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# THE WORLDWIDE INFLUENCE OF THE WORK OF B. V. GNEDENKO

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**Abstract**

We discuss the life and work of B. V. Gnedenko, with particular reference to his pioneering work on extreme-value theory and, with Kolmogorov and others, limit theorems.

**1. Life.**

Boris Vladimirovich Gnedenko was born on 1 January 1912 in Simbirsk, Russia (his family was of Ukrainian origin, and the name Gnedenko is Ukrainian). Simbirsk was also the birthplace of V. I. Lenin (Ulyanov), and its name was changed to Ulyanovsk. The family moved to Kazan when he was three, and he received his schooling there. Partly through being placed in the same class as an older brother, partly through his precocious development, he completed school at the age of 15. He applied to the University of Saratov, but was under the official age limit for entry. He was eventually admitted on the personal recommendation of the then Soviet Minister of Education, Lunacharsky. He graduated in 1930 after three years (reduced from the usual five).

From 1930-34 Gnedenko taught at the Textile Institute in Ivanovo. Here he naturally became interested in the reliability of the machines used in textile manufacture. His first papers are on this topic and date from 1933; reliability theory remained a lifelong interest. He also became interested in probability theory, and attended seminars by A. N. Kolmogorov (1903-87) and A. Ya. Khinchin (1984-1959) in Moscow.

In 1934 Gnedenko was awarded a scholarship that enabled him to begin postgraduate work at the Institute of Mathematics at Moscow State University. He worked at first under both Kolmogorov and Khinchin, but continued under Kolmogorov alone when Khinchin left for Saratov University in 1935. His doctoral thesis was awarded in 1937, for work on infinitely divisible distributions.

He began work as a research assistant in 1937, but was arrested and imprisoned later that year, as a result of incautious remarks to fellow students while on holiday that summer. He was released in 1938, and began lecturing at Moscow State University. He received his higher doctorate in 1942. During the war years of 1941-45 he did war work; he was evacuated from Moscow to the east. In 1945 Gnedenko was elected to the Ukrainian Academy of Sciences. He left Moscow for Kiev, then became a professor at Lvov University. In 1949 Gnedenko returned to Kiev as Head of the Physics, Mathematics and Chemistry Section of the Ukrainian Academy of Sciences and Director of the Kiev Institute of Mathematics. In 1960 he returned to Moscow State University, where he remained for the rest of his life; he was Head of the Department of Probability Theory from 1966, in succession to Kolmogorov.

In 1939 Gnedenko married Natalia Konstantinova; they had two sons, one of whom, Dmitri, is a mathematician.

**2. Early Work: Limit theorems for sums**

Gnedenko's early work was in limit theorems – specifically, the limit theory of sums of independent random variables. This is closely connected with the theory of stochastic processes with stationary independent increments (now generally known in the West as Lévy processes). The study of these processes began with Bruno de Finetti in 1929 and 1930 and continued wtih Kolmogorov in 1932, Lévy in 1934, Khinchin (as Khintchine, as he published this paper in French) in 1937, and Lévy again in 1937, in Ch. VII of his book [L]. The formula specifying the distribution of the general infinitely-divisible law is thus called the *Lévy-Khintchine formula*, while the corresponding processes are called *Lévy processes*; for background and detail, see e.g. the standard works by Bertoin [Be], Sato [Sat] and Applebaum [Ap].

Following a number of papers from 1939 onwards on limit theorems for sums of independent random variables, domains of attraction etc., in 1949 the Russian edition of the famous book by Gnedenko and Kolmogorov appeared, *Limit distributions for sums of independent random variables* [GnKol]. This very important and highly influential book was translated into English in 1954 (the year after the appearance of Doob's book [Do] on stochastic processes, where Lévy processes are discussed in Ch. VIII). The translator, K.-L. Chung, remarks in the Preface '... a certain amount of mathematical maturity, perhaps a touch of single-minded perfectionism, is needed to penetrate the depth and appreciate the classic beauty of this definitive work'. Of its central theme, Chung remarks in the Preface to one of his own books [Ch] that it 'has been called the "central problem" of classical probability theory. Time has marched on and the centre of the stage has shifted, but this topic remains without doubt a crowning achievement.'

