

M2PM3 PROBLEMS 7. 11.3.2010

Q1. Show that for $a > b > 0$,

$$I := \int_0^{2\pi} \frac{\sin^2 \theta \, d\theta}{a + b \cos \theta} = \frac{2\pi}{b^2} (a - \sqrt{a^2 - b^2}).$$

Q2. Find

$$I := \int_0^\infty \frac{x^2}{x^4 + 5x^2 + 6} dx.$$

Q3. Show that for $p, q \geq 0$,

$$\int_{-\infty}^\infty \frac{\cos px - \cos qx}{x^2} dx = -\pi(p - q).$$

Q4. Show that for $a > 0$

$$I := \int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\sqrt{2}\pi}{4a^3}.$$

NHB