m3hw6.tex **Week 6. 21 – 24.2.2017**

17th C.: DESCARTES to NEWTON and LEIBNIZ

Background

We are now well into the 17th C., and what we take for granted – books, universities, the scientific method etc. – is starting to emerge.

Christianity had split a century before, in the Reformation, into Catholicism (centred on Rome, and dominant in S. Europe) and Protestantism (dominant in N. Europe), following Martin Luther (1483-1546) and his *Ninety-Five Theses* (1517)[1]. This led to the Counter-Reformation and the Inquisition, of which Galileo fell foul. It also led (or contributed to) the horrors of the Thirty Years' War (1618-48) in Germany, which devastated Germany and from which it took a long time to recover.[2]

We shall mainly deal with France, Britain and the Low Countries. France had serious internal dissension (the Frondes), then absolute monarchy under Louis XIV (1638-1715, reigned 1661-1715, leading to the French Revolution of 1789). England had the Civil Wars (1642-46 and 1648-49; Interregnum 1649-1660; Restoration 1660). The Low Countries were subjected to the Eighty Years' War (Dutch War of Independence, 1568-1648).

We note that the religious tensions of the time can be seen today in University titles. A University is headed by a Vice-Chancellor (VC); the Chancellor is an honorific figure[3]. Many universities were founded by Papal Bull (order of the Pope). Academics opposed to Catholicism noted that the University Statutes typically did not specify the powers of the Chancellor – who could then be ignored, hence the rise of the VC.

We note here also the translation of the Bible into German (Luther; NT 1522, OT 1534) and English (William Tyndale 1525; Authorised Version – King James Bible – 1611). Latin was no longer used in Protestant services, but survived in Catholic services till the Second Vatican Council of 1962-5.

The Bank of England was founded under William III in 1694, to finance the expansion of the Royal Navy (to counter the French and the Dutch),

The Académie Francaise was founded in 1635 on an initiative of Cardinal Richelieu. The Royal Society was founded in 1662 by King Charles II.

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[1] nailed to the church door at Wittenberg

[2] It is quite arguable that Germany has never fully recovered from the Thirty Years' War. Cf. ancient Greece and the Peloponnesian War.

[3] cf. the Colonel of a Regiment and its Colonel-in-Chief.

Tycho Brahe (1546-1601), Danish astronomer; spent twenty years observing planetary motion.

Johannes Kepler (1517-1630) became Brahe's assistant at the Prague Observatory in 1600, and then spent twenty years analysing Brahe's data.

Astronomia Nova (1609): contains Kepler's Laws:

1. Planets move about the Sun in elliptical orbits with the Sun at one focus.
2. A radius vector from Sun to planet sweeps out equal areas in equal times.

Kepler's third law (1619): $P^2 \propto a^3$ (P the orbit's period, a its semi-axis).

Marin Mersenne (1588-1648).

Mersenne was educated by he Jesuits at La Flêche, where he met Descartes (below), who encouraged him to study mathematics. From 1635 till his death in 1648, he corresponded with the leading scientists and mathematicians of his day, including Descartes and the Pascals (father and son), and organised conferences at which they could meet and discuss.

Mersenne primes are those of the form $M_p := 2^p - 1$ for p prime.

René Descartes (1596-1650) (B 17.2-10)

The Method (1637) (*Discours de la méthode pour bien conduire sa raison en chercher la vérité dans les sciences.*

The Method marks a milestone in the history of philosophy and science. The most important feature is its emphasis on 'systematic doubt' (B, 375).[4]

Descartes believed thought to be the foundation of all knowledge, hence his famous dictum *Cogito, ergo sum* (I think, therefore I am). He had great faith in his 'Cartesian system' of philosophy, which has led to him being called the 'father of modern philosophy'. Its impact helped the change from a religious to a scientific view of the universe. 'Descartes proclaimed explicitly that the essence of science was mathematics' (Kline, 325).

Descartes' views on science are discussed at length, and contrasted with Galileo's, in Kline Ch. 16. It was Galileo, rather than Descartes, who emphasised the importance of experiment and observation, the real essence of the scientific method.

Le Géométrie (Appendix to *La Méthode*.

The only positive contribution of the thirty Years' War to mathematics was the opportunity that winter campaigning in 1619 (under Duke Maximilian of Bavaria, on the Imperialist – 'Catholic' – side) gave Descartes to think about mathematics.

Book I: Geometric solution of quadratics.

