

M3PM16/M4PM16 PROBLEMS 2. 26.1.2012

Q1. For $0 < a < 1$, show that $\exp\{(\log x)^a\}$ grows faster than any power of $\log x$ but more slowly than x .

Q2. Deduce that

(i) PNT with error term

$$\pi(x) - li(x) = O(x \exp\{-(\log x)^a\})$$

(see III.10.2 for $a = 1/2$, and the best known value $a = 3/5 - \epsilon$) is more accurate than

$$\pi(x) - li(x) = O(x/\log^k x)$$

for any k .

(ii) Conclude from Problems 1 Q2 and this that $li(x)$ is more suitable than $x/\log x$, or

$$\frac{x}{\log x} + \dots + \frac{(m-1)!x}{\log^m x},$$

for use in PNT with an error term.

Q3. Write (m, n) for the greatest common divisor (gcd) of natural numbers m, n . For a, b, n natural numbers, show that the (Diophantine) equation

$$ax + by = n$$

has integer solutions x and y iff $(a, b) | n$ ((a, b) divides n).

Q4. If $a | bc$ and $(a, b) = 1$, show that $a | c$.

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