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Problem 1

The adjacency matrix will look like this:

adj_list = [['a','b'], ['a','d'], ['b','a'], ['b','c'], ['b','d'], ['c','b'], ['d','a'], ['d','b']]

Show that it takes O(n(n+m)) time to compute the diameter of the network.

The pseudocode would look something like this:

```
for node in graph:
        for all_other_nodes in graph:
                if edge_between_node1_node2 exists in adj_list:
                        traverse_path.append()
print(max(traverse_path))
```

The first 2 loops equal n^2 since they visit every node twice. Visiting 1 more edge only adds 1 more round to the complexity since we are only concerned with actual edges, not all possible edges; therefore, the complexity only increases by the number of edges. Complexity is there for O(n(n^2 + m)).

Show that it takes O(E[k]) time, on the average, to list the neighbors of a node, where E[k] is the expected value of the node degree.

The degree of the adj_list is 2, since there are 8 edges and 4 nodes. For each node, we only expect to visit 2 other nodes, so it's only going to take O(E[k]) time to list neighbors of a node because there is only on average <k> edges for node.

Show that it takes time O(E[k2]) to list the second-hop neighbors (the neighbors of the neighbors).

To visit neighbors of a neighbor, we would do exactly as we do above, but we would do it twice, therefore, we expect complexity of O([k^2])

The reciprocity of a directed network is defined as the fraction of edges that run in both directions (i.e., I connects to j and j connects to i). For a directed network in which in- and out-degrees are uncorrelated, show that it takes time O(m2/n) to calculate the reciprocity of the network. Why is the restriction to uncorrelated degrees necessary?

Considering the adj_list from above, we look at all nodes that have and edge going to them, and we expect this to be equal to the number of edges divided by the number of nodes; m/n. We then have to look at edges going out of nodes, this is again of complexity m/n. Therefore, the complexity is O((m^2)/n)

Problem 2

A matrix A, of size n x n is nilpontent if A^k yields a matrix of all zeroes

For cyclic, directed matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Squaring, cubing, and raising to power 4 do not yield a matrix of all zeroes

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^4 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Cyclic graphs are never nilpotenet!

For an acyclic, directed matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Raising to power 4 yields a matrix of all zeroes, therefore it is nilpotent.

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Acyclic graphs are nilpotent!

The eigenvalue for nilpotent graphs is always 0, therefore, the eigenvalue for acyclic graphs is always 0

Homework1_problem3

September 20, 2017

A) The network analyzed here is a Protein-Protein Interaction (PPI) network from C. elegans. The dataset comes from Simonis et al., 2009. This dataset was obtained by a high-throughput yeast two-hybrid (Y2H) screening system. Each node in the dataset is a protein and the edges are interactions obtained from the experiment. The network is undirected and is unweighted. This network is only 3-4% complete, Y2H screening is not very sensitive and would need to be run multilpe times in order to accurately uncover PPI of the organism; however even then, some PPI would still not be detectable

In [1]:
```
import networkx as nx
import pandas as pd
import scipy.stats as stats
import matplotlib.pyplot as plt
%matplotlib inline
```

In [2]:
```
dataset = pd.read_csv('wi8.txt', sep='\t')
```

In [3]:
```
dataset = dataset.iloc[:,[0,1]]
```

In [4]:
```
dataset.head()
```

Out[4]:

	#IDA	IDB
0	AC3.10	AC3.3
1	AC3.3	C03A7.14
2	AC3.3	C03A7.4
3	AC3.3	C05D2.1
4	AC3.3	C37C3.6

In [5]:
```
dataset.tail()
```

Out[5]:

	#IDA	IDB
3859	ZK829.7	ZK829.7
3860	ZK849.2	ZK849.2
3861	ZK849.2	ZK938.2
3862	ZK858.4	ZK858.4
3863	ZK945.2	ZK945.2

In [6]:
```
graph = nx.from_pandas_dataframe(dataset, '#IDA', 'IDB')
```

In [7]:
```
graph.number_of_nodes()
```

```
Out[7]: 2528

In [8]: graph.number_of_edges()

Out[8]: 3864

In [9]: # Remove self-loops, k-core cannot be calculated without them removed
        graph.remove_edges_from(graph.selfloop_edges())

In [10]: graph.number_of_nodes()

Out[10]: 2528

In [11]: graph.number_of_edges()

Out[11]: 3706
```

