UNIVERSITÀ DI BOLOGNA

School of Engineering

Master Degree in Automation Engineering

Modeling and Simulation of Mechatronic Systems

SMA Modeling and Control

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Abstract

In this paper, we develop a complete mathematical model of a shape memory alloy (SMA) wire, define a control algorithm and compare its behaviour with the physical plant. The operation of the SMA actuator involves several physical phenomenon that we model with a modular approach. This steps are: heat transfer, phase transformation, stress-strain variation and mechanical model. Hysteresis modeling is one of the main part of the experiment, that model the variation of the internal mechanical structure. The mechanical model is implemented using Simscape, where the ideal SMA force is introduced in a mass-spring-damper system. Once the model is implemented, its parameters are fitted through an optimization tool provided by Simulink. Finally the model is used to develop a position control system. The results obtained from the simulation are in agreement with the experimental data obtained from the real workbench.

Contents

Introduction

Shape Memory Alloys (SMA) is an alloy that can be deformed when cold but if heated it returns to its original shape, the one before the deformation. The crystal structure of the material is changed due to the injection of energy, moving between two phases: Austenite and Martensite. Austenite is the state when the SMA is at high temperature and it is characterized by a cubic crystal structure and more elastic behavior. Martensite is the phase at low temperature, where the material shows a tetragonal crystal structure and more plastic behavior. Heating the wire, SMA is deformed and enters in austenite while if a load is applied to the material in the martensite phase it is possible to detwin it.

In our experiment SMA wire is used as an actuator that moves a mass-spring-dumper system. The proposed model considers the SMA wire to behave as an active spring that changes its equilibrium length and its stiffness to move the passive component of the system. SMA wire is fed with a controlled input voltage that generates a current inside the wire that heats it by the Joule effect. In this way, the Austenite state is reached. Switching off the power supply, the wire starts to cool down slowly returning to the Martensite phase where the mechanical load can stretch back the material to its previous length.

The procedure adopted to analyze and control the electro-mechanical system, composed of the SMA connected in series with a mass-spring-damper system, was to build a model that would emulate the electro-thermal and mechanical behaviors of the SMA itself, and which would then model the mechanical behavior of the system. Once the model of the entire system has been obtained, considering as input to the model the duty cycle required by the microcontroller, and as output the position measured by the position sensor fixed to the reference system of the slide mass, a first estimate of the model parameters was carried out. The model parameters were then refined by collecting data from the system itself. Once we were sure that we had a reliable model, a position control was developed. Finally, the performance of the control was tested again on the real system.

Chapter 1

Heat Transfer and Temperature

In this section, the temperature profile for the SMA wire is generated. The system gains heat energy from electrical power and loses part of it to the environment. The balance of the heat energy governs the temperature of the SMA wire. Lumped parameter analysis is the method used to model this behavior. The internal resistance of the wire to heat conduction is considered negligible compared to the convective heat transfer with the environment. Also ϵ can be neglected This assumption is valid for heat transfer in thin metal samples. The heat transfer equation is the following:

$$\begin{aligned} \rho c \frac{\pi d_0^2 L_0}{4} \frac{dT}{dt} &= vi - \pi d_0 L_0 \left(1 + \frac{\epsilon}{2}\right) h(T - T_{\text{amb}}) \\ &\approx vi - \pi d_0 L_0 h(T - T_{\text{amb}}). \end{aligned}$$

where:

ρ	mass density of SMA wire [kgm^{-3}];
c	specific heat of SMA wire [$\mathrm{Jkg}^{-1}{}^{\circ}\mathrm{C}^{-1}$];
L_0	undeformed SMA wire length in 100% A state [m];
d_0	cross-sectional diameter of undeformed SMA wire [m];
ϵ_0	strain caused by pretension load in 100% A state;
ϵ_r	strain caused by A–M phase transformation;
ϵ	$= \epsilon_0 + \epsilon_r$, total strain;
h	convection heat transfer coefficient [$\mathrm{Wm}^{-2}{}^{\circ}\mathrm{C}^{-1}$];
v	voltage across SMA wire [V];
i	current through SMA wire [A];
T_{amb}	ambient temperature [$^{\circ}$C].

