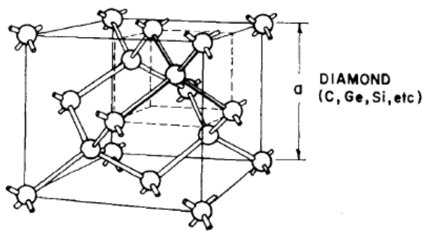
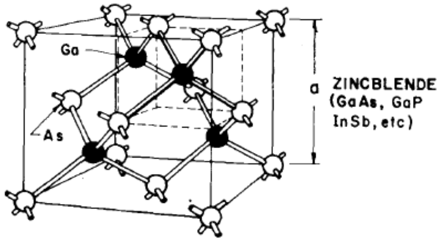
618327-2560

Physics of Electronic Materials and Devices

Dr. Orrathai Watcharakitchakorn

Lecture 4





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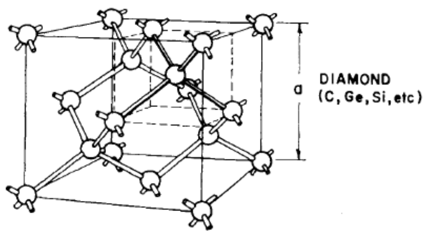
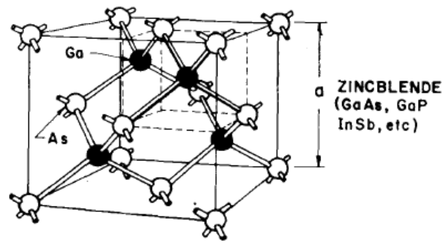




Diffusion process

กระบวนการแพร่ osmosis

แพร่จากความเข้มข้นมาก:สูง → ความเข้มข้นน้อย:ต่ำ

เกิดกระแสการแพร่

- The drift current is the transport of carriers when an electric field is applied.

- There is another important carrier current called "**diffusion current**". (กระแสการแพร่)

- This diffusion current happens as there is a variation of carrier concentration in the material.

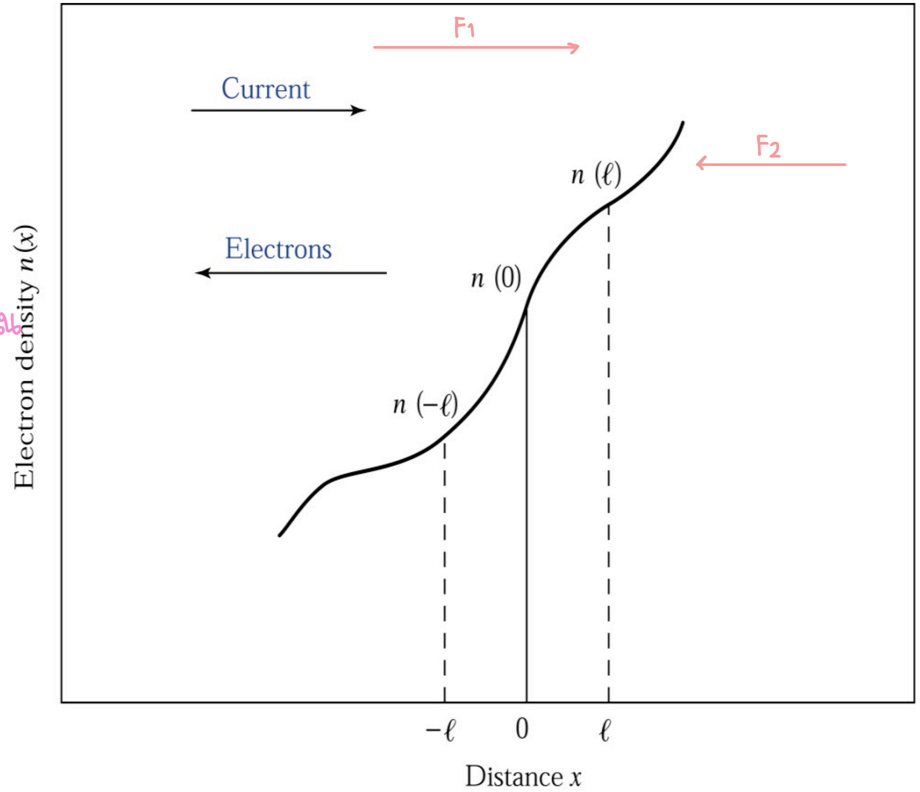
- The carriers will move from a region of high concentration to a region of low concentration.

- This kind of movement is called "**diffusion process**".
  (กระบวนการแพร่)

Diffusion process

กระแสพิจารณาจาก

ความหนาแน่น
ของพาหะ



- An electron density varies in the x-direction under uniform temperature.

