CS7280: Network Science, Fall 2017

Homework-3

November 3, 2017

Due Date: NOVEMBER 15, 5pm

1 Problem 1 (30 points)

Consider an SI-type epidemic spreading on a random graph in which all nodes have the same degree k. The number of network nodes is n. Initially, at time t=0, a small number of c nodes, chosen at random, are infected. The pathogen's transmission rate is β.

1. Show that the probability of infection $i(t)$ of every node increases as $e^{\beta k t}$ for small t (i.e., shortly after the start of the epidemic).
2. Show that the probability of infection $i(t)$ of every node is the same and give the differential equation it satisfies.
3. Show that
$$i(t) = \frac{c\, e^{\beta k t}}{n - c + c\, e^{\beta k t}} \tag{1}$$
4. Compute the time at which the "inflection point" of the epidemic occurs, i.e., the point at which the rate of appearance of new disease cases stops increasing and starts decreasing.

2 Problem 2 (40 points)

In this exercise you will experiment with the following two community detection algorithms:

- **Walktrap:** `https://www-complexnetworks.lip6.fr/~latapy/PP/walktrap.html` We have not discussed the Walktrap algorithm in class. Please make sure that you understand how it works before using it.
- **Louvain:** `http://perso.crans.org/aynaud/communities/`

You can choose a network dataset that you are interested in. Both algorithms are integrated in Network-X.

1 Run the Louvain and Walktrap algorithms on the network. Compare the modularity of the two resulting community sets (one for each algorithm).

2 Compare the two algorithms in terms of the community they assign each node to. Which metric would you choose to make this comparison? (Note: we discussed the Normalized Mutual Information metric in class but there are more metrics you could use. The only requirement is that you justify the metric that you choose to use).

3 Both algorithms have a parameter. The parameter of the Louvain algorithm is called *resolution*, r, while the parameter of Walktrap is tx, the length of the random walk. First, read the research papers that proposed these algorithms to learn more about these two parameters. Then, run Walktrap for $tx = 2, 4, 6, 8, 10$, and Louvain for $r = 5, 10, 15, 20, 25$. Report how these parameter settings affect the modularity and computational speed of each algorithm, and the comparison between the two algorithms.

3 Problem 3 (30 points)

Consider a bipartite network with two types of nodes, Male and Female (M and F). Suppose that a certain Sexually Transmitted Disease (STD) can be transmitted only from M to F (with transmission rate $\beta_{M\to F}$) and from F to M (with transmission rate $\beta_{F\to M}$). Write the equations of the corresponding SI model, assuming the degree-block approximation and that there is no correlation between the node degrees.