POLITECNICO MILANO 1863

FINAL PROJECT

Orbital Mechanics

MSc in Space Engineering
Politecnico di Milano – A.Y. 2020/21 – Group 9

Bardazzi, Niccolò
Carcano, Stefano
Domaschio, Jacopo
Maestro Martínez, José David

CONTENTS

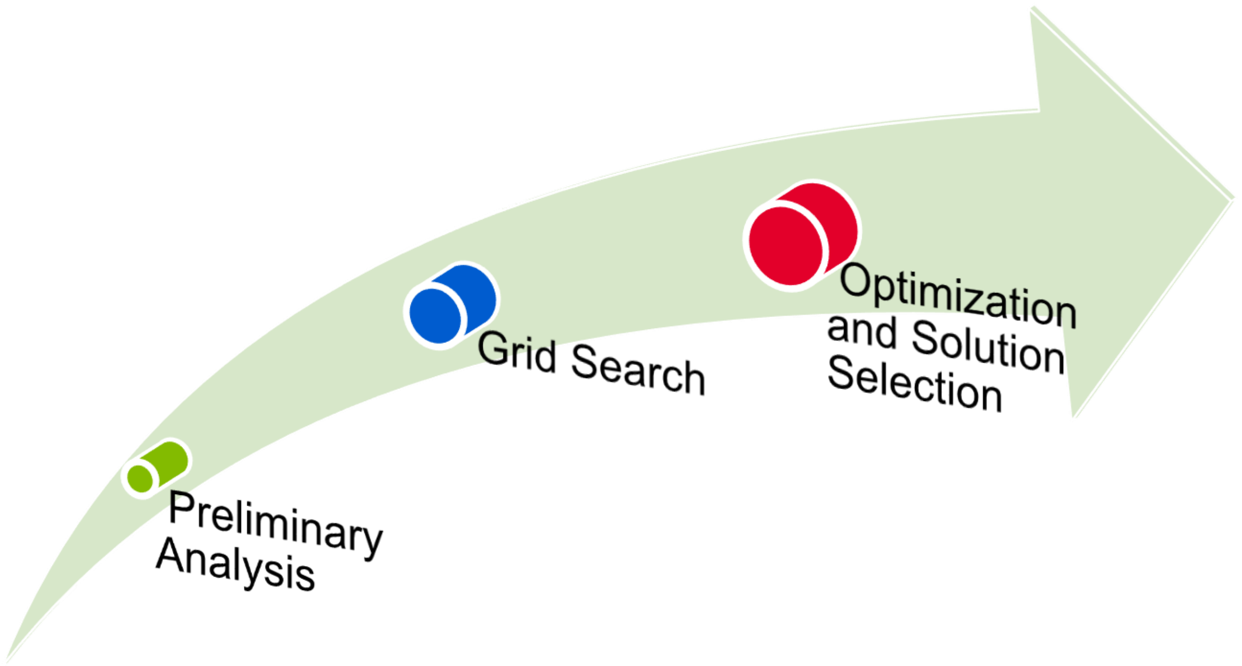
1. Interplanetary Mission

MISSION DESCRIPTION

Departure Planet	Jupiter
Fly-By Planet	Mars
Arrival Planet	Venus
Earliest departure	2029/05/01
Latest arrival	2069/05/01

POLITECNICO MILANO 1863

WORKFLOW



PRELIMINARY ANALYSIS

AIM:

- Understand the behaviour of the problem
- Find reasonable boundaries for the grid search phase
- Comparison with direct transfers

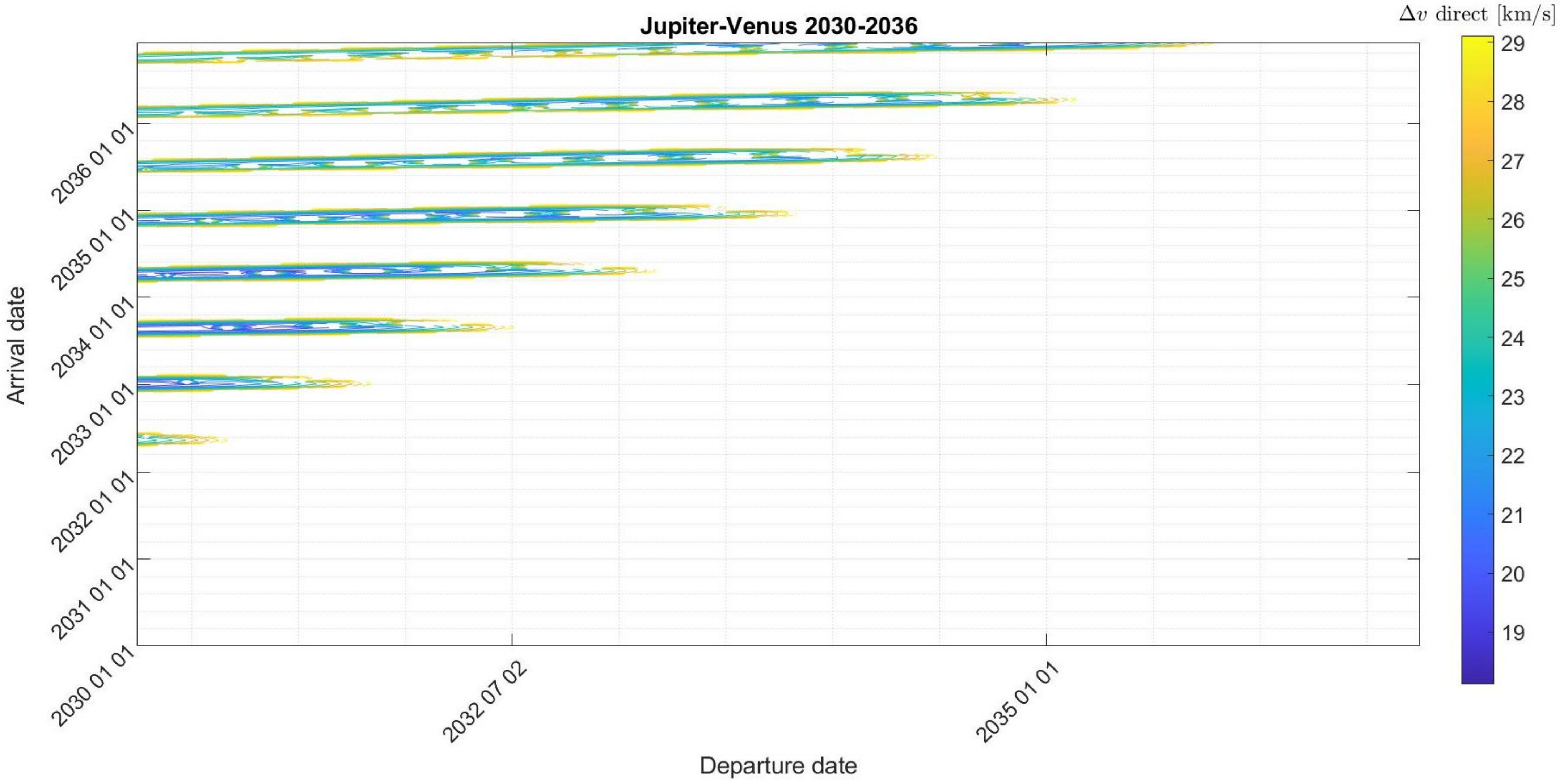
PRELIMINARY ANALYSIS – Hohmann and Parabolic Transfers

Coplanar, Circular and Synchronized Orbits.

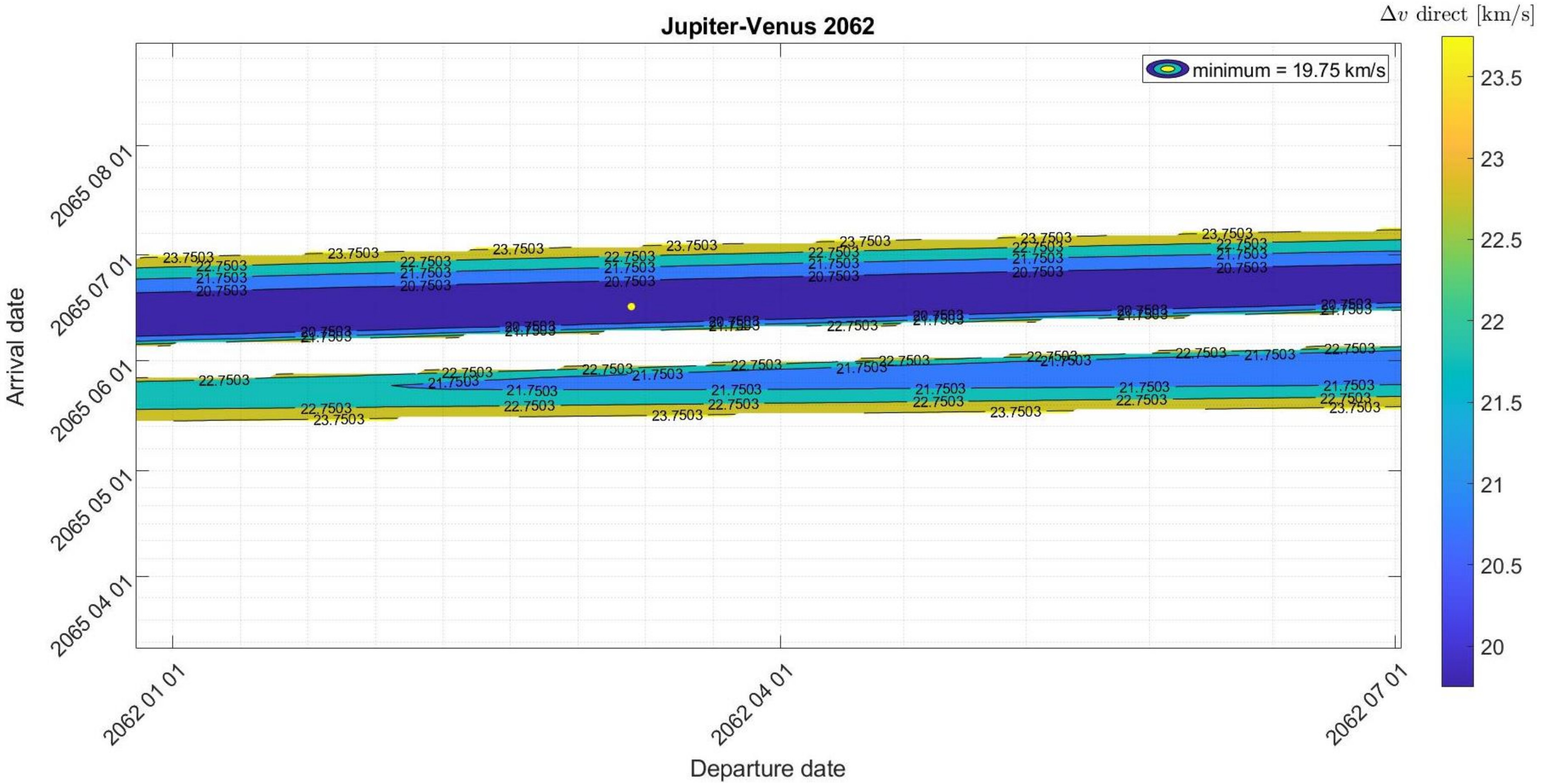
Hohmann transfer	$\Delta v_{departure}$ [km/s]	$\Delta v_{arrival}$ [km/s]	TOF (days)
Jupiter - Mars	4.269	5.882	1126.58
Mars - Venus	5.768	5.763	217.5

Parabolic transfer	TOF (days)
Jupiter - Mars	432
Mars - Venus	75

PRELIMINARY ANALYSIS – Direct Transfers



PRELIMINARY ANALYSIS – Direct Transfers



GRID ANALYSIS

AIM:

- Find a set of non-direct transfers along the whole time span assigned
- Visualize, eventually, a repetition of the pattern
- Find a first solution/initial guess for optimization process (minimizing Δv)

GRID ANALYSIS - Procedure

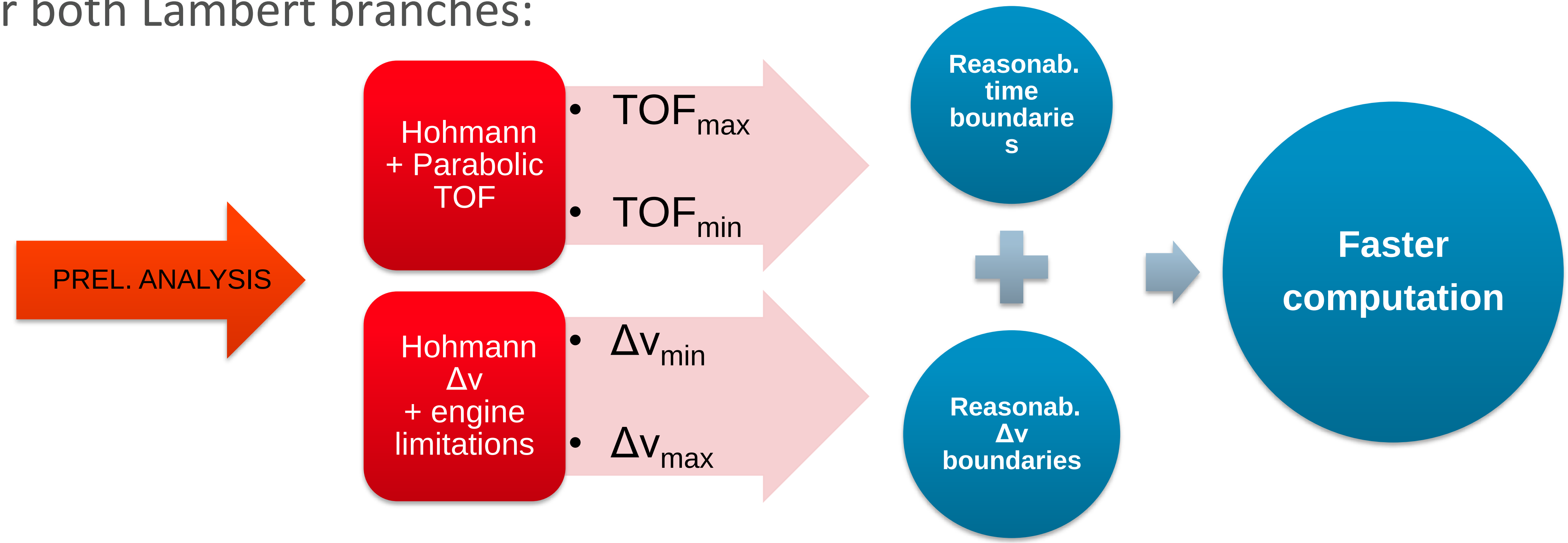
POLITECNICO MILANO 1863

- *Take ten-year-long departure/fly-by/arrival time subwindows*
- *Analize each with 500 iterations (about 1 departure per week)*
- *Patched Conic method: 2 Lambert transfers linked by the fly-by condition*
- *Triple loop to find all possible solutions*

GRID ANALYSIS – Boundaries/Constraints

POLITECNICO MILANO 1863

For both Lambert branches:



- Additional constraint set for minimum altitude above Mars during fly-by: $H_{min} = H_{atm} = 50$ km.