  อุณหภูมิกระจายคงที่สม่ำเสมอ

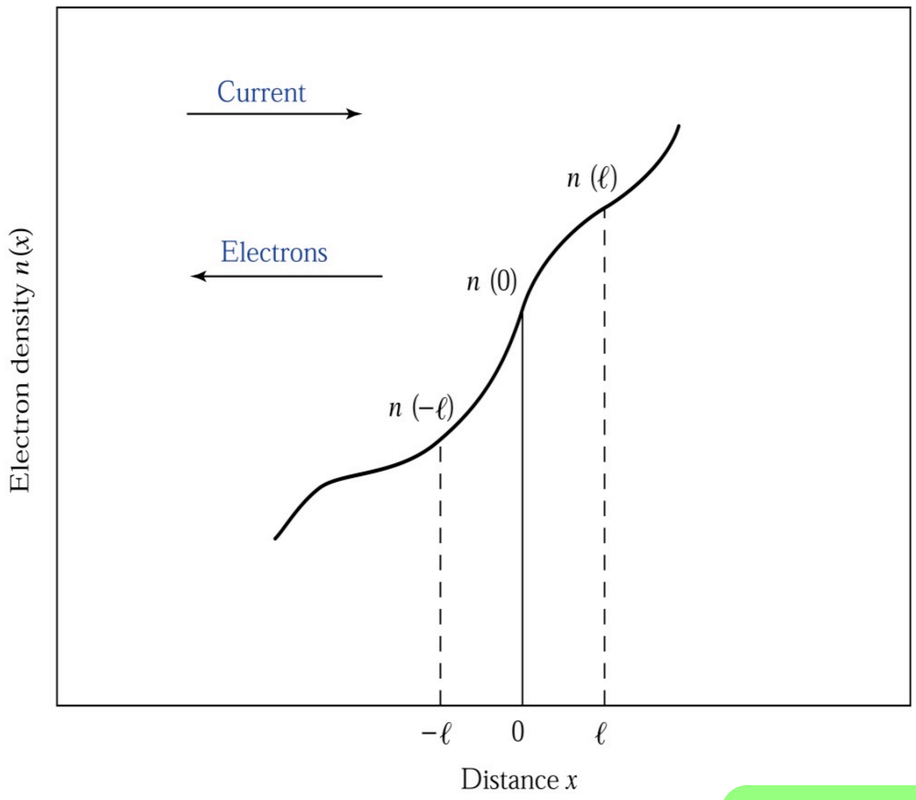
- The average thermal energy of electrons will not vary with x, but the electron density n(x)

  ความเร็วเฉลี่ยเนื่องจากความร้อนของอิเล็กตรอนจะเป็นอิสระกับ x

- The electrons have an average thermal velocity υ_{th} and a mean free path l.

- เพื่อที่จะคำนวณกระแส เราจะต้องกำหนดอัตราไหลสุทธิของอิเล็กตรอนต่อหนึ่งหน่วยเวลาต่อหนึ่งหน่วยพื้นที่ข้ามระนาบที่ $x=0$ โดยที่ l จากรูปคือ mean free path ของอิเล็กตรอน

Diffusion process



- The electron at $x = -l$ have an equal chances of moving left or right.
- The average of electron flow per unit area F_1 of electron crossing plane $x = 0$ from the left is expressed as

  อัตราของการไหลอิเล็กตรอน F_1 ใน

  ทิศทาง $+x$ ที่ $x = o$ (จากทางซ้ายมือ)คือ

$$F_1 = \frac{0.5n(-l)\cdot l}{\tau_c} = 0.5n(-l)\cdot v_{th} \qquad (1.1)$$

Diffusion process

- Likewise, the average of electron flow per unit area F_2 of electron at $x = l$ crossing plan at $x = 0$ from the right is

เช่นเดียวกัน อัตราของการไหลอิเล็กตรอน F_2 ที่ $x = l$ (จากทางขวามือ) คือ

$$F_2 = 0.5n(l) \cdot v_{th} \tag{1.2}$$

- Then, the net rate of electron flow from left to right is given by
- อัตราสุทธิของการไหลอิเล็กตรอนเขียนได้เป็น

$$F = F_1 - F_2 = 0.5v_{th}[n(-l) - n(l)]$$

$$= 0.5v_{th}\left\{\left[n(0) - l\frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right\}$$

อัตราการไหลสุทธิ

$$F = -v_{th} l \frac{dn}{dx} \tag{1.3}$$

Diffusion process

- Therefore, the net rate F can be written as

$$\boxed{F = -D_n \frac{dn}{dx}} \qquad (2)$$

อัตราการไหลสุทธิ

where $D_n = v_{th}.l$ = the diffusion coefficient (diffusivity)

(สัมประสิทธิ์การแพร่อิเล็กตรอน; cm^2/sec)

- A current density caused by the electron diffusion is given by

$$J_e = -qF = qD_n \frac{dn}{dx} \qquad (3)$$

$\frac{dn}{dx}$ คือ เกรเดียนต์ความหนาแน่นอิเล็กตรอน

Example 1

- An n-type semiconductor at T = 300 K, the electron concentration varies linearly from 1×10^{18} to 7×10^{17} cm^{-3} over a distance of 0.1 cm. Calculate the diffusion current density if the electron diffusion coefficient D_n is 22.5 cm^2/s.