B) Below are the NetworkX functions that generated the centrality metrics. The alpha parameters I used for Katz were: 0.1, 0.2, 0.01, 0.001, 0.4

```
In [12]: degree = nx.degree_centrality(graph)
         #The graph is unweighted, so the following calculations of 'in' or 'out' c
         #nx.in_degree_centrality(graph)
         #nx.out_degree_centrality(graph)
         closeness = nx.closeness_centrality(graph)
         harmonic = nx.harmonic_centrality(graph)
         betweenness = nx.betweenness_centrality(graph)
         eigenvector = nx.eigenvector_centrality(graph)
         # Numpy version of katz_centrality did not have max_iteration parameter th
         katz_01 = nx.katz_centrality_numpy(graph, alpha=0.1)
         katz_02 = nx.katz_centrality_numpy(graph, alpha=0.2)
         katz_001 = nx.katz_centrality_numpy(graph, alpha=0.01)
         katz_0001 = nx.katz_centrality_numpy(graph, alpha=0.001)
         katz_04 = nx.katz_centrality_numpy(graph, alpha=0.4)
         pagerank = nx.pagerank_numpy(graph, alpha=0.85)

In [13]: centrality_metrics = {'Degree': degree,
                               'Closeness': closeness,
                               'Harmonic': harmonic,
                               'Betweenness': betweenness,
                               'Eigenvector': eigenvector,
                               'Katz alpha=0.1': katz_01,
                               'Katz alpha=0.1': katz_02,
                               'Katz alpha=0.01': katz_001,
                               'Katz alpha=0.001': katz_0001,
                               'Katz alpha=0.4': katz_04,
                               'Pagerank': pagerank}
```

C) Below are the nodes with the 5 highest scores for each centrality metric.

```
In [14]: for metric_name in centrality_metrics:
             print(metric_name)
             metric = centrality_metrics[metric_name]
             top5 = {key: metric[key] for key in sorted(metric, key=metric.get, rev
             rank = 1
             for key in sorted(top5, key=metric.get, reverse=True):
                 print(rank, "\t",  key, "\t", top5[key])
                 rank+=1
             print("\n")
```

```
Degree
1          R05F9.10          0.03917688959240206
2          T17H7.4          0.02928373565492679
3          T11B7.4          0.02849228333992877
4          T04H1.2          0.02690937870993273
5          K09B11.9          0.024930747922437674



Closeness
1          T17H7.4          0.25277383803636005
2          F14F3.1          0.24703849792540122
3          K09B11.9          0.24566066880780485
4          T11B7.4          0.2455672853807365
5          W05H7.4          0.2452875605904028



Harmonic
1          T17H7.4          735.3273809523757
2          K09B11.9          712.6968253968233
3          T11B7.4          711.3535714285715
4          F14F3.1          703.3714285714277
5          ZK849.2          702.527380952383



Betweenness
1          R05F9.10          0.06642796999808961
2          T17H7.4          0.06232863236435431
3          T04H1.2          0.061452497140486534
4          W02G9.2          0.05979758949660616
5          ZK849.2          0.05925303225666527



Eigenvector
1          T17H7.4          0.282982913154414
2          K09B11.9          0.2679308396039128
3          T11B7.4          0.2644400128976901
4          F14F3.1          0.17925777180247296
5          W05H7.4          0.17902268637792104
```

Katz alpha=0.1

1	W09C2.1	0.372864648115799
2	ZK1053.5	0.1530768047463734
3	M04G12.1	0.14354337547384113
4	C47G2.2	0.09303740584651284
5	T28A11.11	0.08818569110732333