It is well known that Karamata's theory of regular variation provides the natural tools and language for characterizing the domains of attraction of stable laws; see e.g. [BGT], 8.2, 8.3 (the infinitely divisible laws are the limit laws of two-suffix arrays; the self-decomposable laws are the limit laws of one-suffix arrays; the stable laws are the limit laws of random walks – all after centring and scaling). It is also well known that this entered the text-book literature in 1966 in the influential book by Feller [Fe], IX.8, XVII.5. It is less well known that the realisation of the relevance of regular variation to domains of attraction dates from Sakovich in 1956, appropriately enough in the first volume of the journal *Theory of Probability and its Applications* (I learned this from V. M. Zolotarev); see [Sak], [B].

The resulting theory has subsequently been generalised greatly, in several directions; we mention two here. The first is to more general processes such as semi-martingales, where the Lévy-Khintchine characteristics are replaced by characteristics of more general type; for a monograph treatment, see Jacod and Shiryaev [JaSh]. The second is to higher dimensions. The book by Gnedenko and Kolmogorov was used by Meerschaert and Scheffler [MeeSc] as their model for their book on sums of independent random vectors. This book also contains a treatment of multivariate regular variation. This general area is of great topical interest in view of the heavy tails shown by many financial time series and the obvious importance of dependence for financial portfolios in high dimensions; see e.g. Balkema and Embrechts [BaEm]. A general theory of regular variation in any dimension, finite or infinite, has recently been developed by the author and Ostaszewski; see e.g. [BOs].

In his later work, Gnedenko used regular variation, with Feller's book as his reference, for instance in his 1972 [Gn5] paper of 1972 on random sums (sums of a random number of random variables). This was appropriate, as it appeared in the volume commemorating William Feller (1906-70). Random sums were the subject of his last book with V. Yu. Korolev in 1996 [GnKor].

**3. Early work: Extremes**

In 1943 Gnedenko published his famous paper [Gn1] on extreme-value theory (EVT). Here one deals with the *maximum* $M_n := \{X_1, \ldots, X_n\}$ of a sequence of independent and identically distributed (iid) random variables, whereas above one deals with their *sum* $S_n := X_1 + \ldots + X_n$. Here Gnedenko obtains the three possible forms of the limit laws of $M_n$, after suitable centring and norming. These are (in the usual notation and terminology) the *Fréchet* $\Phi_\alpha$ ($\alpha > 0$ – heavy-tailed), *Gumbel* $\Lambda$ (light-tailed) and *Weibull* $\Psi_\alpha$ ($\alpha > 0$ – bounded tail); an alternative parametrisation uses a one-parameter family, with the main cases $\alpha > 0$ and $\alpha < 0$ separated by the Gumbel case $\alpha = 0$. That these were the only possible limits of $M_n$ (to within type, i.e., location and scale) goes back to Fisher and Tippett in 1928 [FiTi]. Gnedenko gives also the domains of attraction, in the Fréchet and Weibull cases (the Gumbel domain of attraction remained open until the work of Gnedenko's pupil Mejzler in 1949 [Mej], Marcus and Pinsky in 1969 [MaPi] and de Haan in 1970 [deH]).

The similarities between sums and maxima become transparent when one notes that for maxima one uses the $n$th power of the *distribution function*, $F$ say ($M_n$ has distribution function $F^n$), whereas for *sums* one uses the $n$th power of the *characteristic function* (CF), $\phi$ say ($S_n$ has CF $\phi^n$). Writing $\bar{F} := 1 - F$, one has as $x \to \infty$

$$\log F^n(a_n x) = n \log(1 - \bar{F}(a_n x)) \sim n\bar{F}(a_n x),$$

and the assumption that $M_n/a_x$ has a limit law $G$ yields

$$n\bar{F}(a_n x) \to \log G(x) \qquad (n \to \infty).$$

This both gives regular variation of $\bar{F}$ and identifies $\log G$ as a power, from which one reads off the functional form of $G$; see e.g. [BGT], 8.13.2.