เปลี่ยนไปเป็นเชิงเส้นจาก 1×10^{18} เป็น 7×10^{17}

ระยะทาง

$0 \longrightarrow 0.1$

$\therefore\ 1\times10^{18} \qquad 7\times10^{17}\ cm^{-3}$

1×10^{18}

$$J = qD_n \frac{dn}{dx}$$

$$J = 1.6\times10^{-19} \times 22.5 \times \left[\frac{(1\times10^{18})-(7\times10^{17})}{0.1-0} \right]$$

$$J = 10.8\ A/cm^2$$

Einstein Relation

- From the conservation of energy for one-dimensional case

$$\frac{1}{2}mv_{th}^2 = \frac{1}{2}kT \qquad (4)$$

we can write

$$D_n = v_{th} l$$

$$= v_{th}\left(v_{th}\tau_c\right)$$

$$= v_{th}^2\left(\frac{\mu_e m_e}{q}\right)$$

$$= \left(\frac{kT}{\cancel{m_e}}\right)\left(\frac{\mu_e \cancel{m_e}}{q}\right)$$

$V_{th}: \sqrt{\frac{kT}{me}}$

$V_{th}^2: \left(\sqrt{\frac{kT}{me}}\right)^2$

$V_{th}^2: \frac{kT}{me}$

Einstein Relation

- Therefore,

สัมประสิทธิ์การแพร่

$$D_n = \left(\frac{kT}{q}\right)\mu_e \qquad (5)$$

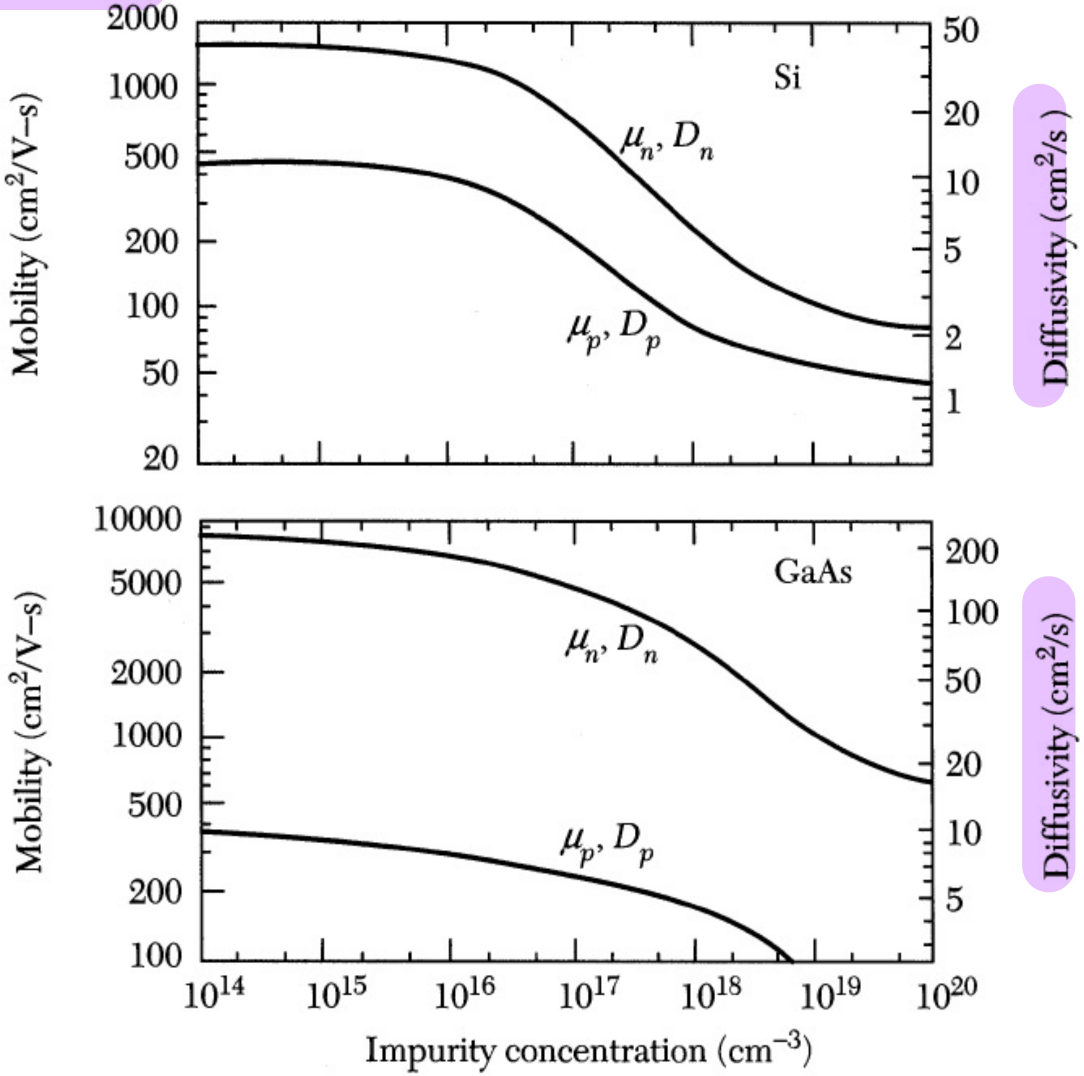
สัมประสิทธิ์ความคล่องตัว

ความสัมพันธ์ของไอสไตน์ บอก ความสัมพันธ์ระหว่าง สปส การแพร่ กับ สปส. ความคล่องตัว

- This is called "**Einstein relation**" (ความสัมพันธ์ของไอสไตน์) since it relates the two constants that describe diffusion and drift transport of carriers, ***diffusivity and mobility***, respectively. (สัมประสิทธิ์การแพร่และความคล่องตัว)

- This Einstein relation can be used with holes as well.