Katz alpha=0.01

1	R05F9.10	0.03894997630910324
2	T17H7.4	0.0349088949355533
3	T11B7.4	0.03425478256076898
4	T04H1.2	0.033292708426794076
5	K09B11.9	0.03267247444477457

Katz alpha=0.001

1	R05F9.10	0.02179813033652241
2	T17H7.4	0.021309829284867826
3	T11B7.4	0.021267468212742786
4	T04H1.2	0.021186671880009625
5	K09B11.9	0.021090613554937552

Katz alpha=0.4

1	C45G9.7	0.15475355606047345
2	F35G2.2	0.10382252057988557
3	T08G5.5	0.1007606078255713
4	F38A6.1	0.10048560934388334
5	K08F8.2	0.10041202435590722

Pagerank

1	R05F9.10	0.013611198823769256
2	Y40B10A.2	0.007559704049684905
3	T11B7.4	0.006897137033939696
4	T04H1.2	0.006822929910931026
5	T17H7.4	0.006755427172061979

D) Comparison of the Kendall rank correlation coefficient (tau) between all 11 centrality metrics analyzed. The centrality metrics with the highest Tau's were between closeness, harmonic, and eigenvector, all had a Tau of over 0.9. This makes sense because eigenvector centrality is high when the degrees of a nodes neighbors are high; harmonic centrality is high when a node has a lot of neighbors, and those nieghbors have a lot of neighbors, and closeness centrality is high for a node when it can reach anyother node over the shortest paths. All three of these centrality metrics have similar definitions and so they will rank nodes similarly. Katz centrality metrics with alpha parameters of 0.01 and 0.001 had the highest tau; this can be easily be explained since all the edges in the graph for a particular node would score similarly, and therefore would be similar between theses two centrality metrics.

The lowest correlation was between harmonic and katz with an alpha parameter of 0.01; the guessing I did of choosing the alpha parameter really had messed with the centrality metrics and this particular metric probably did a very poor job of ranking nodes by importance.

```
In [15]: for i in range(9):
             for j in range(i+1,9):
                 print(list(centrality_metrics.keys())[i], "-", list(centrality_met
                 x1 = list(list(centrality_metrics.values())[i].values())
                 x2 = list(list(centrality_metrics.values())[j].values())
                 tau, p_value = stats.kendalltau(x1, x2)
                 print("tau:", tau)
                 print("\n")
```

```
Degree - Closeness
tau: 0.449946078409


Degree - Harmonic
tau: 0.452605858609


Degree - Betweenness
tau: 0.883800191159


Degree - Eigenvector
tau: 0.41883944141


Degree - Katz alpha=0.1
tau: 0.0265427674887


Degree - Katz alpha=0.01
tau: 0.80996242635


Degree - Katz alpha=0.001
tau: 0.814195258478
```

```
Degree - Katz alpha=0.4
tau: 0.0127900122072

Closeness - Harmonic
tau: 0.979776972576

Closeness - Betweenness
tau: 0.40784417932

Closeness - Eigenvector
tau: 0.912511597026

Closeness - Katz alpha=0.1
tau: -0.156359374957

Closeness - Katz alpha=0.01
tau: 0.619003736749

Closeness - Katz alpha=0.001
tau: 0.612342235494

Closeness - Katz alpha=0.4
tau: -0.0314113082004

Harmonic - Betweenness
tau: 0.410341990616

Harmonic - Eigenvector
tau: 0.915541094857

Harmonic - Katz alpha=0.1
tau: -0.157755165286

Harmonic - Katz alpha=0.01
tau: 0.628545806028
```

