

M3PM16/M4PM16 PROBLEMS 8. 14.3.2014

Q1 *Sum-function of ϕ* . Show that

$$\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x).$$

Q2 *$Q(x)$ and PNT*.

Given PNT in the form

$$M(x) := \sum_{n \leq x} \mu(n) = o(x),$$

show that we can improve the error term $O(\cdot)$ in Problems 7 Q4(iii) to $o(\cdot)$:

$$Q(x) = \frac{6}{\pi^2} x + o(\sqrt{x}).$$

(Use Problems 7 Q3 and Dirichlet's hyperbola identity with $a = u$, $b = \nu$, letting first $x \rightarrow \infty$ and then $y \rightarrow \infty$.)

Note. One can prove more, from PNT with remainder (Ch. IV): $o(\sqrt{x})$ can be improved to $O(x \exp\{-\frac{1}{2}c\sqrt{\log x}\})$, where $O(x \exp\{-c\sqrt{\log x}\})$ is the remainder term in PNT (MV, 6.2.1 Ex. 19(d) p.186).

NHB