ullprob4b.tex

PROBLEMS 4b. 19.4.2016

Q1. The *moment-generating function* $M_X(t)$ of a random variable X is defined by $M_X(t) := E[e^{tX}]$. If X has the normal distributiion $N(\mu, \sigma^2)$, with density

$$f(x) := \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{1}{2}(x-\mu)^2/\sigma^2\},$$

show (by completing the square, or otherwise) that

$$M_X(t) = \exp\{\mu t + \frac{1}{2}\sigma^2 t^2\}.$$

Q2. The *lognormal distribution* $logN(\mu, \sigma^2)$ is defined as the distribution of $X := e^Y$, where Y is $N(\mu, \sigma^2)$.
(i) By using the result of Q1, or otherwise, show that X has mean

$$E[X] = \exp\{\mu + \frac{1}{2}\sigma^2\}.$$

(ii) Explain without proof why the prices of stocks in the Black-Scholes model are log-normally distributed.

Q3. *The exponential martingale for Brownian motion.*
If $B = (B_t)$ is Brownian motion and θ is a parameter, show that $M = (M_t)$, with

$$M_t := \exp\{\theta B_t - \frac{1}{2}\theta^2 t\},$$

is a martingale.

NHB