# Lecture 6: Diversification

# Lecture Outline

- Diversification
  - Independent risky assets
  - Correlated risky assets
  - The difference between safety and familiarity
- Active versus passive investing
  - Passive investing
  - The iron law of active management
  - Diversified active investing
  - Active investing to express values
  - More to follow in the next lecture….

# Diversification

# A Single Risky Asset

- Suppose there is a risky asset that will either deliver a 20% return over the next year, or a −10% return. These two outcomes are equally likely.
- The **mean return** is the probability-weighted average of the two returns:

$$\overline{R} = (0.5 \times 0.2) + (0.5 \times -0.1) = 0.1 - 0.05 = 0.05 = 5\%.$$

- If the asset does well, the return is 15% above the mean; if it does badly, it is 15% below the mean. We say the **demeaned return** is either 15% or −15%.

# Risk of a Single Risky Asset

- The demeaned return is either 15% or −15%.
- The **variance** of the return (sometimes written $\sigma^2$) is the probability-weighted average of the squared demeaned return:

$$\mathrm{Var}(R) = (0.5 \times (0.15)^2) + (0.5 \times (-0.15)^2) = (0.15)^2 = 0.0225.$$

- The **standard deviation** of the return (sometimes written $\sigma$) is the square root of the variance:

$$\mathrm{S.d.}(R) = \sqrt{0.0225} = 0.15 = 15\%.$$

- In the case of a coin toss (a bet with two equally likely outcomes), the standard deviation equals the absolute value of the deviations from the mean.

# Two Independent Risky Assets

- Now suppose that instead of one risky asset that is equally likely to deliver a 20% return or a −10% return, there are two.
- The good and bad outcomes are determined by a separate coin toss for each asset, that is, these assets are independent of one another.

| | Asset 1 20% | Asset 1 −10% | Unconditional |
|---|---|---|---|
| Asset 2 20% | 0.25 | 0.25 | 0.5 |
| Asset 2 −10% | 0.25 | 0.25 | 0.5 |
| Unconditional | 0.5 | 0.5 | |

- The joint probability of two good outcomes or two bad outcomes is 0.25, and the joint probability of one good and one bad outcome is 0.5.

# An Equal-Weighted Portfolio

- With two assets, you can form a **portfolio** that contains both of them. If you divide your risky investment equally between the two assets, your portfolio return $R_p$ is an equal-weighted average of the two assets' returns: $R_p = 0.5R_1 + 0.5R_2$.
- $R_p$ is 20% with probability 0.25, 5% with probability 0.5, and $-10\%$ with probability 0.25.
- The mean return on your portfolio is unchanged:

$$\overline{R}_p = (0.25 \times 0.2) + (0.5 \times 0.05) + (0.25 \times -0.1) = 0.05 = 5\%.$$

- Then your demeaned portfolio return (subtracting the mean) is 15% with probability 0.25, 0% with probability 0.5, and $-15\%$ with probability 0.25.

# Risk Reduction in a Portfolio

- If you divide your risky investment equally between the two assets, your demeaned portfolio return is 15% with probability 0.25, 0% with probability 0.5, and −15% with probability 0.25.
- The variance of your portfolio return is one-half smaller:

$$\begin{aligned}\mathrm{Var}(R_p) &= (0.25 \times (0.15)^2) + (0.5 \times 0^2) + (0.25 \times (-0.15)^2) \\ &= 0.5 \times (0.15)^2 = 0.01125.\end{aligned}$$

- The standard deviation of your return is divided by the square root of two:

$$\mathrm{S.d.}(R_p) = \sqrt{0.01125} = 0.106 = 10.6\% = 15\%/\sqrt{2}.$$

# Risk Reduction in a Portfolio

- The variance of your portfolio return is one-half smaller:

$$\begin{aligned}\mathrm{Var}(R_p) &= (0.25 \times (0.15)^2) + (0.5 \times 0^2) + (0.25 \times (-0.15)^2) \\ &= 0.5 \times (0.15)^2 = 0.01125.\end{aligned}$$

- You now have a more attractive risky portfolio that has the same mean return and half the variance as either individual asset.
  - Hence the standard rule of risktaking says you should invest twice as much in it.

# Correlated Risky Assets

# Assets That Move Together

- In the above example, diversification works particularly well because the investments are independent of one another.
  - There is no tendency for one to do well when the other does well.
- In practice, many assets tend to move together because their returns are all influenced by common factors such as the performance of the US economy.
- The tendency to move together can be measured using the concepts of **covariance** and **correlation**.