In order to get the temperature profile, the heat transfer obtained is integrated. To perform the hysteresis transformation but also the heat transfer too, temperature is needed.

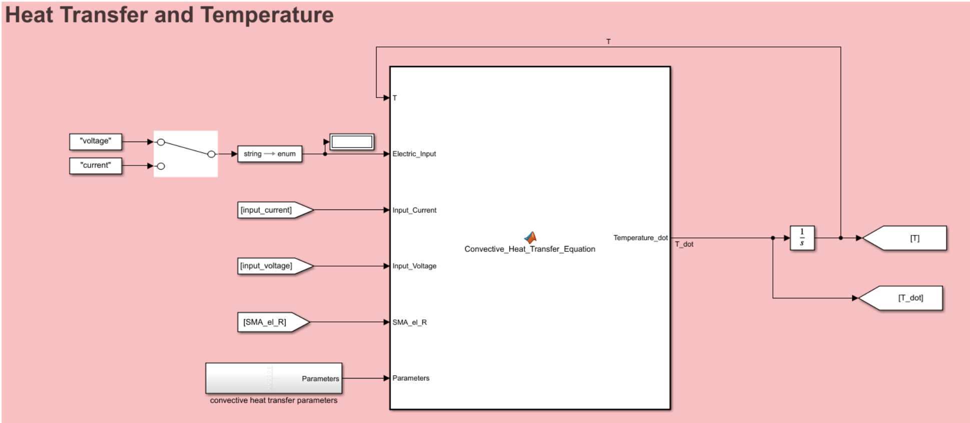


Figure 1.1: Heat trasfer and temperature Simulink block

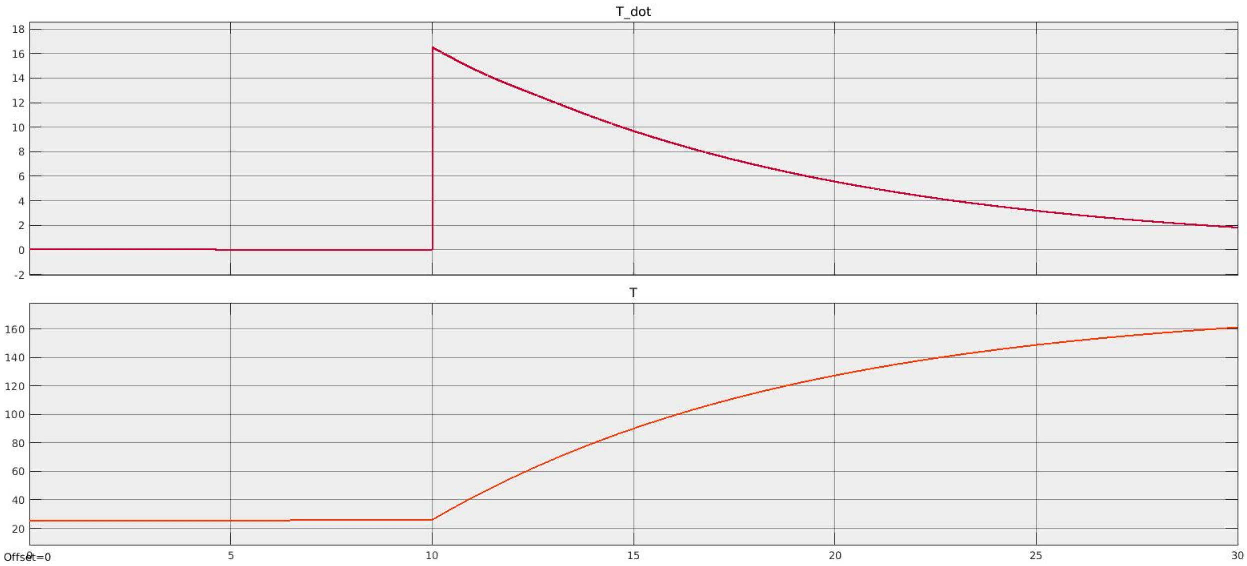


Figure 1.2: Heat trasfer and temperature profile in response to a step

Chapter 2

Hysteresis Model

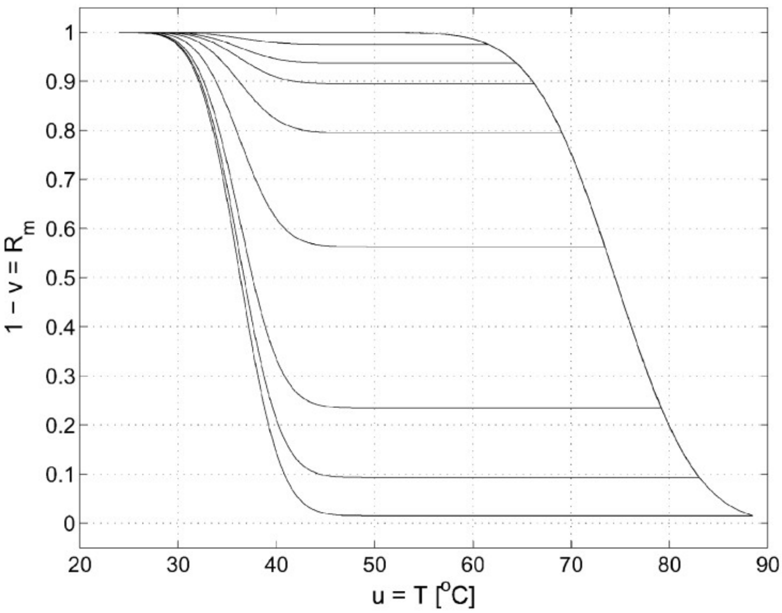


Figure 2.1: fig1:Martensite fraction-temperature hysteresis

The A-M transformation, which is governed by the relationship between R_m and T, is **hysteric** and its typical shape is illustrated in fig2.1. The differential hysteresis model proposed by Likhachev, which implements a Duhem differential model, is studied and applied in this experiment. A complete A-M transformation is called a major hysteresis loop while the incomplete transformation is referred to as a minor loop. Anyway, both show the same shape.

$$\begin{cases} \frac{dR_m}{dT} = \begin{cases} \frac{h_-(T)+R_m-1}{h_+(T)-h_-(T)} \, g_+(T), & \dot{T} \geq 0 \\ \frac{h_+(T)+R_m-1}{h_-(T)-h_+(T)} \, g_-(T), & \dot{T} < 0 \end{cases} \\ R_m(0) = 1 \end{cases}$$

where for the slope function a Gaussian probability density functions is proposed

$$g_{i_{+/-}}(u) = \frac{n_{i_{+/-}}}{\sigma_{+/-}\sqrt{2\pi}} \exp\left(-\frac{(u-\mu_{+/-})^2}{2\sigma_{+/-}^2}\right)$$

The scaling constants $n_{i_{+-}}$ are the values that allow the model to simulate also the minor cycle:

$$n_{1_-}(u,v) = \frac{b}{a+b} = \frac{h_{1_-}(u) - h_+(u)}{h_-(u) - h_+(u)} = \frac{v - h_+(u)}{h_-(u) - h_+(u)}$$

