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RESISTANCE MODELLING OF SMA WIRE ACTUATORS

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ABSTRACT

Shape Memory Alloys (SMAs) are well suited for use as linear actuators. In particular, NiTi wires as linear SMA actuators offer the advantages of light weight, a simple actuation mechanism and high force-to-weight ratio. An additional advantage is that the electrical resistance of the NiTi wires varies intrinsically as the material undergoes a phase change between martensite and austenite, and so can be used as a sensing element in controlling the actuator system. In the past, models of the resistance behaviour have been used with some success in controller design. These models are often simplified, assuming a linear change between austenite and martensite resistances. More complex models may include non-linear dependence on phase transformation, but most neglect to account for the impact of changes in temperature and stress. The focus of this work is the investigation of a more detailed electrical resistance model that accounts for these effects during Joule heating of NiTi wires. The model takes into account the effects of temperature, stress, and martensite phase fraction. Data obtained from controlled experimental trials using a 0.254mm diameter Flexinol wire is used to identify model parameters, and the model is simulated and validated experimentally. The eventual goal is to use the model for simulation during controller synthesis, and also incorporate it into online controllers. Hence, the inversion of the model and its application in controller design is also discussed.

Keywords: Shape Memory Alloy, Resistance feedback, Controller design, SMA modelling

INTRODUCTION

Materials made of Shape Memory Alloy (SMA) exhibit unique mechanical properties when the temperature of these materials is varied. Specifically, Flexinol wires, a commercially available NiTi Shape Memory Alloy, have the unique property of the shape memory effect (SME) – that is, the NiTi wire contracts to a pre-determined, stress-dependent length when the wire is heated. This "high-temperature" wire length can be up to 8% shorter than the wire length at low temperature. Specifically, the NiTi wire transitions from a "martensite" crystalline phase to an "austenite" phase when it is heated from a low to a high temperature; and vice-versa when it is cooled. It is this phase transformation that affects the mechanical changes in the material which is seen most easily through a contraction of the wire. There has been an increasing amount of research interest in exploiting this extraordinary behaviour to make next generation linear actuators using SMAs.

In [1], it was reported that the shape memory effect also causes changes in up to 23 internal properties of the SMA material, one of them being the electrical resistivity of the alloy. The relative ease and low cost of sensing the electrical resistance motivates its use as a self-sensing mechanism for actuators made from SMA wires However, the complex dependence of the resistance on temperature and stress is generally not well understood and limits its applicability as a feedback signal. Thus, a model of the resistance behaviour of SMAs is highly desirable and is the focus of this research.

SMA RESISTANCE MODEL BACKGROUND

The resistance behaviour of SMAs was first examined from a controller design perspective in [2], where the authors argued that the ease and cost-effectiveness of measuring electrical resistance (ER) make it attractive to be used as a feedback quantity. The authors of [2] found that the SMA wire exhibits a relatively high ER value in the low temperature martensite phase (M-phase) and a lower ER value in the high temperature austenite phase (A-phase), and that during the phase transformation the ER variation between these two values is approximately proportional to the strain of the wire. Furthermore, the authors reported very little hysteresis in the resistance–strain relationship. These findings allowed the authors in [2] to use the normalized resistance feedback to calculate the corresponding strain and design a position control system for the SMA actuator and apply it with a high degree of success in an active endoscope [2]. It was discovered in more recent literature [3,4] that the resistance behaviour of commercially available NiTi SMA wires is more complex than observed in [2], and therefore, the methods developed in [2] are not readily applicable to SMA actuator controller design using today's commercially available SMA products.

In [5], knowledge of the resistance behaviour of SMA wires was used to shorten the heating time of a wire actuator. The investigation of SMA wire resistance behaviour in [5] showed that the SMA in A-phase consistently exhibited a low electrical resistivity compared to the M-phase, despite the temperature hystereses present in NiTi SMAs. Therefore, it is possible to detect the complete transformation from M- to A-phase by measuring the resistance and comparing it with a threshold resistance value. Finally, [5] showed that a higher current than the maximum safe current could be applied to heat the wire until the resistance drops below the threshold resistance, after which a lower, safe current could be supplied to maintain the wire in A-phase. It was shown in [5] that using this heating technique the actuation time could be improved by a factor of two. Cooling time was unaffected, since it is a largely passive process that cannot be hastened without some sort of cooling system.