GRID ANALYSIS - Results

POLITECNICO MILANO 1863

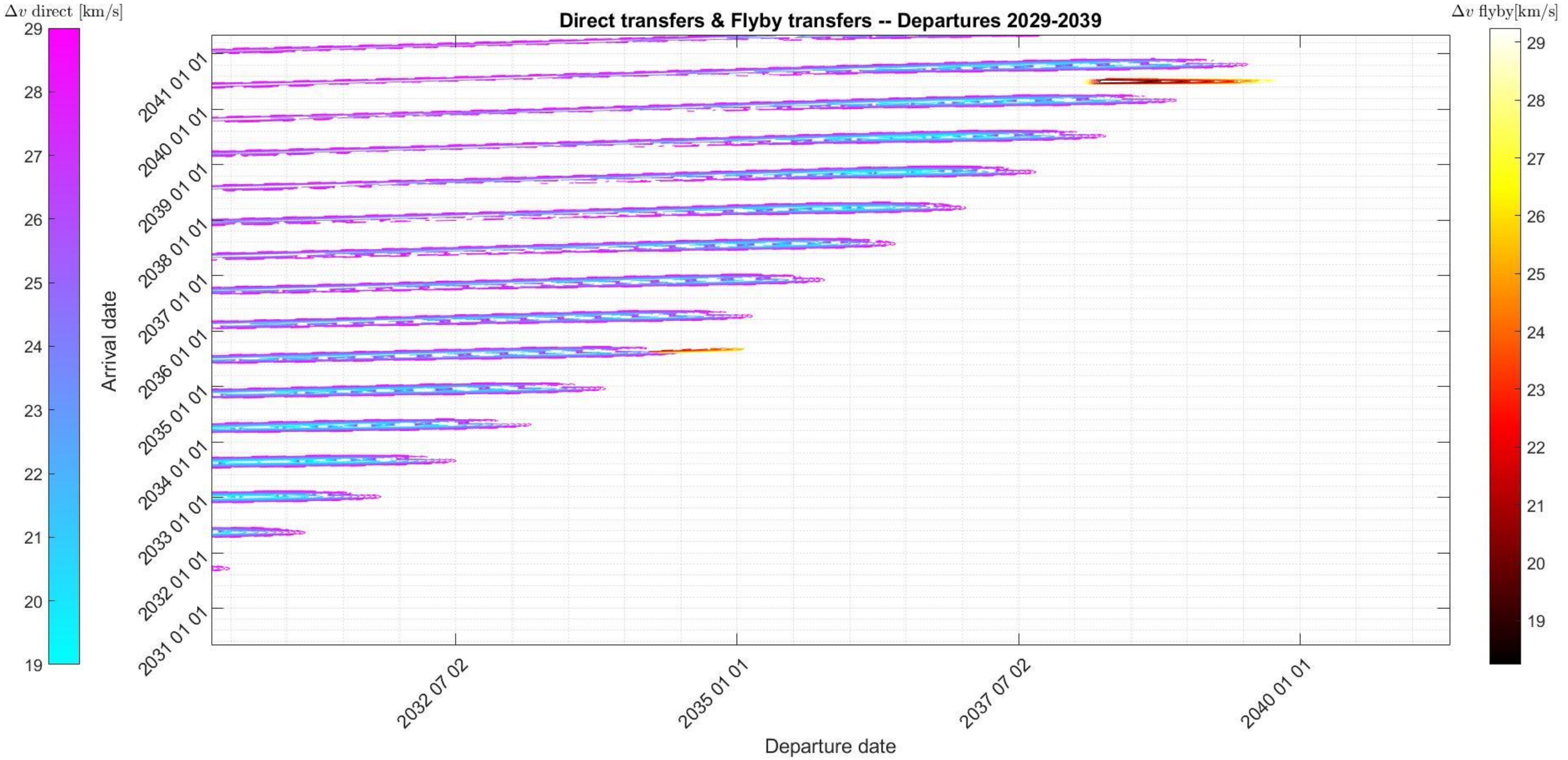
Dv	TOF	Rp	Departure	Flyby	Arrival
17.685	1137.53	3454.338	16-May-2062 08:37:59	10-Mar-2065 10:31:56	26-Jun-2065 21:24:36
17.762	1138.41	3454.338	16-May-2062 08:37:59	10-Mar-2065 10:31:56	27-Jun-2065 18:28:29
17.165	1130.51	3796.134	16-May-2062 08:37:59	11-Mar-2065 12:36:15	19-Jun-2065 20:53:31
17.168	1131.39	3796.134	16-May-2062 08:37:59	11-Mar-2065 12:36:15	20-Jun-2065 17:57:24
17.183	1132.27	3796.134	16-May-2062 08:37:59	11-Mar-2065 12:36:15	21-Jun-2065 15:01:17
17.208	1133.14	3796.134	16-May-2062 08:37:59	11-Mar-2065 12:36:15	22-Jun-2065 12:05:10
17.239	1134.02	3551.994	16-May-2062 08:37:59	11-Mar-2065 12:36:15	23-Jun-2065 09:09:03
17.283	1134.90	3551.994	16-May-2062 08:37:59	11-Mar-2065 12:36:15	24-Jun-2065 06:12:56
17.336	1135.78	3551.994	16-May-2062 08:37:59	11-Mar-2065 12:36:15	25-Jun-2065 03:16:50
17.399	1136.65	3551.994	16-May-2062 08:37:59	11-Mar-2065 12:36:15	26-Jun-2065 00:20:43
17.467	1137.53	3454.338	16-May-2062 08:37:59	11-Mar-2065 12:36:15	26-Jun-2065 21:24:36
16.931	1131.39	3796.134	16-May-2062 08:37:59	12-Mar-2065 14:40:34	20-Jun-2065 17:57:24
16.948	1132.27	3796.134	16-May-2062 08:37:59	12-Mar-2065 14:40:34	21-Jun-2065 15:01:17
16.975	1133.14	3796.134	16-May-2062 08:37:59	12-Mar-2065 14:40:34	22-Jun-2065 12:05:10
17.010	1134.02	3551.994	16-May-2062 08:37:59	12-Mar-2065 14:40:34	23-Jun-2065 09:09:03
17.057	1134.90	3551.994	16-May-2062 08:37:59	12-Mar-2065 14:40:34	24-Jun-2065 06:12:56
17.113	1135.78	3551.994	16-May-2062 08:37:59	12-Mar-2065 14:40:34	25-Jun-2065 03:16:50
17.177	1136.65	3551.994	16-May-2062 08:37:59	12-Mar-2065 14:40:34	26-Jun-2065 00:20:43
17.248	1137.53	3454.338	16-May-2062 08:37:59	12-Mar-2065 14:40:34	26-Jun-2065 21:24:36
16.740	1133.14	3551.994	16-May-2062 08:37:59	13-Mar-2065 16:44:53	22-Jun-2065 12:05:10
16.780	1134.02	3551.994	16-May-2062 08:37:59	13-Mar-2065 16:44:53	23-Jun-2065 09:09:03
16.830	1134.90	3551.994	16-May-2062 08:37:59	13-Mar-2065 16:44:53	24-Jun-2065 06:12:56
16.888	1135.78	3551.994	16-May-2062 08:37:59	13-Mar-2065 16:44:53	25-Jun-2065 03:16:50
16.954	1136.65	3454.338	16-May-2062 08:37:59	13-Mar-2065 16:44:53	26-Jun-2065 00:20:43
17.029	1137.53	3454.338	16-May-2062 08:37:59	13-Mar-2065 16:44:53	26-Jun-2065 21:24:36
16.550	1134.02	3551.994	16-May-2062 08:37:59	14-Mar-2065 18:49:12	23-Jun-2065 09:09:03
16.602	1134.90	3551.994	16-May-2062 08:37:59	14-Mar-2065 18:49:12	24-Jun-2065 06:12:56
16.663	1135.78	3551.994	16-May-2062 08:37:59	14-Mar-2065 18:49:12	25-Jun-2065 03:16:50

- Δv: mission cost (km/s);
- TOF = time of flight (days);
- R_p = minimum radius of flyby hyperbolas (km).

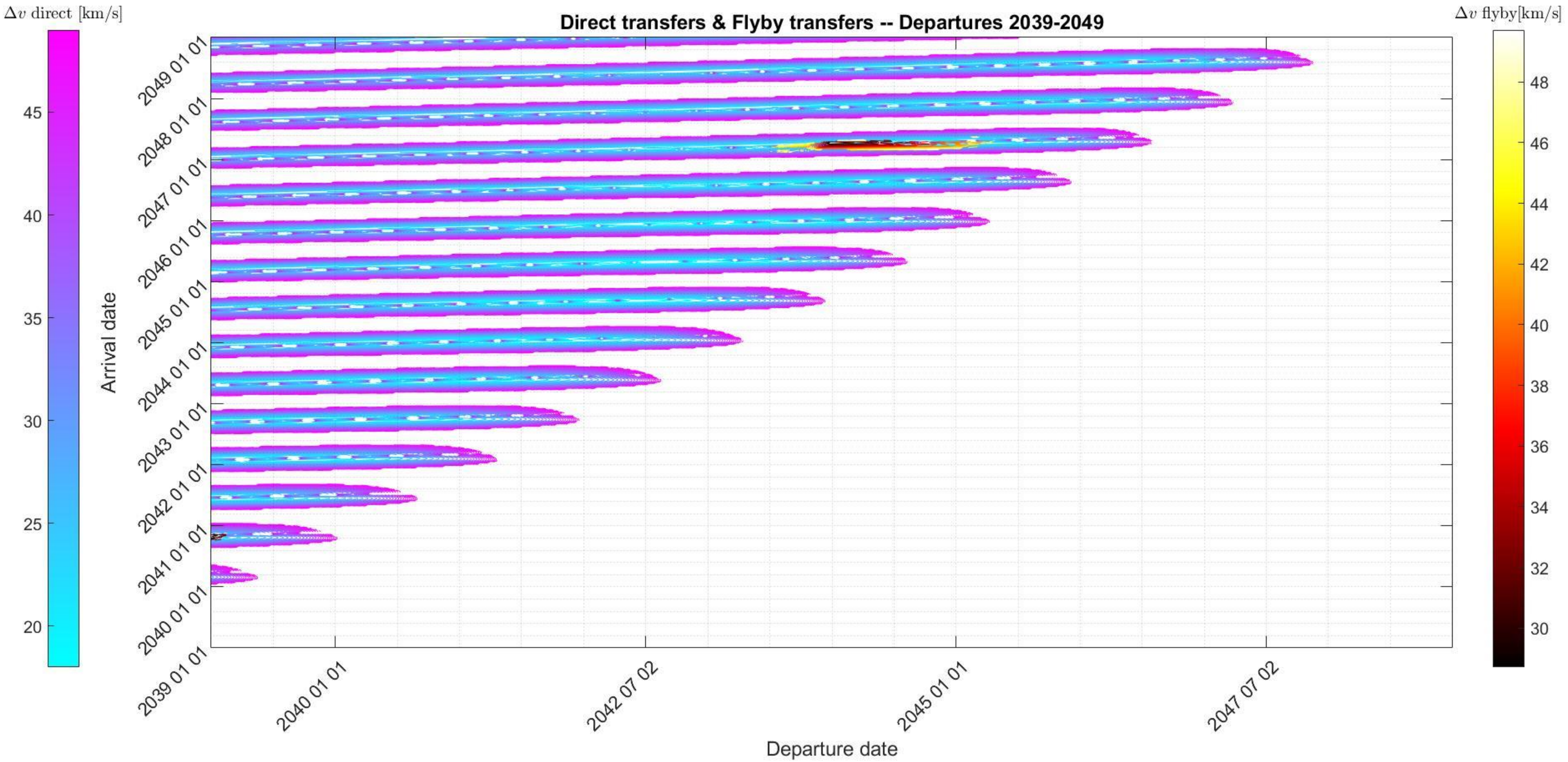
$R_p = R_{Mars} + H_{flyby.}$

R_{Mars} = 3390 km.