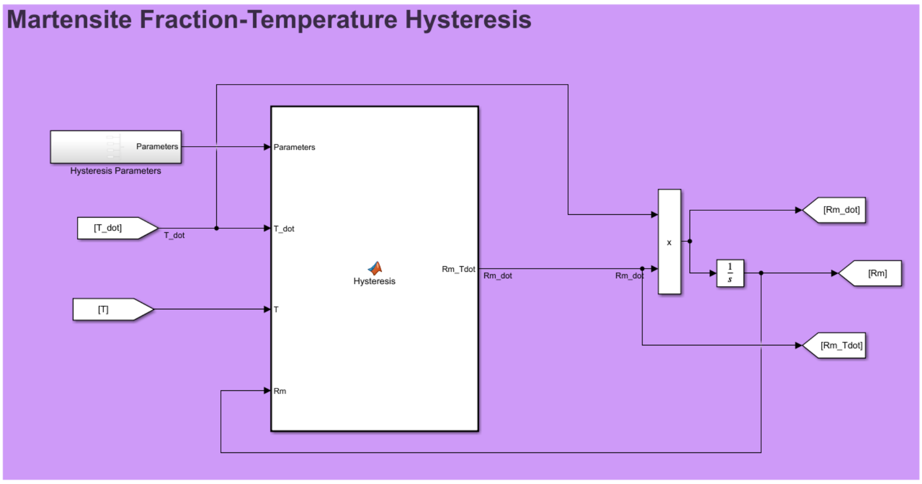


Figure 2.2: Rm Hysteresis Simulink block

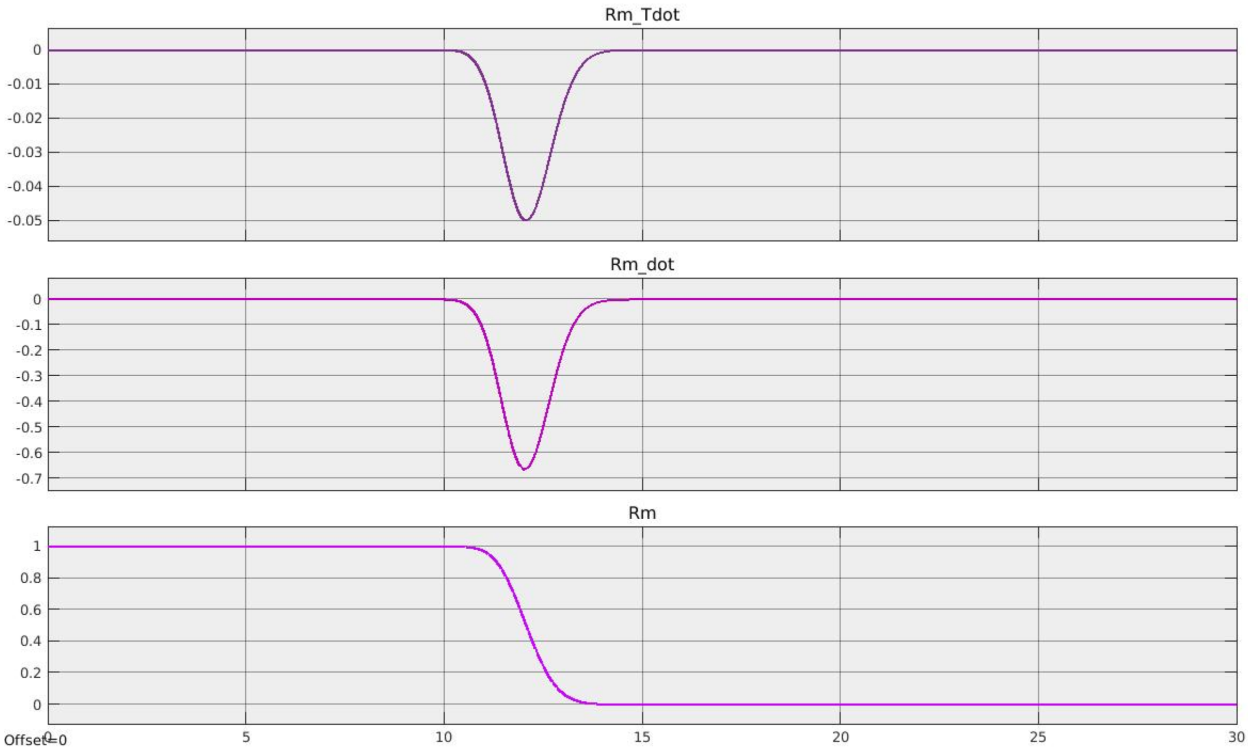


Figure 2.3: A-M transformation in response to a step

Chapter 3

SMA Mechanical Force

In this section, starting from the obtained result on the temperature and R_m hysteresis, the SMA force is modeled.
Firstly the strain is approximated to change quadratically, depending on the value of R_m :

$$\epsilon = \epsilon_0 + k_1 R_m + k_2 R_m^2$$

where the pretension load stress ϵ_0 can be neglected.