One overlooked aspect of the resistance behaviour of SMA wires is that neither [2] nor [5] had attempted to investigate how wire stress affects the resistance response as all experiments in [2] and [5] were performed at constant load levels. Additionally, the electrical resistance was assumed to be constant in the stable M- and A-phase temperature regions. In [3] and [4] experiments were performed to study the effects of varying stresses and temperatures on the SMA wire's resistance response. It was shown that 1) the resistivity change is approximately linear with the temperature change in both M- and A-phase temperature regions and 2) increased stress has the effect of both increasing the absolute resistivity values and shifting the transformation temperatures (i.e. A_s, A_f, M_s, M_f) upwards.

Additional investigation by the authors of [3] allows the resistance behaviour of SMA wire actuators to be characterised in great detail for a wide range of temperature and stress levels. The findings in [3] can be summarised as follows:

1. In pure M- and A-phases, the wire resistance is not constant but rather increases linearly with temperature. This behaviour is the same as other (perhaps single-phase) alloys where the material's resistivity and temperature are related via a positive coefficient. It is also shown that wire resistivity increases more rapidly as temperature changes in M-phase than in A-phase.

2. In the range of temperature where the phase transformation takes place, the overall resistance characteristic is a weighted combination of the temperature-dependant resistance behaviour of each phase. The combined effect is that as the wire is heated, the resistance first increases linearly to a maximum, then drops to a minimum, finally increases linearly again with further heating; and

3. Increasing wire stress causes an up-shift of the transformation temperatures, A_s, A_f, M_s, M_f, as well as the absolute resistivity values of the SMA wire in M-phase. The result is an increase in both the maximum/minimum peak resistances as well as the temperatures at which those peaks are achieved.

Based on these key results, [3] presented a novel framework for interpreting the resistance behaviour of SMA wires. From a control design perspective, a mathematical model enables the ability to test controller designs that use resistance as feedback in simulation. If the model is invertible, a model which explicitly accounts for variations in stress and temperature also allows the possibility to identify variations in the operating temperature and stress, based on actual resistance. These qualities motivate the investigations presented in this paper, where the resistance behaviour of a 0.254mm diameter Flexinol wire is characterised using a modified version of the model from [3] expressed as Equation 1:

$$R_{total} = R_M \xi_M + R_A(1 - \xi_M) \tag{1}$$

where R_M and R_A are the resistance functions of the M- and A- phases respectively; while ξ_M represents the phase fraction of martensite in the SMA. Furthermore, R_M and R_A are calculated as functions of temperature and stress as shown in Equations 2 and 3:

$$R_M = R_{0M} + (T - T_{0M})\frac{\partial R_M}{\partial T} + \sigma\frac{\partial R_M}{\partial \sigma} \tag{2}$$

$$R_A = R_{0A} + (T - T_{0A})\frac{\partial R_A}{\partial T} + \sigma\frac{\partial R_A}{\partial \sigma} \tag{3}$$

where T is the wire temperature and σ is the wire stress, and all other terms are parameters summarized in Table 1, which need to be experimentally determined. There are eight parameters that need to be identified.

Table 1 – Parameters in Equations 2 & 3

parameter	description
R_{0M} (R_{0A})	Nominal wire resistance in martensite (austenite) phase
T_{0M} (T_{0A})	Temperature at which the nominal resistances were measured
$\partial R_M/\partial T$ ($\partial R_A/\partial T$)	Linear temperature dependence of martensite (austenite) resistance
$\partial R_M/\partial \sigma$ ($\partial R_A/\partial \sigma$)	Linear stress dependence of martensite (austenite) resistance

