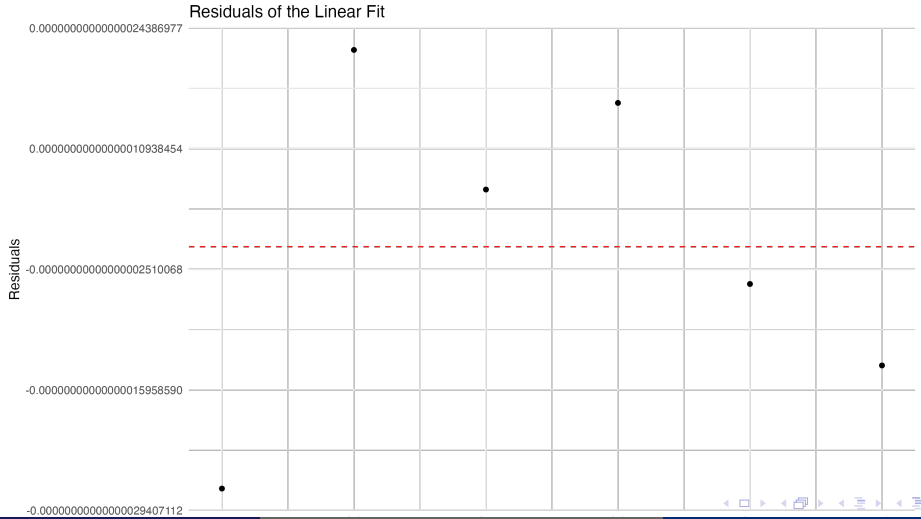

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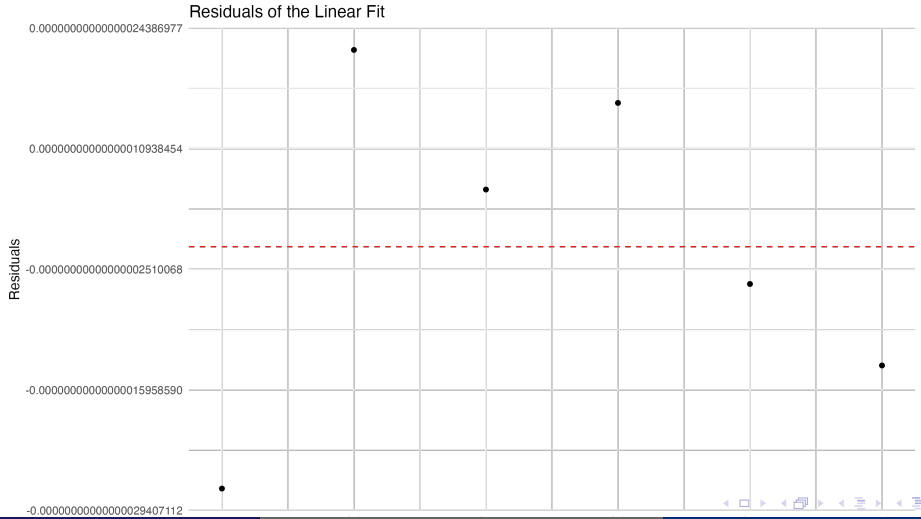

The Equation of the Fitted Line

Once we have our fitted line, we can express it mathematically:

Equation

$$y = a + b \cdot x$$

Where:

- y is the predicted size,
- a is the y-intercept,
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This equation allows us to make predictions about mouse size based on weight.

Evaluating the Fit with R^2

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How do we assess the quality of our predictions? The coefficient of determination, R^2, quantifies how much of the variation in the dependent variable can be explained by the independent variable.

Mathematical Formulation

$$R^2 = \frac{SS_{mean} - SS_{fit}}{SS_{mean}}$$

Where:

- SS_{mean} is the total sum of squares around the mean,
- SS_{fit} is the sum of squares around the fitted line.

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Let's calculate R^2 using specific values:

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This indicates that 60% of the variation in mouse size can be explained by mouse weight.

Exploring Different Scenarios for R^2

What happens when the relationship changes?

1. **Perfect Prediction:** If knowing mouse weight perfectly predicts size, $R^2 = 1$.
2. **No Prediction:** If mouse weight does not help at all, $R^2 = 0$.

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These scenarios illustrate the extremes of model performance.

Moving to Multiple Variables

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What if we want to include more predictors, such as tail length? In this case, we fit a plane instead of a line, using multiple dimensions.

Equation for the Plane

$$y = a + b_1 \cdot x_1 + b_2 \cdot x_2$$

Where x_1 is weight and x_2 is tail length.

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Understanding p-values in Regression

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Mathematical Formulation

$$F = \frac{\text{Variance explained}}{\text{Variance not explained}}$$

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This process helps us determine the significance of our findings.

Summary of Key Concepts

- **Linear Regression**: A method to predict one variable based on another.
- **Least Squares**: A technique to minimize the sum of squared residuals.
- R^2: A measure of how much variance is explained by the model.
- **p-value**: Indicates the statistical significance of the relationship.

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Exercise 4

Describe the process of calculating a p-value in the context of linear regression.