The mechanical force generated by the tension of the wire SMA can be modeled as an elastic force, where both the stiffness and the equilibrium length of the wire vary, according to its state variable R_m.
The equilibrium length is strictly dependent from the value of R_m and it has a different value depending to the transformation occurring in the wire:

$$L_{eq} = L_{init} - (1 + \epsilon) l_0$$

$$L_0 = (1 + \epsilon) l_0$$

with l_0 being the undeformed SMA wire length in A state and L_{init} the length of the wire in M state.
The stiffness coefficient of the spring is affected by the change of phase, since the material change its behaviour from elastic to plastic. For $0 < R_m < 1$

$$K_{wire} = \frac{(E_m R_m + (1 - R_m) E_a) A_{wire}}{L_0}$$

In conclusion the equation for the SMA force is:

$$SMA_{force} = K_{wire}(Delta_L - L_{eq})$$

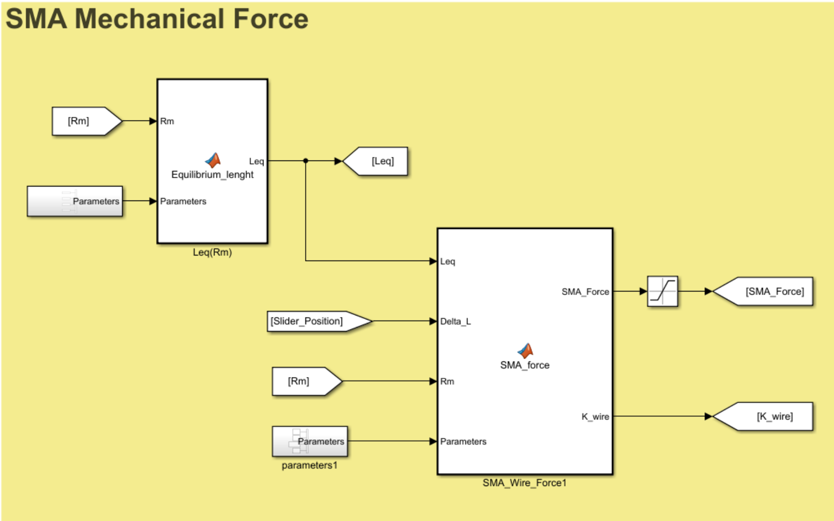


Figure 3.1: SMA mechanical force Simulink block

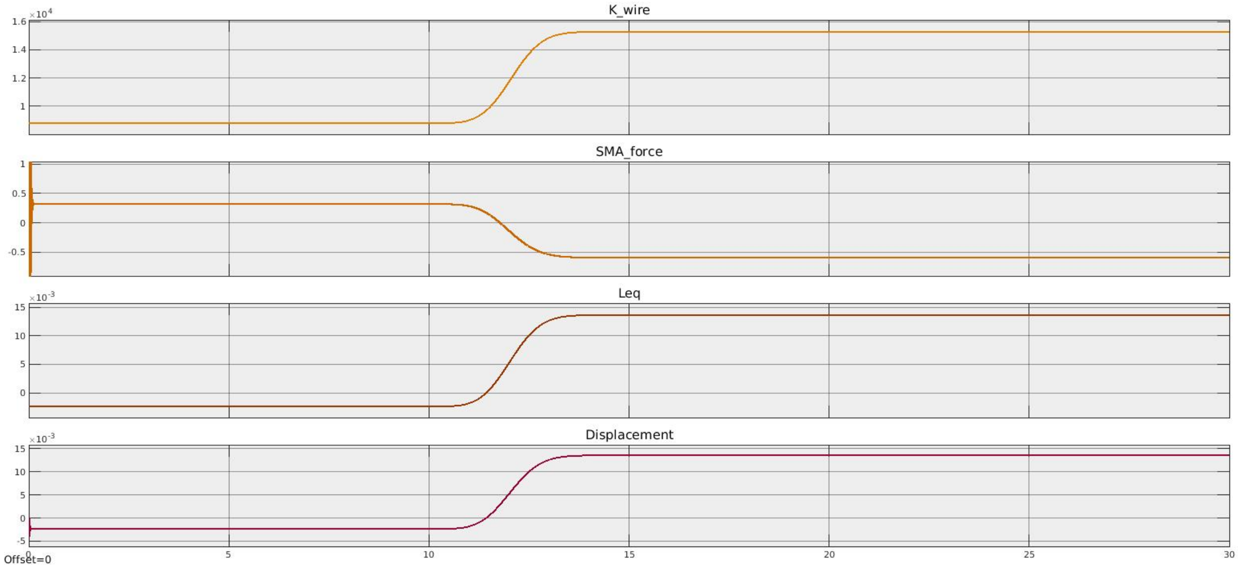


Figure 3.2: K_{wire}, SMA_{force}, L_{eq}, Displacement profile in response to a step