Measurable quantities used to identify these parameters are the resistance of the SMA wire and its stress. The measured wire strain from constant-stress experimental trials is used to infer the martensite phase fraction. Since there are no sensors on the experimental jig that can measure the wire temperature, it is estimated using the first-order convection equation (Equation 4) where the power supplied to the wire is calculated with knowledge of the supply voltage and SMA resistance (Equation 5):

$$CV\rho\frac{dT}{dt} = P(t) - hA(\Delta T(t)) \tag{4}$$

$$P(t) = V_s(t)^2 R(t) \tag{5}$$

where

ρ = material density = 6.45×10^3 kg/m^3 [6]
C = specific heat = 465.2 J/(kg °C) [6]
V = wire volume = 1.434×10^{-8} m^3
h = convection heat transfer coefficient = 131W/(m^2 °C)
A = wire surface area = 2.258×10^{-4} m^2
$P(t)$ = power consumed by SMA (W)
$\Delta T(t)$ = Wire temperature above ambient (°C)
$V_s(t)$ = source voltage (V)
$R(t)$ = measured wire resistance(Ω)

Note that the parameter h is dependent on the characteristics of the air flow around the SMA wire, modelled thermodynamically as a horizontal cylinder. The literature shows much variation in the convection coefficient. For example, in [7], h = 70W/(m^2 °C) for an SMA wire while in [8] h = 150W/(m^2 °C). The discrepancy can often be attributed to different experimental conditions. To determine an appropriate h in the current work, the average value h = 110 W/(m^2 °C) is used as a starting point then iteratively adjusted to provide correspondence between the resistance-temperature and strain-temperature data from the experimental trials with estimated temperature data. Using this method, h = 131W/(m^2 °C).

In this paper, the experiments are designed to identify resistance behaviour during heating, so the model which is identified will be valid for the M→A transformation. Experimental procedures similar to the one presented here can be used also to model the resistance during cooling; however, the experiment design will need to be adjusted appropriately.