[4]Otherwise put: the essence of a scientific attitude is *constructive scepticism*.

Book II. Conics: conics as curves having equations of the second degree.
Book III: Solution of equations. Descartes' Rule of Signs.

Modern analytical geometry, or 'Cartesian geometry', is only loosely derived from Descartes' *La Géométrie.* The strengths of his book lie in its demonstration of the power of algebraic methods in geometry. Its weaknesses include failure to exploit graphical methods (curve-tracing, etc.), reluctance to use negative coordinates, and absence of standard rectangular ('Cartesian') axes (Descartes used oblique axes, to be found in Apollonius).
La Dioptrique (Appendix to *La Méthode.* Treatment of the rainbow; refraction. See e.g.
C. B. BOYER, *The rainbow: From myth to mathematics*, Princeton UP, 1987 (1st ed. 1959), Ch. VIII.
Girard Desargues (1591-1661) and *Projective Geometry* (B 17.21, 22)

Desargues' first important book was *La Perspective* (1636). This led him on to his introduction of projective geometry in his *Brouillon projet (d'une atteinte aux evenements des rencontres d'une cone avec un plan)* (1639) *Rough draft (of an attempt to deal with the outcome of a meeting of a cone with a plane).* As background, recall:
(i) Many results in geometry concern only *incidence properties* (whether lines meet, point lie on a line, etc.); these are preserved under projection.
(ii) Often one has to qualify statements because of exceptional cases involving 'infinity' – e.g., two lines in a plane meet in a point (unless parallel).
Also relevant was the growing awareness of *perspective* (vanishing point = 'point at infinity').

All this led to Projective Geometry, in which it is incidence properties rather than metrical ones that count. Here one works *projectively*, using *homogeneous coordinates* (in which point in a plane has three coordinates rather than two, determined up to a constant multiple). Use of projective methods is a powerful tool which allows much simplification, but involves a thorough-going change of viewpoint.

Desargues introduced projective geometry in his pioneering book of 1639, the *Brouillon Projet.* But it had little impact at the time: it was too far ahead of its time, too badly written, and too few copies were distributed.

Projective methods in geometry, together with analytic (= coordinate) and synthetic (= classical) methods, complete the main tools needed to treat the geometric problems studied up to that time.
Conics. Projective methods allow a simple interpretation of conics as sections of circular cones by planes: every conic is a projection of a circle, and

every projection of a circle is a conic.

Projective geometry is of great practical importance: it is the basis of computer graphics, hence of virtual reality etc.

Blaise Pascal (1623-1662) (B 17.23-25)

Essay pour les coniques (1640) (one page!) Pascal's theorem (on hexagons inscribed in a conic), inspired by Desargues' work.

La logique, ou l'art de penser (1662). This book, written by Pascal (and/or his associates) at Port Royal, was the most successful logic book of its time, and was still in use in the 19th C. in Oxford and Edinburgh. The concluding pages of the Port Royal Logic contain what is apparently the first mention of 'probability' as something measurable.

Traité du triangle arithmétique (1665, posth.). As we have seen, 'Pascal's triangle' was known long before Pascal to the Chinese and the Arabs; Stifel introduced the term 'binomial coefficient' around 1544. However, Pascal's book of 1665 developed the theory of the arithmetic triangle, and had such an impact that it has been known as 'Pascal's triangle' ever since. See e.g.

A. W. F. Edwards, *Pascal's arithmetic triangle*, Griffin/OUP, 1987.

Probability. The date for the beginning of probability as a subject is taken as 1654, when Pascal wrote a letter to Fermat on de Méré's problem.

Pierre de Fermat (1601-1665), lawyer and amateur mathematician.

Ad locos planos et solidos isagoge (Introduction to plane and solid loci), written by 1636; published 1679, posth. Fermat recognised that an (algebraic) equation linking x and y represents a curve or locus in the (x, y)-plane. Written a year before Descartes *Geometry*, this insight went beyond Descartes. Fermat also found the canonical forms of the equations of the conics.

Number theory. Fermat was the founder of modern Number Theory, influenced by Diophantus' *Arithmetica* (translated 1621).

Fermat's little theorem: if p is prime, $a^p \equiv a \pmod{p}$. See e.g.

