

M4PM16 MASTERY QUESTION, 2013

Prove Chebyshev's Theorem: If

$$\ell := \liminf \frac{\pi(x)}{x/\log x}, \quad L := \limsup \frac{\pi(x)}{x/\log x},$$

then

$$\ell \leq 1 \leq L.$$

In particular, if the limit exists, it is 1 (as in PNT).

Hint: Use Mertens' Second Theorem.

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