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Mixtures of Normal Distributions: Application to Bursa Malaysia Stock Market Indices

[1]*Zetty Ain Kamaruzzaman*, [1]*Zaidi Isa and* [2]*Mohd Tahir Ismail*

[1]School of Mathematical Sciences, Faculty of Science and Technology,
Universiti Kebangsaan Malaysia, 43600 UKM Bangi Selangor, Malaysia
[2]School of Mathematical Sciences, Universiti Sains Malaysia,
11800 USM Penang, Pulau Pinang, Malaysia

Abstract: In this paper, mixture of Normal distributions is proposed to accommodate the non-normality and asymmetry characteristics of financial time series data as found in the distribution of monthly rates of returns for three indices of Bursa Malaysia Index Series namely the FTSE Bursa Malaysia Composite Index (FBM KLCI), the Finance Index and the Industrial Index from July 1990 until July 2010. We also present the most commonly used Maximum Likelihood Estimation (MLE) via the EM algorithm to fit the two-component mixture of Normal distribution using data sets on logarithmic stock returns of Bursa Malaysia indices.

Key words: Bursa Malaysia stock market indices % Behaviour of financial time series % Mixture of Normal distributions % EM algorithm

INTRODUCTION

Mixtures of normal distributions are well recognized in empirical finance. There exists a long history of modelling asset returns with a mixture of normal (see, for example, Press [1], Praetz [2], Clark [3], Blattberg and Gonedes [4], Kon [5]). The use of normal mixtures to handle fat tails was first considered by Newcomb [6]. Gridgeman [7] proves that a mixture of normal is leptokurtic, when all regimes have the same mean. Mixtures of normal are flexible to accommodate various shapes of continuous distributions and able to capture leptokurtic, skewed and multimodal characteristics of financial time series data. Normal mixtures are also appropriate to accommodate certain discontinuities in stock returns such as the 'weekend effect', the 'turn-of-the month effect' and the 'January effect' (Klar and Meintanis [8]). A good introduction to the theory and applications of mixture distributions can be found in Everitt and Hand [9], Titterington *et al*. [10], McLachlan and Basford [11], Lindsay [12], McLachlan and Peel [13] and Frühwirth-Schnatter [14].

Fitting mixture distributions can be handled by variety of techniques, this includes graphical methods, the method of moments, maximum likelihood and Bayesian approaches (see Titterington *et al*. [10] for an exhaustive review of these methods). Now extensive advances have been introduced in the fitting of mixture models especially via the maximum likelihood method. Among all, the maximum likelihood method becomes the first preference due to the existence of an associated statistical theory. The key property of the EM algorithm has been established by Dempster *et al*. [15]. The EM algorithm is a popular tool for simplifying maximum likelihood problems in the context of a mixture model. The EM algorithm has become the method of choice for estimating the parameters of a mixture model, since its formulation leads to straightforward estimators (Picard [16]).

The paper is structured as follows. We present the case study and their properties in Section 2. In Section 3, we provide discussion on how mixture distributions accommodate with non-normality and asymmetry characteristics of financial time series. In Section 4, we present the most commonly used Maximum Likelihood Estimation (MLE) via the EM algorithm to fit the two-component mixture of Normal distribution. Lastly, Section 5 concludes.

Corresponding Author: Zetty Ain Kamaruzzaman, School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia.

Case Study

Data: The data sets used in this paper are monthly closing prices from July 1990 to July 2010 for 3 Malaysian stock market indices namely Composite Index, Financial Index and Industrial Index obtained from DataStream[1]. The series is denominated in Malaysian Ringgit (MYR). In total, we have 241 observations per index.

The FTSE Bursa Malaysia Kuala Lumpur Composite Index is formerly known as the Kuala Lumpur Composite Index (KLCI) when it adopts the FTSE global index standard starting from 6 July 2009 onwards[2]. The FTSE Bursa Malaysia KLCI comprises the largest 30 companies listed on the Main Board[3]. Meanwhile, Financial Index comprises of 38 companies and Industrial Index comprises of 24 companies as at 30 December 2010.

Prior to analysis, all the series are analyzed in return, which is the first difference of natural algorithms multiplied by 100. This is done to express things in percentage terms.

Empirical Properties of Return: The 3 stock market indices of Bursa Malaysia are as listed in Table 1. This table also provides descriptive statistics of the time series. As can be seen, the Composite Index and Financial Index distribution of return rates are positively skewed while Industrial Index is negatively skewed. Moreover, the kurtosis (which equals 3 for normal distribution) shows values greater than 3, indicating heavier tails and higher peak than the normal. The Jarque-Bera test rejects the null hypothesis of normality for each of the 3 stock market indices.

Figure 1 depicts the histogram for the Malaysia's stock return rates and the corresponding normal curve with the same mean and standard deviation. It can easily be seen that the empirical distribution is higher peak and has heavier tails than the normal distribution. Also note that the return distributions with thicker tails have a thinner and higher peak in the center compared to normal distribution.

Mixture of Distributions: One possible alternative explanation of departures from normality is mixture of distributions. Fama [17] claims that "the most popular approach to explain long-tailed distributions is to hypothesize that the distribution of price changes is a mixture of several normal distributions with possibly the same mean, but substantially different variances".

