17 Glossary

bijection one-to-one correspondence. If a function f is bijective, it has an inverse f^{-1}; more formally, a function is bijective if it is both injective and surjective.

binary operation an operation acting on two elements.

cardinality size of a group; number of elements. Also known as order.

closure the property that operation on elements of a group always produces an element of that same group.

collinear on the same line.

complex conjugate the complex conjugate of the complex number $z = a + bi$ has the same real part as z but a negated imaginary part; $\overline{z} = a - bi$.

DeMoivre's theorem

$$(r_1(\cos\theta + i\sin\theta))(r_2(\cos\phi + i\sin\phi)) = (r_1 r_2)(\cos(\theta+\phi) + i\sin(\theta+\phi)).$$

In other words, when multiplying two complex numbers, we add their angles and multiply their magnitudes.

dihedral group the group of rotational and reflective symmetries of a regular polygon.

eigenvalue the scalar multiple that is associated with the eigenvector.

eigenvector a vector which when operated on by a given matrix gives a scalar multiple of itself.

Euler's totient function a function $\phi(n)$ that tells how many numbers are relatively prime to n. Formally defined mathematically as: for prime factorization of an integer $n = {p_1}^{n_1}{p_2}^{n_2}{p_3}^{n_3}\cdots{p_k}^{n_k}$, $\phi(n)$ (symbol for Euler's totient function) is equal to $\phi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2})\cdots(1-\frac{1}{p_k})$.

generating set a set of elements which can generate a group by repeatedly applying the group operator to these elements.

generator element that can generate the entire group by a series of operations.

group a set of elements, finite or infinite, formed by a certain binary operation that satisfies the four fundamental properties: closure, associativity, identity, and invertibility.

identity element an element I, when acted on another element A via a group's binary operation $\cdot$, gives A; $I \cdot A = A$ for all A.

image output of a transformation given a preimage.

injective function a function that maps distinct elements of its domain to distinct elements of its codomain.

isometry a linear transformation preserving length.

isomorphism a bijection, or one-to-one correspondence, between two groups which preserves the group's structure.

linear mapping a mapping in which all lines are mapped to lines and the origin remains fixed.

linearly independent (of an eigenvector) two vectors that are not multiples of each other; having different directions.

matrix decomposition decomposing a transformation matrix into simpler transformations.

order size of a group; number of elements. Also known as cardinality.

period number of times an element of a group has to be operated on itself to yield an identity element of that group.

permutation an order of things in which they can be arranged.

preimage input of a transformation.

shear a linear transformation where all points along a particular line remain fixed, while other points are shifted parallel to the fixed line by a distance proportional to their perpendicular distance from the fixed line.

surjective function a function that has every values of its codomain pointed at by at least one element in the domain.

transportation matrix a square matrix connecting vertices of a graph, sometimes known as an adjacency matrix.

unit vector a vector with a magnitude of 1.