# Two Risky Assets That Move Together

- Now consider a more general situation in which the risky assets can deliver similar outcomes either more frequently or less frequently than in the independent case.
- The probability of two good or two bad outcomes is increased by a factor of $\rho$, and by the symmetry of the two assets we can fill in the rest of the table.

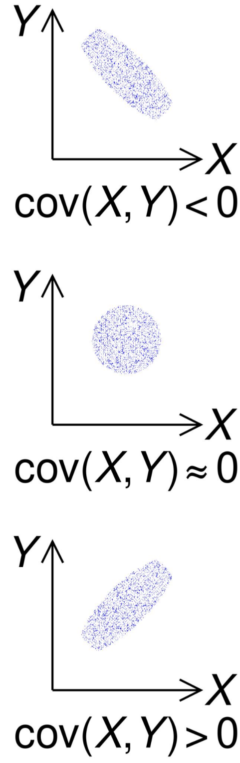
| | Asset 1 20% | Asset 1 $-10\%$ | Unconditional |
|---|---|---|---|
| Asset 2 20% | $0.25(1+\rho)$ | $0.25(1-\rho)$ | 0.5 |
| Asset 2 $-10\%$ | $0.25(1-\rho)$ | $0.25(1+\rho)$ | 0.5 |
| Unconditional | 0.5 | 0.5 | |

- Since probabilities can never be negative, we must have $-1 < \rho < 1$.

# Covariance

- **Covariance** is defined as the probability-weighted average of the product of the demeaned returns for asset 1 and asset 2.

$$\begin{aligned}\mathrm{Cov}(R_1, R_2) &= 0.5(1+\rho)(0.15)^2 + 0.5(1-\rho)(-0.15)(0.15) \\ &= \rho(0.15)^2.\end{aligned}$$



# Correlation

- **Correlation** is defined as covariance divided by the product of the standard deviations of asset 1 and asset 2.
- In this case, since asset 1 and asset 2 have the same standard deviation, correlation equals covariance divided by the variance of the asset return:

$$\mathrm{Corr}(R_1, R_2) = \frac{\mathrm{Cov}(R_1, R_2)}{\mathrm{S.d.}(R_1)\mathrm{S.d.}(R_2)} = \frac{\mathrm{Cov}(R_1, R_2)}{(0.15)^2} = \rho.$$

# Properties of Correlation

$$\text{Corr}(R_1, R_2) = \frac{\text{Cov}(R_1, R_2)}{\text{S.d.}(R_1)\text{S.d.}(R_2)} = \frac{\text{Cov}(R_1, R_2)}{(0.15)^2} = \rho.$$

- We have $-1 < \text{Corr}(R_1, R_2) < 1$.
- The case we considered before of independent returns has zero covariance and correlation.
- If the two assets move perfectly together, correlation $\rho = 1$ and covariance $=$ variance.
- If the two assets move perfectly opposite one another, correlation $\rho = -1$ and covariance $= -$variance.

# Risk Reduction with Two Correlated Assets

- If you divide your money equally between the two assets, your demeaned portfolio return is 15% with probability $0.25(1+\rho)$, 0% with probability $0.5(1-\rho)$, and $-15\%$ with probability $0.25(1+\rho)$.
- The variance of your portfolio return is now:

$$\begin{aligned} \mathrm{Var}(R_p) &= 0.5(1+\rho)(0.15)^2 + 0.5(1-\rho)(0^2) \\ &= 0.5(1+\rho)(0.15)^2. \end{aligned}$$

# Risk Reduction with Two Correlated Assets

$$\begin{aligned}\mathrm{Var}(R_p) &= 0.5(1+\rho)(0.15)^2 + 0.5(1-\rho)(0^2) \\ &= 0.5(1+\rho)(0.15)^2.\end{aligned}$$

- The case we considered before of independent returns has correlation $\rho = 0$, and the variance of the portfolio return is cut in half.
- If the two assets move perfectly together, correlation $\rho = 1$ and diversification does not reduce the variance of the portfolio return at all. This is the worst case for diversification.
- If the two assets move perfectly opposite one another, correlation $\rho = -1$ and the variance of the diversified portfolio is zero. We say that one asset is a **hedge** for the other which is the best case for diversification.