GRID ANALYSIS – Porkchop plots

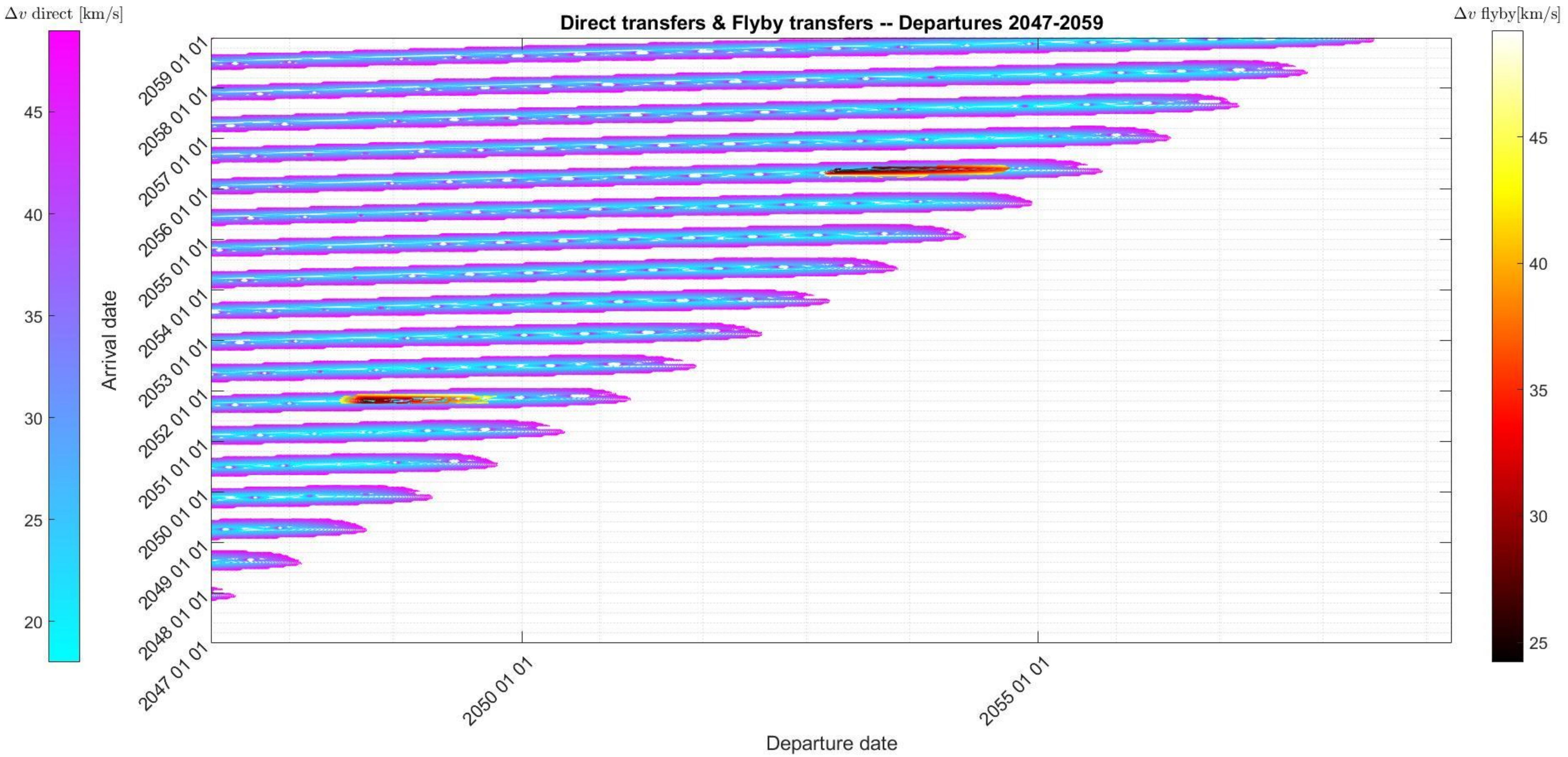


GRID ANALYSIS - Porkchop plots

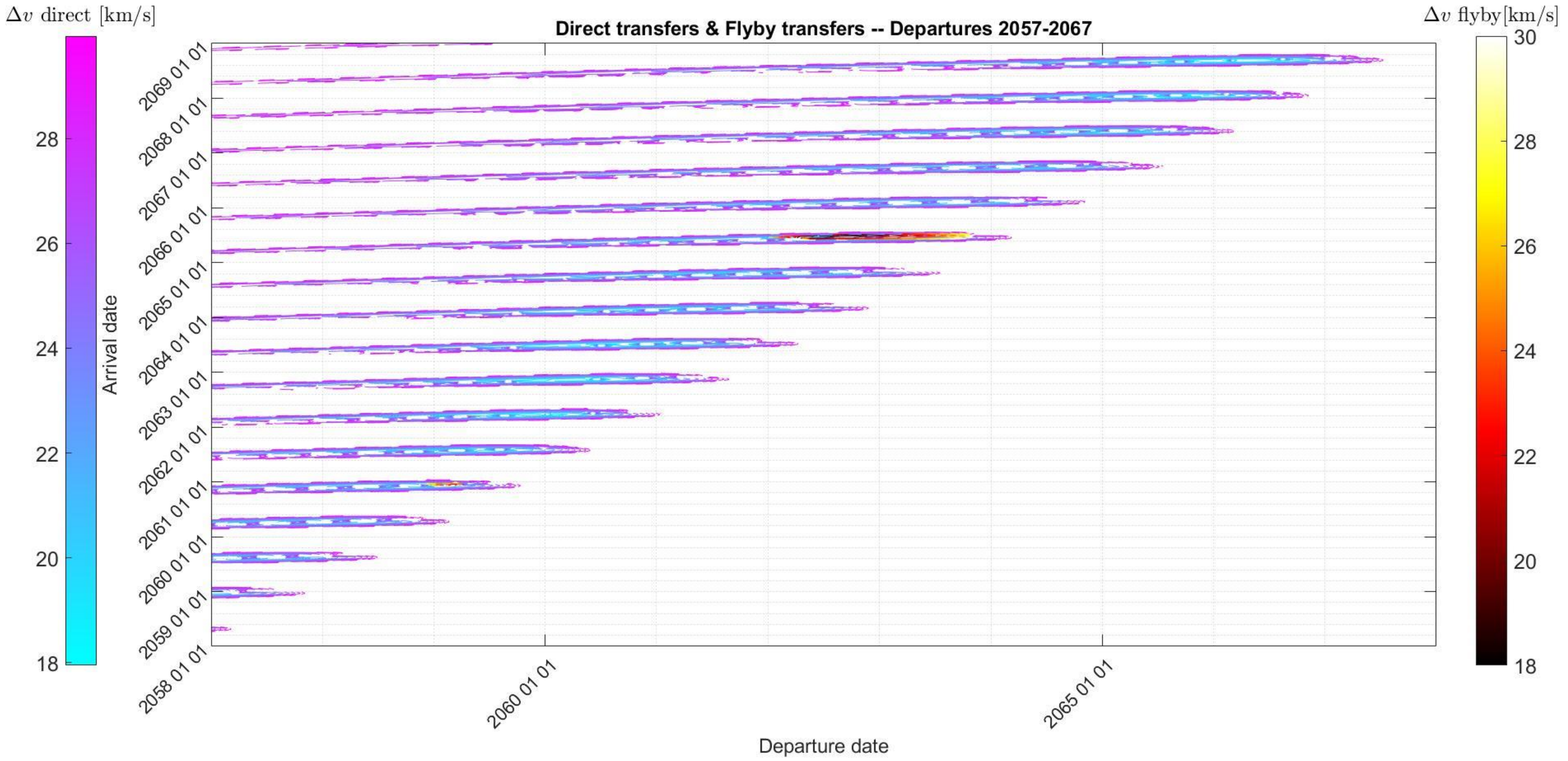


GRID ANALYSIS - Porkchop plots

POLITECNICO MILANO 1863



GRID ANALYSIS - Porkchop plots



GRID ANALYSIS - Best Results

POLITECNICO MILANO 1863

Best solution for each time window:

Departure Window	Δv_{min} [km/s]	Departure date	Flyby date	Arrival date
2037/04 - 2038/12	18.2421	2037/09/22	2040/07/12	2040/10/23
2048/01 - 2049/05	24.7768	2048/07/04	2051/07/04	2051/10/27
2059/01 - 2059/08	20.6513	2059/03/03	2060/08/30	2060/12/11
2062/03 - 2063/02	16.6023	2062/05/16	2065/03/15	2065/06/27

POLITECNICO MILANO 1863

OPTIMIZATION and SOLUTION SELECTION

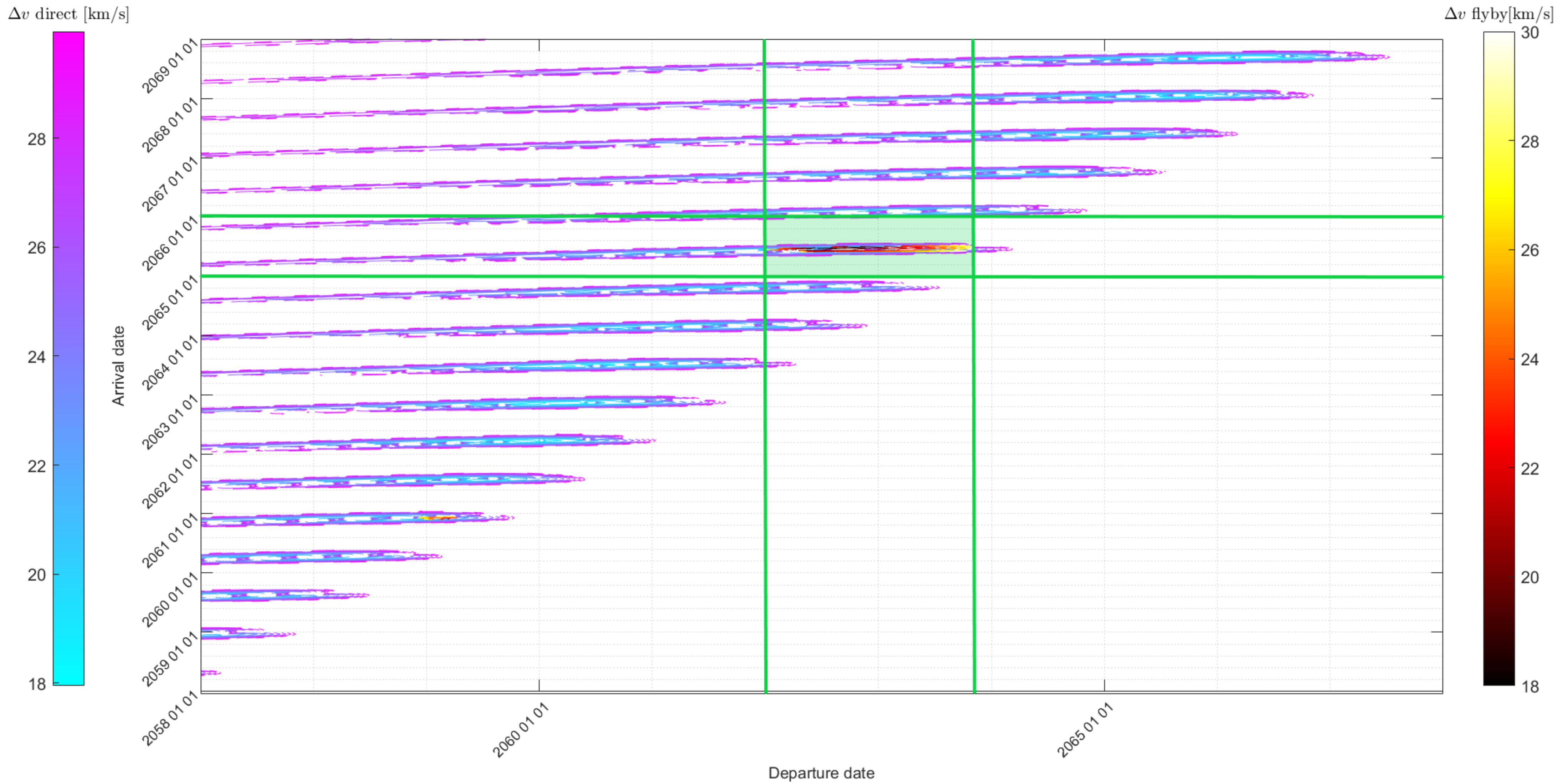
1. > Identification of promising regions from grid search

2. > Finer grid search
   > *GlobalSearch* algorithm
   > Genetic algorithm

3. > Solution selection
   > Transfer analysis

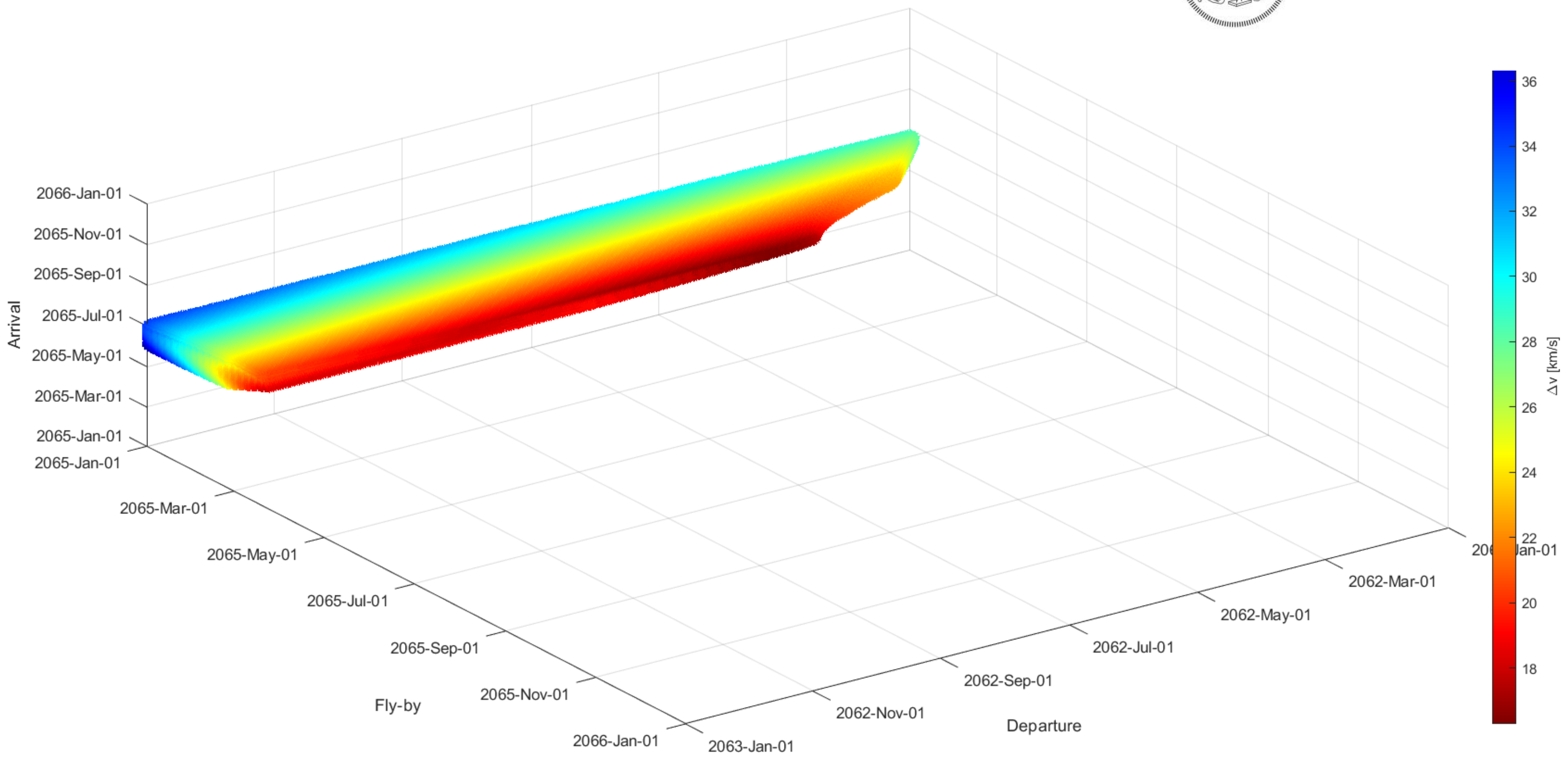
OPTIMIZATION – Grid search refinement

POLITECNICO MILANO 1863



OPTIMIZATION – Grid search refinement

POLITECNICO MILANO 1863



Δv [km/s]	Departure date	Fly-by date	Arrival date
17.855	05-Jul-2037	09-Jul-2040	19-Oct-2040
16.291	06-May-2062	15-Mar-2065	24-Jun-2065

