MATL480 RESIT EXAMINATION 2016-17

Five questions, 20 marks each

Q1. (i) Discuss the types of risk facing financial institutions, including: market risk, credit risk, operational risk, liquidity risk, model risk.
(ii) Comment briefly on stress testing by financial regulators.

Q2. The 'doubling strategy' is: when betting on tossing a fair coin, respond to losing by doubling the stakes.
(i) Find the distribution of the number N of losses before the first win.
(ii) Show that $N < \infty$ a.s.; deduce that one is certain to win eventually.
(iii) Find one's eventual fortune (capital at one's eventual win).
(iv) Find the probability generating function $P(s)$ of N, its mean $E[N]$, and the mean length of time the game lasts.
(v) Why is this nevertheless an impossible strategy to use in practice?

Q3. (i) Given a random variable Y with $E[|Y|] < \infty$ and a σ-field $\mathcal{C}$, define the conditional expectation $E[Y|\mathcal{C}]$ of Y with respect to $\mathcal{C}$.
(ii) What happens if $\mathcal{C}$ is the trivial σ-field? What happens if $\mathcal{C}$ is the whole σ-field?
(iii) Show that if $\mathcal{B}$, $\mathcal{C}$ are σ-fields with $\mathcal{B} \subset \mathcal{C}$, then applying both conditional expectations is the same as applying just one of them, either way round. Which one?
(iv) Show that the expectation of a conditional expectation is just the expectation.
(v) Explain what is meant by saying that a conditional expectation is a projection.

Q4. (i) Explain briefly, without proofs, the discrete Black-Scholes formula, the continuous Black-Scholes formula, and the relationship between them.
(ii) Neither formula involves the mean return rate μ on the stock: why not?
(iii) Describe briefly how to find the value and the continuation region of an American put option.

Q5. (i) Give the stochastic differential equation of geometric Brownian motion, and its interpretation in terms of the stock-price dynamics of the Black-Scholes model.

(ii) Show how this stochastic differential equation changes when we discount by the riskless interest rate r.
(iii) Show how it changes further when we apply Girsanov's theorem to change to the equivalent martingale (or risk-neutral) measure.
(iv) What is the role of the representation theorem for Brownian martingales here?

N. H. Bingham