smfsoln6

SOLUTIONS 6: NON-PARAMETRICS. 2.3.2017

Q1. *Brownian bridge covariance*

$$B_0(t) := B_t - tB_1. \qquad E[B_0(t)] = 0.$$

For $s, t \in [0,1]$,

$$\begin{aligned} cov(B_0(s), B_0(t)) &= E[(B_s - sB_1)(B_t - tB_1)] \\ &= E[B_s B_t] - tE[B_s, B_1] - sE[B_t B_1] + stE[B_1^2] \\ &= \min(s,t) - st - st + st \\ &= \min(s,t) - st. \end{aligned}$$

Q2. *Sample median*

If (at most) half the sample goes off to infinity (in either direction), (at least) the other half remains. So the sample median, though changed, does not go off to infinity. So the breakdown point (supremum of the fraction the estimator will thus tolerate) is a half.

Sample quartiles; Semi-inter-quartile range (SIQ).

The upper quartile has breakdown point 1/4: at most a quarter of the sample can go off to $+\infty$ without dragging the upper quartile with it (going off to $-\infty$ is harmless here). Similarly, the lower quartile also has breakdown point 1/4. So, the SIQ, which involves them both, has breakdown point 1/2.

Background:

N. H. BINGHAM, The sample mid-range and interquartiles. *Statistics and Probability Letters* **27** (1996), 131-136.

NHB