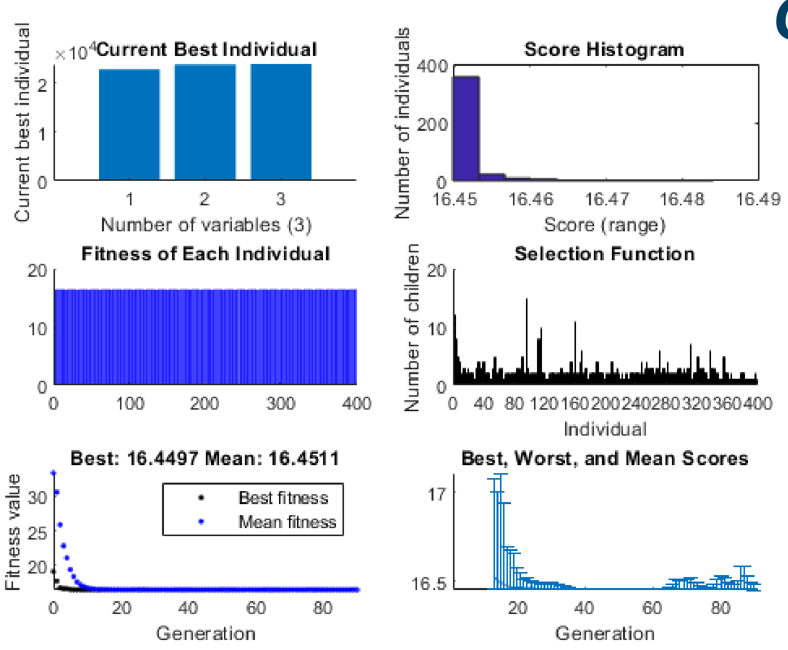
OPTIMIZATION

GlobalSearch results

- The result of the grid search is used as an initial guess

Δv [km/s]	Departure date	Fly-by date	Arrival date
17.473	21-Jan-2037	04-Jul-2040	19-Oct-2040
16.283	06-May-2062	15-Mar-2065	23-Jun-2065

Genetic Algorithm results



- PopulationSize = 300
- MaxGenerations = 90
- CrossoverFraction = 0.9

Δv [km/s]	Departure date	Fly-by date	Arrival date
17.850	09-Jul-2037	08-Jul-2040	18-Oct-2040
16.450	01-Jul-2062	17-Mar-2065	23-Jun-2065

SOLUTION SELECTION

The solution providing the lowest Δv is the one computed with the *GlobalSearch* algorithm

Optimal solution & fly-by characteristics	
Departure date	06-May-2062, 14:34:16
Fly-by date	15-Mar-2065, 17:29:56
Arrival date	23-Jun-2065, 13:27:34
Total Δv [km/s]	16.2833
Δv required at departure [km/s]	5.9303
Overall fly-by Δv [km/s]	2.8400
Δv required for powered fly-by [km/s]	0.000049
Δv required at arrival [km/s]	10.3530
Impact parameter (aiming radius) [km]	3735.6435
Fly-by altitude [km]	162.0937
Time spent inside Mars' SOI [hours]	21.79
SOI radius at fly-by [km]	592176.0555

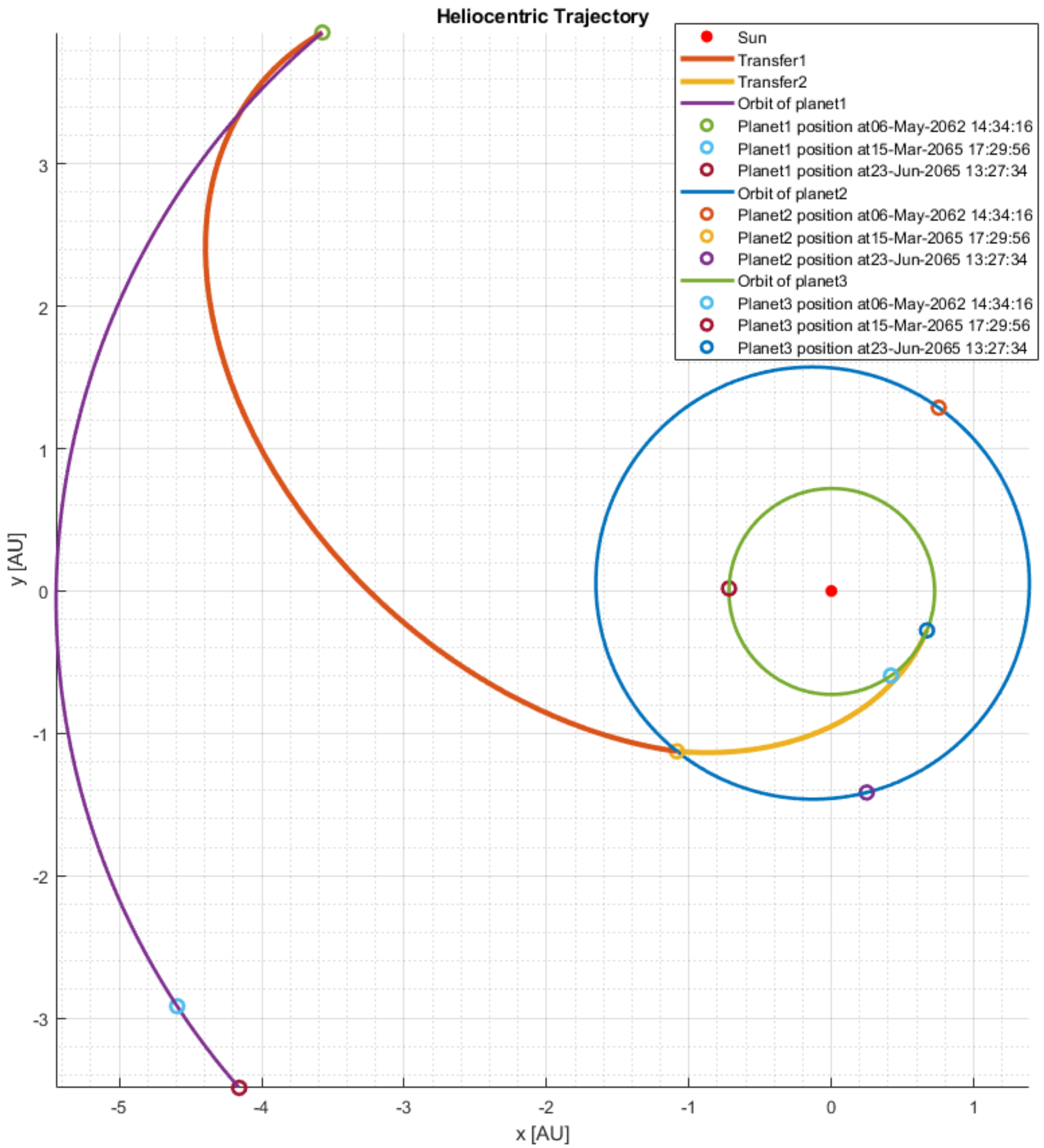
SOLUTION SELECTION - *Analysis*

POLITECNICO MILANO 1863

Pros	Cons
• Low total Δ**v** (lower than direct transfer) • Fast transfer time (~ 3 years) • Low fly-by cost	• Late departure • Low fly-by altitude means smaller safety margin

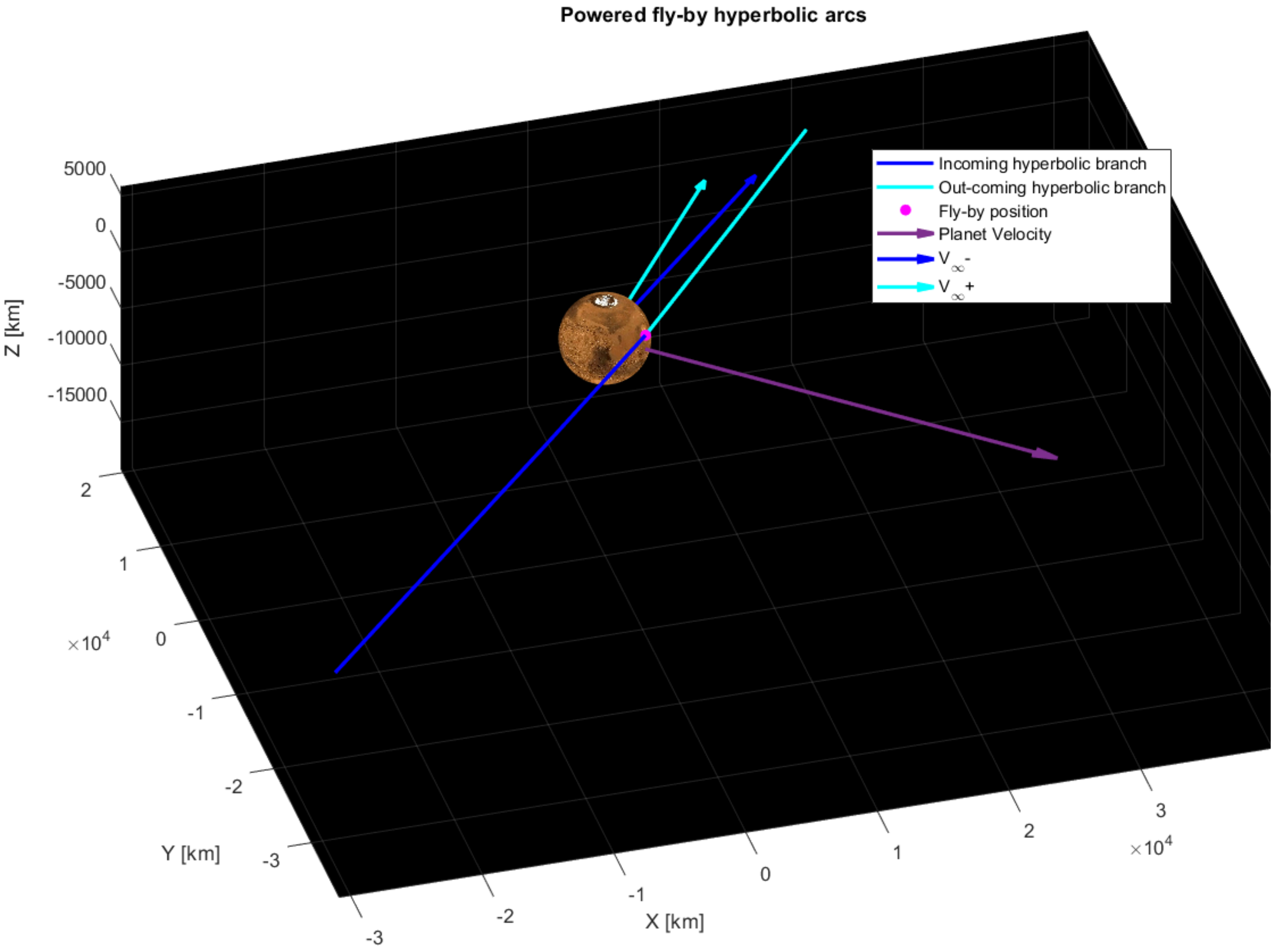
SOLUTION SELECTION - Analysis

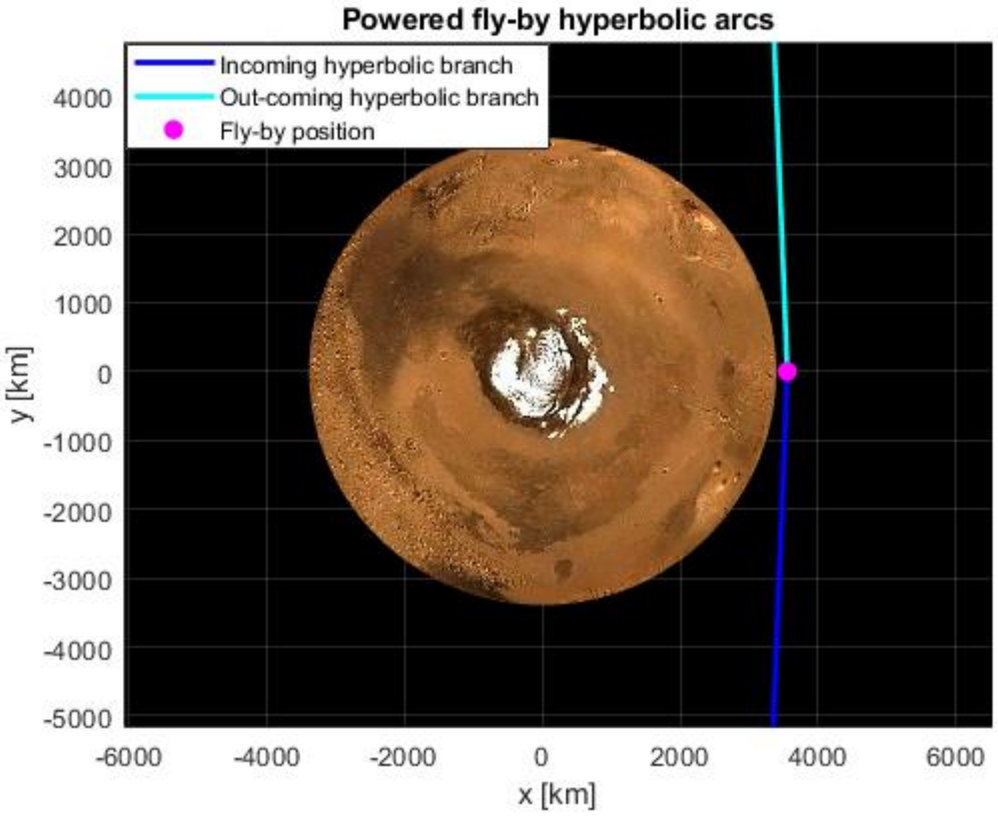
POLITECNICO MILANO 1863



- Similar in shape to a Hohmann transfer
- Very smooth transition between the different trajectories