# Risk Reduction with Two Correlated Assets

- What if the two assets do not have the same variance?
- What if you don't divide your money equally between the two assets?
- The general formula for the variance of your portfolio return is

$$\begin{aligned}
\mathrm{Var}(R_p) &= w_1^2\mathrm{Var}(R_1) + w_2^2\mathrm{Var}(R_2) + 2w_1w_2\mathrm{Cov}(R_1, R_2) \\
&= w_1^2\mathrm{Var}(R_1) + w_2^2\mathrm{Var}(R_2) \\
&\quad +2w_1w_2\mathrm{S.d.}(R_1)\mathrm{S.d.}(R_2)\mathrm{Corr}(R_1, R_2),
\end{aligned}$$

and this is increasing in the correlation of the two assets whenever you have some money invested in each one ($w_1 > 0$ and $w_2 > 0$).

# Relation to Market “Beta”

An often used term in finance is a stock’s “beta”

This basically measures the correlation of an individual asset’s returns with movements in the overall market.

- A beta<1 means that the stock is less volatile than the overall market – this might be a company that is more “recession-proof”
- A beta>1 means that the stock is more volatile than the market, may be a growth stock
- A beta<0 means the stock price moves inversely with the market – it is countercyclical!

Formula

$$\beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)}$$

$\beta_i$ = market beta of asset i

$\mathrm{Cov}$ = covariance

$\mathrm{Var}$ = variance

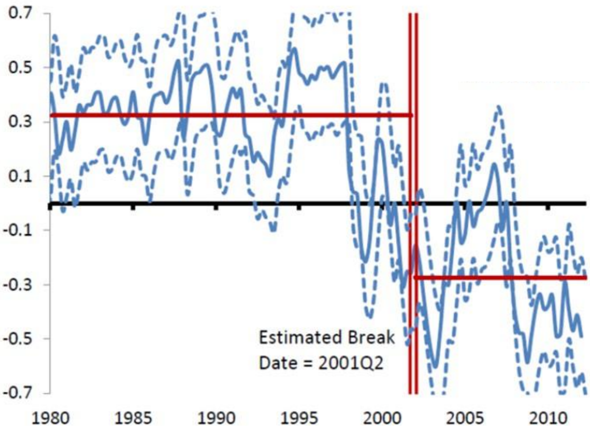
$r_m$ = average expected rate of return on the market

$r_i$ = expected return on an asset i

# The Correlation of Bonds and Stocks

- Correlations can change over time, and that changes the potential for risk reduction through diversification.
- A relevant example is the correlation between bond and stock returns.
  - This correlation was **positive** in the late 20$^{th}$ Century, when the Fed raised interest rates to fight inflation (bad for bonds!) and caused recessions (bad for stocks!)
  - In the early 21$^{st}$ Century it has been **negative**, as the Fed has lowered interest rates (good for bonds!) to fight recessions (global financial crisis, COVID pandemic, bad for stocks!)
  - Bond-stock diversification has worked well in the last few years, but will it keep working in the 2020s?

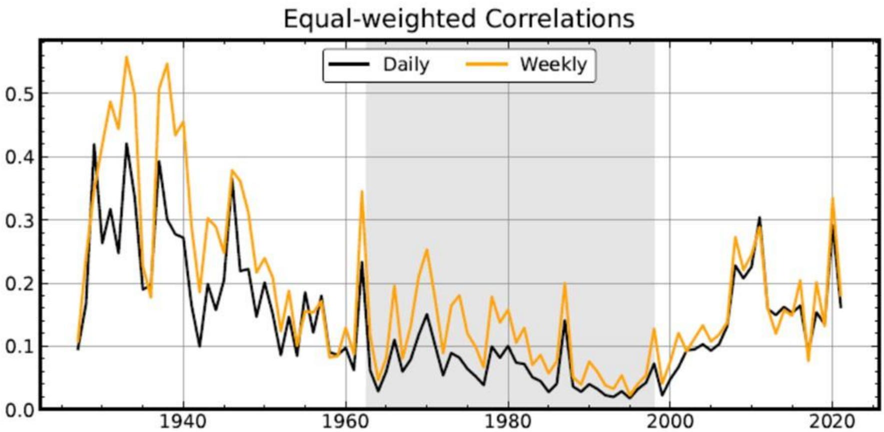
# The Correlation of Bonds and Stocks




Source: Campbell, Pflueger, and Viceira, “Macroeconomic Drivers of Bond and Equity Risks”, *Journal of Political Economy* 2020.