Diffusivities



สปส.การแพร่

Mobilities and diffusivities in Si and GaAs at 300 K as a function of impurity concentration.

Example 2

มาหะ:ข้างน้อย

ฉูกฉีดไปสาร n

- Minority carriers (holes)are injected into a homogeneous n-type semiconductor sample at one point. An electric field of 50 V/cm is applied across the sample, and the field moves these minority carriers a distance of 1 cm in 100 μs. Find the drift velocity and the diffusivity of the minority carriers. T = 300 K.

ตกคร่อม N-type

ประจุเคลื่อนที่ ใน

VD

$$D_p = \frac{1.38\times10^{-23}\,\frac{J}{K}\cdot\text{u} \times 300}{1.6\times10^{-19}\,C} \times 200\ cm^2/V\cdot s \quad ; \quad D_p = \left[\frac{kT}{m}\right]\mu_h$$

$$D_p = 5.175\ cm^2/s \ \#$$

$$; V_D = \frac{s}{t} = \frac{1cm}{100\mu} = 10^4\ cm/s \ \#$$

$$; V_D = \mu_h E \qquad \frac{cm}{s}\times\frac{cm}{V}$$

$$\mu_h = \frac{V_D}{E} = \frac{10^4\ cm/s}{50\ V/cm} = 200\ cm^2/V\cdot s$$

Diffusion process

- For conclusion, when an electric field is applied in addition to a concentration gradient, both drift current and diffusion current will flow.

  กระแสการ drift

  และ กระแสการแพร่

- The total current density due to electron movement can be written as

$$J_e = q\left[\underbrace{\left(\mu_e nE\right)}_{\text{drift current}} + \underbrace{D_n\frac{dn}{dx}}_{\text{diffusion current}}\right] \qquad (6)$$

$\bar{e}$ (+)

where n = electron density

Diffusion process

The total conduction current density is given by

$$J_{total} = J_e + J_h \qquad (7)$$

where J_h is the hole current.

สมการของ hole

$$J_h = q\mu_h pE \boxed{- qD_h \frac{dp}{dx}} \qquad (8)$$

hole -

คล้ายกับ 6 ต่างกันตรงการพิจารณา e^-
(เรา พิจารณาที่ hole

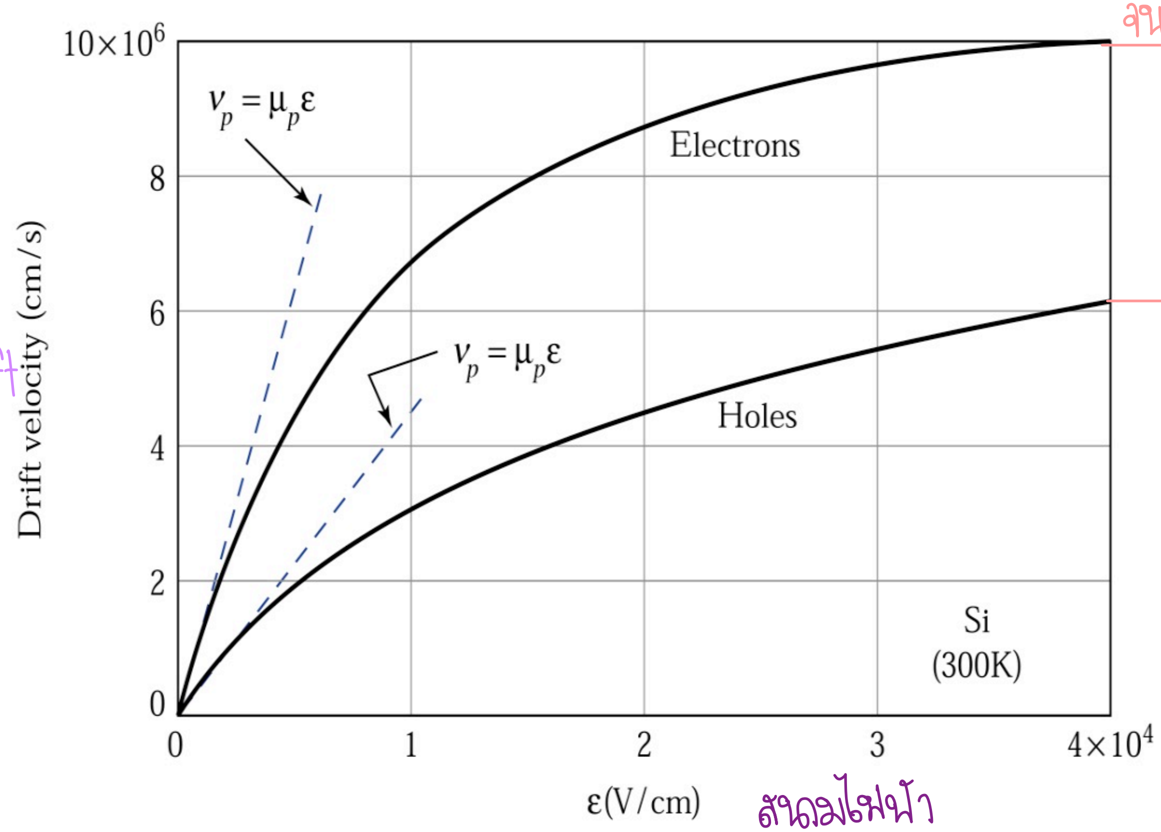
where p is hole density.