SOLUTION SELECTION - Analysis

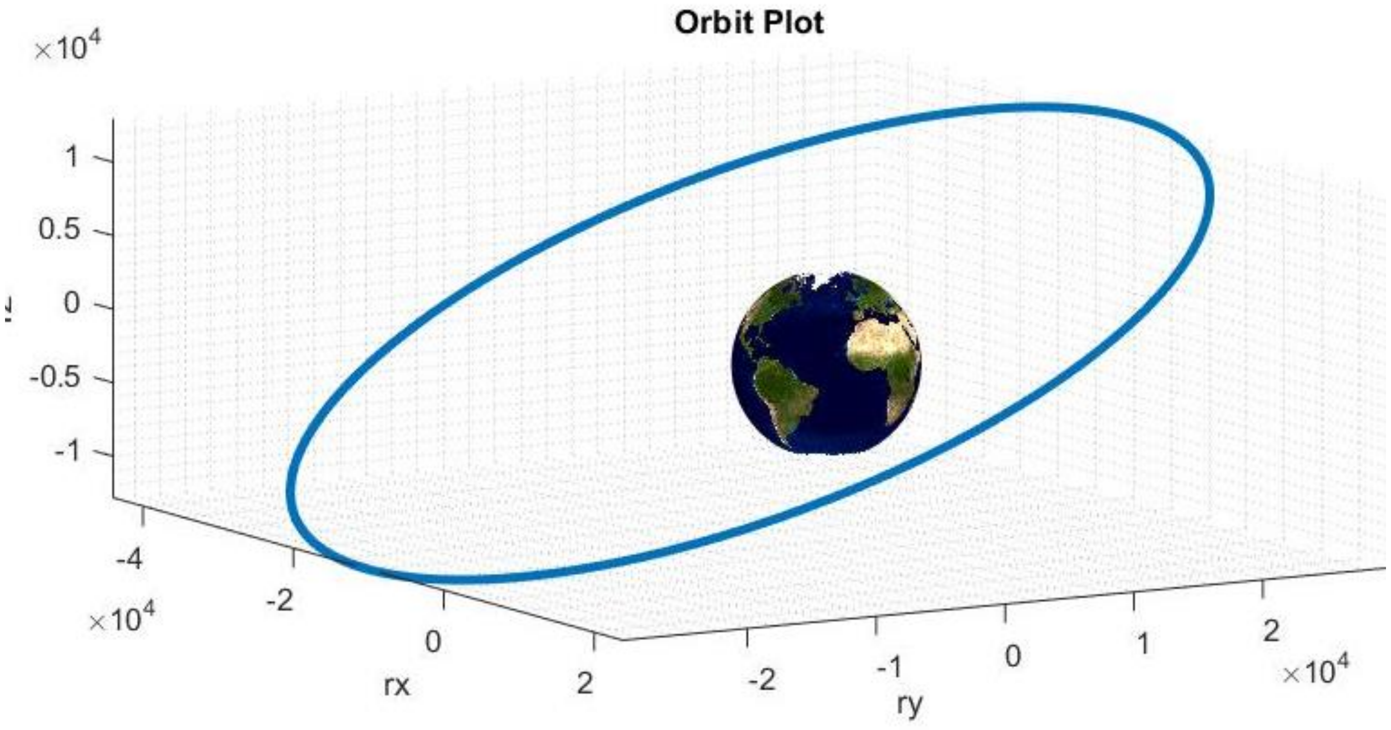




2. Planetary Explorer

MISSION DESCRIPTION

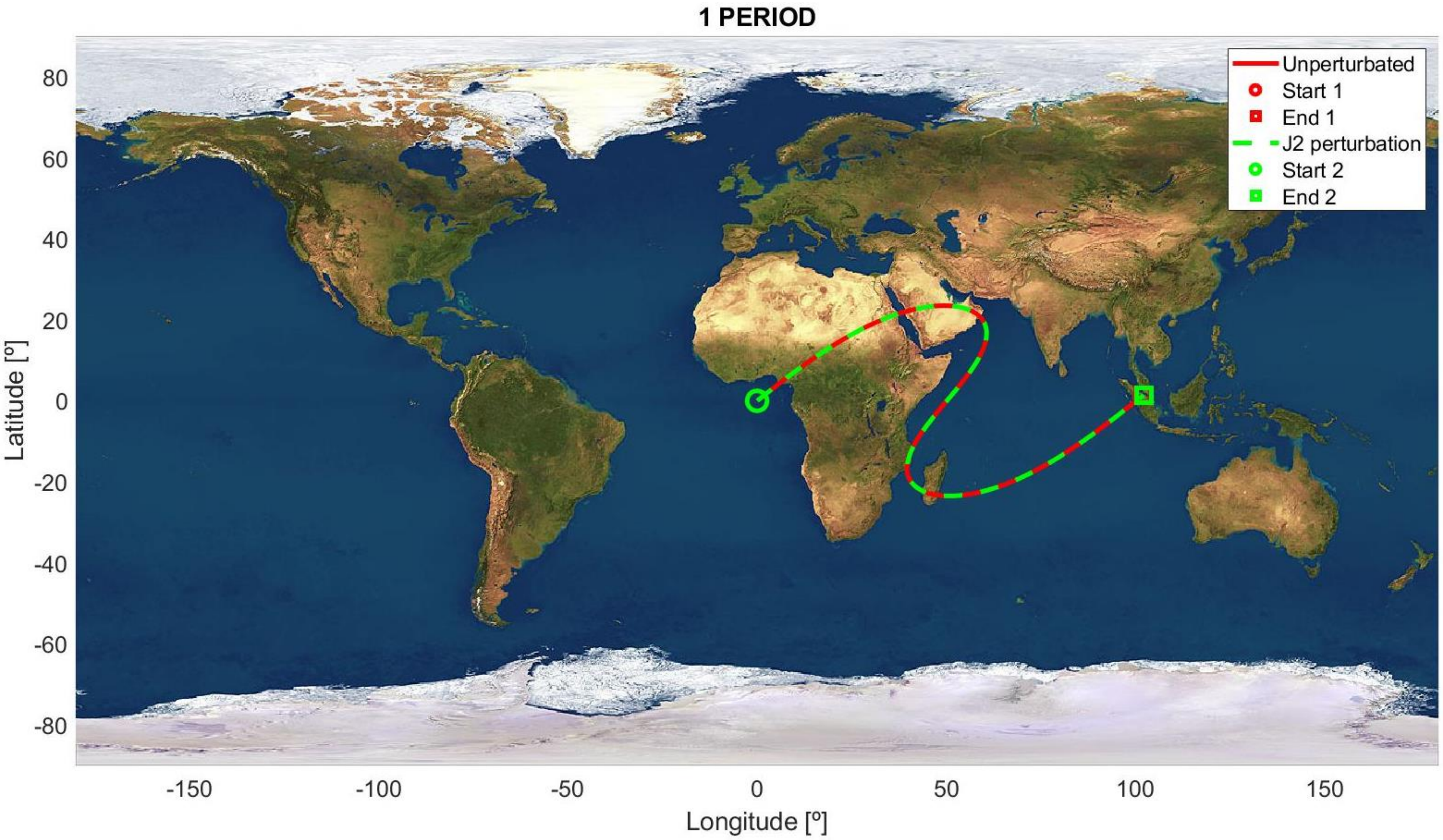
Parameter	Value
a[km] (assigned)	$3.3923 \cdot 10^4$
e (assigned)	0.2991
i[deg] (assigned)	23.2497
Perturbation (assigned)	Moon
Ω[deg]	0
ω[deg]	0
θ_0[deg]	0



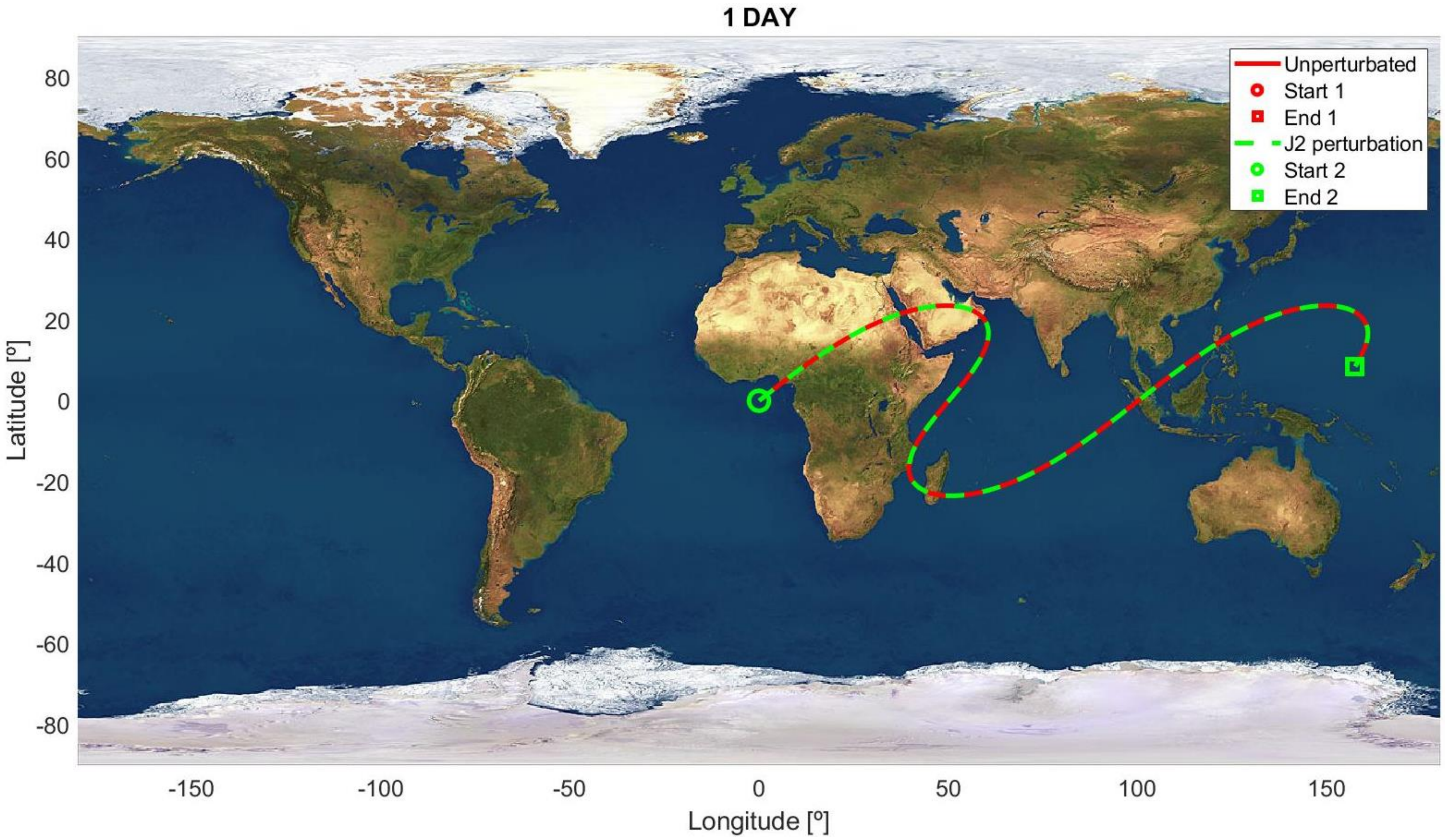
GROUND TRACK DESIGN – 1 PERIOD

$$T = 17\,h\,16\,\text{min}\,20.2\,s$$

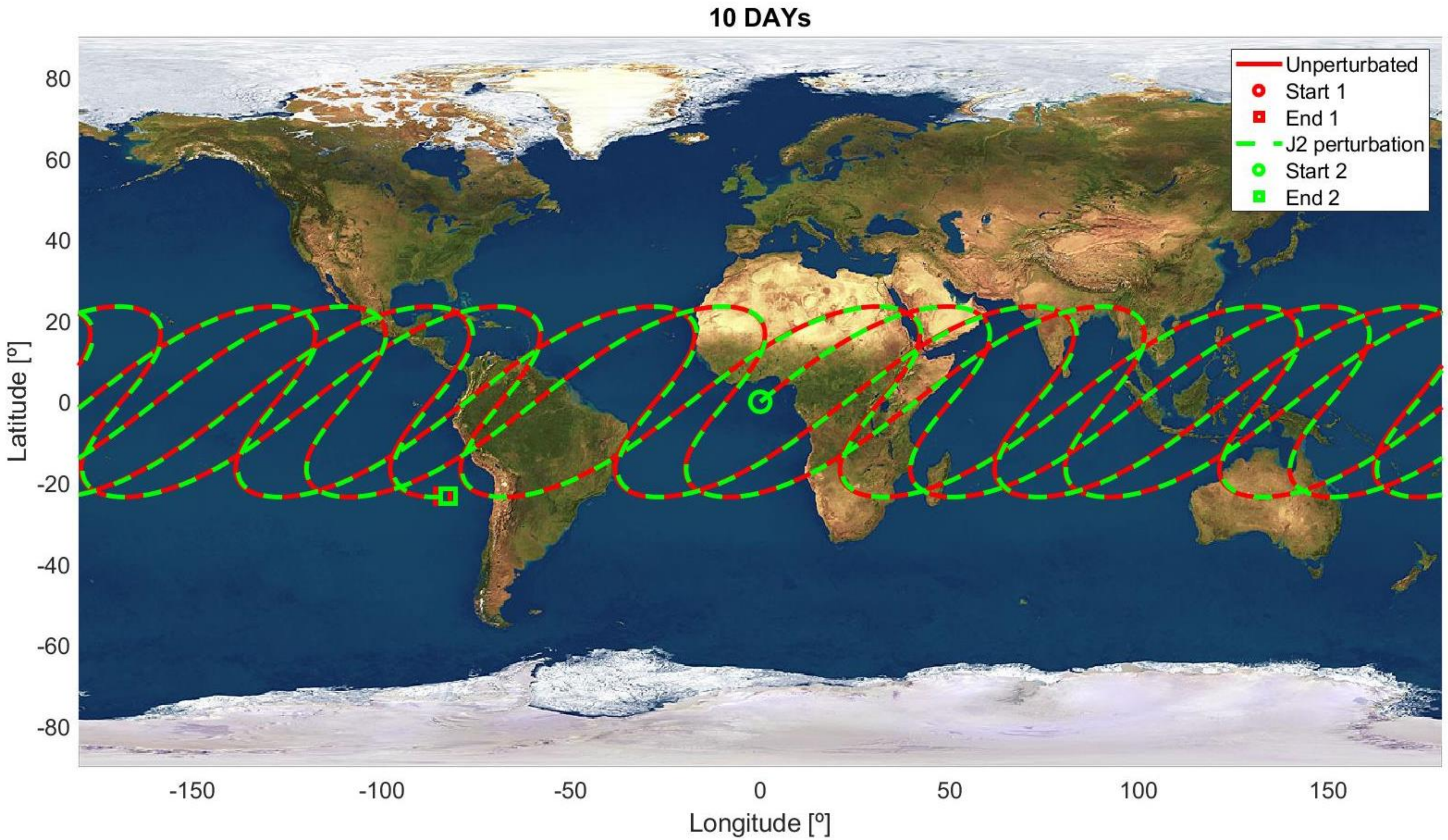
$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$