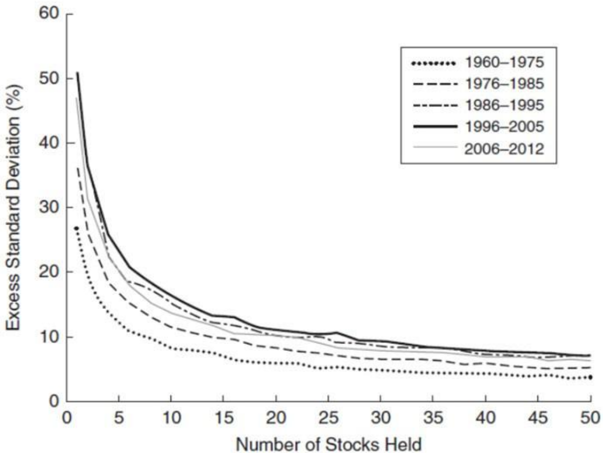
Correlation between daily returns on 5-year nominal Treasury bonds and a value-weighted stock index, measured in each quarter. The solid blue line is the point estimate, and the dashed blue lines indicate the range of uncertainty.

# Average Correlation Across US Stocks




Source: Campbell, Lettau, Malkiel, and Xu, "Idiosyncratic Equity Risk Two Decades Later", *Critical Finance Review*, 2023,
Figure 5

# Implied Benefits of Diversification




Suppose you pick stocks randomly and put them in a portfolio with equal weights. How much extra risk are you taking above the risk of an equally weighted portfolio of all stocks in the market?

Source: Campbell, *Financial Decisions and Markets: A Course in Asset Pricing,* Princeton University Press 2018, Figure 2.4

# Familiarity vs Safety

# The Familiar Seems Safe…

- Many people feel they can pick stocks without taking risk because they know certain companies are safe investments.
  - “I work for a good company, I would know if there were a problem with it.”
  - “I work in tech so tech stocks are not risky for me.”
  - “I see the company trucks on the street so I know it’s a solid business.”
  - “I have faith in the USA.”

# … But It Isn’t …

- The problem: any company, sector, or even country can have unexpected bad news.
    - People who worked for Enron and put their retirement savings in its stock were financially devastated by the company’s accounting scandal.
- Investing in the stock of your employer is particularly dangerous because your salary depends on the company too.

# So How to Diversify?

- Invest across multiple asset classes, including at least bonds, domestic equity, and foreign equity.
    - Within each asset class, diversify broadly.
- Hold a value-weighted index (a passive strategy).
- OR, if you deviate from this (an active strategy), do so with a clear rationale, a diversified approach, and sensitivity to trading costs and active management fees.

# Passive Investing

# Value-Weighted Index

- Within any asset class, we can add up all the dollars invested in individual members of the class.
  - Example: imagine an asset class containing only Apple and Tesla stock.
  - The **market capitalization** of Apple (the total value of all its shares) is $2.29t = $2,290b.
  - The market capitalization of Tesla is $760b.
  - The total invested in both companies is $3,050b, of which 75% is Apple and 25% is Tesla.
- A value-weighted (VW) index holds the companies in proportion to the total dollars invested.
  - In the example, 75% Apple and 25% Tesla.

# Examples of VW Indexes

- Most indexes that are widely followed today are value-weighted.
  - S&P 500 is calculated by Standard and Poor's value weighting 500 large US companies selected by S&P.
  - Wilshire 5000 index value weights many more US companies.
  - Nasdaq Composite index value weights stocks listed on the Nasdaq (primarily technology stocks).
  - Morgan Stanley Capital International (MSCI) has international VW equity indexes.
  - Bloomberg Barclays has VW bond indexes.
- An exception is the Dow Jones index which is price-weighted for historical reasons.

# Passive Investing

- If you hold a VW index for an asset class, you earn the same return as the average dollar invested in that asset class.
- This is called **passive investing** because
  - Once you buy the VW index, minimal trading is needed to maintain the position: price changes adjust the weights automatically, so you only need to trade when companies issue or repurchase shares, new companies enter the asset class, or old ones drop out.
  - By holding the VW index, you guarantee yourself average performance and are not actively seeking to do better than that.

# Diversified Active Investing

# Active Investing

- The alternative to passive investing is **active investing**, picking a portfolio with the intention of outperforming the VW index.
- The **iron law of active investing** says that every dollar of active outperformance is accompanied by a dollar of active underperformance (because the passive investor earns the average return and not everyone can be better than average!)
- So active investing should only be attempted if you have a convincing theory of how you will win and who will lose.
  - *"If you've been playing poker for half an hour and you still don't know who the patsy is, you're the patsy."* Warren Buffett.