Table 1: Summary Statistics

	Bursa Malaysia stock market indices		
Statistics	Composite Index	Financial Index	Industrial Index
Mean	0.3237	0.6021	0.3233
Median	0.6119	0.8037	0.6474
Maximum	28.2488	40.3887	22.5572
Minimum	-24.8089	-32.3621	-23.1723
Std. Dev.	6.8727	9.3636	5.9313
Skewness	0.0749	0.3479	-0.3980
Kurtosis	5.2621	6.5048	5.4303
Jarque-Bera	51.3942	127.6802	65.4024
P-value	0.0000	0.0000	0.0000

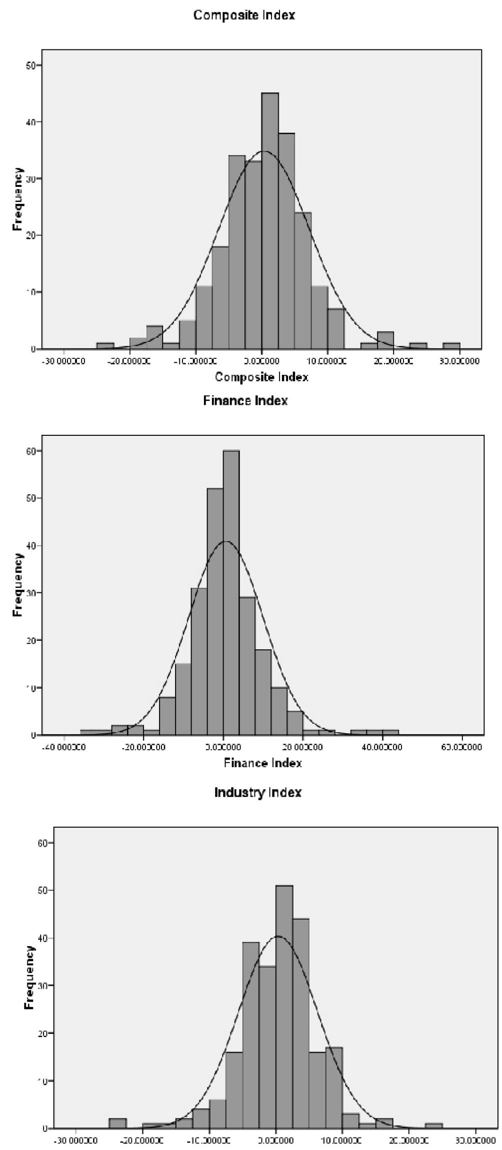

Fig. 1: Empirical distribution of the Malaysian indices

[1]Data sets was obtained from DataStream (www.datastream.com) access through UKM library on 15th July 2010.

[2]http://www.bursamalaysia.com/website/bm/market_information/index_components.html. Access on 30th December 2010.

[3]http://www.bursamalaysia.com/website/bm/market_information/fbm_klci.html. Access on 30th December 2010.

Recent studies of stock returns tend to use mixture of normal distributions. Under the assumption of normal distribution, the log return is normally distributed with mean μ and variance F^2 i.e. r_t - $N(\mu, F^2)$. Advantages of mixture of normal include that they maintain the tractability of normal, have finite higher order moments and can capture the excess kurtosis [18]. Besides, the mixture of normal has two other advantages. One is that it can capture the structural change not only in the variance but also in the mean. The other advantage is that it can be asymmetric [19].

The general form of the CDF of a normal mixture can be represented as

$$F(x) = \sum_{i=1}^{K} p_i \Phi\left(\frac{x - m_i}{s_i}\right) \qquad (1)$$

Where M is the cumulative density function of $N(0, 1)$. The probability density function of a mixture of normal is therefore given by

$$f(x) = \sum_{i=1}^{K} p_i f\left(x; m_i, s_i\right) \qquad (2)$$

Where

$$f\left(x; m_i, s_i\right) = \frac{1}{\sqrt{2p}\, s_i} e^{\frac{-(x-m_i)^2}{2s_i^2}}$$

$$\sum_{i=1}^{K} p_i = 1 \quad and \quad 0 \le p_i \le 1$$

for i = 1,2,...,K. Thus, in a normal mixture, the return distribution is approximated by a mixture of normal each of which have unique mean μ_i and standard deviation F_i and weight (or probabilities or mixing parameter) B_i [20].

Dias *et al*. [21] discuss how skewness and excessive kurtosis in financial time series can be deal using finite mixtures of normal distributions? and illustrate why mixture models provide a flexible way of dealing with skewness and kurtosis?

A mixture of two normal distributions is given by

$$f\left(x_t\right) = p f_1\left(m_1, s_1^2\right) + (1-p) f_2\left(m_2, s_2^2\right) \qquad (3)$$

Where $N(x_i; \mu_i, F_i^2)$ is the pdf of a normal distribution with mean μ_i and variance F_i^2. The five parameters (B, μ_1, μ_2, F_1^2, F_2^2 of mixture distribution allow a very flexible definition of departures from symmetry and normality. The weight is the probability B which the first regime occurs while the second regime occurs with probability 1–B.

Therefore, mixture distributions have the ability to deal with skewness and kurtosis in analyzing financial time series. By using mixture distributions, we can obtain densities with higher peaks and heavier tails than normal distribution.

Fitting a Mixture Normal Distribution to Data

The EM Algorithm for Two Component Mixture Model: The EM algorithm is a popular tool for simplifying difficult maximum likelihood problems. The EM algorithm has shown great performance in practice where it has the ability to deal with missing data, unobserved variables and mixture density problems.

The EM algorithm will find the expected value as well as the current parameter estimates at the E step and maximizes the expectation at the M step. By repeating the E and M step, the algorithm will converge to a local maximum of the likelihood function.

In this section, we describe a simple mixture model for density estimation and the associated EM algorithm for carrying out maximum likelihood estimation.

