

Some F Squares [January 2024 Jane Street Puzzle] – Approach

Problem: <https://www.janestreet.com/puzzles/some-f-squares-index/> (credit to Jane Street)

Solution by Nicholas Patel

For each F-pentomino of enlargement N , there are N consecutive rows/columns that the pentomino contributes $3N^2$ to.

Thus, the largest possible pentomino must satisfy $3N^2 \leq \max \text{sum across } N \text{ consecutive rows/columns}$. This gives a max size of $N=3$, since there are no 4 consecutive rows or columns with a sum of at least 48.

Now, the grid is 17×17 with a total of 20 regions. Let R be the sum of each region.

Let c be the unknown column sum and r be the unknown row sum (note only 1 row and 1 column have unknown sums). Then, the total sum is $458 + c = 441 + r = 20R$

Since the smallest region is 8 squares, $R \leq 3 \cdot 8 = 24$. Hence, $20R \leq 480$

This immediately gives that $(r, c) = (19, 2)$ or $(39, 22)$.

Now $c=2$ is impossible because that column contains 2 squares in a region with 8 squares; even if those 2 squares summed to 2 (i.e. and all other column values were zero), the max the 8-square region could sum to is $2 + 6 \cdot 3 = 20$, in which case $20R = 400 < 460$, giving a contradiction.

Thus the only possible solution is $(r, c) = (39, 22)$ and $R = 24$.

Hence, we can immediately fill in all 8-square regions with all 3s.

The F-pentomino of enlargement 3 in the upper left part of the grid looks like an easy guess. Using that there are 3 consecutive rows/columns with sum ≥ 27 in the long part helps to orient them. Similar logic follows for the one in the bottom right of the grid; the column sum of 49 is a big help here. Another useful observation is that 12-square regions with no 3s must be all 2s.

14		1			1	1	3	3	3					1	1			
24	1	1	1	1	1		3	3	3			2	2	2	2	1	1	
24	1				1		3	3	3	2	2	2	2	2	2	1		
39	3	3	3	3	3	3	3	3	3	2	2			2	2	2	2	
43	3	3	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2	
39	3	3	3	3	3	3	3	3	3	2	2	2	2	2	2			
22		1			3	3	3			2	2	2	2		2	2		
23		1	1	3	3	3				2	2	2	2		2	2		
29	1	1	1	3	3	3	2	2	3	3	3			2	2			
28		1	1	1			2	2	3	3	3	2	2	2	2	2	2	
34		1	2	2	2	2	2	2	3	3	3	2	2	2	2	2	2	
36	1	1	2	2	2	2	2	2	3	3	3	3	3	3		2	2	
29		1	1	1	2	2			3	3	3	3	3	3	3	2	2	
26		1	1	1	2	2			3	3	3	3	3	3	1			
26		1	1	1	1	3	3	3	3	3	3		1	1	1	1		
24			1	1		3	3	3	3	3	3	1	1	1		1		
20			1			3	3	3	3	3	3			1				
		13	20	22	28	30	36	35	39	49	39	39	22	23	32	23	17	13
		Areas of connected groups of empty squares:																
		1	2	7	1	2	3	1	2	8	6	2	1	2	2	2	2	
		4	7	3	1	2	8											
		Product = 346816512																

Finally, we arrive at the answer of **346816512**.