G. H. HARDY & E. M. WRIGHT, *An introduction to the theory of numbers*, 6th ed., OUP, Ch. VI.

Fermat's last theorem: if $n > 2$, $x^n + y^n = z^n$ has no solution x, y, z in non-zero integers ($n = 2$: Pythagorean triples!). Proved in 1995 by Wiles.

Fermat's Principle: light travels along pasths of *shortest time*. In a homogeneous medium, this reduces to *Heron's Principle* : light travels along paths of *shortest distance*. By the 17th C., light was believed to have finite speed. The speed of light was first measured by the Danish astronomer Römer in 1676, after Fermat's death. Fermat's Principle is a precursor of the Calculus of Variations (below).

Christiaan Huygens (1629-1695).

Huygens was Dutch, a pupil of Frans van Schooten (1615-1660), Professor of Mathematics at Leiden. Huygens moved from the Netherlands to Paris in 1668, when the Académie des Sciences was founded.

Traité de la lumière (publ. 1690; read to the French Academy, 1678).

Huygens advocated the *wave theory of light*; the work also covers reflection and refraction, and polarisation. [5]

Pendulum clock, 1656 (Galileo is said to have proposed this in 1641).

Horologium oscillatorium (1673): centripetal force for circular motion. Principle of Conservation of Energy. This book was an important precursor of Newton's *Principia*.

Telescopes: Observation of the rings of Saturn.

Thermometers: suggested 0 and 100 for freezing and boiling points – the *Centigrade* scale [6].

Huygens' Principle: Light travels along paths of shortest time. Kline (579-582) discusses the history of this idea, from the Greeks (Heron) through Fermat and Huygens to Euler.

The cycloid. Huygens knew from his work on the pendulum clock that the period of a pendulum is only approximately constant, but depends on the displacement. He sought a curve such that a particle sliding on it has *exactly* the same period regardless of displacement, and showed that this was a *cycloid*.[7]

Envelopes and wavefronts. His work on the wave theory of light led Huygens to regard propagation of light as a continuous process of 'ripple formation': subsidiary wavelets released at time t from, through their envelope, the wavefront at time $t + dt$. For background, see e.g.

J. L. SYNGE, *Geometrical optics: An introduction to Hamilton's method*. Cambridge Tracts in Math. 37, CUP, 1937.

De ratiociniis in ludo aleae (1657) (On reasoning in dice games). This was the first important book on Probability Theory (recall Cardano's *De ludo*

[5] Huygens' wave theory of light was opposed by Newton's corpuscular theory of light. The wave theory was later revived by Thomas Young in 1801, and by Fresnel. It was not until Einstein's work of 1905 on the photon, and later Quantum Mechanics, that it was realised (wave-particle duality) that both theories are correct, indeed complementary.

[6] long before Anders Celsius (1707-1744) in 1742

[7] The locus of a point on a circle rolling without slipping on a plane. The cycloid is also often used for the arches of bridges.

aleae was written c. 1526 but only published in 1663).
Gravitation. Huygens opposed Newton's Law of Gravity, which he regarded as unphysical ('action at a distance') and a mathematical artefact.[8] For historical background, see e.g.
Sir Edmund WHITTAKER, *A history of the theories of the aether and electricity.* Dover, 1989 (Nelson, 1910/1951), Volume I, Ch. 1: The theory of the aether to the death of Newton.

John Wallis (1616-1703), Savilian Professor of Geometry at Oxford (1649).

A pupil of Oughtred, Wallis was the greatest British mathematician before Newton.
Tractatus de sectionibus conicis (1665). This book popularised Cartesian methods in England, as Schooten's (written before 1655, publ. 1659) has on the Continent. Wallis advocated use of coordinate (analytic) and algebraic methods in geometry – the reverse of the Greek approach, using geometric methods in algebra. He used the canonical forms for the conics.
Arithmetica infinitorum (1655). This was a (rather non-rigorous) work on calculus – or rather, the 'pre-calculus' of the time. It contains (in effect) $\int x^n dx = x^{n+1}/(n+1)\ n \neq -1)$, use of the symbol ∞ for infinity, and *Wallis' product* for π (for details, see e.g. my webpage, M2P3, Handout, 'Infinite products for sin, cos and tan; Wallis' product).
Treatise on algebra, both historical and practical (MS 1676; publ. 1685). If m/n is a fraction in its lowest terms, the decimal expansion of m/n terminates or recurs after at most $n-1$ places. 'Newton-Raphson method' for iterative solution of equations (Newton, *De analysi*, below; Joseph Raphson, *Analysis aequationum universalis*, 1690).

