smfprob5(1718).tex

SMF PROBLEMS 5. 9.11.2017

Q1. In the regression model
$$y = A\beta + \epsilon$$
(data y an n-vector, the design matrix A an $n \times p$ matrix of constants, β a p-vector of parameters, ϵ an n-vector of errors with independent $N(0, \sigma^2)$ components), show that the maximum-likelihood estimators, and also the least-squares estimators, are
$$\hat{\beta} = (A^T A)^{-1} A^T y.$$

Show also that (in the notation of lectures)
$$Py = A\hat{\beta}.$$

Q2. The $AR(p)$ process (X_t) is given by
$$X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p}, \qquad (\epsilon_t) \quad WN(\sigma^2).$$
(i) State without proof the condition for stationarity.
(ii) Derive the Yule-Walker equations for the autocorrelation (ρ_k).
(iii) State the general solution of the Yule-Walker equations.

Q3. The $MA(1)$ process (X_t) is given by
$$X_t = \epsilon_t + \theta\epsilon_{t-1}, \qquad |\theta| < 1, \qquad (\epsilon_t) \quad WN(\sigma^2).$$
Find
(i) the variance $\gamma_0 = var X_t$,
(ii) the autocovariance $\gamma_k = cov(X_t, X_{t+k})$,
(iii) the autocorrelation $\rho_k = corr(X_t, X_{t+k})$.

Q4. The time-series model is given by
$$X_t = X_{t-1} - \frac{1}{4}X_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}, \qquad (\epsilon_t) \quad WN(\sigma^2).$$
(i) Classify (X_t) within the $ARIMA$ class.
(ii) Show that (X_t) is stationary and invertible.

NHB