Chapter 4

Mechanical Model

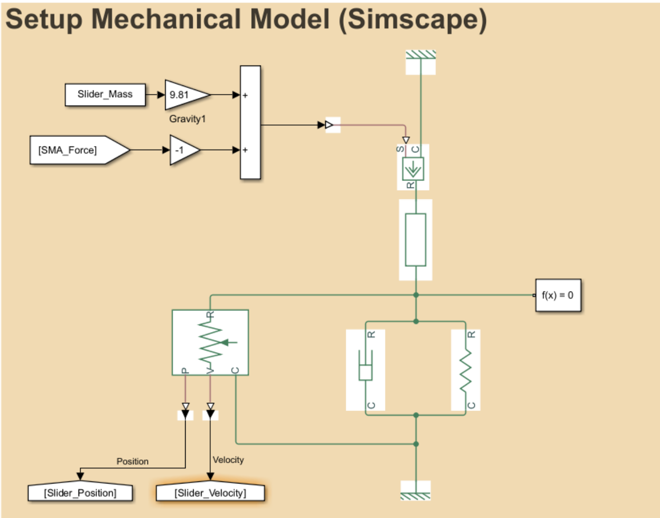


Figure 4.1: Mechanical Model Simulink block

The SMA force is injected in a mass-spring-dumper system. Having modeled an ideal force as a spring force, this is sufficient to simulate a SMA wire as a spring. The equation that govern the system is:

$$m\ddot{x} = SMA_{force} - mg - c\dot{x} - kx$$

Both the SMA wire tension and the gravitational force acting on the system are added together with their proper sign, and the resultant force is applied then to the mass-spring-damper system modelled in Simscape, through an ideal force source.
To measure the position of the slider, a translational motion sensor is placed in parallel to the spring and damper, similarly as it is in the real system.

Chapter 5

Control

Parameters estimation

The next stage is to set the right value of the parameters.
Considering the model in open-loop with input signal the duty cycle of the PWM signal generated by the microcontroller, and as output the computed slider position. Experimental data were saved feeding the system with a square and a triangular input signal of several amplitudes.

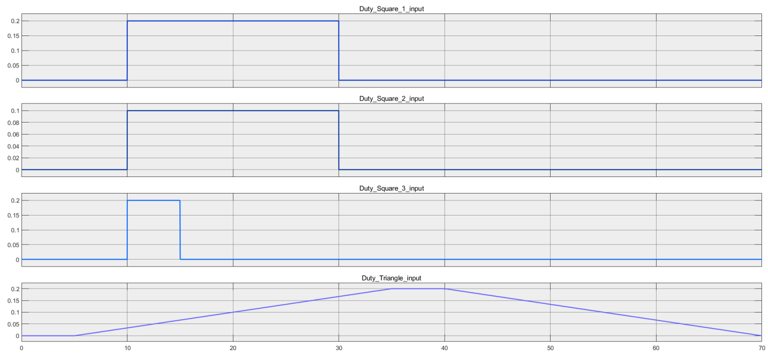


Figure 5.1: Duty input used to refine the parameters

Then the parameters of the model were changed to make its behaviour matching the real one. Starting from that result, the model parameters were fitted through a Simulink optimization toolbox. Not all the parameters were allowed to change and they were optimized ranging just in a reasonable interval of values, interval that depends on physical and known condition. For example, the mass of the slider is assumed to be known with a value of 0.035g but allow to change in a range of ±20% due to error measurement.