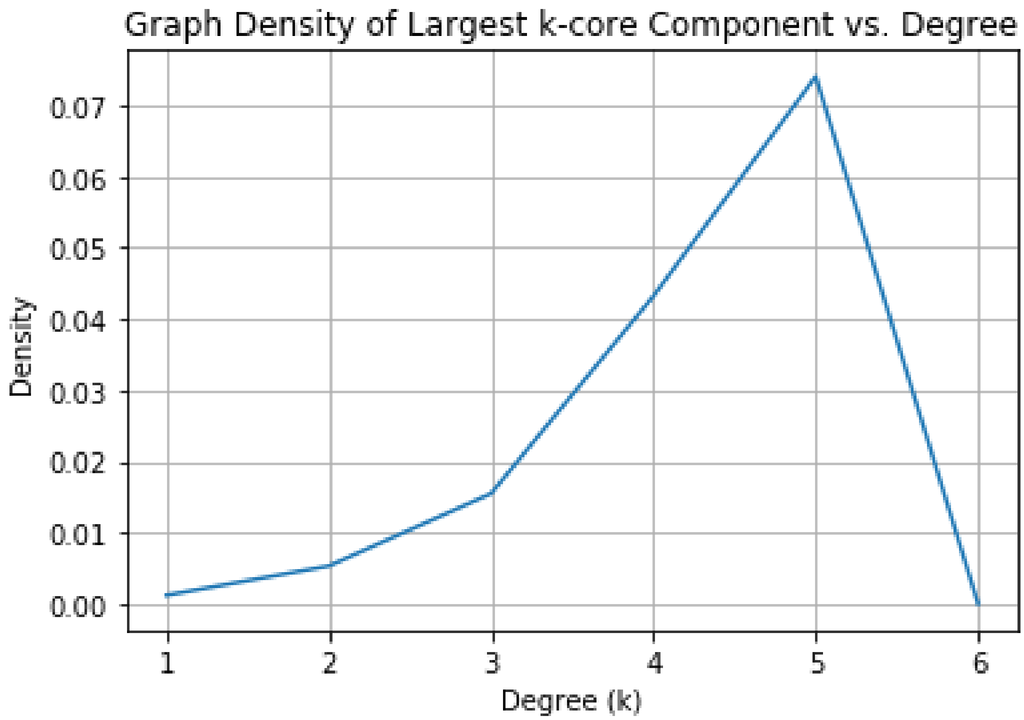
```
Harmonic - Katz alpha=0.001
tau: 0.621817809638


Harmonic - Katz alpha=0.4
tau: -0.0340615274689


Betweenness - Eigenvector
tau: 0.375260978408


Betweenness - Katz alpha=0.1
tau: 0.0248599140178


Betweenness - Katz alpha=0.01
tau: 0.71712574218


Betweenness - Katz alpha=0.001
tau: 0.719072482537


Betweenness - Katz alpha=0.4
tau: 0.0136941730486


Eigenvector - Katz alpha=0.1
tau: -0.154670775861


Eigenvector - Katz alpha=0.01
tau: 0.588784702304


Eigenvector - Katz alpha=0.001
tau: 0.582102375466


Eigenvector - Katz alpha=0.4
tau: -0.0321186594312


Katz alpha=0.1 - Katz alpha=0.01
tau: -0.0389097881557
```

```
Katz alpha=0.1 - Katz alpha=0.001
tau: -0.0364314188578


Katz alpha=0.1 - Katz alpha=0.4
tau: 0.0348347571789


Katz alpha=0.01 - Katz alpha=0.001
tau: 0.990901420886


Katz alpha=0.01 - Katz alpha=0.4
tau: -0.0171007776557


Katz alpha=0.001 - Katz alpha=0.4
tau: -0.0166715786499
```

```
In [16]: x_axis = list(range(1,7))
         y_axis = []
         for i in range(1,7):
             y_axis.append(nx.density(nx.k_core(graph, k=i)))
```

