## Some F Squares [January 2024 Jane Street Puzzle] - Approach

Problem: <a href="https://www.janestreet.com/puzzles/some-f-squares-index/">https://www.janestreet.com/puzzles/some-f-squares-index/</a> (credit to Jane Street)

Solution by Nicholas Patel

For each F-pentomino of enlargement N, there are N consecutive rows/columns that the pentomino contributes 3N^2 to.

Thus, the largest possible pentomino must satisfy 3N^2 <= max sum across N consecutive rows/columns This gives a max size of N=3, since there are no 4 consecutive rows or columns with a sum of at least 48.

Now, the grid is 17x17 with a total of 20 regions. Let R be the sum of each region.

Let c be the unknown column sum and r be the unknown row sum (note only 1 row and 1 column have unknown sums). Then, the total sum is 458 + c = 441 + r = 20R

Since the smallest region is 8 squares,  $R \le 3*8 = 24$ . Hence,  $20R \le 480$ 

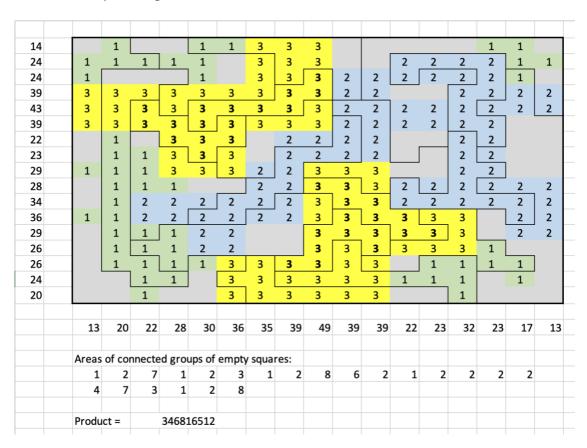
This immediately gives that (r,c) = (19,2) or (39, 22).

Now c=2 is impossible because that column contains 2 squares in a region with 8 squares; even if those 2 squares summed to 2 (i.e. and all other column values were zero), the max the 8-square region could sum to is 2 + 6\*3 = 20, in which case 20R = 400 < 460, giving a contradiction.

Thus the only possible solution is (r,c) = (39,22) and R = 24.

Hence, we can immediately fill in all 8-square regions with all 3s.

The F-pentomino of enlargement 3 in the upper left part of the grid looks like an easy guess. Using that there are 3 consecutive rows/columns with sum >= 27 in the long part helps to orient them. Similar logic follows for the one in the bottom right of the grid; the column sum of 49 is a big help here. Another useful observation is that 12-square regions with no 3s must be all 2s.



Finally, we arrive at the answer of **346816512**.