Diffusion process

- At a very high electric field, the drift velocity will saturated where it approaches the thermal velocity.

จนถึง Vth ความเร็วอิ่มตัว

ความเร็วdrift



สนามไฟฟ้า

Electron as a wave

- L. de Broglie said electrons of momentum p exhibits wavelike properties that are characterized by wavelength λ.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (9)$$

, p = mv

ค่าคงที่ Planck's

where h = Planck's constant = 6.62×10^{-34} J.s

m = electron mass

v = speed of electron

Schrödinger's equation

- Kinetic energy + Potential energy = Total energy
- Schrodinger's equation describes a wave equation in 3 dimensions is written as

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,t)+V(r)\psi(r,t)=i\hbar\frac{\partial}{\partial t}\psi(r,t) \qquad (10)$$

where $\hbar = h/2\pi$

∇^2 = Laplacian operator in rectangular coordinate

$$= \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

ψ = wave function

V = potential term

Schrödinger's equation

To solve the equation, we assume

$$\psi(r,t) = \psi(r).\phi(t) \tag{11}$$

and substitute it in Schrödinger's equation. We have

$$-\frac{\hbar^2}{2m}\phi(t)\nabla^2\psi(r) + V(r)\psi(r)\phi(t) = i\hbar\psi(r)\frac{d\phi(t)}{dt} \tag{12}$$

Divide both sides by $\psi(r).\phi(t)$, we have

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\nabla^2\psi(r)}{\psi(r)}}_{f^n \text{ ของตำแหน่ง } x} + V(r) = \underbrace{i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt}}_{f^n \text{ ของเวลา}(t)} \tag{13}$$

A:B

C

Schrödinger's equation

- Both can be equal only if they are separately equal to a constant ***E*** as
  - *Time dependent case:*

$$i\hbar \frac{d\phi(t)}{dt} = E\phi(t) \qquad (14)$$

  - *Position dependent: (Time independent)*

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r) \qquad (15)$$

Schrödinger's equation

Consider time dependent part, we can solve the equation as,

$$\phi(t) = \exp\left\{\left(-\frac{iE}{\hbar}\right)t\right\}$$

where $E = h\nu = h\omega/2\pi = \hbar\omega$ $\quad \frac{h}{2\pi}\cdot w = \hbar w$

Therefore ;

$$\phi(t) = \exp\left(-i\omega t\right) \tag{16}$$

Schrödinger's equation

- For position dependent, to make it simple, assume that electrons can move in only one dimension (x-direction).

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \qquad (17)$$

$$\frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}\left[E-V(x)\right]\psi(x)=0 \qquad (18)$$

$\hbar$ คือ modify ของ planck

where $|\psi(x)|^2$ is probability of finding electron.

Schrödinger's equation

- Now let us consider the ***4*** different cases for *V(x)*

<u>Case 1</u>: The electron as a free particle ($V = 0$)

equation (18) becomes

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi(x) = 0 \qquad (19)$$

General solution of this equation is

$$\psi(x) = Ae^{+ikx} + Be^{-ikx} \qquad (20)$$

Schrödinger's equation

- From $\psi(r,t) = \psi(r).\phi(t)$, the general solution of Schrodinger's equation in this case is

$$\psi(x,t) = \left(Ae^{+ikx} + B^{-ikx} \right) e^{-i\omega t} \qquad (21)$$

where A and B are amplitudes of forward and backward propagating waves and k is related to E by