Isaac Barrow (1630-1667); Professor of Geometry, Gresham College, London, 1662; Lucasian Professor of Geometry, Cambridge, 1664.

Barrow is chiefly remembered for his influence on Newton, in whose favour he magnanimously resigned his Cambridge chair. He was a political conservative, disapproving of the Royal Society for 'infringing on the universities' prerogatives', and a mathematical conservative, disliking algebra.
Lectiones geometricae (1670). This work all but anticipates the differential calculus of Newton and Leibniz.

[8] This was prescient, as such things are still topical in Physics, e.g. quantum entanglement (cf. the lack of fit between General Relativity and Quantum Mechanics).

(Lord) *William Brouncker* (1620-1684), first President of the Royal Society. Continued fraction for π (related to Wallis' product):

$$\pi/4 = \cfrac{1}{1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{\ddots}}}}$$

Other notable founding Fellows of the Royal Society included:
Robert Boyle (1627-91). Boyle's Law $PV = kT$); mercury thermometer; temperature of blood. *The Sceptical Chemist*, 1661.
Sir Christopher Wren (1632-1723), Professor of Astronomy at Gresham College and architect of St. Paul's Cathedral.

Sir Isaac Newton (1642-1727).

Newton was born on Christmas Day 1642, in Woolsthorpe, Lincs. He matriculated at Trinity College, Cambridge in 1661, was elected to a scholarship there in 1664 and a fellowship in 1667. He succeeded Barrow as Lucasian Professor of Mathematics at Cambridge in 1669, and was elected FRS in 1672.

The value of the books cited earlier may be judged from their impact on Newton as a student. He read Oughtred's *Clavis*, van Schooten's *Geometria a Renato Des Cartes*, Kepler's *Optics* and Wallis' *Arithmetica infinitorum*. He attended lectures by Barrow, and knew the work of Viète, Galileo, Fermat and Huygens.

Following an outbreak of plague, Trinity was closed during 1665-6, and Newton went home. The foundations of his best work were done during this period: the binomial theorem; the calculus; the Law of Gravity; decomposition of white light into colours.

After his unhappy first experiences of publication (below), Newton withdrew as much as possible from mathematical and scientific life. He continued to lecture on mathematics at Cambridge (though even this lapsed later – as was not uncommon in those days). His main interests during the 1670s and early 1680s were alchemy (= chemistry) and theology.

Newton became Member of Parliament for Cambridge University in 1689 (following the attempts under James II (reigned 1685-8) to re-catholicise England)[9], Warden of the Mint in 1696 and Master of the Mint in 1699. He

[9]The Universities of Oxford and Cambridge had MPs till 1950.

was President of the Royal Society from 1703 till his death. He was knighted by Queen Anne in 1705, and is buried in Westminster Abbey; see his statue and memorial stone there, and his statue in the square at Grantham, near his birthplace.

Optics

Newton constructed a reflecting telescope with his own hands (unaware that he had been anticipated in this by James Gregory (1638-75), *Optica promota*, 1663), and presented it to the Royal Society in 1671.

Newton showed, in his *Experimentum crucis* of 1666, how white light can be split by prisms into the colours of the rainbow. His work on this was published in No. 80 of the *Philosophical Transactions of the Royal Society* ('Phil. Trans.') in 1672.

Newton advocated the *corpuscular theory of light*. His work was attacked by Huygens, who favoured the *wave theory of light*, and by *Robert Hooke* (1635-1703, Curator of the RS from 1665), in his *Micrographia* of 1665. Newton's paper on light was refereed and criticised by Hooke. Newton was hypersensitive to criticism, and reacted to this by refusing to publish subsequently. As a result, his best work was published long after it was done. Predictably, this led to priority disputes (most notably with Leibniz over the calculus, below).

Opticks, 1704. Newton's work on optics was eventually published in book form (note the archaic spelling).

Newton gave a full treatment of the theory of the rainbow using differential calculus, following less complete treatments by Descartes and Huygens.

Analysis

Series. Although Newton found the Binomial Theorem

$$(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n, \qquad |x| < 1, \qquad \binom{a}{n} := a(a-1)\ldots(a-n+1)/n!$$

in his 'plague years', 1665-6, he never published it. The result was published by David Gregory (1627-1720), elder brother of James and Savilian Professor of Astronomy at Oxford (1691), in 1688. Again, Newton found 'Taylor's Theorem', eventually published by Brook Taylor (1685-1731) in *Methodus incrementorum directa et inversa* in 1715.