EXPERIMENTAL SETUP

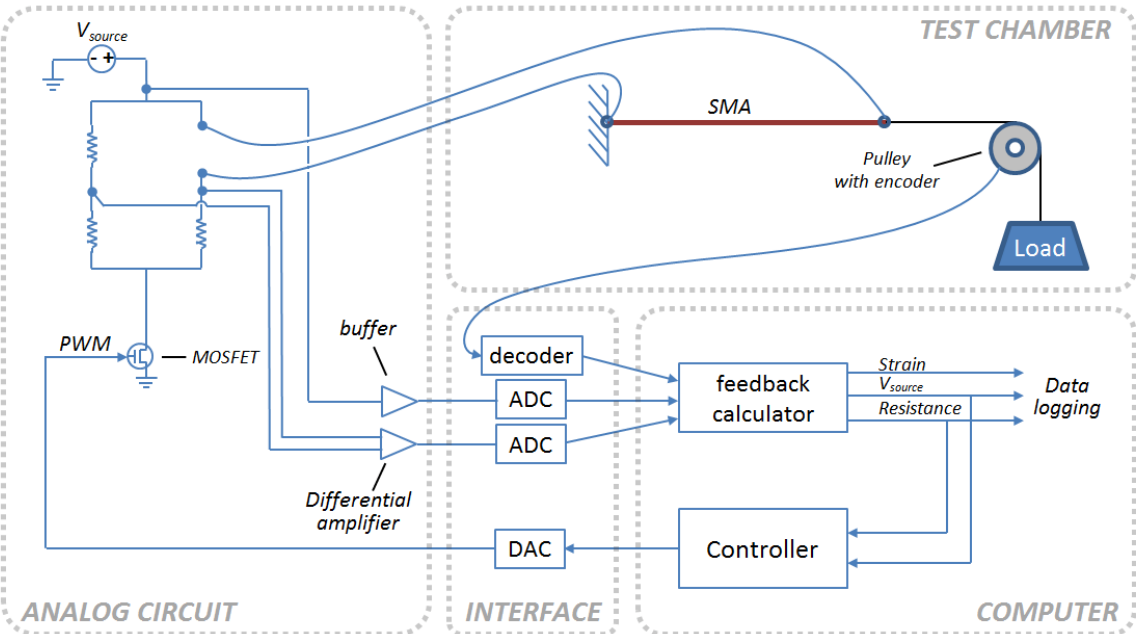

Figure 1 – Diagram of the complete experimental platform. The SMA wire is electrically connected as one of the loads in a Wheatstone bridge circuit. The encoder signal and voltage measurements from the bridge circuit are sent to the computer via an interface. The computer uses these signals to calculate the wire strain, resistance and source voltage. A controller may be implemented to control the duty cycle of a PWM waveform that modulates the energy flow into the SMA.

A complete diagram of the experimental setup is shown in **Fehler! Verweisquelle konnte nicht gefunden werden.**. On the experimental testing jig, a 283mm long SMA actuator wire is held horizontally with one end fixed and the other end fastened to a high strength wire connected to a weight via a pulley mechanism. Joule heating of the SMA wire is accomplished by connecting the two ends of the SMA wire to a power supply (in this case, a 12V DC power supply). The energy flow into the SMA wire is controlled by modulating the supply current with a PWM waveform using a MOSFET. Finally, the entire testing jig is contained within a closed chamber so that the operating temperature (measured using a thermocouple placed 8cm away from the SMA wire) can be accurately controlled to within ±2.5°C. The chamber also prevents outside factors from affecting the temperature and convection characteristics of air inside the chamber.

An H5S-1024 shaft encoder is used to measure the pulley's rotation, and thus indirectly the SMA wire strain, while the resistance of the SMA wire is calculated using voltage measurements from the Wheatstone bridge circuit. The Wheatstone bridge is advantageous to four-point resistance sensing because the bridge circuit allows even the smallest resistance *variations* to be precisely measured. Given limited space, the operation of the bridge circuit is omitted here but can be found in any electrical engineering textbook. The encoder signal and voltage measurements from the bridge circuit are sent via the Quanser MultiQ PCI data acquisition interface to a computer, which then calculates the wire contraction, resistance and source voltage using the measured data. The resistance and source voltage signals could be used as feedback for a closed-loop controller implemented on the computer to control the duty cycle of the PWM signal that modulates the current flow through the SMA wire.

TESTING PROCEDURE

The effect of the SMA wire temperature on the wire resistance is identified by running a set of trials, each with a different PWM duty cycle, and all at the same stress. Given the form of Equation 4, it can be conjectured that during heating the wire temperature would reach different steady-state values above the ambient temperature for different input powers, so the resistance-temperature (R-T) data taken at steady-state from all trials can be used to generate an R-T curve. The effect of stress on the resistance behaviour is determined by changing the load attached to the wire and repeating the entire set of trials.

Since the experimental trials generate data only for temperatures above the ambient temperature, a low ambient temperature was used for these trials so that the model being identified is applicable to most real-world applications where the operating temperature is higher than that of these trials. Thus, the ambient temperature in the test chamber is maintained at -30°C during these trials, which is the minimum temperature that could be achieved in the test chamber.

As illustrated in Figure 2, each run is divided into two phases: the wire is heated by the supply current using a PWM duty cycle for the first 60 seconds of the trial (phase "A"), followed by 90 seconds of applying 0% duty cycle (phase "B") so that the wire is allowed to cool completely to the ambient temperature prior to the next trial run. For each set of trial runs with the same stress, the range of duty cycles for phase A is 1.5% to 17.5% in 1% increments then 17% to 35% in 0.5% increments. The order of these trial runs is randomized to prevent data skewing that may result from incomplete cooling in-between trials.