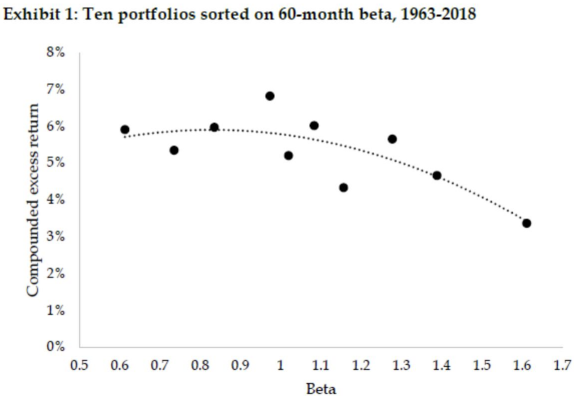
# The Least Bad Active Strategies

- Some active strategies have both theory and long-term historical evidence to support them.
  1. Low-risk stocks
  2. Value stocks
  3. Small-cap stocks
- Critically, all of these strategies are consistent with broad diversification and can be implemented cheaply.
- However, their recent performance has been poor.

# Low-Risk Investing (1)

- The risk of a stock can be measured by its volatility (standard deviation of return) or its **market beta** (the covariance of its return with a VW stock index, divided by the variance of the VW index).
  - A stock's market beta tells you how an extra dollar invested in that stock will contribute to the risk of a portfolio that is initially passively invested in the VW index.
  - In practice, low-volatility stocks also tend to have low market betas.
- There is long-term historical evidence that low-risk stocks have almost as high an average return as high-risk stocks, so you can
  - Lower risk without reducing return by holding the same dollar value of low-risk stocks, or
  - Increase return without increasing risk by investing more dollars in low-risk stocks.

# Low-Risk Investing (2)




Source:
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3442749

"[Based on] historical return series for US stock portfolios sorted on 60-month market beta, with data going back to July 1963…we plot the performance of ten decile portfolios sorted on beta… The graphs clearly show a flat, or even slightly negative relation between risk and return."

# Low-Risk Investing (3)

- Why does low-risk investing work?
  - Some aggressive investors are keen to get the highest return they can but are not able to borrow to buy stocks.
  - Once all their money is in the stock market, the only way they can get a higher return is to buy high-risk stocks.
  - They drive up the price of high-risk stocks so that these stocks offer too low a return in relation to the risk.
  - If you are a less aggressive investor who would otherwise hold cash, you can do better by reducing your cash holding and investing in low-risk stocks.
- Caveat: low-risk investing does not always beat passive investing.
  - Low-risk stocks did particularly badly in 2020 because these are often physical businesses such as utilities.

# Value Investing (1)

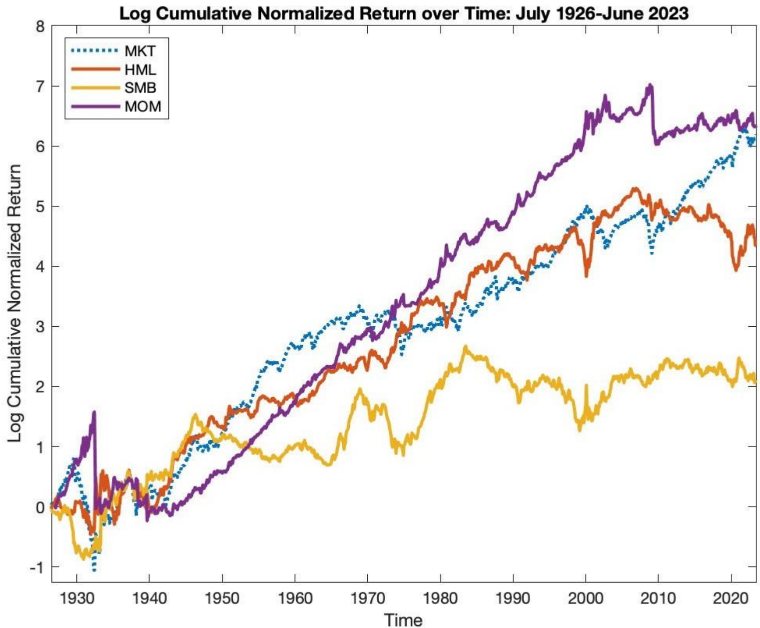
- **Value stocks** have low market prices relative to accounting measures of a company's **fundamental value** such as earnings or **book value** (cumulated past investments made by the company).
- **Growth stocks** have high market prices relative to accounting measures of fundamental value.
- There is long-term historical evidence that value stocks beat growth stocks and have somewhat lower risk.
  - Value investing was first popularized by Benjamin Graham in the 1930s.
  - Warren Buffett is one of the most famous practitioners of this approach today.