Remark 1: (Estimation of the mixture normal pdf). With two mixture components, the log likelihood is:

$$\log L = \sum\nolimits_{t=1}^{T} \ln f\left(x_t; m_1, m_2, s_1^2, s_2^2, p\right)$$

Where $f(\ )$ is the pdf in (3). A numerical optimization method could be used to maximize this likelihood function [22].

We model X as a mixture of two normal distributions:

$$\begin{aligned} &Model\ 1: X_1 \sim N\left(m_1, s_1^2\right), \\ &Model\ 2: X_2 \sim N\left(m_2, s_2^2\right), \\ &\qquad X = \Delta X_1 + (1-\Delta) X_2 \end{aligned} \qquad (4)$$

Where) , {0,1} with Pr () = 1) = B.

Generate a) , {0,1} with probability B. Depending on the outcome, deliver either X_1 or X_2.

Let $N_2(y)$ denote the normal density with parameters $2 = (\mu, F^2)$. Then the density of X is

$$g_X\,(x_t) = B N_{21}\,(x_t) + (1-B)\, N_{22}\,(x_t) \qquad (5)$$

Now suppose we want to fit this model to the data by maximum likelihood. The parameters are

$$q = (p, q_1, q_2) = \left(p, m_1, s_1^{\,2}, m_2, s_2^{\,2}\right) \qquad (6)$$

The log-likelihood is

$$\ell(q; \mathrm{Z}) = \sum_{t=1}^{T} \log\left[p f_{q_1}(x_t) + (1-p) f_{q_2}(x_t)\right] \qquad (7)$$

Direct maximization of (7) is quite difficult numerically. This is due to the sum of terms inside the logarithm. Using a simpler approach, we consider unobserved latent variables $)_i$ taking values 0 or 1 as in (4): if $)_i = 1$ then X_t comes from model 1, otherwise it comes from model 2.

Suppose we knew the values of the $)_i$'s. Then the log-likelihood would be

$$\ell_0(q; \mathrm{Z}, \Delta) =$$
$$\sum_{t=1}^{T}\sum_{i=1}^{K}\left[\Delta_i \log f_{q_1}(x_t) + (1-\Delta_i)\log f_{q_2}(x_t)\right]$$
$$+\sum_{i=1}^{K}\left[\Delta_i \log p + (1-\Delta_i)\log(1-p)\right] \qquad (8)$$

The maximum likelihood estimates of μ_1 and $F_1^{\,2}$ would be the sample mean and variance of the data with $)_i = 1$, while μ_2 and $F_2^{\,2}$ when $)_i = 0$. The estimate of B would be the proportion of $)_i = 1$.

Since the values of the $)_i$'s are unknown, we proceed in an iterative fashion, substituting for each $)_i$ in (8), its expected value

$$(_i\,(2) = E()_i|2, \mathrm{Z}) = \Pr()_i = 1|2, \mathrm{Z}) \qquad (9)$$

also called the responsibility of model 1 for observation *i*.

We used a procedure called the EM algorithm, given in Algorithm 1 for the special case of normal mixtures by Hastie *et al*. [23]. In the Expectation (E) step, the current estimates of the parameters are used to assign responsibilities according to the relative density of the training points under each model. Next, in the Maximization (M) step, these responsibilities are used in weighted maximum-likelihood fits to update the estimates of the parameters Hastie *et al*. [23].

Algorithm 1: EM algorithm for two-component normal mixture Hastie *et al*. [23], Söderlind [22].

Take initial guesses for the parameters μ_1, μ_2, $F_1^{\,2}$, μ_2, $F_2^{\,2}$.

We choose $\mu_1 = x_1$ and $\mu_2 = x_2$ as the initial guesses for μ_1 and μ_2 Söderlind [22]. Alternatively, other way to construct initial guesses for μ_1 and μ_2 is to choose two of the x_t at random Hastie *et al*. [23].

Both $F_1^{\,2}$ and $F_2^{\,2}$ can be set equal to the overall sample variance $\sum_{t=1}^{T}\frac{(x_t-\bar{x})^2}{T}$. The mixing proportion B can be started at the value 0.5.

Expectation (E) Step.
Calculate the responsibilities

$$g_t = \frac{p f\left(x_t; m_2, s_2^{\,2}\right)}{p f_1\left(x_t; m_1, s_1^{\,2}\right) + (1-p) f_2\left(x_t; m_2, s_2^{\,2}\right)}$$
$$\text{for } t = 1, \ldots, T.$$

Maximization (M) Step.
Calculate the weighted means and variances

$$m_1 = \frac{\sum_{t=1}^{T}(1-g_t)x_t}{\sum_{t=1}^{T}(1-g_t)}, \quad s_1^{\,2} = \frac{\sum_{t=1}^{T}(1-g_t)(x_t-m_1)^2}{\sum_{t=1}^{T}(1-g_t)},$$
$$m_2 = \frac{\sum_{t=1}^{T}g_t x_t}{\sum_{t=1}^{T}g_t}, \quad s_2^{\,2} = \frac{\sum_{t=1}^{T}g_t(x_t-m_2)^2}{\sum_{t=1}^{T}g_t}, \text{ and}$$
$$p = \sum_{t=1}^{T}\frac{g_t}{T}$$

Iterate over steps 2 and 3 until the parameter values converge.

For the initial values, we decided to have the following values. Table 2 depicts the initial values for the EM algorithm.

