M4A22/M5A22 MATHEMATICAL FINANCE: MASTERY QUESTION 2014

The Ornstein-Uhlenbeck stochastic differential equation is

$$dV = -\beta V dt + \sigma dW, \qquad (OU)$$

with W standard Brownian motion.
(i) Interpret (OU) physically. **[2]**
(ii) Solve (OU) to obtain

$$V_t = v_0 e^{-\beta t} + \sigma e^{-\beta t} \int_0^t e^{\beta u} dW_u. \qquad [4]$$

(iii) By using the Itô isometry, or otherwise, show that V_t has distribution $N(v_0 e^{-\beta t}, \sigma^2(1 - e^{-2\beta t})/(2\beta))$. **[4]**
(iv) By (iii) and independence of Brownian increments, or otherwise, show that the covariance is

$$cov(V_t, V_{t+u}) = \sigma^2 e^{-\beta u}(1 - 2e^{-2\beta t})/(2\beta) \qquad (u \geq 0). \qquad [4]$$

(v) Show that V is Markov. **[3]**
(vi) What is the financial relevance of this model? **[3]**

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