GROUND TRACK DESIGN – 1 DAY



GROUND TRACK DESIGN – 10 DAYS



GROUND TRACK DESIGN – REPEATING GROUND TRACK

$$(k = 7, m = 5, T_{day} = 23\,h, 56\,min, 4.0916\,s)$$

a) No perturbation orbit design

$$a_{RGT} = \left\{ \left(\frac{m}{k\omega_E} \right)^2 \mu \right\}^{1/3}$$

$$a_{RGT} = 33692\,km$$

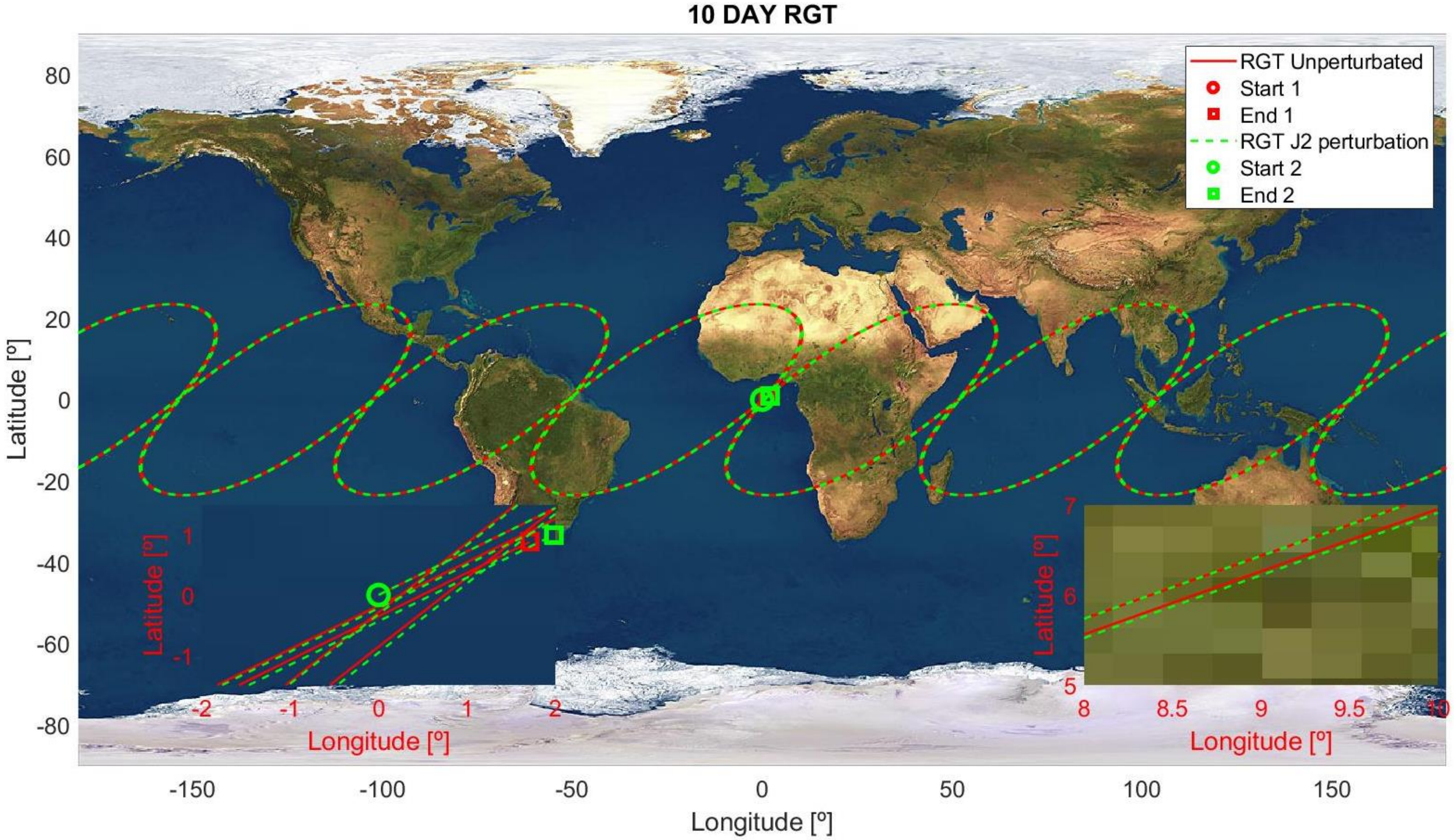
b) J_2 accounting design

$$n = \sqrt{\frac{\mu}{a_{RGT}^3}} = \frac{k(\omega_E - \dot{\omega})}{m} - \dot{\omega} - \dot{M}_0$$

$$\begin{cases} \dot{\Omega} = & -\left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \cos i \\ \dot{\omega} = & -\left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \left(\frac{5}{2} \sin^2 i - 2 \right) \\ \dot{M}_0 = & -\left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \left(1 - \frac{3}{2} \sin^2 i \right) \end{cases}$$

$$a_{RGT\,J2} = 33691\,km$$

REPEATING GROUND TRACK DESIGN – 10 DAYS

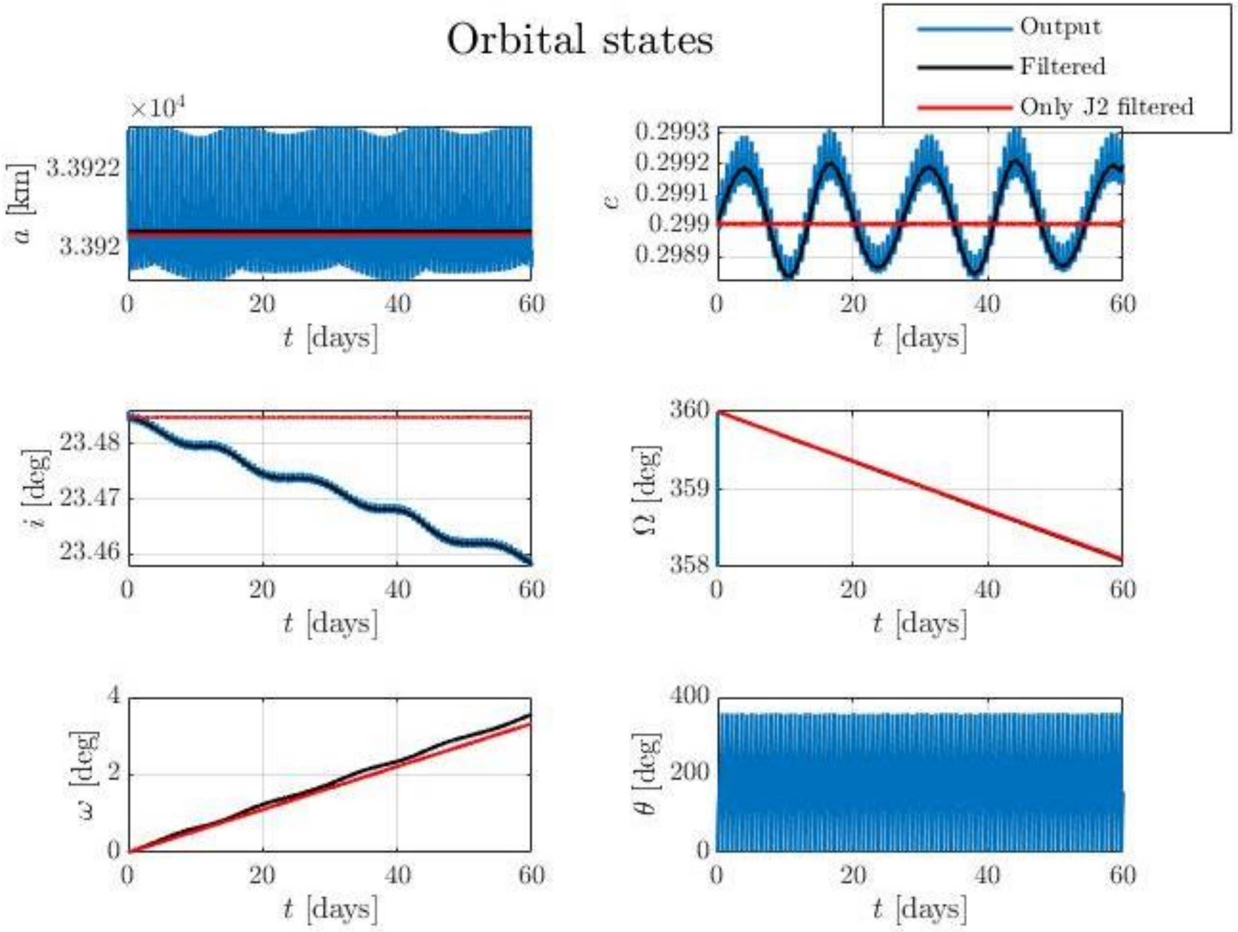


MOON PERTURBATION – 60 DAYS

$$\mathbf{a} = \frac{\mu_E}{r^3}\mathbf{r} + \frac{\mu_M}{|\, r_M - r \,|^3}(\mathbf{r_M} - \mathbf{r}) - \frac{\mu_M}{r_M^3}\mathbf{r_M}$$

$$\boxed{r << r_M}$$

$$\mathbf{a_M} = \frac{\mu_M}{r_M^3}\left(-\mathbf{r} + 3\frac{\mathbf{r_M}\cdot\mathbf{r}}{r_M^2}\mathbf{r_M}\right)$$