After the EM algorithm, our final maximum likelihood estimates for unknown parameters are as described in Table 3. This table provides the estimates of the five parameters of a two component normal mixture as well as the normal mixture model for the three stock market indices of Bursa Malaysia. For Composite Index, 27.07% of the returns follow the first normal distribution and 72.92% follow the second normal distribution. 38.68% of the returns of Finance Index follow the first normal distribution and 61.32% follow the second normal distribution. Meanwhile for Industry Index, 19.64% of the returns follow the first normal distribution and 80.36% follow the second normal distribution.

Figure 2 depicts the mixture normal distribution for the three indices. It shows that mixture normal distribution can accommodate leptokurtic as well as skewed in the data as the distribution has thicker tails and higher peak. From Table 3 and Figure 2, the two indices (Composite Index and Industry Index) indicate that the first normal is a low mean high variance regime and the second normal is a high mean low variance regime.

Table 2: Initial values for the EM algorithm

	Composite Index	Finance Index	Industry Index
$B^{(0)}$	0.5	0.5	0.5
$\mu_1^{(0)}$	-10.7651	-10.7736	-11.9791
$\mu_2^{(0)}$	-1.8481	-6.0206	0.5584
$F_1^{2(0)}$	47.2335	87.6776	35.1808
$F_2^{2(0)}$	47.2335	87.6776	35.1808

Table 3: EM estimation for Normal mixtures

Composite Index

	Normal 1	Normal 2
Weight $\hat{p}$	0.2708	0.7292
Mean $\hat{m}$	-4.4542	2.0979
Variance $\hat{s}^2$	43.4384	36.7481

Model:

$f(x_t) = 0.2708N_1\ (-4.4542, 43.4384) + 0.7292N_2\ (2.0979, 36.7481)$

Finance Index

	Normal 1	Normal 2
Weight $\hat{p}$	0.3868	0.6132
Mean $\hat{m}$	-1.9839	2.2331
Variance $\hat{s}^2$	79.9652	85.0682

Model:

$f(x_t) = 0.3868N_1\ (-1.9839, 79.9652) + 0.6132N_2\ (2.2331, 85.0682)$

Industry Index

	Normal 1	Normal 2
Weight $\hat{p}$	0.1964	0.8036
Mean $\hat{m}$	-5.9841	1.8652
Variance $\hat{s}^2$	35.8676	22.7276

Model:

$f(x_t) = 0.1964N_1\ (-5.9841, 35.8676) + 0.8036N_2\ (1.8652, 22.7276)$

However, the Finance Index indicates that the first normal is a low mean (-1.9839) low variance (79.9652) regime and the second normal is a high mean (2.2331) high variance (85.0682) regime. Meanwhile, the weights indicate that the second regime is the more prevalent regime for the three stock market indices of Bursa Malaysia.

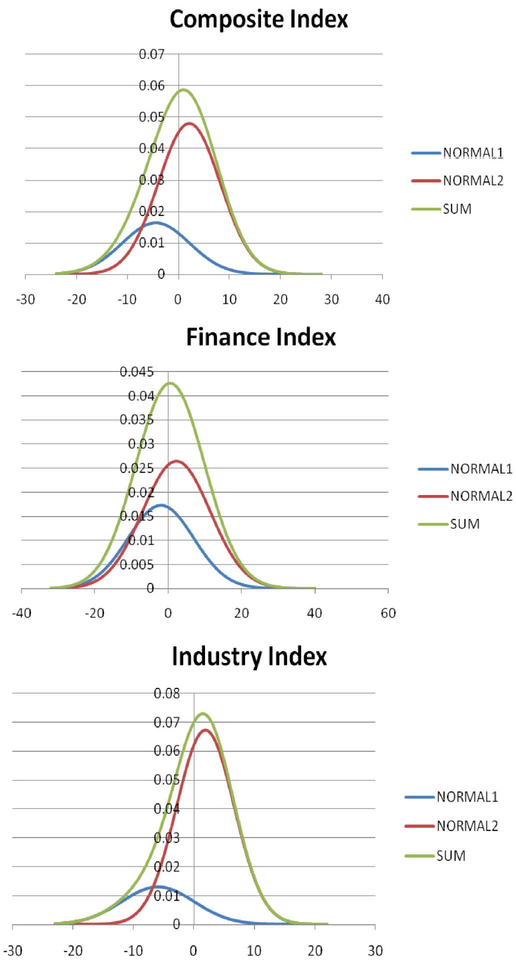

Fig. 2: Mixture of normal distributions

CONCLUSION

In this paper, we found that mixture distribution can accommodate leptokurtic as well as skewed in the data. Mixture of normal distribution is proposed to accommodate the non-normality and asymmetric characteristics of financial time series data as found in the distribution of monthly rates of returns for three indices of Bursa Malaysia Index Series namely the FTSE Bursa Malaysia Composite Index (FBM KLCI), the Finance Index and the Industrial Index from July 1990 until July 2010. Lastly, we fit the two component mixture normal distribution to data sets using the EM algorithm.

