

PROBLEMS 3. 24.10.2013

Q1. Show that Stirling's formula gives

$$\Gamma(x+a) \sim x^a \Gamma(x) \quad (x \rightarrow \infty).$$

Q2. You may quote that the *Student t* statistic

$$t := \sqrt{n-1}(\bar{X} - \mu)/S$$

has distribution $t(n-1)$, where $t(r)$ has density

$$f(x) = \frac{\Gamma(\frac{1}{2}r + \frac{1}{2})}{\sqrt{\pi r} \Gamma(\frac{1}{2}r)} \left(1 + \frac{x^2}{r}\right)^{-\frac{1}{2}(r+1)}.$$

Show that as $n \rightarrow \infty$

$$t(n) \rightarrow \Phi = N(0, 1)$$

(i) by using Q1 and the density above;

(ii) by using the Law of Large Numbers and the definition of the Student t -statistic.

Q3. By considering the quadratic

$$Q(\lambda) := \frac{1}{n} \sum_{i=1}^n (\lambda(x_i - \bar{x}) + (y_i - \bar{y}))^2,$$

show that the sample correlation coefficient r satisfies

(i) $-1 \leq r \leq 1$

(ii) $r = \pm 1$ iff there is a linear relationship between x_i and y_i ,

$$ax_i + by_i = c \quad (i = 1 \dots n).$$

Q4. By considering the quadratic

$$Q(\lambda) := E[(\lambda(x - \bar{x}) + (y - \bar{y}))^2],$$

show that the population correlation coefficient ρ satisfies

(i) $-1 \leq \rho \leq 1$,

(ii) $\rho = \pm 1$ iff there is a linear relationship between x and y , $ax + by = c$ with probability 1.

NHB