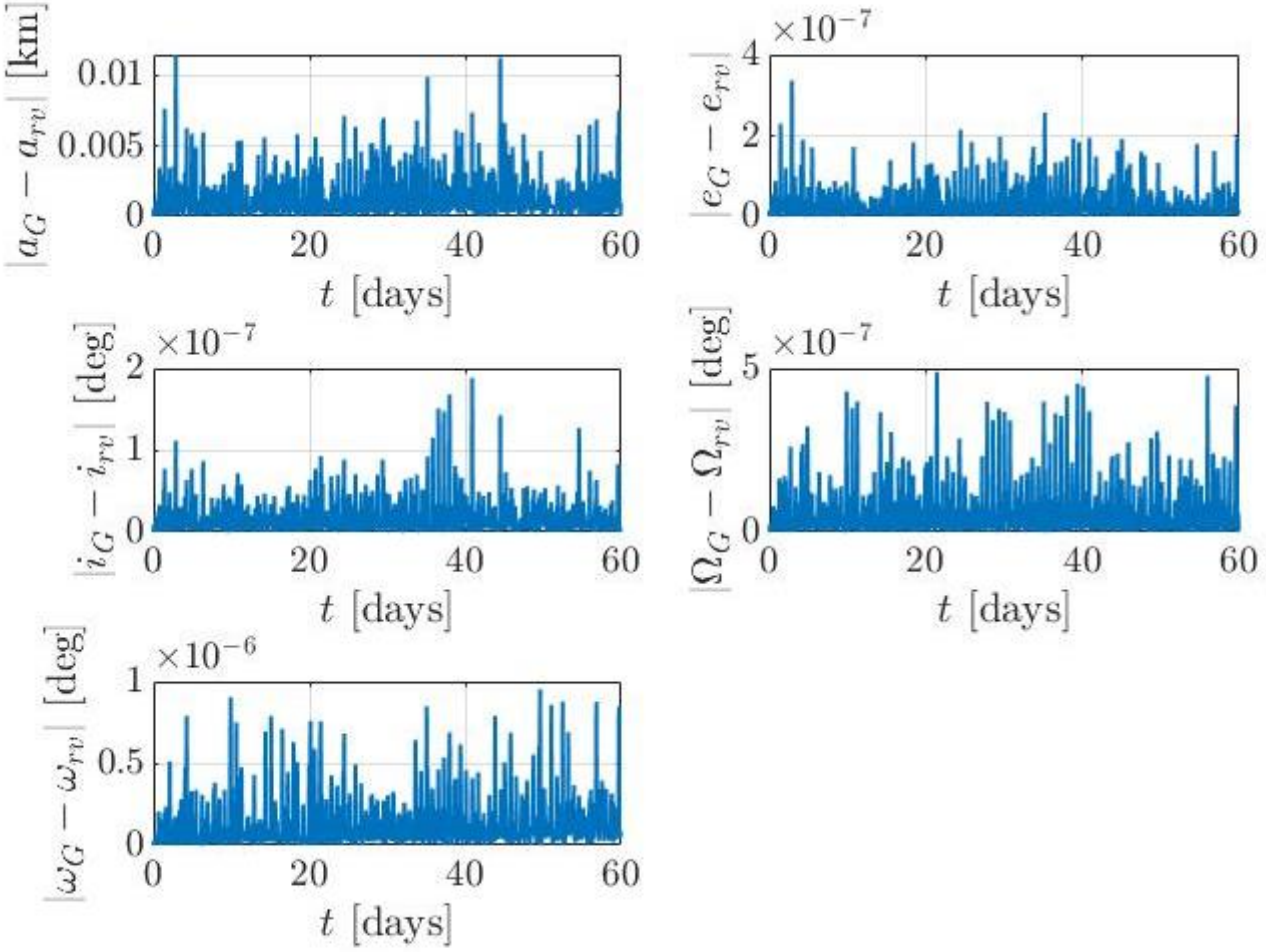
ORBITAL ELEMENTS COMPARISON - 60 DAYS

KEPLER

$$t_{CPU} = 0.3972\ s$$
$$steps = 11000$$

GAUSS

$$t_{CPU} = 0.9240\ s$$
$$steps = 6400$$

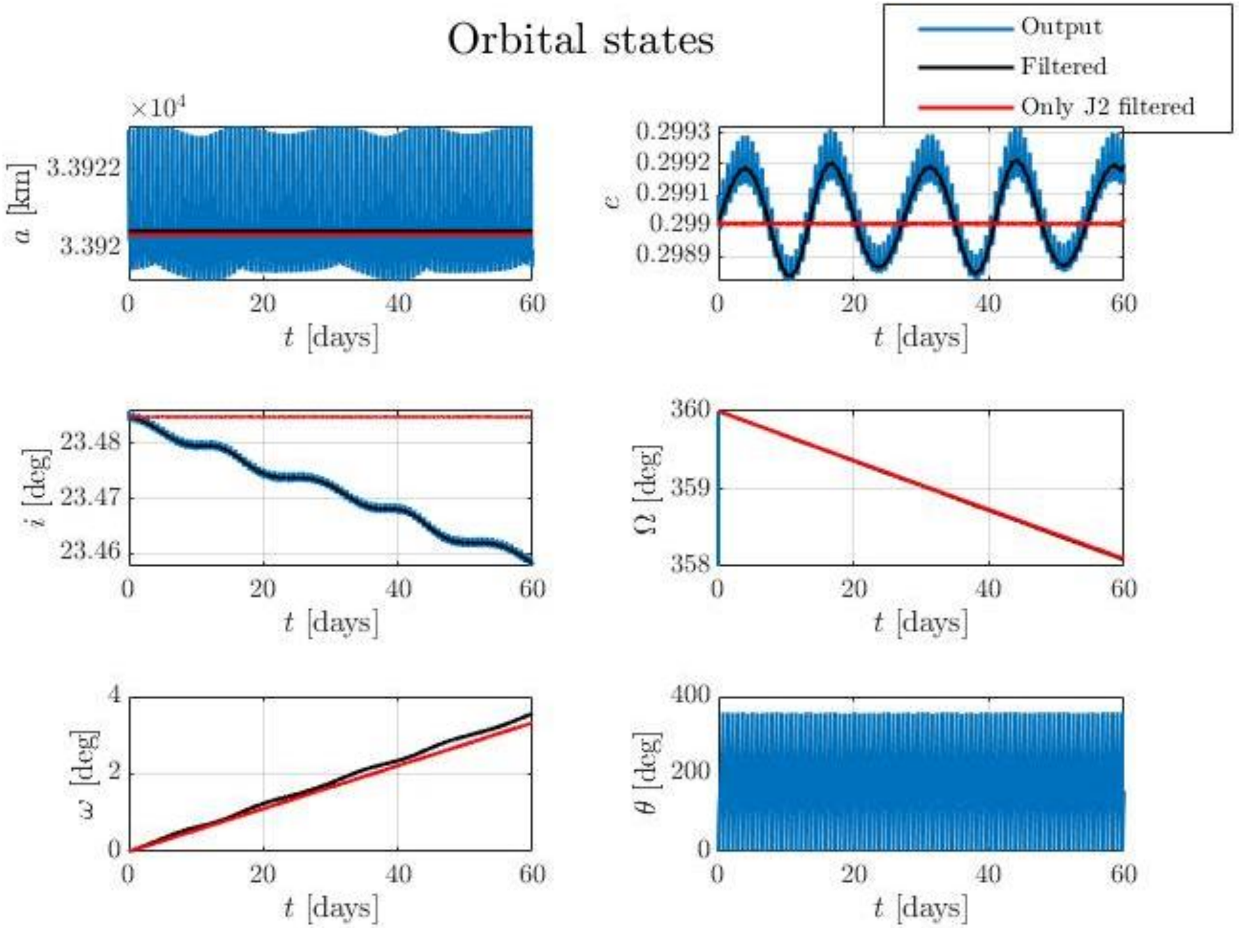


ORBITAL ELEMENTS EVOLUTION – 60 DAYS

Filter used: moving average (low-pass) with cut-off $= \frac{1}{T}$

J2 effect : $\begin{cases} \Omega \\ \omega \end{cases}$

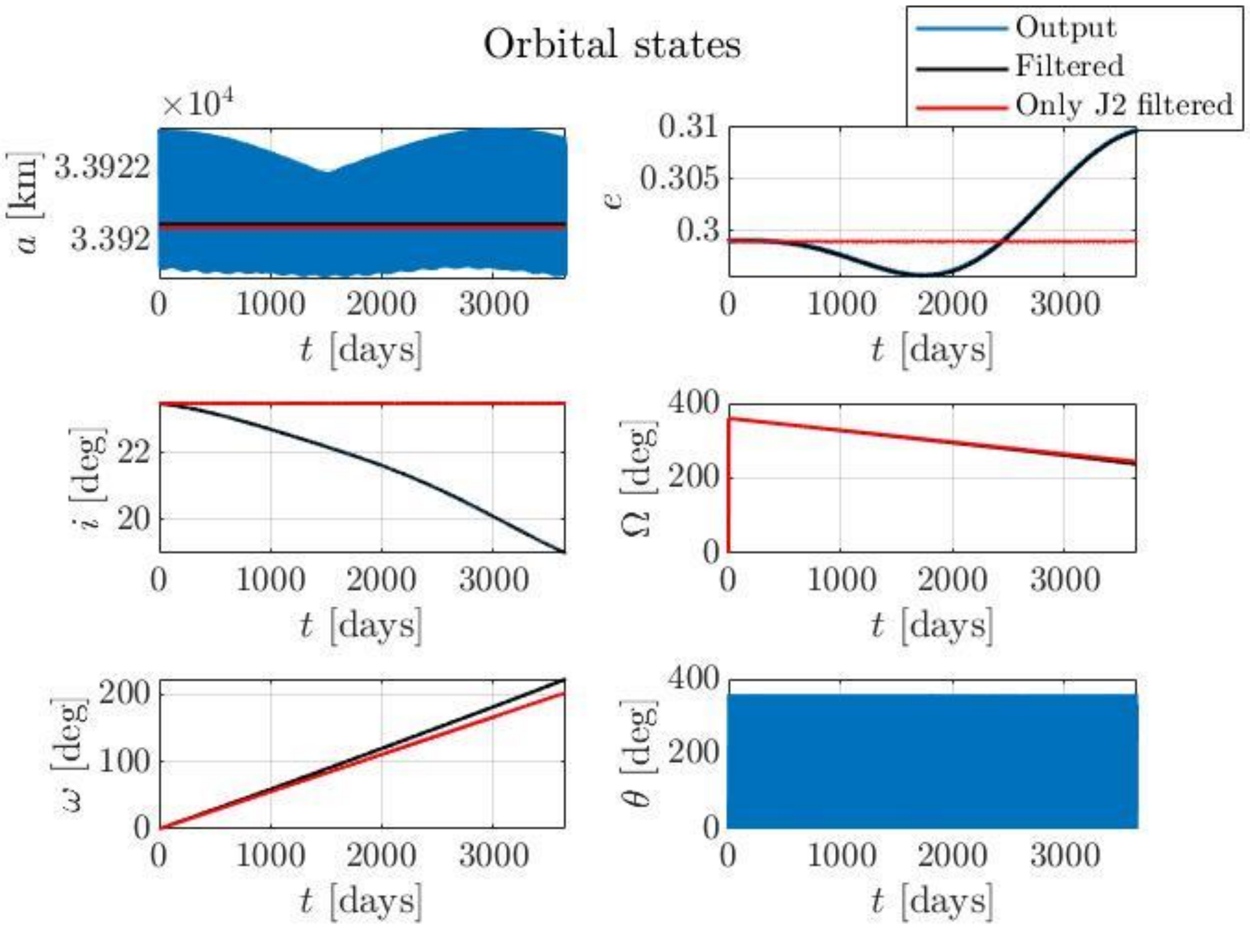
Moon – 60 days effect: $\begin{cases} e \\ i \\ \omega \end{cases}$



ORBITAL ELEMENTS EVOLUTION-Secular effect

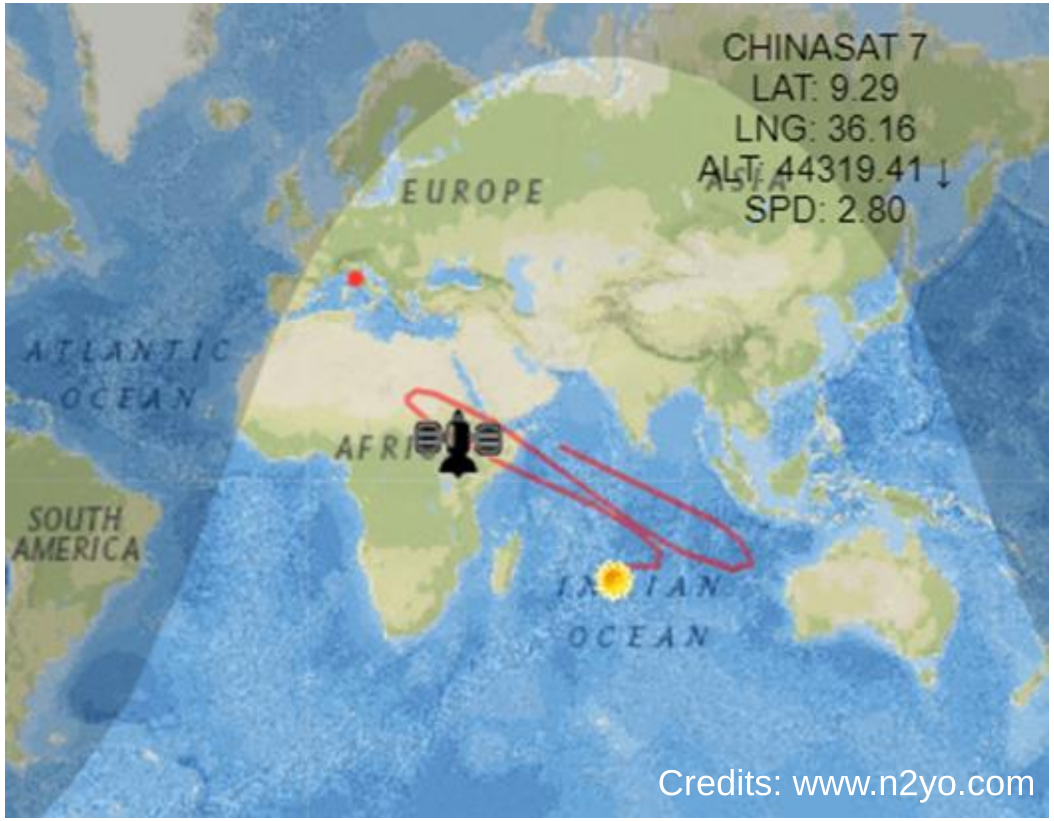
POLITECNICO MILANO 1863

Moon – Disposal time effects : $\begin{cases} \omega, e, i \\ \Omega \end{cases}$

