

PROBLEMS 8. 28.11.2011

Q1. Find the Fourier series on $[0, 2\pi]$ of the function

$$f(x) := x \quad (0 \leq x < \pi), \quad 0 \quad (\pi \leq x \leq 2\pi).$$

Q2. Show that

$$f(x) = \frac{1}{2}e^{-|x|}$$

has Fourier transform $1/(1+t^2)$:

$$\int_{-\infty}^{\infty} \frac{1}{2}e^{-|x|}e^{ixt}dx = 1/(1+t^2)$$

(replace x in $\int_{-\infty}^0$ by $-x$, combine the two integrals over \int_0^{∞} , and integrate by parts twice).

Q3. By the Fourier Integral Theorem, or otherwise, show that

$$\int_{-\infty}^{\infty} \frac{e^{-ixt}dx}{\pi(1+x^2)} = e^{-|t|}.$$

NHB