Newton did pioneering work on the use of infinite series in analysis, doing much to make their use regarded as legitimate (recall that the ancient Greeks had avoided the use of infinite processes where possible).

Calculus. Newton found the cornerstones of calculus – differentiation, integration and the 'Fundamental Theorem of Calculus' linking the two – in 1665-6 also. His first published account is in the *Principia* (1687, below). But he wrote three earlier accounts:
De analysi (per aequationes numero terminorum infinitas (MS 1669, publ. 1711).
De methodis fluxionum (MS 1671, publ. 1736, posth., *Method of Fluxions*);
Tractatus de quadratura curvarum (MS 1676, publ. 1704, as an appendix to the *Opticks*).
Newton's notation and terminology have not stood the test of time. It survives only in the dot notation in Mechanics for a time derivative, $\dot{x}(t) := dx(t)/dt$, but the standard calculus notation and terminology is that of Leibniz.
Dynamics; Celestial Mechanics

The great challenge of the new astronomy was to explain Kepler's Laws, which had been arrived at empirically. It was suspected that an inverse square law of attraction was the key by several people, including Hooke and Halley (Edmond Halley (1656-1742); 2nd Astronomer Royal (after Flamsteed), 1720-42). In 1679 Newton and Hooke corresponded on this. In 1684, Halley went to Cambridge (where Newton lived as a recluse, having nothing to do with the Royal Society because of his antipathy to Hooke). Halley asked Newton what the orbit of a body was under the inverse square law, and Newton replied that it was an ellipse. On being pressed as to how he knew, he replied that he had calculated it (long before), but could not find the proof among his papers. In November 1684 Newton wrote an unpublished manuscript *De motu (corporum in gyrum)*, on the link between Kepler's laws and the inverse square law. Halley put the fear of being beaten by others (perhaps Hooke) into Newton's mind, hoping that this would provoke Newton into writing his results up into book form. In this Halley was successful. Newton's great work, his *Principia*, was published in 1687, at Halley's expense.

Newton was reluctant to publish his work on dynamics, partly because of his morbid fear of controversy (above), partly because he needed to develop his ideas on force (culminating in his Laws of Motion), partly because of the mathematical difficulties, which were severe with the tools then available. For example, we know that a rigid body in motion behaves like a point mass at its centre of gravity (centre of mass, centroid). But this needs Integral Calculus, on which Newton was still working.

Principia, 1687
Philosophiae naturalis principia mathematica, 1st ed. 1687, 2nd ed. 1713, 3rd ed. 1726.

Newton's *Principia* is the most famous mathematical book ever published, and rightly so. It triggered the Scientific Revolution, and so helped to usher in the modern world.

In the Preface, one find:

Newton's Laws of Motion
Law I (inertia): A body continues in its state of rest or uniform motion in a straight line unless force is applied to change it.
Law II: Force = Mass × acceleration; $F = ma$.
Law III: To every action there is an equal and opposite reaction.[10]
Book I. Calculus (which Newton called the Method of Fluxions).
Inverse Square Law of Gravity: every particle of mass in the Universe attracts every other, with a force $F = cm_1m_2/r^2$, where m_i are the masses and r is their distance apart; c is a universal constant, the *gravitational constant*.
Deduction of Kepler's Laws from the Inverse Square Law.
Deduction of the Inverse Square Law from motion along a conic (so from Kepler's First Law).
Centripetal force.
Motion of bodies along surfaces; pendula (following Huygens).
Effective concentration of the mass of a spherical body at the centre.
Three-body problem (motion of three bodies, each attracting the other two); approximate results. This is motivated by the Sun-Earth-Moon dynamical system. The problem is still open (and no doubt insoluble).
Book II.
Motion of bodies in resisting media; hydrodynamics. Wave motion; sound waves; velocity of sound in air. Planetary motion in a vacuum.
Book III (On the System of the World).
Calculation of the Sun's and planets' masses from the Earth's mass.
Average density of the Earth shown to be between 5 and 6 times that of water (the modern figure is c. 5.5; recall that we know know the Earth's core to be molten iron).
Earth not a sphere but an oblate spheroid. Precession of the equinoxes.
Tides (primary cause: the Moon; secondary cause: the Sun (spring and neap

[10]Note the analogy with double-entry book-keeping: every transaction has a debit effect one way and a credit effect the other.

tides)). Why there are *two* tides a day.
Orbits of comets. Motion of the moon; determination of longitude.
Numerical Analysis: Newton's divided-difference interpolation formula (Lemma v, case (ii)).