# Value Investing (2)

- There are several academic theories about why value investing works.
  - The simplest story is that high market prices reflect both the future prospects of a company and irrational enthusiasm of some investors for that company.
  - Hence many growth stocks are overpriced stocks and you should underweight them.
- Caveat: value stocks do not always beat growth stocks.
  - The value strategy has had a “lost decade” since 2010 and performed disastrously in 2020.

# Small-Cap Investing

- Small-cap stocks have low market capitalization (a low total dollar value of their shares).
- There is long-term historical evidence that these stocks deliver high returns relative to their risk.
- Again there are several theories why this is the case.
  - Perhaps fewer investors are familiar with these firms or comfortable investing in them.
  - Or a small-cap firm may have low market cap because it is out of fashion and undervalued.
- Caveat: small-cap stocks do not always beat large-cap stocks.
  - The late 1990s and the late 2010s were both periods of higher large-cap returns.



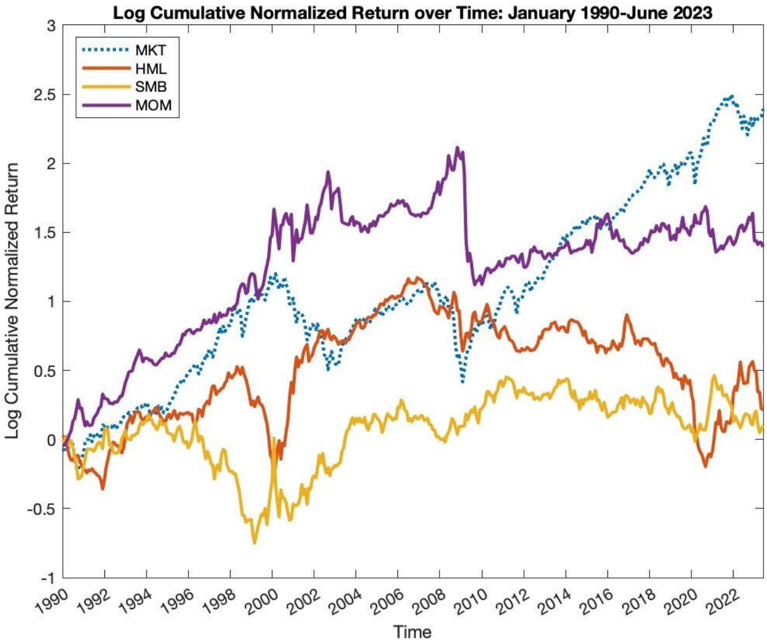

Source: Campbell, *Financial Decisions and Markets: A Course in Asset Pricing,* Princeton University Press 2018, Figure 3.7 (updated and normalized to a common standard deviation).

MKT = stock market relative to Treasury bills

HML = value stocks relative to growth stocks.

SMB = small-cap stocks relative to large-cap stocks.

MOM = stocks that have recently gone up relative to stocks that have recently gone down.




Source: Campbell, *Financial Decisions and Markets: A Course in Asset Pricing,* Princeton University Press 2018, Figure 3.8 (updated and normalized to a common standard deviation).

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## Active Investing to Express Values (1)

- One other reason to invest actively is to avoid investing in (**divest** from) companies whose business model is inconsistent with your values, e.g
  - Tobacco companies
  - Gun manufacturers
  - Private prison operators
  - Fossil fuel producers.
- This is often called **ESG investing** since you are paying attention to **environmental, social, and governance (ESG)** criteria.

# Active Investing to Express Values (2)

Two key points about ESG investing are:

1. You should expect ESG investing to lower your average return.
   - The point of divestment is to drive down the prices of “bad” companies, raising their cost of capital; but this implies that “bad” stocks will deliver high returns relative to “good” stocks.
2. However the return cost of ESG investing will likely be modest if you continue to diversify your portfolio among the remaining available stocks.