

SOLUTIONS 10. 14.12.2011

Q1.

$$\begin{aligned}
\operatorname{div}(\mathcal{F} \times \mathcal{G}) &= D_1(\mathcal{F} \times \mathcal{G})_1 + D_2(\mathcal{F} \times \mathcal{G})_2 + D_3(\mathcal{F} \times \mathcal{G})_3 \\
&= D_1[F_2G_3 - F_3G_2] + D_2[F_3G_1 - F_1G_3] + D_3[F_1G_2 - F_2G_1] \\
&= (D_1F_2)G_3 + F_2(D_1G_3) - (D_1F_3)G_2 - F_3(D_1G_2) \\
&\quad + (D_2F_3)G_1 + F_3(D_2G_1) - (D_2F_1)G_3 - F_1(D_2G_3) \\
&\quad + (D_3F_1)G_2 + F_1(D_3G_2) - (D_3F_2)G_1 - F_2(D_3G_1) \\
&= G_1[D_2F_3 - D_3F_2] + G_2[D_3F_1 - D_1F_3] + G_3[D_1F_2 - D_2F_1] \\
&= \mathcal{G} \cdot (\nabla \times \mathcal{F}) - \mathcal{F} \cdot (\nabla \times \mathcal{G}).
\end{aligned}$$

Q2.

$$\begin{aligned}
\nabla \times (f\mathbf{F}) &= [D_2(fF)_3 - D_3(fF)_2]\mathbf{i} + [D_3(fF)_1 - D_1(fF)_3]\mathbf{j} + [D_1(fF)_2 - D_2(fF)_1]\mathbf{k} \\
&= [(D_2f)F_3 + fD_2F_3 - (D_3f)F_2 - fD_3F_2]\mathbf{i} \\
&\quad + [(D_3f)F_1 + fD_3F_1 - (D_1f)F_3 - fD_1F_3]\mathbf{j} \\
&\quad + [(D_1f)F_2 + fD_1F_2 - (D_2f)F_1 - fD_2F_1]\mathbf{k} \\
&= f[(D_2F_3 - D_3F_2)\mathbf{i} + (D_3F_1 - D_1F_3)\mathbf{j} + (D_1F_2 - D_2F_1)\mathbf{k}] \\
&\quad + [(\nabla f)_2F_3 - (\nabla f)_3F_2]\mathbf{i} + [(\nabla f)_3F_1 - (\nabla f)_1F_3]\mathbf{j} + [(\nabla f)_1F_2 - (\nabla f)_2F_1]\mathbf{k} \\
&= f(\nabla \times \mathbf{F}) + (\nabla f) \times \mathbf{F}.
\end{aligned}$$

NHB