M2PM3 COMPLEX ANALYSIS: ASSESSED COURSEWORK 1, 2011

Set Mon 31.1.2011, Deadline 2pm Wed 9 Feb 2011; 20 marks

Q1 [**5**]. Write T_n for the nth Tchebycheff polynomial of the first kind.
(i) Find the bounds of T_n on the real interval $[-1, 1]$ [**1**].
(ii) On which sets in the complex plane is T_n bounded? [**4**]

Q2 [**5**]. For an nth root of unity $\omega = e^{2\pi ik/n}$ ($k = 0, 1, \ldots, n-1$), the *order* of ω is the least natural number r with $\omega^r = 1$. Find the orders of all nth roots of unity.

Q3 [**4**]. Find the radius of convergence of each of the following power series:
(i) $\sum_{n=0}^{\infty} z^{2^n}$ [**1**];
(ii) $\sum_{n=0}^{\infty} z^{2^n}/n!$ [**2**];
(iii) $\sum_{n=0}^{\infty} n! z^{2^n}$ [**1**]
(you may quote Stirling's formula: $n! \sim \sqrt{2\pi} e^{-n} n^{n+\frac{1}{2}}$ as $n \to \infty$).

Q4 [**6**]. For each of the following functions, find (a) the first three terms of its power series expansion about the origin, and (b) the radius of convergence of this series (you are not asked for the general term of the series):
(i) $z/(e^z - 1)$ [**1,1**];
(ii) $1/\cos z$ [**1,1**];
(iii) $1/\mathrm{ch}z$ [**1,1**].

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