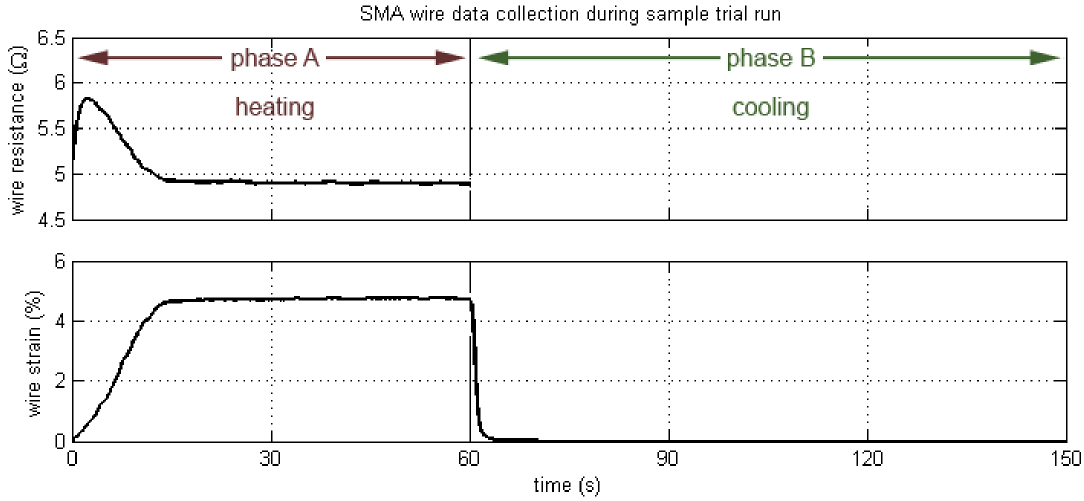

Figure 2 – Diagram of the two phases during a typical trial run. Phase A: the heating duty cycle is turned on to heat the wire for 60s. Phase B: the PWM duty cycle is set to 0% (turned off) to allow complete cooling.

The wire resistance, supply voltage and wire strain data are collected during phases A and are used to generate the heating resistance-temperature curve. The resistance and supply voltage can only be determined accurately during the "ON" part of each PWM cycle, which is why data during phase "B" are discarded.

Finally, the set of trial runs is repeated three more times at different loads, for a total of four trial sets. The load and corresponding stress for each set is summarized in **Table 2**. These stresses correspond to the typical range of operating stress levels of the SMA wire actuator.

Table 2 – Stresses for 4 trial sets

Set #	1	2	3	4
Load (g)	460	652	920	1375
Equiv. stress (MPa)	89	126	178	266

RESULTS, MODEL IDENTIFICATION AND SIMULATION

Results from a single set of trial-runs at the same stress

Experimental data shows that for most trials, the steady-state condition in phase A is reached within about 20 seconds. Since the air in the testing chamber is cooled using a fan and heat exchanger combination, fluctuations in the convection characteristics would likely be present, and therefore, at steady-state there are still small variations in the SMAs wire resistance. Thus, the steady-state resistance, source voltage, wire strain and estimated wire temperature values are taken by averaging the values in the last 10 seconds of phase A of each trial. Figure 3 shows the wire temperature estimated using data from a sample trial run, while Figure 4 shows the R-T and ε-T graphs obtained from a single set of trial runs at a constant stress of 178MPa.