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Supplementary Material

Bursa Malaysia Stock Market Indices Data Sets

	COMPOSITE	FINANCE	INDUSTRY	RET_COMP	RET_FIN	RET_IND
1990M07	616.7000	2850.640	1225.930	NA	NA	NA
1990M08	553.7600	2559.490	1087.530	-10.76513	-10.77355	-11.97907
1990M09	543.6200	2409.940	1093.620	-1.848091	-6.020617	0.558422
1990M10	471.2200	2054.950	948.4300	-14.29254	-15.93503	-14.24406
1990M11	472.8500	1999.880	939.9500	0.345314	-2.716434	-0.898130
1990M12	508.1100	2099.360	1016.690	7.191975	4.854536	7.848085
1991M01	481.4400	2055.880	965.3800	-5.391635	-2.092856	-5.178573
1991M02	527.6000	2324.880	1048.000	9.155681	12.29644	8.211706
1991M03	594.2300	2610.930	1151.280	11.89280	11.60381	9.399078
1991M04	577.2800	2526.390	1119.100	-2.893903	-3.291507	-2.834958
1991M05	587.1200	2435.330	1130.160	1.690181	-3.670914	0.983442
1991M06	627.3100	2505.120	1181.070	6.621161	2.825437	4.406159
1991M07	598.6000	2392.450	1130.430	-4.684724	-4.601869	-4.382272
1991M08	569.5800	2247.010	1065.930	-4.969435	-6.271750	-5.875043
1991M09	544.4600	2141.430	1000.860	-4.510477	-4.812661	-6.298803
1991M10	518.8000	2051.990	970.7900	-4.827603	-4.266378	-3.050474

	COMPOSITE	FINANCE	INDUSTRY	RET_COMP	RET_FIN	RET_IND
1991M11	553.2100	2203.390	1040.540	6.421922	7.118703	6.938492
1991M12	537.8500	2123.130	1037.370	-2.815797	-3.710567	-0.305115
1992M01	546.6300	2110.570	1048.820	1.619245	-0.593336	1.097706
1992M02	595.7500	2254.870	1148.030	8.604896	6.613427	9.038171
1992M03	595.2200	2316.540	1128.750	-0.089003	2.698237	-1.693660
1992M04	564.5100	2195.700	1042.640	-5.297299	-5.357379	-7.935487
1992M05	593.0300	2222.400	1072.620	4.928689	1.208679	2.834830
1992M06	590.5500	2246.070	1029.180	-0.419068	1.059433	-4.134189
1992M07	617.9400	2326.880	1051.630	4.533706	3.534629	2.157897
1992M08	581.9400	2265.350	964.2000	-6.002402	-2.679904	-8.679788
1992M09	583.7400	2282.600	996.6300	0.308833	0.758587	3.308085
1992M10	606.5600	2487.500	1052.640	3.834797	8.596305	5.467699
1992M11	650.6200	2776.650	1136.340	7.012210	10.99670	7.651128
1992M12	632.5400	2676.840	1080.010	-2.818229	-3.660817	-5.084227
1993M01	619.8100	2595.100	1049.610	-2.033048	-3.101194	-2.855163
1993M02	635.4200	2852.690	1148.410	2.487322	9.463736	8.995971
1993M03	638.0500	2899.150	1136.130	0.413045	1.615518	-1.075063
1993M04	660.6000	3203.770	1172.270	3.473186	9.991065	3.131429
1993M05	720.0000	3709.520	1282.260	8.610270	14.65742	8.968211
1993M06	743.1600	4014.190	1319.660	3.166015	7.893310	2.874998
1993M07	719.3000	3708.320	1242.420	-3.263285	-7.925664	-6.031304
1993M08	771.9100	4416.840	1332.500	7.058945	17.48456	6.999579
1993M09	817.7800	4757.750	1433.340	5.772539	7.435036	7.295051
1993M10	919.0300	5374.090	1509.400	11.67254	12.18144	5.170484
1993M11	972.4700	6069.110	1618.590	5.652046	12.16227	6.984318
1993M12	1092.660	7131.480	1731.950	11.65311	16.13068	6.769254
1994M01	1134.140	7593.400	1783.450	3.725956	6.276066	2.930175
1994M02	1093.300	7257.050	1740.450	-3.667401	-4.530604	-2.440599
1994M03	1046.480	7029.720	1720.720	-4.376849	-3.182653	-1.140089
1994M04	1021.300	6655.190	1651.160	-2.435583	-5.474987	-4.126474
1994M05	1004.550	6544.690	1625.720	-1.653665	-1.674297	-1.552728
1994M06	1013.490	6353.100	1603.840	0.886014	-2.971115	-1.355004
1994M07	1010.800	6344.590	1607.920	-0.265772	-0.134040	0.254066
1994M08	1105.290	7310.190	1794.850	8.936565	14.16668	10.99800
1994M09	1173.840	8283.290	1965.200	6.017268	12.49710	9.067257
1994M10	1128.450	8200.360	1941.120	-3.943542	-1.006218	-1.232890
1994M11	1044.310	7473.910	1819.340	-7.748863	-9.275977	-6.479133
1994M12	937.1500	6637.800	1643.140	-10.82683	-11.86377	-10.18648
1995M01	898.5000	6097.240	1596.750	-4.211665	-8.494437	-2.863873
1995M02	972.9900	6728.580	1726.150	7.964710	9.852792	7.792318
1995M03	929.7100	6408.040	1717.930	-4.550110	-4.881067	-0.477342
1995M04	965.9700	6702.880	1759.950	3.826007	4.498383	2.416532
1995M05	1013.940	7279.150	1811.770	4.846623	8.247681	2.901887
1995M06	1049.970	7573.300	1851.080	3.491786	3.961481	2.146498
1995M07	1074.170	8164.710	1925.130	2.278668	7.519230	3.922425
1995M08	1048.250	8084.790	1933.240	-2.442616	-0.983669	0.420385
1995M09	993.2000	7567.280	1841.070	-5.394533	-6.615083	-4.885043
1995M10	963.1300	7256.620	1762.590	-3.074366	-4.191953	-4.356261
1995M11	883.9600	6431.350	1647.450	-8.577658	-12.07297	-6.755568
1995M12	996.2600	7500.480	1799.370	11.95965	15.37825	8.820797
1996M01	1061.770	8213.210	1925.770	6.368434	9.077682	6.788928
1996M02	1069.960	8209.430	1922.400	0.768394	-0.046034	-0.175148
1996M03	1109.000	8483.430	1971.260	3.583744	3.283136	2.509853
1996M04	1150.240	8824.790	2062.980	3.651191	3.944996	4.547861
1996M05	1163.990	9027.990	2073.260	1.188314	2.276494	0.497071
1996M06	1125.480	8694.300	2034.460	-3.364415	-3.766211	-1.889182
1996M07	1138.170	9009.300	2104.200	1.121210	3.558974	3.370492
1996M08	1102.560	8621.980	2077.070	-3.178696	-4.394262	-1.297710
1996M09	1117.550	8917.620	2114.210	1.350404	3.371434	1.772297
1996M10	1148.850	9233.300	2185.460	2.762265	3.478742	3.314511
1996M11	1189.430	9521.040	2224.450	3.471276	3.068757	1.768336
1996M12	1188.450	9852.550	2284.150	-0.082426	3.422622	2.648427
1997M01	1211.040	10227.65	2330.350	1.882956	3.736453	2.002451
1997M02	1246.910	11016.98	2335.290	2.918900	7.434288	0.211761
1997M03	1242.470	11177.42	2324.350	-0.356716	1.445795	-0.469565
1997M04	1101.090	9535.010	2068.890	-12.08007	-15.89254	-11.64282