$$E = \frac{\hbar^2 k^2}{2m} = \frac{1}{2} m v^2 \qquad (22)$$

E_k

$E_k = \frac{\hbar^2 k^2}{2m}$

$\sqrt{\frac{E_k 2m}{\hbar^2}} = \sqrt{k^2}$

$\frac{\sqrt{E_k 2m}}{\hbar} = k$

$\frac{2\pi}{\lambda}$

$\lambda \frac{\sqrt{E_k 2m}}{\hbar} = 2\pi$

$\lambda = \frac{2\pi \hbar}{\sqrt{E_k 2m}}$

Schrödinger's equation

$$E_k = \frac{\hbar^2 k^2}{2m} = \frac{1}{2}\underline{mv^2}$$

$mv^2 = p^2$

$$E_k = \frac{\hbar^2 k^2}{2m} = \frac{1}{2}\frac{p^2}{m}$$

$E_k = \frac{p^2}{2m}$

$p^2 = E_k \cdot 2m$

$p = \sqrt{E_k 2m}$

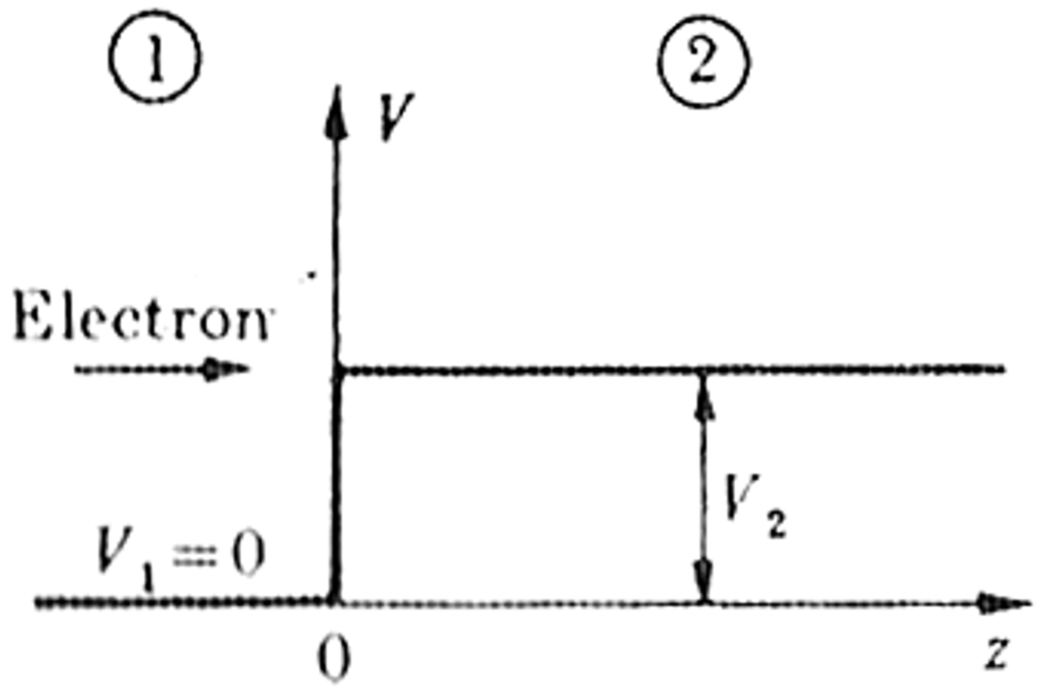
$$p = \hbar k \qquad (23)$$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{E_k 2m}}$

This equation is called "de Broglie's relation".

Schrödinger's equation

- **Case 2**: Step potential barrier (1 dimension)



Schrödinger's equation

For *region 1*, the solution is already known as

$$\psi_1(x) = Ae^{+ik_1x} + Be^{-ik_1x} \tag{24.1}$$

$$k_1^2 = \frac{2mE}{\hbar^2} \qquad k_1 = \sqrt{\frac{2mE}{\hbar^2}} \tag{24.2}$$

For *region 2*, an equation (18) becomes $\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0$

$$\frac{d^2\psi_2(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V_2]\psi_2(x) = 0$$

Schrödinger's equation

- General solution:

$$\psi_2(x) = Ce^{ik_2 x} + De^{-ik_2 x} \qquad (25.1)$$

เลขคลื่นบริเวณที่ 2

$$\underline{k_2^2} = \frac{2m(E - V_2)}{\hbar^2} \qquad (25.2)$$

$k_2 : \sqrt{\dfrac{2m(E-V_2)}{\hbar^2}}$

$E > V_2$; k_2 : real

$E < V_2$; k_2 : จินตภาพ

- Since there is no wave incident from *region* ***2***, *D* must be zero.