A similar argument but with CFs in place of distribution functions leads both to regular variation of $1 - \phi(.)$ at the origin and to the log-CF of the limit being a power. This yields both the domain of attraction of a stable law and the CF of a stable law. These results go back to [GnKol] above; textbook treatments using regular variation as above are in [Fe], [BGT] 8.3.2. Relevant here is the question of passing from regular variation of the CF $\phi$ at the origin to regular variation of the tails of $F$. This is a *Tauberian* question; see [Pi]. One can avoid both regular variation and Tauberian aspects, as in [GnKol], but matters simplify with them.

Also in his 1943 paper is his result on *relative stability* of $M_n$: the condition for the existence of constants $a_n$ such that $M_n/a_n \to 1$ in probability (a weak law of large numbers). The idea of relative stability goes back to work of Khinchin in 1935 – after Karamata's work on regular variation of 1930-31, but before its extension from continuous to measurable functions in

1949, and its introduction into probability theory mentioned above.

The whole area of EVT has proved enormously important, in both theory and practice. One area of application is in engineering, textiles and the like – the strength of a chain is that of its weakest link; it is the strongest gust of wind that tests the ability of a building to withstand wind, etc. No doubt Gnedenko's early technical experience in the Textile Institute in Ivanovo played its part in motivating his important work in this area. The field received another massive boost as a result of the disastrous flooding in the Netherlands (and to a lesser extent, the UK) on the night of 31 January – 1 February 1953. This led to a special focus on the mathematics of EVT in the Netherlands, and in particular to the extensive works of L. de Haan in this field, from 1970 on.

A quite different application of EVT lies in the actuarial, insurance and financial fields. In insurance, it is the big risks that are potentially lethal to the insurance company, which may seek to hedge their exposure by seeking *reinsurance*. This of course leads immediately to modern versions of Juvenal's famous question *quis custodiet ipsos custodes?* (who guards the guards? – who polices the police?): who insures the insurers? – who reinsures the reinsurers? One classic instance of this was the Lloyds of London insurance scandal of 1988-96 (we note in passing that Lloyds of London has a long history pre-dating the concept of limited liability and the legislation needed for the concept of the limited liability company, one of the foundation stones of modern capitalism; Lloyds members – called 'names' – had unlimited liability, and many of them were forced into personal bankruptcy).

The relevance of EVT has been seen very clearly in the fallout from the financial crisis of 2007 (US), 2008 (UK), ...., Greece (2012), ...., and in particular on what it revealed about the *risk management* of leading financial institutions. Inquiries after the event into some of the most prominent major corporate failures revealed a systematic underestimate of the dangers of dependence between different stock prices. Under normal circumstances, one can diversify (in the manner of Markowitz), and balance one's portfolio by including lots of negative correlation between the different risky assets one holds. In crisis conditions, when the market as a whole experiences a disastrous fall, this gives one no protection. The need to improve risk management in the light of these dangers is now widely recognised, taken very seriously by the financial regulatory authorities, and is highly topical today; see [BaEm] for some of the mathematics relevant here.

Extensions of EVT to higher dimensions are of great practical as well as theoretical importance. In finance, the dimensionality is the number of assets one chooses to hold in one's portfolio. In the planning of flood defences, sea walls and dykes, it is the number of stations at which the sea height is recorded. But here, an infinite-dimensional treatment is really necessary (using stochastic processes rather than random vectors): one significant breach in the sea wall, anywhere along the coastline of the Netherlands (for example) is enough to trigger a major disaster.

As an afterword to Gnedenko's work on EVT: in the 1992 conversation with Gnedenko by Singpurwalla and Smith [SiSm], the question of the influence of the Fisher-Tippett paper on Gnedenko's 1943 paper [Gn1] was raised repeatedly on p. 276-277 (after a preliminary editorial insertion on p.276, in which the Fisher-Tippett result is described as 'without rigorous proof'). The eight questions on this point, and their answers, form an interesting exchange (without, in this writer's opinion, casting light on the matter). Note that the second interviewer, R. L. Smith, is a noted specialist in EVT.