	COMPOSITE	FINANCE	INDUSTRY	RET_COMP	RET_FIN	RET_IND
1997M05	1072.490	9210.960	2017.030	-2.631755	-3.457621	-2.538610
1997M06	1078.380	9202.340	2064.670	0.547687	-0.093628	2.334427
1997M07	1015.500	8579.420	1984.220	-6.007881	-7.009149	-3.974452
1997M08	918.5400	7816.450	1876.590	-10.03509	-9.313582	-5.576959
1997M09	852.0800	6818.620	1747.400	-7.510503	-13.65734	-7.132733
1997M10	822.5900	6295.430	1671.920	-3.522252	-7.983313	-4.415630
1997M11	677.4700	4691.590	1326.110	-19.40926	-29.40524	-23.17228
1997M12	574.9200	3394.480	1118.970	-16.41344	-32.36210	-16.98412
1998M01	536.6800	2725.430	1065.450	-6.882889	-21.95244	-4.901137
1998M02	685.5000	4081.700	1263.870	24.47465	40.38874	17.07812
1998M03	708.1600	4015.770	1275.770	3.252156	-1.628446	0.937148
1998M04	664.8400	3761.630	1215.630	-6.312365	-6.537674	-4.828746
1998M05	560.5000	2908.350	1079.600	-17.07172	-25.72665	-11.86719
1998M06	472.3700	2612.840	950.3500	-17.10667	-10.71482	-12.75155
1998M07	420.3300	2142.550	865.1700	-11.67225	-19.84410	-9.390432
1998M08	327.9800	1646.260	686.6800	-24.80875	-26.34907	-23.10576
1998M09	393.2400	2420.000	752.1900	18.14675	38.52615	9.112056
1998M10	384.2200	2380.820	778.0500	-2.320481	-1.632257	3.380184
1998M11	465.0800	2995.670	878.2600	19.09941	22.97229	12.11519
1998M12	541.0100	3638.990	1023.230	15.12283	19.45383	15.27769
1999M01	593.3400	3756.580	1078.790	9.232983	3.180280	5.287575
1999M02	570.7900	3722.330	1039.640	-3.874622	-0.915915	-3.696554
1999M03	528.7900	3442.390	984.1600	-7.642999	-7.818382	-5.484129
1999M04	578.1100	3951.170	1023.320	8.917278	13.78457	3.901904
1999M05	766.8200	5641.690	1282.260	28.24879	35.61719	22.55719
1999M06	782.9800	5720.400	1342.000	2.085506	1.385507	4.553689
1999M07	851.2100	6888.400	1469.390	8.355171	18.58001	9.068631
1999M08	772.8800	5977.290	1339.340	-9.653507	-14.18715	-9.267039
1999M09	729.0500	5574.080	1305.410	-5.838148	-6.984001	-2.565979
1999M10	738.0500	6008.900	1272.630	1.226926	7.511442	-2.543154
1999M11	721.5200	5735.520	1267.950	-2.265148	-4.656329	-0.368420
1999M12	769.9400	6055.540	1319.380	6.495249	5.429514	3.976051
2000M01	928.2400	7836.600	1509.060	18.69777	25.78315	13.43250
2000M02	998.2900	8615.360	1633.230	7.275350	9.474159	7.907271
2000M03	932.0500	7818.710	1568.790	-6.865735	-9.702708	-4.025503
2000M04	931.2100	7919.900	1596.120	-0.090165	1.285900	1.727106
2000M05	912.6600	7544.330	1589.420	-2.012140	-4.858229	-0.420651
2000M06	862.1200	6910.290	1513.690	-5.696894	-8.778468	-4.881879
2000M07	856.2400	6895.140	1488.300	-0.684376	-0.219479	-1.691585
2000M08	800.2300	6555.560	1405.750	-6.765152	-5.050327	-5.706356
2000M09	747.1700	5958.480	1358.900	-6.860645	-9.549813	-3.389542
2000M10	750.0400	6036.640	1308.070	0.383380	1.303215	-3.812278
2000M11	745.2500	5674.630	1351.250	-0.640681	-6.184220	3.247732
2000M12	712.5000	5286.660	1285.850	-4.493982	-7.081870	-4.961011
2001M01	678.9000	5065.580	1233.500	-4.830607	-4.271802	-4.156432
2001M02	714.5400	5304.450	1247.140	5.116514	4.607745	1.099727
2001M03	682.3300	4954.010	1213.580	-4.612557	-6.834874	-2.727826
2001M04	581.3700	4048.950	1123.350	-16.01260	-20.17398	-7.725938
2001M05	566.6900	4102.740	1130.970	-2.557497	1.319745	0.676038
2001M06	600.8600	4495.050	1154.360	5.854955	9.132175	2.047041
2001M07	621.0300	4482.350	1172.470	3.301743	-0.282933	1.556656
2001M08	657.7200	4880.790	1262.370	5.739992	8.515963	7.387827
2001M09	644.5300	4695.980	1222.140	-2.025794	-3.860027	-3.238749
2001M10	611.3200	4560.790	1212.740	-5.290082	-2.921097	-0.772116
2001M11	612.7200	4599.670	1221.740	0.228751	0.848871	0.739381
2001M12	664.1200	4945.850	1284.910	8.055479	7.256428	5.041260
2002M01	698.4500	5278.530	1333.150	5.040074	6.509881	3.685589
2002M02	714.8100	5697.860	1359.490	2.315318	7.644302	1.956507
2002M03	751.6400	6069.880	1402.840	5.024071	6.324817	3.138912
2002M04	776.2100	6204.200	1466.630	3.216562	2.188765	4.446850
2002M05	785.5100	6264.370	1508.400	1.191009	0.965154	2.808223
2002M06	748.5100	6064.300	1460.220	-4.824862	-3.245891	-3.246238
2002M07	739.8500	6093.350	1435.540	-1.163710	0.477889	-1.704602
2002M08	729.9900	5967.290	1433.620	-1.341663	-2.090512	-0.133837
2002M09	688.2900	5498.470	1377.540	-5.882057	-8.182302	-3.990341
2002M10	630.0100	5002.160	1309.560	-8.847457	-9.460005	-5.060810