Schrödinger's equation

- By applying a boundary condition, *solutions must be continuous* at x = 0 with

$$\psi_1(0) = \psi_2(0)$$

$$\frac{d\psi_1(0)}{dx} = \frac{d\psi_2(0)}{dx}$$

- From (24) and (25), we have

$$A + B = C$$

$$ik_1(A - B) = ik_2 C$$

$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2} \qquad (26.1)$$

$$\frac{C}{A} = \frac{2k_1}{k_1 + k_2} \qquad (26.2)$$

Schrödinger's equation

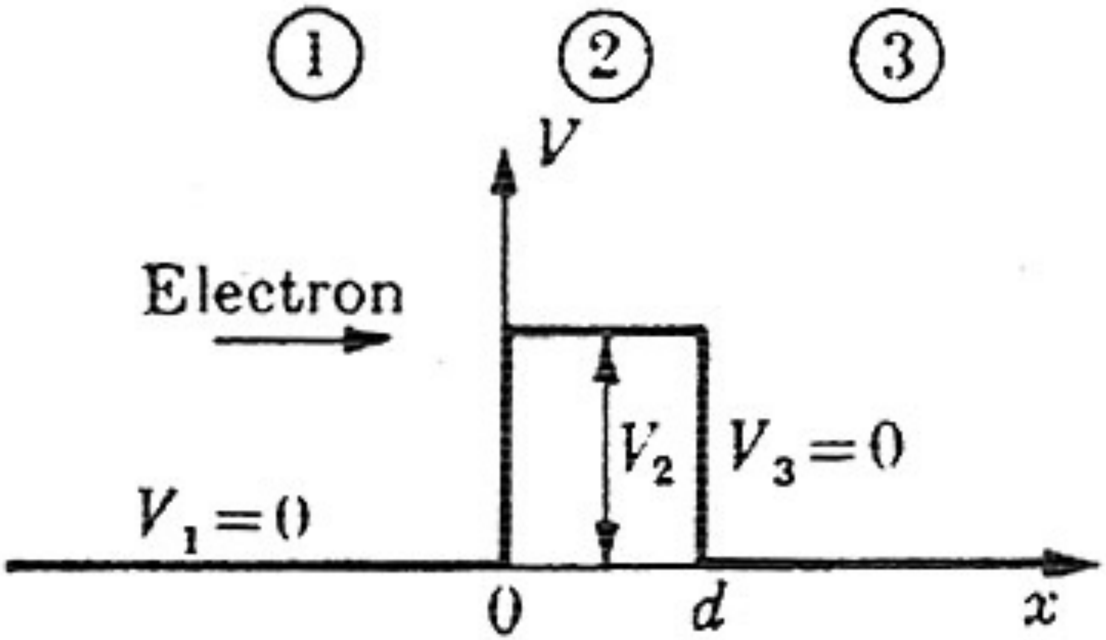
Now let us consider ***2*** cases:

1. **$\underline{E > V_2}$**: In this case k_2 is real , k_2 and k_1 are different leading *B/A* finite. This means an electron could be seen in both *region 1* and *2*. It goes over the barrier and solution is oscillatory in space in both regions.

2. **$\underline{E < V_2}$**: In this case k_2 is imaginary. The solution shows that an electron decays exponentially in *region 2*. An electron may penetrate the potential barrier.

Schrödinger's equation

- **Case 3**: Finite width potential barrier (1 dimension)



Schrödinger's equation

- We are interested in the case of **$E < V_2$**. In this case, solutions for *region 1* and *region 2* are the same as previous case. Now we turn our interest to *region 3*. At *region 3*,

$$\psi_3(x) = Fe^{ik_3 x} \qquad (27.1)$$

$$k_3^2 = \frac{2mE}{\hbar^2} \qquad (27.2)$$

Schrödinger's equation

- Consider transmittance, ***T*** is a ratio of energy in transmitted wave in *region 3* to energy in incident wave in *region 1*.

$$T = \frac{\text{Energy in transmitted wave in region 3}}{\text{Energy in incident wave in region 1}}$$

$$\boxed{T = \frac{|F|^2}{|A|^2} \approx \exp\left\{-2d\sqrt{\frac{2m(V_2 - E)}{\hbar^2}}\right\}} \qquad (28)$$

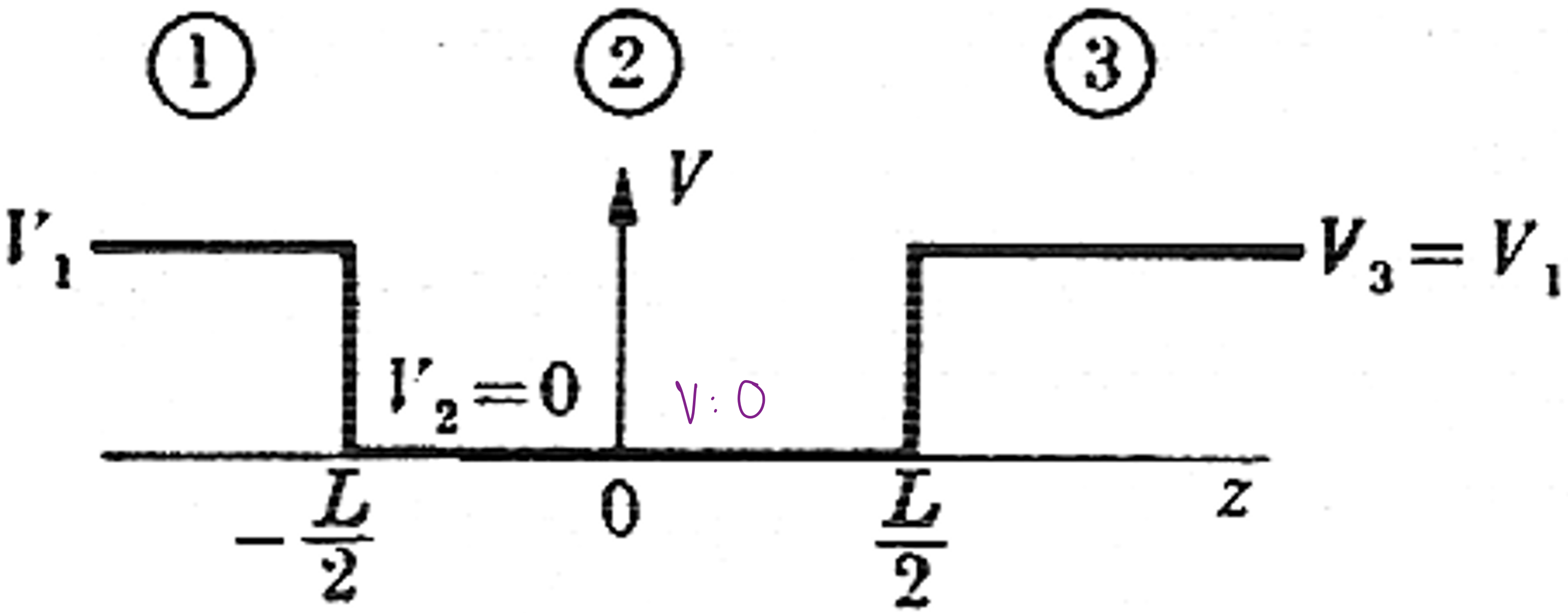
- This is called "Tunneling probability".