Later years

In his work at the Mint, and as PRS, Newton was an efficient and careful administrator. He did much to restore both the coinage and the scientific standing of the Royal Society.

Overall assessment

Newton was (with Archimedes and Gauss) one of the three greatest mathematicians of all time. He was (with Einstein) one of the two greatest physicists of all time. Combining these two, there is a strong case for regarding Newton as the greatest scientist of all time. He was undoubtedly the greatest British scientist of all time. His fame – particularly in Britain – needs no emphasising; we merely note e.g. the well-known story of the apple; his face on the old £1 note; Alexander Pope's epitaph:

Nature, and Nature's Laws, lay hid in Night.
God said, Let Newton be! and All was Light.[11]

Newton's name is commemorated in, e.g., the Isaac Newton Institute for the Mathematical Sciences (INIMS) in Cambridge (next to the Centre for Mathematical Sciences).

Newton called himself a philosopher (seeker after the truth), rather than a scientist or a mathematician. His fascination with theology may seem odd today, but religion played a much more dominant role in life in those days than now. He is known to have been a unitarian (one who believed in one God rather than the Trinity), which would have been perceived as heresy in his day. This would have been incompatible with his Fellowship of Trinity and Cambridge chair, so he had to conceal it, at the cost of living a lie. This partly explains his difficult personality.

Gottfried Wilhelm Leibniz (1646-1716)
Nova methodus pro maximus et minimis, idemque tangentibus, qua nec irrationales quantitates moratur (1684) (A new method for maxima and minima, and for tangents, not obstructed by irrational quantities).

Leibniz was a polymath, who studied mathematics with Huygens. His

[11] It could not last. The devil, shouting 'Ho,
Let Einstein be!' restored the status quo.

Nova methodus was his account of the differential calculus, using the modern notation dy/dx, etc. His account of the integral calculus, and the Fundamental Theorem of Calculus linking the two, appeared in *Acta Eruditorum* in 1686. The notation $\int f(x)dx$ is due to him (the integral sign is an elongated S, for Summe = sum, German).

Leibniz' Theorem: writing D for d/dx, $D^n(fg) = \sum_{r=0}^{n} \binom{n}{r} D^r f D^{n-r} g$.

The priority dispute

Leibniz visited London in 1673, was elected a Fellow of the Royal Society, and knew its Secretary, Oldenburg, and Librarian, Collins. It was because he could have seen *De analysi* in MS, at the Royal Society or in one of the copies circulating on the Continent, that Newton accused Leibniz of plagiarism, a charge Leibniz indignantly denied. The quarrel between Newton and Leibniz broadened to a quarrel between their followers in England and on the Continent. This was doubly unfortunate, because not only was Leibniz almost certainly blameless (unlike Newton, who wilfully delayed publication of his work), but Leibniz' notation was much superior. It blighted British mathematics for much of the 1700s, cutting it off from Continental progress.

It is also fair to point out that Newton engaged in acrimonious quarrels with (in addition to Leibniz) Hooke and Flamsteed, the first Astronomer Royal. He was not above abusing his position in pursuing such quarrels. For instance, although we have many fine portraits of Newton (who enjoyed sitting for artists), we do not know what Hooke looked like: Newton, as President of the Royal Society, had all portraits of Hooke destroyed. Again, Newton had anonymous articles supporting him against Leibniz published in *Phil. Trans. Roy. Soc.*, again an abuse of his power as President.

One moral of this unfortunate quarrel – recognised today – is promptness in publication. A modern scientist recognises that prompt writing up and submission of his work is part of his professional duty.

Other work

Leibniz was a lawyer, diplomat and courtier as well as a mathematician. He wrote *Theodicy* in 1710, an attempt to reconcile a benevolent, omnipotent God with the existence of evil in the world. Such Leibnizian optimism was satirised by Voltaire (1694-1778) in his *Candide* of 1759. The ridiculous Dr Pangloss, whose slogan is 'All is for the best in this best of all possible worlds', is based on Leibniz. *Candide* was influential in the Enlightenment.

J. E. HOFMANN, *Leibniz in Paris 1672-1676: His growth to mathematical maturity*, CUP, 1974.