	COMPOSITE	FINANCE	INDUSTRY	RET_COMP	RET_FIN	RET_IND
2002M11	646.1500	5047.310	1353.930	2.529598	0.898561	3.332027
2002M12	632.2000	4913.300	1345.760	-2.182588	-2.690961	-0.605257
2003M01	661.4100	5118.190	1369.680	4.516812	4.085505	1.761823
2003M02	656.9500	4949.120	1387.940	-0.676601	-3.359108	1.324350
2003M03	628.5500	4805.390	1364.660	-4.419233	-2.947158	-1.691532
2003M04	624.2100	4787.140	1369.300	-0.692873	-0.380505	0.339435
2003M05	635.7800	4863.090	1416.770	1.836574	1.574088	3.407997
2003M06	689.9400	5489.410	1466.330	8.175205	12.11467	3.438305
2003M07	727.1100	5981.290	1530.780	5.247314	8.581548	4.301473
2003M08	727.4600	6006.160	1573.970	0.048124	0.414935	2.782368
2003M09	740.9400	6072.410	1638.230	1.836064	1.096995	4.001530
2003M10	782.6300	6540.520	1710.210	5.474039	7.426111	4.299978
2003M11	792.2300	6447.250	1783.510	1.219171	-1.436299	4.196716
2003M12	791.9400	6454.580	1777.720	-0.036612	0.113627	-0.325169
2004M01	815.9800	6744.430	1805.380	2.990421	4.392702	1.543945
2004M02	825.9100	6957.320	1822.850	1.209596	3.107736	0.963011
2004M03	884.5500	7564.040	1933.540	6.859323	8.361110	5.895131
2004M04	866.2900	7197.400	1906.300	-2.085932	-4.968559	-1.418833
2004M05	793.9700	6531.450	1779.540	-8.717405	-9.709088	-6.880929
2004M06	817.6100	6657.680	1804.800	2.933977	1.914210	1.409488
2004M07	845.0200	6998.430	1852.850	3.297485	4.991476	2.627521
2004M08	815.6200	6754.040	1781.640	-3.541174	-3.554499	-3.919071
2004M09	856.7100	6992.240	1835.730	4.915091	3.466012	2.990793
2004M10	852.1600	6954.530	1844.260	-0.532517	-0.540772	0.463589
2004M11	882.2900	7113.230	1921.140	3.474650	2.256318	4.084065
2004M12	897.0200	7438.270	1926.720	1.655736	4.468186	0.290032
2005M01	929.7400	7802.520	2008.570	3.582682	4.780846	4.160395
2005M02	920.3100	7797.780	2003.050	-1.019441	-0.060768	-0.275201
2005M03	901.6400	7653.980	1933.240	-2.049524	-1.861330	-3.547367
2005M04	877.2300	7177.640	1945.240	-2.744611	-6.425514	0.618801
2005M05	891.3600	7105.070	2006.210	1.597917	-1.016202	3.086201
2005M06	892.6400	7111.410	1968.820	0.143498	0.089192	-1.881299
2005M07	921.0600	7384.660	2025.110	3.134182	3.770434	2.818964
2005M08	937.0400	7475.160	2046.650	1.720079	1.218065	1.058029
2005M09	914.7400	7438.970	1981.780	-2.408610	-0.485313	-3.220888
2005M10	925.5900	7387.580	2033.660	1.179150	-0.693219	2.584169
2005M11	896.1900	7143.940	2014.380	-3.227893	-3.353577	-0.952567
2005M12	898.8000	7258.320	2013.170	0.290810	1.588395	-0.060086
2006M01	911.9000	7300.160	2030.340	1.446979	0.574787	0.849267
2006M02	925.5100	7437.400	2050.010	1.481460	1.862506	0.964140
2006M03	924.3700	7422.100	2051.360	-0.123251	-0.205929	0.065832
2006M04	938.3200	7633.500	2015.480	1.497862	2.808442	-1.764561
2006M05	966.0500	7791.270	1993.450	2.912455	2.045742	-1.099057
2006M06	893.2300	7174.640	1863.100	-7.837149	-8.245129	-6.762504
2006M07	913.6300	7300.310	1885.200	2.258157	1.736423	1.179215
2006M08	946.0000	7489.120	2002.210	3.481689	2.553449	6.021765
2006M09	955.3100	7631.180	2013.150	0.979333	1.879119	0.544909
2006M10	983.5400	7924.630	2048.680	2.912241	3.773314	1.749502
2006M11	1030.090	8307.190	2152.770	4.624315	4.714577	4.955970
2006M12	1075.470	8474.030	2175.570	4.311160	1.988479	1.053531
2007M01	1119.330	8834.820	2271.760	3.997252	4.169454	4.326416
2007M02	1245.640	10321.10	2439.030	10.69192	15.54896	7.104556
2007M03	1166.380	9637.240	2287.120	-6.574452	-6.855558	-6.430704
2007M04	1308.200	10492.80	2560.840	11.47472	8.505455	11.30419
2007M05	1359.590	10940.70	2633.480	3.853104	4.180047	2.797084
2007M06	1357.180	10928.30	2624.520	-0.177417	-0.113403	-0.340814
2007M07	1384.720	11237.90	2651.650	2.008894	2.793624	1.028407
2007M08	1288.340	10315.90	2531.830	-7.214339	-8.560560	-4.623973
2007M09	1289.500	10478.60	2490.880	0.089998	1.564869	-1.630630
2007M10	1375.250	10964.90	2714.360	6.438099	4.536418	8.592014
2007M11	1384.580	10894.80	2721.730	0.676131	-0.641365	0.271151
2007M12	1403.410	10712.90	2900.420	1.350814	-1.683699	6.358785
2008M01	1507.040	11510.30	3135.810	7.124247	7.179366	7.803196
2008M02	1436.100	10836.10	3022.050	-4.821636	-6.035913	-3.695210
2008M03	1194.840	9359.270	2525.810	-18.39188	-14.65159	-17.93736
2008M04	1233.430	9729.050	2557.300	3.178662	3.874896	1.239021