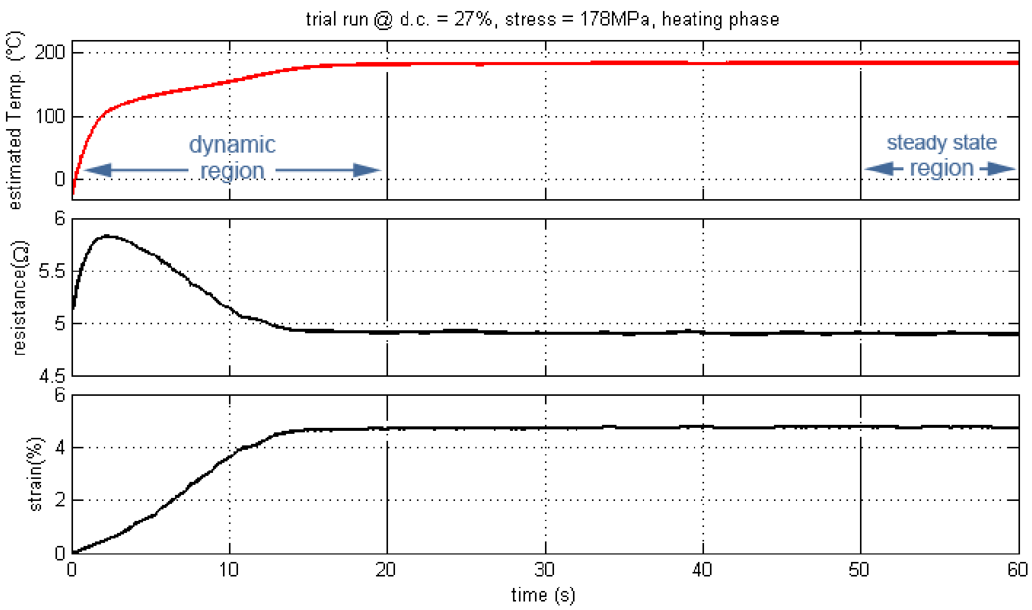

Figure 3 – Information from a sample trial run is used in Equation 4 to estimate the wire temperature (top plot). The initial transient region is the "dynamic" region and the last 10 seconds of the trial is the "steady state" region.

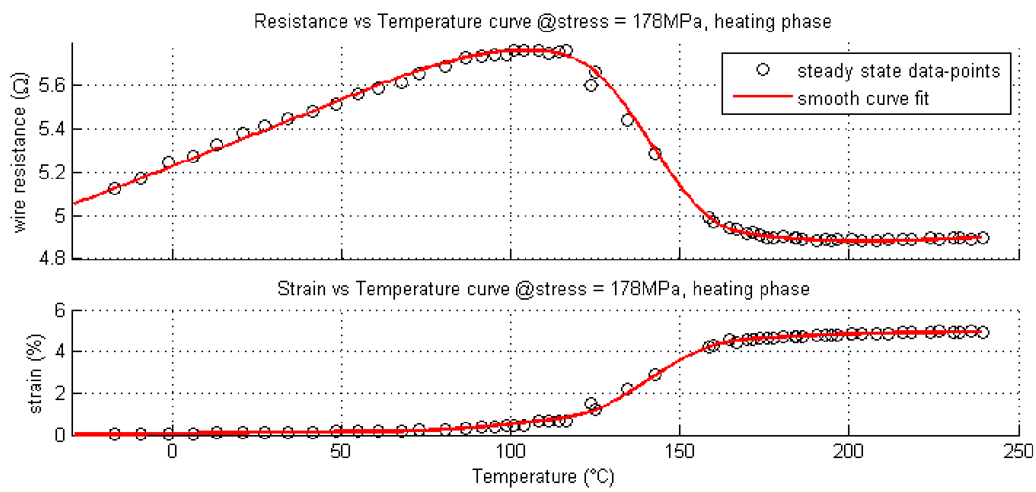

Figure 4 – The "steady-state" data-points from all trials in the 178MPa trial-set are used to generate curve-fits for *R*-*T* and ε-*T* functions.

<u>Resistance behaviour at multiple stress levels</u>

Using the steady-state data obtained from all four sets of experiments, a family of *R*-*T* curves at different stresses is obtained, shown in Figure 5-a.