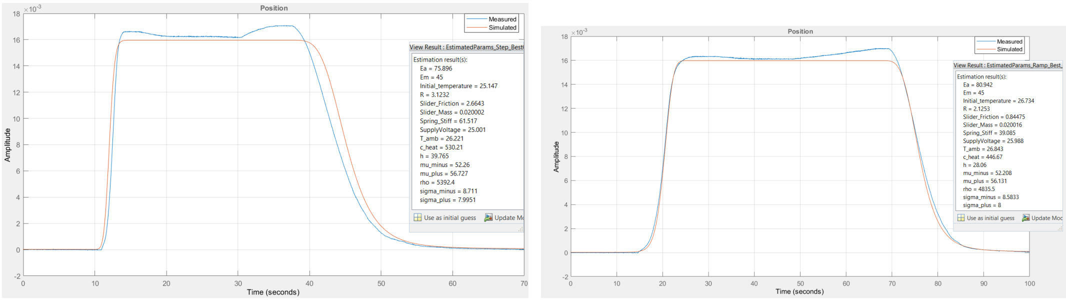


Figure 5.2: Comparison between real displacement and model displacement in response to a Square input (left) and to a Triangular input(right) and relative parameters

Having performed more than one experiment on the real platform and having saved that results, allowed us to use a set of data as training set and others for testing. The obtained parameters for the square and triangular signal show different values, because the system adapt them to follow the reference signal as precisely as it can, thus, a weighted average of the two results was performed (giving more importance to the step instead of the triangular input 0.8-0.2). This makes the system not adapted on a specific experiment but to follow the real system behaviour, with the aim to avoid over-fitting. In this way system response is more general to different input.

PID tuning

To develop a position control system, a PID controller was added in the model, controlled by a feedback from the slider position signal. After tuning the proportional, integral and derivative value of the PID, the controller was ready to be tested also on the workbench.
Random gaussian noise was also added to the feedback signal to simulate a noise affecting the sensor. Results and comments are discussed in the next paragraph.

