Somewhat Square Sudoku [January 2025 Jane Street Puzzle] – Solution by Nicholas Patel

First, we must choose 9 digits from 0-9 (i.e. one of ten 10 combinations), and this set must include 0, 2 and 5.

Notice that 5 can never be a common factor since the last column can only contain up to one 5 and one 0 (i.e. so at least 7 numbers are not divisible by 5). Similarly, 2 can never be a common factor since at best the numbers end in 0, 2, 4, 6 and 8 (in which case the remaining four numbers would be odd). We could force all numbers to be divisible by 9 via selecting appropriate digits, since the row sums are the same. 0 + 1 + 2 + ... + 9 = 45 = 0 mod 9, so for the sum excluding one digit to be divisible by 9 the only option is to remove 9. That is, all the numbers are divisible by 9 iff the digits are 0, 1, 2, ... 8. All the numbers are divisible by 3 (but not 9) iff we exclude digits 3 or 6. Still though we're not really getting anywhere.

Let's use another property: if all rows are divisible by some number then so is their sum.
The sum can be expressed as 45*111,111,111 – d*111,111,111 = 3^2 x 37 x 333667 x (45 – d) where d is the digit excluded. Thus, the GCD must be a factor of this value.

We can try one prime factor at a time to identify which ones cannot be factors of the GCD.
Let's first try to rule out 333667. Consider the set of all multiples of 333667 that are less than 10^9 (padded with leading 0s to reach 9 digits if necessary). Only 108 multiples of 333667 with all distinct digits exist, and in fact we can find a valid solution:

3 9 5 0 6 1 7 2 8
0 6 1 7 2 8 3 9 5
7 2 8 3 9 5 0 6 1
9 5 0 6 1 7 2 8 3
2 8 3 9 5 0 6 1 7
6 1 7 2 8 3 9 5 0
8 3 9 5 0 6 1 7 2
5 0 6 1 7 2 8 3 9
1 7 2 8 3 9 5 0 6

Thus, the max possible GCD is at least 333667. Also, since. 333667 > 9 x 37 x 45, the max GCD must be divisible by 333667. This implies that all row numbers must belong to the set of 108 we found earlier. Constraining on the excluded digit d, we find it cannot be 0, 1, 2, 3, 5, 6, 7, 8 or 8, thus d=4. So, the GCD must be a factor of 37 x 333667 x 41 (we can eliminate the 3^2 since d is not 3, 6 or 9). There is no solution consisting of multiples of 333667 x 41, but there is with multiples of 37 x 333667 (the one shown above). Therefore, this is an optimal solution, and a valid answer is **283950617**.