Schrödinger's equation

- k_3 is real, so that $|\psi_3|^2$ is not zero.
- Thus, there is a probability that electron crosses the barrier and appear at other side.
- Since the particle does not go over the barrier due to $\mathbf{E < V_2}$, this mechanism that particle penetrates the barrier is called "tunneling".

Schrödinger's equation

- **Case 4**: Finite potential well (1 dimension)



Schrödinger's equation

Consider *2* cases

1. **E > V$_1$**: Solutions in all regions are similar to those previous cases that is particle travels oscillatory everywhere.

2. **E < V$_1$**: ***Region 1*** ($x < -L/2$, $V = V_1$) , Solution must be exponentially decaying,

$$\psi_1(x) = Ae^{ik_1 x} \qquad (29.1)$$

$$\boxed{k_1^2 = \frac{2m(V-E)}{\hbar^2}} \quad \text{ตอนขั่น} \qquad (29.2)$$

Schrödinger's equation

- *Region 2* (V = 0), Free particle case,

$$\psi_2(x) = B\sin(k_2 x) + C\cos(k_2 x) \qquad (30.1)$$

$$k_2^2 = \frac{2mE}{\hbar^2} \qquad (30.2)$$

ตอนช่วงเป็น 0

- *Region 3* ($x > L/2$, $V_3 = V_1$), Solution is again decaying,

$$k = \frac{\sqrt{2m(V-E)}}{\sqrt{\hbar^2}}$$

$$\psi_3(x) = De^{ik_1 x} \qquad (31.1)$$

$$k = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$k_3^2 = \frac{2m(V-E)}{\hbar^2} = k_1^2 \qquad (31.2)$$

$k_1 \cdot k_3$ เพราะขั้นๆ
เท่ากัน

Schrödinger's equation

Applying boundary conditions:

at x = -L/2

$$\psi_1(x) = \psi_2(x)$$

$$\frac{d\psi_1(x)}{dx} = \frac{d\psi_2(x)}{dx}$$

at x = L/2

$$\psi_2(x) = \psi_3(x)$$

$$\frac{d\psi_2(x)}{dx} = \frac{d\psi_3(x)}{dx}$$

Schrödinger's equation

It can solve only at $x = L/2$ due to its symmetrical.

$$C\cos\left(\frac{k_2 L}{2}\right) = De^{-\frac{ik_3 L}{2}}$$

$$-Ck_2\sin\left(\frac{k_2 L}{2}\right) = -Dk_3 e^{-\frac{ik_3 L}{2}} \tag{32}$$

By solving (32), this leads to

$$k_2\tan\left(\frac{k_2 L}{2}\right) = k_3 \tag{33}$$

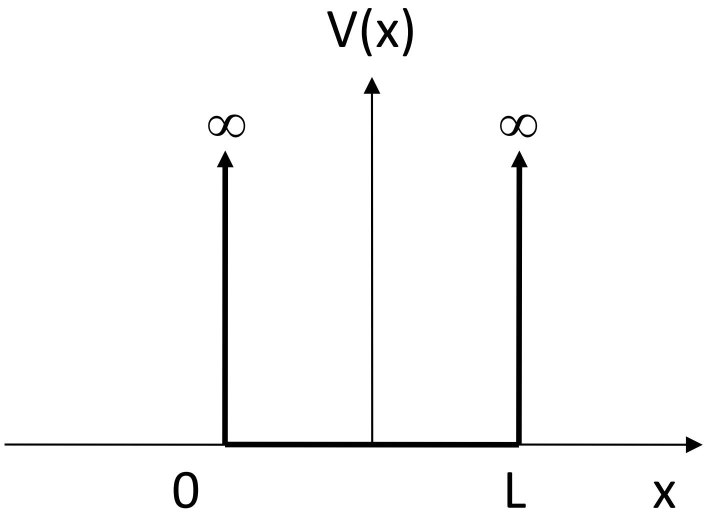
Schrödinger's equation

- Substitute (33) into (30.2) and (31.2), we have

$$\sqrt{E}\tan\left(\sqrt{\frac{mL^2E}{2\hbar^2}}\right)=\sqrt{V_1-E} \qquad (34)$$

Schrödinger's equation

- If we consider in the case of $V \rightarrow \infty$ or infinite potential well.



Schrödinger's equation

- There is impossible that a particle can penetrate an infinite barrier. This brings $\psi(x) = 0$ at $x = 0$ and $x = L$. Applying the boundary conditions, we first have $B = 0$ and

$$k_2 L = n\pi; \text{ where n = integer}$$

$$k_2 = \frac{n\pi}{L} \qquad (35)$$

- We can solve for energy E,

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad (36)$$