Final project
Catching a target with the Extended Kalman filter

The objectives of this assignment are to encourage you to think creatively and critically to extract a useful signal from noisy experimental data, find best estimation method of a dynamical process and make forecast of its future development.

This assignment is to be done in groups of 3-4 students, and only one document is submitted for the group. You may also freely talk with students in other groups, but the final documents that you submit must be done only by your group.

Formulation

1. Airplane A is flying on a height of ~5000 meters. Airplane B is flying on a roughly the same height and trying to catch the airplane A. Airplane B is flying faster then the airplane A. Please answer at which distance from the observer and in how many seconds the distance between Airplane A and B would be minimal (Airplane B catches airplane A). What is this minimal distance?

2. There are radar measurements of range D, azimuth β and elevation angle ε for your availability.
   files/folder/Final projects/Project 6/data/ trajectory_data.mat
   The file contains two variables z1 and z2 corresponding to the radar measurements of two airplanes A and B. Measurements are available every second.

   The format of z1 and z2 in Matlab:
   First column – measurements of the range D in meters
   Second column – measurements of the azimuth β in radians
   Third column – measurements of the elevation angle ε in radians. Check the lecture on Extended Kalman filter to check the elevation angle.

   The format of txt data
   Element in the first column - measurements of the range D in meters
   Element in the second column - measurements of the azimuth β in radians
   Element in the third column – measurements of the elevation angle ε in radians. Check the lecture on Extended Kalman filter to check the elevation angle.

3. Solve the problem by developing the **Extended Kalman filter** for two trajectories**.**

   Construct a tracking filter for both datasets.

   State vector $X_i = \begin{vmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \\ z_i \end{vmatrix}$

   (a) Let's assume that the true trajectory X_i of the airplane A is described by a motion with normally distributed unbiased random acceleration a_i with variance $\sigma_a^2 = 0.02$ for both a_i^x, a_i^y and we describe the dynamics of the z component (height) with the random walk model, where the variance of state noise is $\sigma_a^2 = 1$ for a_i^z. The same for the trajectory of the airplane B with $\sigma_a^2 = 0.01$ for a_i^x, a_i^y and $\sigma_a^2 = 1$ for a_i^z.

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$

$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$

$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$

$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

$$T = 1$$

$$z_i = z_{i-1} + a_{i-1}^z \mathrm{T}$$

(b) The measurement vector consists of range D, azimuth β and the elevation angle ε, thus the measurement equation is nonlinear.

(c) From a prior information you know that the variance of measurement noise of range D is $\sigma_D^2 = 200^2$ (in meters), the variance of measurement noise of azimuth β is $\sigma_\beta^2 = 0.01^2$ (in radians), and the variance of measurement noise of elevation angle ε is $\sigma_\varepsilon^2 = 0.01^2$ (in radians).

(d) In this case covariance matrix of state noise would be represented as.

$$Q = G \begin{vmatrix} (\sigma_a^{\mathrm{x}})^2 & 0 & 0 \\ 0 & (\sigma_a^{\mathrm{y}})^2 & 0 \\ 0 & 0 & (\sigma_a^{\mathrm{z}})^2 \end{vmatrix} G^T$$

It has the same form for 2 filters, but the variances are different according to the information above.

(e) Initial filtration error covariance matrix $\mathrm{P}_{0,0}$.
Diagonal elements are 10^5, non-diagonal are zero.

(f) Observation function in this case is

$$h(X_i) = \begin{vmatrix} \sqrt{x_i^2 + y_i^2 + z_i^2} \\ arctg\left(\frac{x_i}{y_i}\right) \\ arcsin\left(\frac{z_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}}\right) \end{vmatrix}$$

For the implementation of Kalman filter you need to know the derivative with respect to the extrapolation point

$$\frac{dh(\widehat{X}_{i+1,i})}{dX_{i+1}} = \begin{vmatrix} \frac{x_{i+1,i}}{\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2}} & 0 & \frac{y_{i+1,i}}{\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2}} & 0 & \frac{z_{i+1,i}}{\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2}} \\ \frac{y_{i+1,i}}{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2} & 0 & -\frac{x_{i+1,i}}{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2} & 0 & 0 \\ -\frac{x_{i+1,i}z_{i+1,i}}{(x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2)\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2}} & 0 & -\frac{y_{i+1,i}z_{i+1,i}}{(x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2)\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2}} & 0 & \frac{\sqrt{x_{i+1,i}^2 + y_{i+1,i}^2}}{x_{i+1,i}^2 + y_{i+1,i}^2 + z_{i+1,i}^2} \end{vmatrix}$$

4. Develop two filters for both trajectories.
   The distance between two aircrafts is then determined as
   $$Dist = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2}$$

   Here, x, y and z are the components of filtered state vector.

   Plot the resulted distance to answer the question of this project. Compare it with the distance derived directly from the non-filtered measurements of range D, azimuth β and the elevation angle ε. For that you need to make the coordinate transformation of measurements from polar to Cartesian coordinate system to get pseudo-measurements of x, y and z and calculate the distance between airplane A and B again. What is the difference between filtered and non-filtered distance? What is the scatter of non-filtered distance around the minimal distance between aircraft A and B? How strongly the filtration reduces the level of uncertainty of minimal distance between A and B?

5. Contact the group of students working on project 7. They are doing the same, but developing one joint filter, instead of doing 2 independent filters using coordinate transformation of measurements. accuracy of predictions with your and their approach.

October 19 and October 21

1. Present your results with charts in 15 minutes.
   The presentation should include the problem formulation, why it is important, nice figures, grounds why the chosen method is the best method (visual analysis, quantitative criteria, simplicity of implementation, and any other arguments). Which regularities are found. Discuss what are the risks of obtained estimations and conclusions about the process. Make general conclusions about the efficiency of method.

   Try **to share** with the audience a practical and useful idea behind the project, **to make the overall exchange of practical and efficient tools and approaches and for what they can be applied.**

2. Submit the final version of your project to canvas.