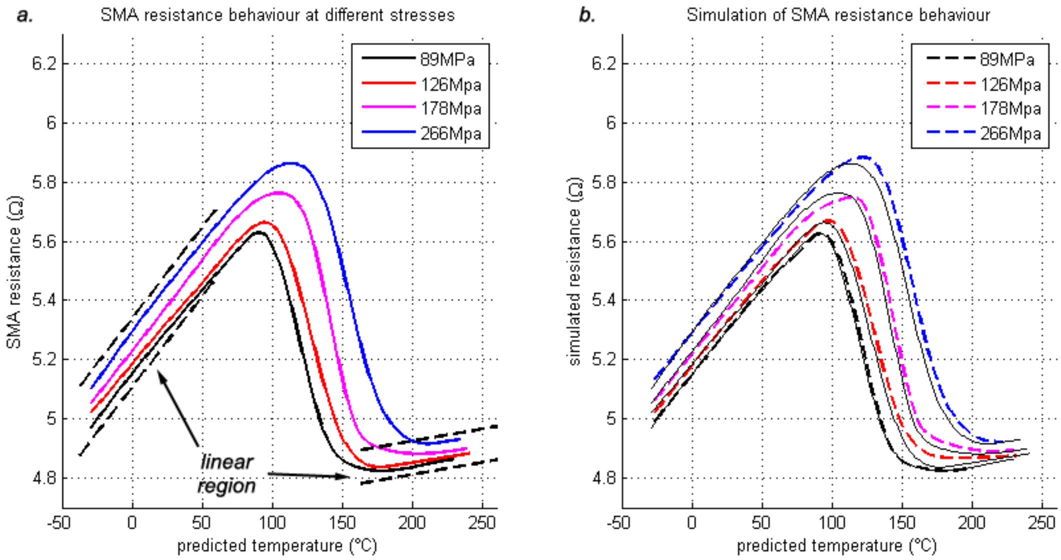

Figure 5 – a) family of R-T curves at different stresses, data obtained experimentally. The dashed lines indicate regions where linear resistance change is observed. b) simulated resistance response. Dotted lines are simulated curves while the thin solid lines are the experimental R-T curves identical to those in a).

It can be seen qualitatively that the experimental result corroborates with the work of [3] in that 1) resistance change vs. temperature change is approximately linear in both the M- and A- phases; 2) the resistance rate of change $\partial R/\partial T$ in M-phase is larger than in A-phase; 3) a resistance maximum and a minimum are observed between the linear regions; and 4) increasing wire stress causes an overall upshift in the measured resistances as well as the temperatures at which the resistance maximum and minimum occur.

By examining the resistance response in the linear M- and A-phase regions, the parameters in Equations 2 & 3 can be identified. They are presented in Table 3. Using these parameters, the SMA wire resistance is simulated for varying stresses and shown in Figure 5-b. In the simulation, the experimentally generated ε-T is used to infer the martensite phase fraction, and it can be seen that the simulation matches experimental data to a reasonable degree of accuracy. Furthermore, the simulated resistance maxima are close to their experimental counterparts, while noticeable discrepancies exist between the experimental and simulated resistance minima for the 126MPa and 178MPa stresses.

Table 3 – Parameter values

parameter	value	parameter	value
R_{0M}	5.08 Ω	R_{0A}	4.82 Ω
T_{0M}	0 °C	T_{0A}	225 °C
$\partial R_M/\partial T$	6.14 mΩ/°C	$\partial R_A/\partial T$	0.802 mΩ/°C
$\partial R_M/\partial \sigma$	0.821mΩ/MPa	$\partial R_A/\partial \sigma$	0.390 mΩ/MPa

It can be observed that there is a one-to-one-to-one correspondence between the wire's resistance, temperature and stress, which are most easily seen at temperatures below the onset of the A-phase. This means that given any two of the three quantities, the remaining quantity can be calculated using an inverse model. Moreover, for each stress level there is a unique resistance peak at a unique temperature, so if the resistance peak can be detected and identified accurately then both wire stress and temperature would be known instantly. This may allow controllers to be designed to use resistance feedback to gain knowledge of previously unknown temperature and stress information.

CONCLUSIONS AND RECOMMENDED FUTURE WORK

A mathematical model that describes the electrical resistance behaviour of a NiTi wire actuator during heating is formulated and its parameters are identified using experimental data. The model is a modified version of a resistivity model from [3], where the inputs are the martensite phase fraction, wire temperature, and wire stress. Using this model and knowledge of the wire's transformation temperatures, it is possible to predict how the resistance of the wire will change during heating, given information on the wire's temperature and stress. It is recommended that similar experiments should be performed that focus on characterizing the resistance behaviour during cooling. It is also recommended that a suitable model for the phase fraction of SMAs should be used to simulate the phase fraction so the entire SMA system can be simulated and not just the resistance.

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