	COMPOSITE	FINANCE	INDUSTRY	RET_COMP	RET_FIN	RET_IND
2008M05	1287.740	9969.860	2707.870	4.308984	2.445029	5.721033
2008M06	1229.350	9258.140	2605.090	-4.640317	-7.406338	-3.869512
2008M07	1144.000	8587.210	2407.140	-7.195468	-7.522928	-7.902790
2008M08	1109.430	8746.280	2308.760	-3.068452	1.835458	-4.172874
2008M09	1044.030	8639.410	2190.170	-6.075815	-1.229417	-5.273142
2008M10	966.0600	7497.080	2206.790	-7.761756	-14.18207	0.755980
2008M11	881.6500	6760.930	2118.250	-9.143079	-10.33532	-4.094869
2008M12	852.2700	6634.350	2028.570	-3.389177	-1.889976	-4.325916
2009M01	913.4600	7244.730	2128.270	6.933621	8.801361	4.797833
2009M02	909.8400	7182.290	2132.380	-0.397083	-0.865603	0.192928
2009M03	843.4500	6141.010	2042.610	-7.576814	-15.66290	-4.301032
2009M04	953.7100	7244.350	2230.990	12.28590	16.52326	8.821703
2009M05	1011.990	7995.210	2242.960	5.931433	9.862076	0.535099
2009M06	1090.150	8729.830	2349.250	7.439661	8.790328	4.629971
2009M07	1079.630	8698.480	2374.370	-0.969691	-0.359760	1.063601
2009M08	1188.570	9650.190	2609.050	9.613251	10.38293	9.425403
2009M09	1203.360	9827.200	2624.590	1.236674	1.817645	0.593852
2009M10	1246.840	10528.24	2667.120	3.549471	6.890712	1.607455
2009M11	1270.960	10927.30	2697.980	1.916017	3.720307	1.150411
2009M12	1265.450	10926.26	2677.940	-0.434473	-0.009518	-0.745550
2010M01	1294.710	11296.78	2718.080	2.285894	3.334866	1.487791
2010M02	1253.390	10946.04	2588.070	-3.243485	-3.153998	-4.901332
2010M03	1311.200	11664.37	2643.490	4.509087	6.356115	2.118759
2010M04	1335.890	12027.53	2767.800	1.865499	3.065929	4.595277
2010M05	1339.300	12111.05	2700.700	0.254935	0.692007	-2.454178
2010M06	1297.160	11654.49	2597.510	-3.196983	-3.842675	-3.895770
2010M07	1341.080	12091.59	2663.270	3.329800	3.681866	2.500140

Notes:
COMPOSITE: Composite Index
FINANCE: Financial Index
INDUSTRY: Industrial Index
RET_COMP: Return Composite
RET_FIN: Return Finance
RET_IND: Return Industry