

## MATH482 MATHEMATICAL FINANCE

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Liverpool University, Monday 28 January – Monday 13 May 2013

### *Books*

Our main text will be Ch. 1-6 of

[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., CUP, 2004.

Relevant background material can be found on links on my Imperial College homepage (Imperial College > Mathematics Department > Staff > Staff List > Bingham > Homepage), e.g.

[SP] Stochastic Processes [30 hours, MSc, Mathematical Finance];

[SA] Stochastic Analysis [20 hours, MSc].

[LTCC] Measure-theoretic probability theory. London Taught Course Centre [10 hours; MSc]. For background:

[PfS] Probability for Statistics;

[SMF] Statistical Methods for Finance.

Just as important as the technical mathematics, you need to think about the systemic faults at the geofinancial/economic/political level thrown up by the crisis of 2007 on (Credit Crunch, etc.). Any prospective employer in the financial services industry should ask you questions about this, and your views on it, in interview. Some of my views are in

N. H. BINGHAM: The Crash of 2008: A mathematician's view. *Significance* **5** no. 4 (2008), 173-175.

We shall make systematic use of *conditioning* (informally: using what we know). For background, see e.g.

[BF] N. H. BINGHAM and J. M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), Springer, 2010.

Books for reference include:

[DP] Avinash K. DIXIT and Robert S. PINDYCK: *Investment under uncertainty*. Princeton University press, 1994.

[CR] John C. COX and Mark RUBINSTEIN: *Options markets*. Prentice Hall, 1985.

[PS] G. PESKIR and A. N. SHIRYAEV: *Optimal stopping and free-boundary*

*problems*. Birkhäuser, 2006.

[E] Alison ETHERIDGE: *A course in financial calculus*, CUP, 2002 [a classroom revision of [BR] below];

[BR] Martin BAXTER and Andrew RENNIE: *Financial calculus: an introduction to derivatives pricing*, CUP, 1996.

[WHD] WILMOTT, P., HOWISON, S. & DEWYNNE, J. (1995): *The mathematics of financial derivatives: A student introduction*, Oxford Financial Press [‘PDE with everything’];

[H1] HULL, J. (1995): *Introduction to futures and options markets* (2nd ed), Prentice-Hall, (‘baby Hull’), or

[H2] HULL, J. (1993): *Options, futures and other derivative securities* (2nd ed.), Prentice-Hall (‘Hull’).

*Background and general interest*

[B1] Peter L. BERNSTEIN: *Capital ideas: The improbable origins of modern Wall Street*. New York: The Free Press, 1992.

[B2] Peter L. BERNSTEIN: *Against the Gods: The remarkable story of risk*. Wiley, 1996.

[S] Robert L. SHILLER: *Irrational exuberance*. Princeton Univ. Press, 2000.

[G] Alan GREENSPAN, *The age of turbulence*. Penguin, 2007.

I thoroughly recommend [G] – but get the latest edition of it that you can. The author was Chairman of the US Federal Reserve (Fed) 1987-2006. His views up to 2007 were largely Panglossian optimism (markets know best, and are self-correcting, etc.). The ongoing problems since have forced a re-think; see the epilogues to later editions, his evidence to the House Committee, etc.

*Mathematics, for reference*

[D] J. L. DOOB: *Stochastic processes*, Wiley, 1953.

[N] J. NEVEU: *Discrete-parameter martingales*, North-Holland, 1975.

[KS] KARATZAS, I. & SHREVE, S. (1988): *Brownian motion and stochastic calculus*. Graduate Texts in Math. **113**, Springer.

[RY] REVUZ, D. & YOR, M. (1999): *Continuous martingales and Brownian motion*. Grundlehren der math. Wiss. **293**, Springer, 3rd ed. (1st ed. 1991, 2nd ed. 1994.).

[RW1] ROGERS, L. C. G. & WILLIAMS, D. (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundation*, 2nd ed., Wiley (1st ed. 1970).

[RW2] ROGERS, L. C. G. & WILLIAMS, D. (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*. Wiley.

## CONTENTS

### **I. ECONOMIC BACKGROUND** [2 weeks].

- §1. Time value of money; discounting.
- §2. Economics and finance; utility
- §3. Brief history of mathematical finance.
- §4. Markets and options.
- §5. Portfolios and hedging.
- §6. Arbitrage.
- §7. Put-call parity.
- §8. An example.
- §9. Complements

### **II. PROBABILITY BACKGROUND** [ $1\frac{1}{2}$ weeks].

- §1. Measure.
- §2. Integral.
- §3. Probability.
- §4. Equivalent measures and Radon-Nikodym derivatives.
- §5. Conditional expectations.
- §6. Properties of conditional expectations.

### **III. STOCHASTIC PROCESSES IN DISCRETE TIME** [1 week].

- §1. Filtrations and information flow.
- §2. Discrete-parameter stochastic processes.
- §3. Discrete-parameter martingales.
- §4. Martingale convergence.
- §5. Martingale transforms.
- §6. Stopping times and optional stopping.
- §7. The Snell envelope and optimal stopping.
- §8. Doob decomposition.
- §9. Examples.

### **IV. MATHEMATICAL FINANCE IN DISCRETE TIME** [ $2\frac{1}{2}$ weeks].

- §1. The model.
- §2. Viability: existence of equivalent martingale measures (EMMs).
- §3. Complete markets: uniqueness of equivalent martingale measures.
- §4. The Fundamental Theorem of Asset Pricing: Risk-Neutral Valuation.
- §5. European options. The discrete Black-Scholes formula.
- §6. Continuous-time limit of the binomial model.
- §7. More on European options.
- §8. American options.
- §9. American options: Infinite time-horizon.

## **V. STOCHASTIC PROCESSES IN CONTINUOUS TIME** [ $1\frac{1}{2}$ weeks].

- §1. Filtrations; finite-dimensional distributions.
- §2. Classes of processes.
- §3. Brownian motion.
- §4. Quadratic variation of Brownian motion.
- §5. Stochastic integrals; Itô calculus.
- §6. Stochastic differential equations; Itô's Lemma.

## **VI. MATHEMATICAL FINANCE IN CONTINUOUS TIME** [ $2\frac{1}{2}$ weeks].

- §1. Geometric Brownian motion and asset prices.
- §2. The Black-Scholes model and the Black-Scholes PDE.
- §3. The Feynman-Kac formula and the Black-Scholes formula.
- §4. Girsanov's theorem and change of measure.
- §5. American options; exotic options
- §6. Real options (Investment options)
- §7. Extensions.

### **Division of Time**

Week 1: I, §1-4 [28 Jan]

Week 2: I, §5-6 [4 Feb]

Week 3: II, §1-5 [11 Feb]

Week 4: II, §5-6; III, 1-6 [18 Feb]

Week 5: III, §6-9; IV, 1-2 [25 Feb]

Week 6: IV, §3-6 [4 March]

Week 7: IV, §7-9 [11 March]

Week 8: V, §1-4 [18 March]

Easter break

Week 9: V, §5-6; VI, 1-2 [15 April]

Week 10: VI, §2-3 [22 April]

Week 11: VI, §4-7; Postscript [29 April]

Week 12: [6 May] – Bank Holiday] – No lectures

Revision Week [13 May]

*Format and Examination.* Lectures are 4 hours per week on Mondays 11-1 and 3-5pm. There will be 11 handouts, of 12 pages each. I aim to cover about 4 pages per hour, and have time for questions, say 4-5pm. There will be Problems each week, with Solutions the week after. The syllabus is unchanged, and the exam will be of standard format. But as the lecturer (and so the style) is new, I will set a Mock Exam with Solutions. NHB