**4. Reliability; queueing**

From his earliest work in technical areas and the textile industry, Gnedenko had always been interested in reliability. He wrote, with Yu. K. Belyaev and A. D. Soloviev, the book *Mathematical methods in reliability theory* [GnBeSo], which appeared in 1966 (the same authors spoke on reliability theory in the Fifth Berkeley Symposium on Mathematical Statistics and Probability in 1965; the proceedings appeared in 1967).

Also in 1966, Gnedenko published, with I. N. Kovalenko, *Introduction to queueing theory* [GnKov] (English translation 1968). This was widely praised. In 1969, J. W. Cohen, in his standard work *The single-server queue* [Co], comments on the authors' use there of the 'method of suplementary variables' (increasing the dimensionality so as to obtain the Markov property).

Queueing theory and reliability theory are good examples of areas of probability theory directly motivated by, and applicable to, the demands of industry and the economy. We note that Gnedenko's teacher Khinchin published a monograph [Kh] on queueing theory (or in the Russian original, the theory of mass service) in 1955 (the English translation appeared in 1960, after Khinchin's death). Naturally, the growth of applied probability was stimulated in the USSR by the demands of industrialization; the founding in 1956 of the journal *Theory of Probability and its Applications* was noted earlier. The needs of Applied Probability were recognized in the West also

– witness, for instance, the foundation in the UK by J. Gani of the *Journal of Applied Probability* in 1964 (followed by its sister journal *Advances in Applied Probability* in 1969).

**5. Random sums**

As we have mentioned, this topic is summarised in [GnKor]. One result here is the *Gnedenko transfer theorem* (Gnedenko and Fahim [GnFa] in 1969, completed by Gnedenko's student Domokos Szász [Sz] in 1972), linking the possible limit laws of random sums (of independent random variables) and of non-random sums. For instance, Gnedenko's 1970 paper [Gn5] shows how the law on the positive half-line with Laplace-Stieltjes transform $1/(1+Cs^{\gamma})$ ($C > 0$, $0 < \gamma \leq 1$) arises in this context.

**6. Statistics and empiricals**

Following the work of Kolmogorov and Smirnov on empiricals for one sample (Kolmogorov found the limit law in 1933; Smirnov tabulated this in 1948), and the associated non-parametric goodness-of-fit test, Gnedenko and others wrote a number of papers in the early 1950s on various aspects of empiricals, including two-sample goodness-of-fit tests. In his paper [GnKoSk] with Korolyuk and Skorohod, he makes accessible asymptotic expansions for the Kolmogorov-Smirnov distribution (due to Chan Li-Tsian) previously accessible only in Chinese. Some of their results may be found in the standard work by Shorack and Wellner [ShWe].

**7. History of mathematics**

Gnedenko wrote a history of mathematics in Russia before the war, but it appeared after the war in 1946 [Gn2]. He wrote a number of papers in this area later, some studies of Marx and mathematical education, a history of the Department of Probability at Moscow State University, etc. He also wrote a large number of obituaries and appreciations of mathematicians: an obituary of his teacher Khinchin, for instance [Gn3]; pieces on the 60th and 70th birthdays of his teacher Kolmogorov, and many such pieces for other mathematical colleagues. He also wrote on the historical evolution of the probability concept, and on such major figures from the history of science as M. V. Lomonosov.

**8. Books**

We have already mentioned the monographs [GnKol], [GnBeSo], [GnKov] and [GnKor], and the historical book [Gn2]. In addition, Gnedenko wrote two classics: his elementary book with Khinchin [GnKh] in 1945, and his text [Gn4] in 1969. The influence of Gnedenko's books can be seen from the fact that a number of them ran to several (or many) editions, and were translated into several (or many) languages.

**9. Some reminiscences**

Gnedenko travelled quite widely in the West – for example, he spoke at the last three of the Berkeley Symposia. He was frequently an invited speaker at conferences on probability and stochastic processes, where I heard him speak a number of times (very well). His manner was always very gentlemanly, and I got the impression that he enjoyed his status as (what the English call) a Grand Old Man.