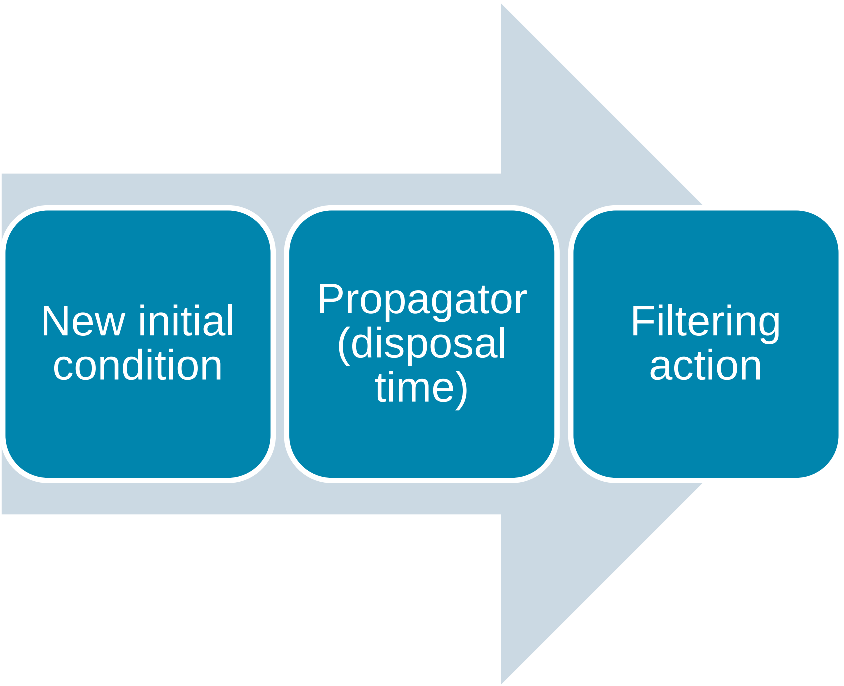


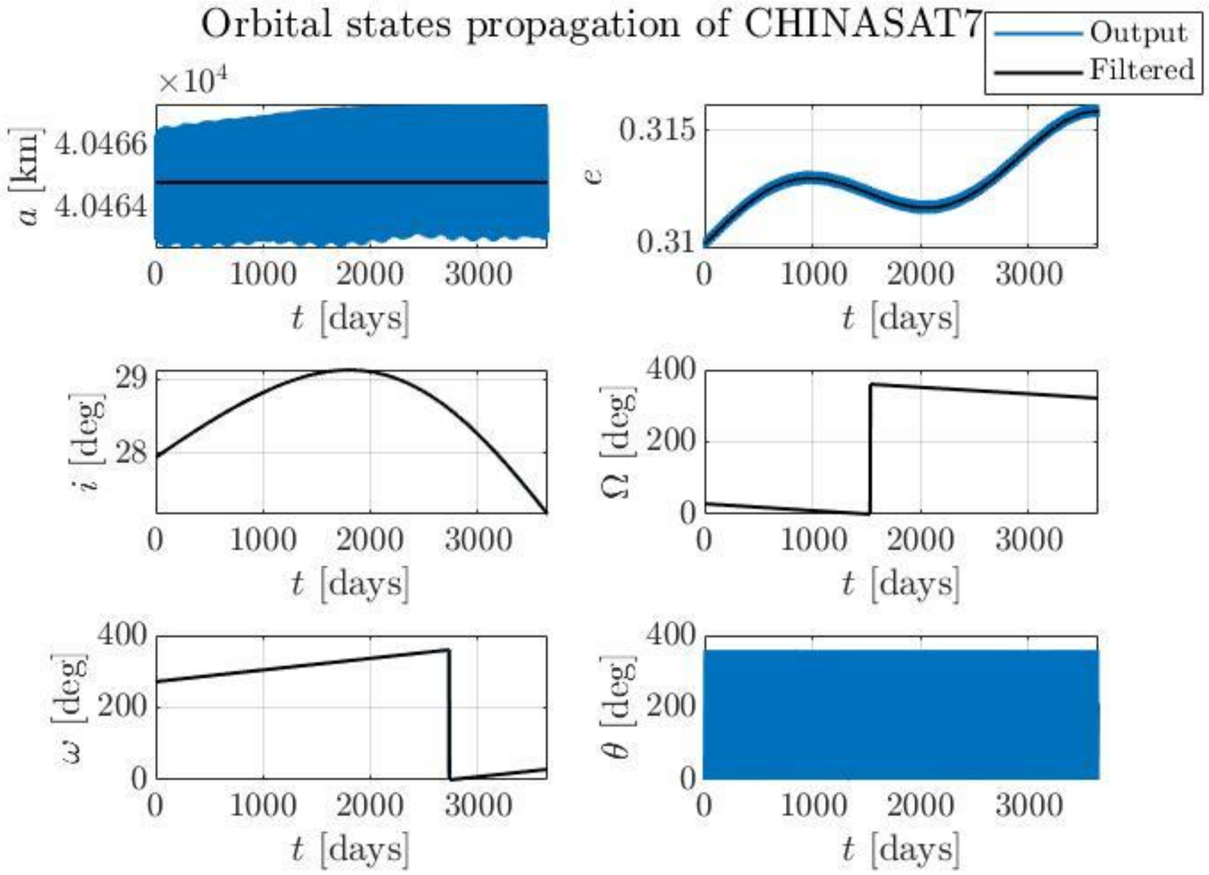
COMPARISON WITH REAL DATA

CHINASAT7
Geostationary satellite
NORAD ID: 24282
Perigee: 21,724.5 km
Apogee: 46,462.5 km
Inclination: 20.3 °
Period: 1,350.1 minutes
Semi major axis: 40464 km
Launch date: August 18,1996
Source: People's Republic of China (PRC)
Launch site: Xichang Space Center, China (XSC)



COMPARISON WITH REAL DATA-Propagation

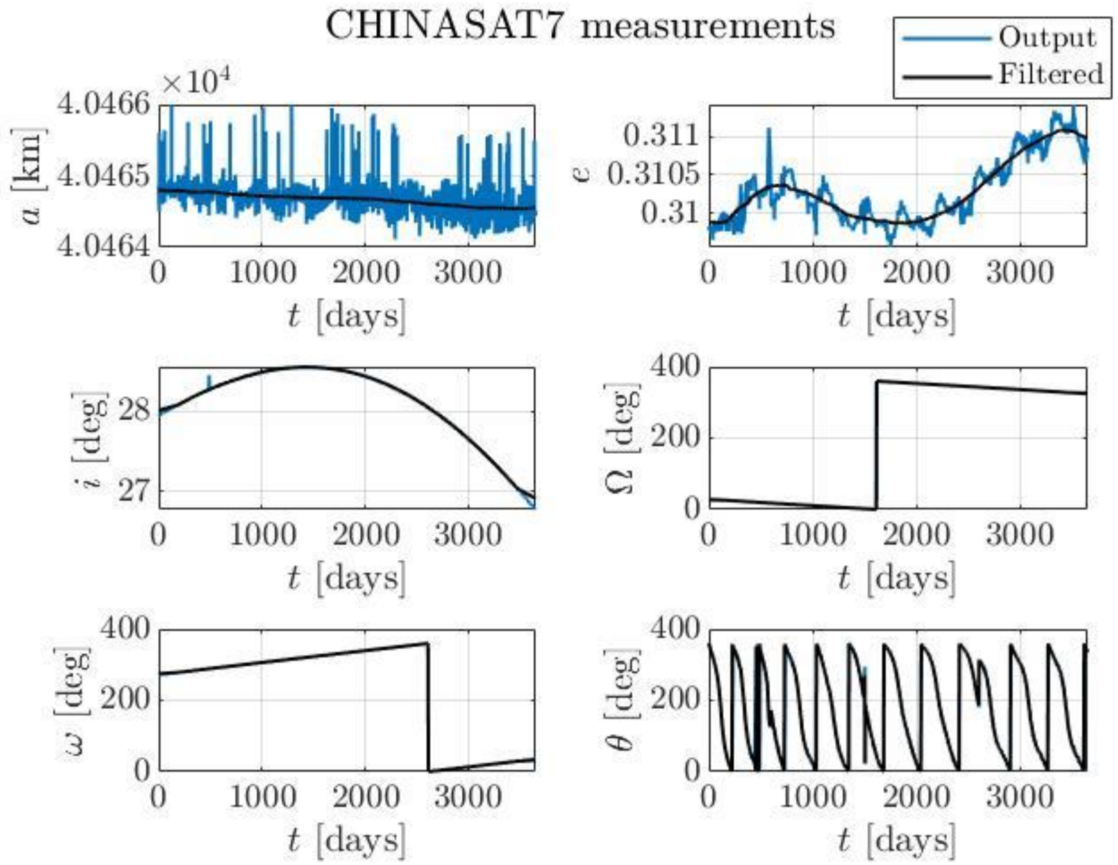




COMPARISON WITH REAL DATA-TLE Data

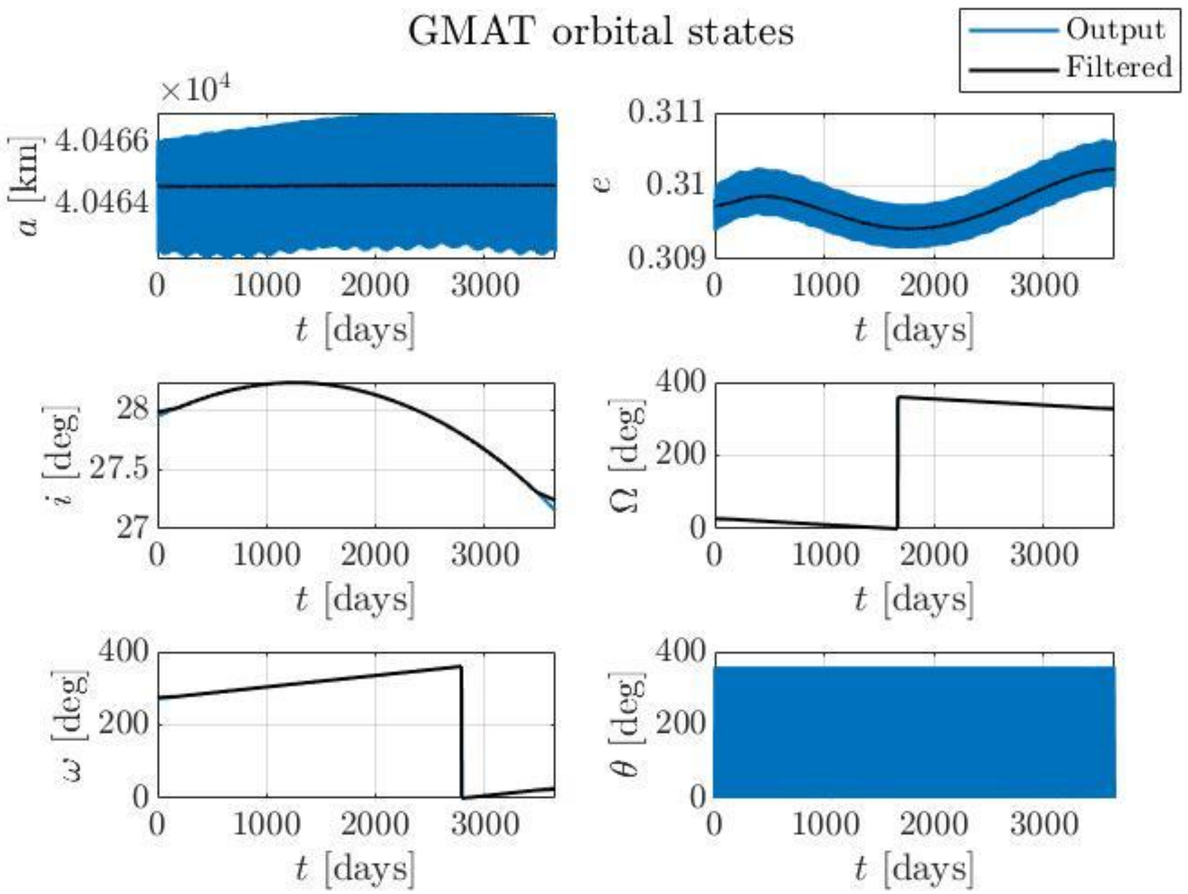
- Reconstruction of data from Two-Line Element

- Adding moving average filter based on # points



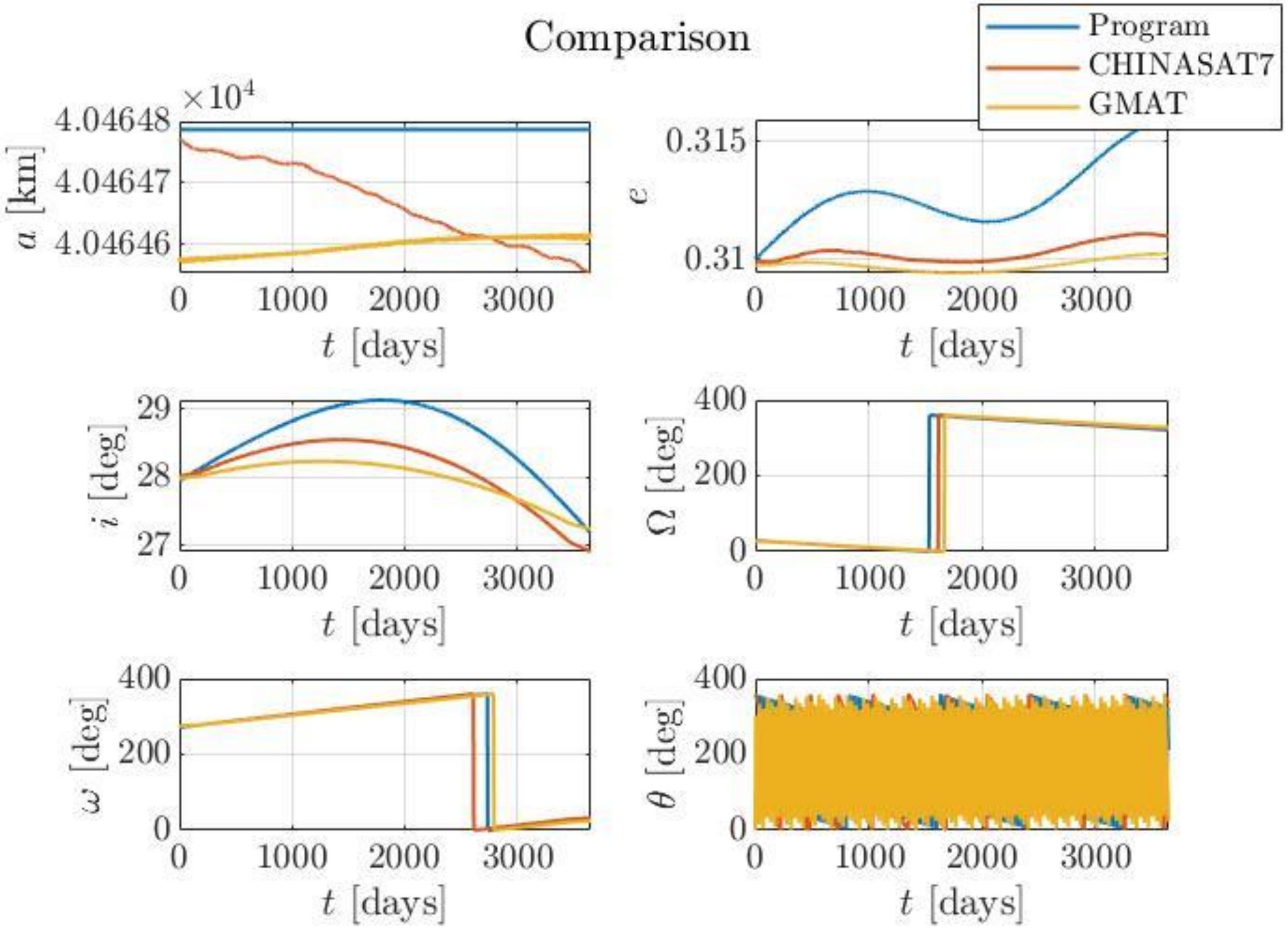
COMPARISON WITH REAL DATA-GMAT

Additional comparison with GMAT results



COMPARISON WITH REAL DATA-Comparison

- *Numerical error in semi_major axis*
- *Quantitative error on eccentricity and inclination*
- *Similarities on RAAN and argument of perigee*



Credits: Airbus