E) The nodes that are in the 5-core component, the largest component have low metrics in all the centrality metrics except for harmonic centrality. The nodes with the 5 highest ranks by that centrality scored aboce 55, which is crazy.

```
In [17]: plt.plot(x_axis, y_axis)
         plt.xlabel('Degree (k)')
         plt.ylabel('Density')
         plt.title('Graph Density of Largest k-core Component vs. Degree')
         plt.grid(True)
         plt.show()
```




```
In [18]: five_component = nx.k_core(graph, k=5)
```

```
In [19]: degree = nx.degree_centrality(five_component)
         closeness = nx.closeness_centrality(five_component)
         harmonic = nx.harmonic_centrality(five_component)
         betweenness = nx.betweenness_centrality(five_component)
         eigenvector = nx.eigenvector_centrality(five_component)
         katz_01 = nx.katz_centrality_numpy(five_component, alpha=0.1)
         katz_02 = nx.katz_centrality_numpy(five_component, alpha=0.2)
         katz_001 = nx.katz_centrality_numpy(five_component, alpha=0.01)
         katz_0001 = nx.katz_centrality_numpy(five_component, alpha=0.001)
         katz_04 = nx.katz_centrality_numpy(five_component, alpha=0.4)
         pagerank = nx.pagerank_numpy(five_component, alpha=0.85)
```

```
In [20]: centrality_metrics = {'Degree': degree,
                               'Closeness': closeness,
                               'Harmonic': harmonic,
                               'Betweenness': betweenness,
                               'Eigenvector': eigenvector,
                               'Katz alpha=0.1': katz_01,
                               'Katz alpha=0.1': katz_02,
                               'Katz alpha=0.01': katz_001,
                               'Katz alpha=0.001': katz_0001,
                               'Katz alpha=0.4': katz_04,
                               'Pagerank': pagerank}
```

```
In [21]: for metric_name in centrality_metrics:
             print(metric_name)
             metric = centrality_metrics[metric_name]
             top5 = {key: metric[key] for key in sorted(metric, key=metric.get, rev
             rank = 1
             for key in sorted(top5, key=metric.get, reverse=True):
                 print(rank, "\t",  key, "\t", top5[key])
                 rank+=1
             print("\n")
```

```
Degree
1          K09B11.9         0.24074074074074073
2          T17H7.4          0.19444444444444442
3          F14F3.1          0.18518518518518517
4          ZK121.2          0.17592592592592593
5          T11B7.4          0.16666666666666666



Closeness
1          K09B11.9         0.5046728971962616
2          F14F3.1          0.4864864864864865
3          T17H7.4          0.4778761061946903
4          ZK121.2          0.47577092511013214
5          T11B7.4          0.4675324675324675



Harmonic
1          K09B11.9         63.00000000000006
2          F14F3.1          59.66666666666673
3          T17H7.4          59.416666666666735
4          ZK121.2          58.66666666666673
5          T11B7.4          57.50000000000008



Betweenness
1          K09B11.9         0.10875526983230903
2          F14F3.1          0.08677546399658279
3          T17H7.4          0.07544623639851308
4          T04H1.2          0.06479263186428481
5          ZK849.2          0.059716112104435275



Eigenvector
1          K09B11.9         0.29583954510987864
2          W05H7.4          0.2438907221730043
3          ZK121.2          0.23352419179098816
4          T17H7.4          0.22570297725368121
5          F14F3.1          0.22357805661960395
```

Katz alpha=0.1

1	C06A5.9	0.25997417136429135
2	F27C1.6	0.1743611197580395
3	H06I04.1	0.16673645386365288
4	F01G10.2	0.1659090049165122
5	R02F2.5	0.16207597132583587

Katz alpha=0.01

1	K09B11.9	0.11303571931063659
2	T17H7.4	0.10824089521920223
3	F14F3.1	0.10738490379277811
4	ZK121.2	0.10650408405118213
5	T11B7.4	0.10538366663169679

Katz alpha=0.001

1	K09B11.9	0.0975065565181868
2	T17H7.4	0.09702780961555434
3	F14F3.1	0.09693299162105348
4	ZK121.2	0.09683797486056203
5	T11B7.4	0.09674039248389046

Katz alpha=0.4

1	R05F9.10	0.24139311518919715
2	B0507.1	0.22041493011799637
3	F43G9.11	0.21790454513107282
4	F38A6.1	0.18382407122124933
5	F32B4.4	0.17907670471405848

Pagerank

1	K09B11.9	0.02661033878863133
2	T17H7.4	0.02191465413535914
3	F14F3.1	0.02060371518605548
4	ZK121.2	0.019477268787831955
5	T11B7.4	0.018504070155469544