An interesting brief tribute to Gnedenko is included in the book *Leading personalities in statistical sciences* by Johnson and Kotz [JoKo, 244-246]. This contains the reply Gnedenko gave in an interview in honour of his eightieth birthday [Shi]. He was asked "What are your goals, as one carrying the torch to continue Kolmogorov's activities?" He replied "My first goal is not to allow the outstanding Kolmogorov School to vanish. Times are very difficult – the country is in a deep economic and political crisis ..." He emphasises the need to retain the services of the younger generation of scientists, rather than see them seek more profitable employment elsewhere. Of course, large numbers of excellent Soviet-trained mathematicians did leave for the West – to the great benefit of mathematics in the West, and to the sad loss of mathematics in Russia. Nevertheless, crises pass (eventually – we in the West have our own ongoing crises!), and this Gnedenko Centenary Conference has shown clearly the continuing vitality of the Kolmogorov School, whose distinguished member it celebrates.

I close with three anecdotes taken from [SiSm]. The first relates to Gnedenko's 1937 arrest and imprisonment.

"Smith: So how long were you in prison altogether?

Gnedenko: Six months. I was very weak but I went back to Moscow, where Kolmogorov and Khinchin made my rehabilitation in the department possible, against the opposition of some of the faculty. At that time many of the professors were arrested, including almost all the specialists in mechanics.

Singpurwalla: Why was that?

Gnedenko: Nobody knows. They worked in a special institute within the university.

Singpurwalla: You said that one of the reasons you were arrested was that you were a member of a counter-revolutionary organisation. Does that make you a Czarist?
Gnedenko: It was enough to be anything. All it took was one person to denounce you as a counter-revolutionary. If I had said anything against Kolmogorov, that would have been enough to arrest him.
Singpurwalla: And were you a counter-revolutionary?
Gnedenko: Yes, naturally.
Ushakov: All Russians are counter-revolutionary in some sense. "

The second relates to Gnedenko's wonderful teachers, Kolmogorov and Khinchin. Both were great mathematicians; Kolmogorov was the greater (the greatest in probability – Kolmogorov is to probability as Fisher is to statistics and Gauss is to mathematics). But, God in His wisdom distributes His gifts unequally, no one gets everything, and Khinchin had an abundance of a quality Kolmogorov lacked – the ability to communicate clearly in public. Khinchin was by all accounts a wonderful lecturer (and a wonderful author, as those who know, use and love his books will agree). Kolmogorov was not. (I heard Kolmogorov only once, at the International Congress of Mathematicians in Nice in 1970, speaking in French on information theory. The occasion was unforgettable as a piece of theatre, and was an experience I will always treasure, but was not particularly successful as an exercise in communication.) In his early years, Gnedenko complained to Khinchin that he could only understand 50 per cent of what Kolmogorov said. Khinchin replied "But that is wonderful – I only understand 30 per cent".

The third anecdote, given by Professor Igor Ushakov in the Postscript to [SiSm], needs a little explanation. Gnedenko's arrest and imprisonment in 1937 resulted from incautious comments to a fellow student while on holiday. When arrested, Gnedenko was relentlessly interrogated – about his links with Kolmogorov – and pressed to say something – anything – that could be used against Kolmogorov. This period was at the height of the purges under Stalin (see any standard work on this period, e.g. [De], 368; those executed included Zinoviev, Kamenev, Tukhachevsky, Bukharin, ...). Had Gnedenko said *anything* to satisfy his interrogators, this would undoubtedly have been used against Kolmogorov. Gnedenko said nothing. Many years later, at a banquet following Soloviev's thesis defence (A. D. Soloviev took his PhD in 1955 at Moscow State University, under A. O. Gelfond), "Kolmogorov raised a toast to Boris Gnedenko as an honest and brave man. When I privately enquired as to the reason behind this toast, Kolmogorov answered with a slight smile, 'In a very difficult time Boris suffered torment to save his friend.' At that time I did not know who the friend was. It was only later on that Natalia Gnedenko, Gnedenko's wife, told me that the friend in question was Kolmogorov himself. However, she added 'Please never say this to anybody, because Boris and Andrei don't like to advertise this and show the public their deep feelings for one another'." Thanks to Ushakov, this has been in the public domain for twenty years now, so one is allowed to repeat it – and where and when better to do so than here and now. Gnedenko was a great mathematician – and a great man. Let us honour his memory.

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