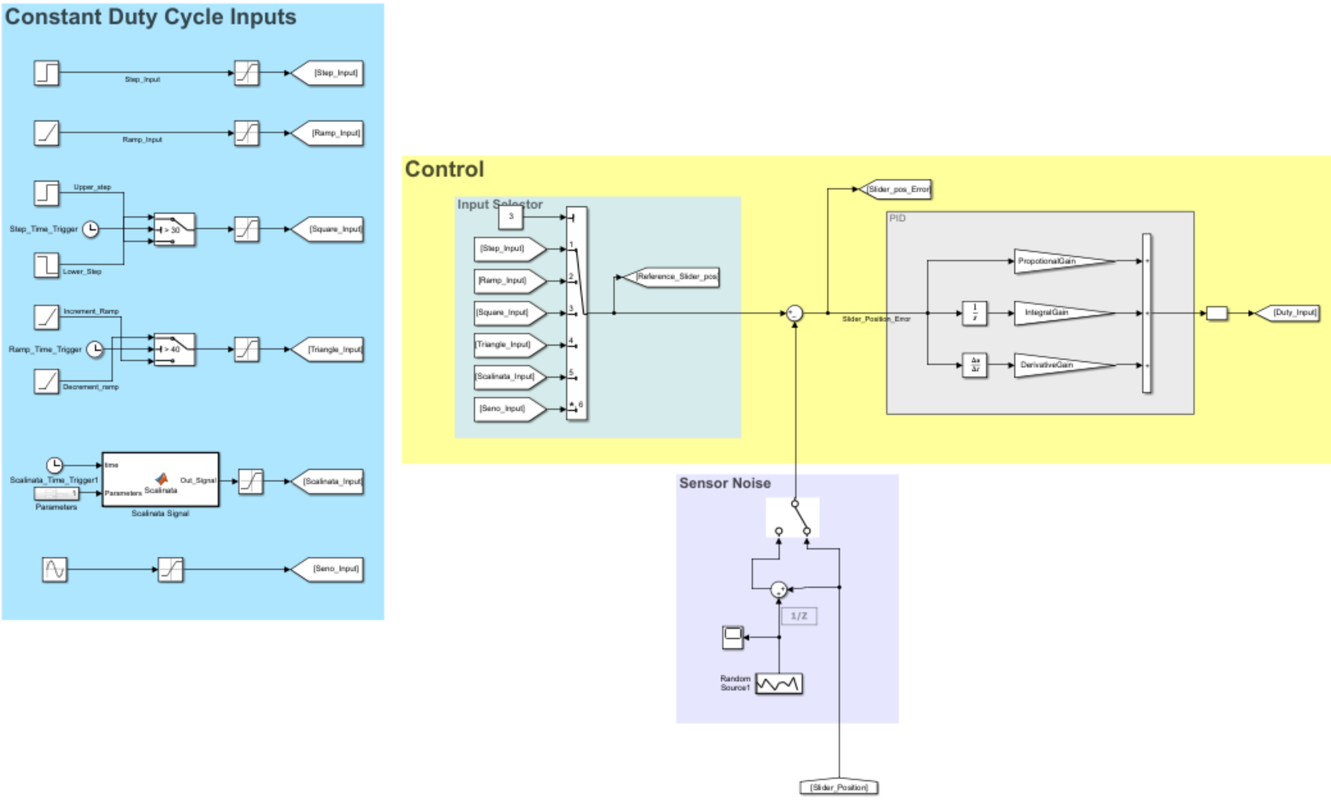


Figure 5.3: Control Simulink block

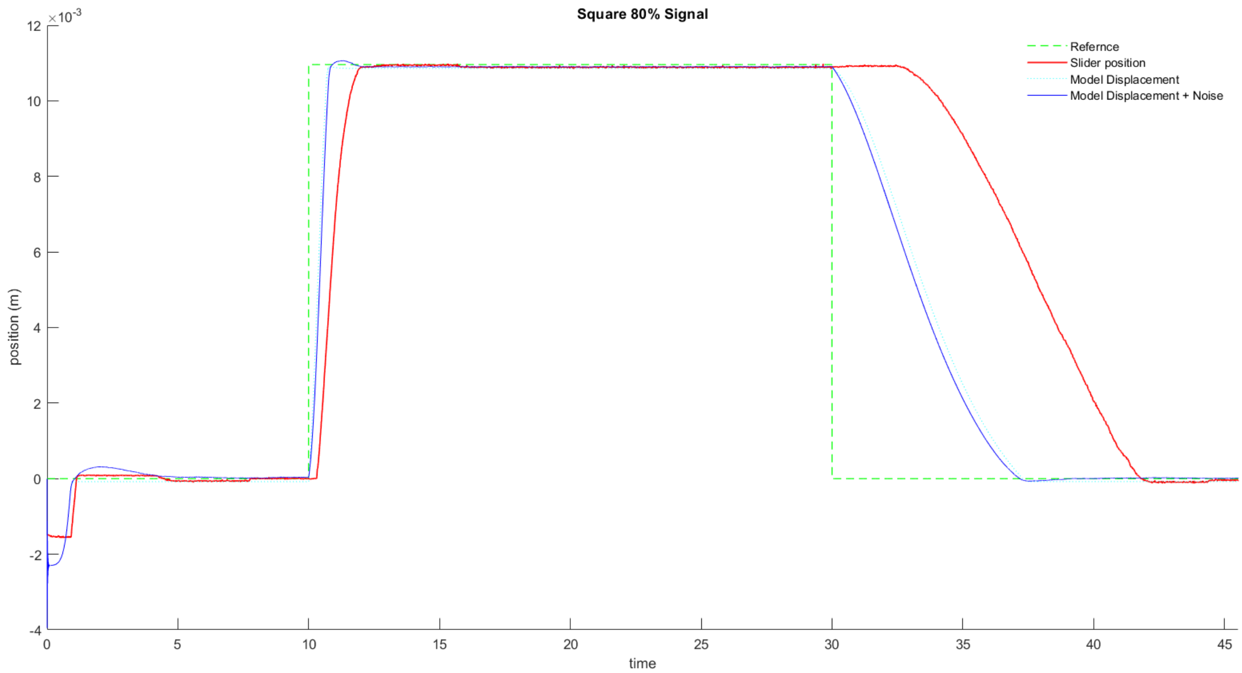
Final Results and Conclusions

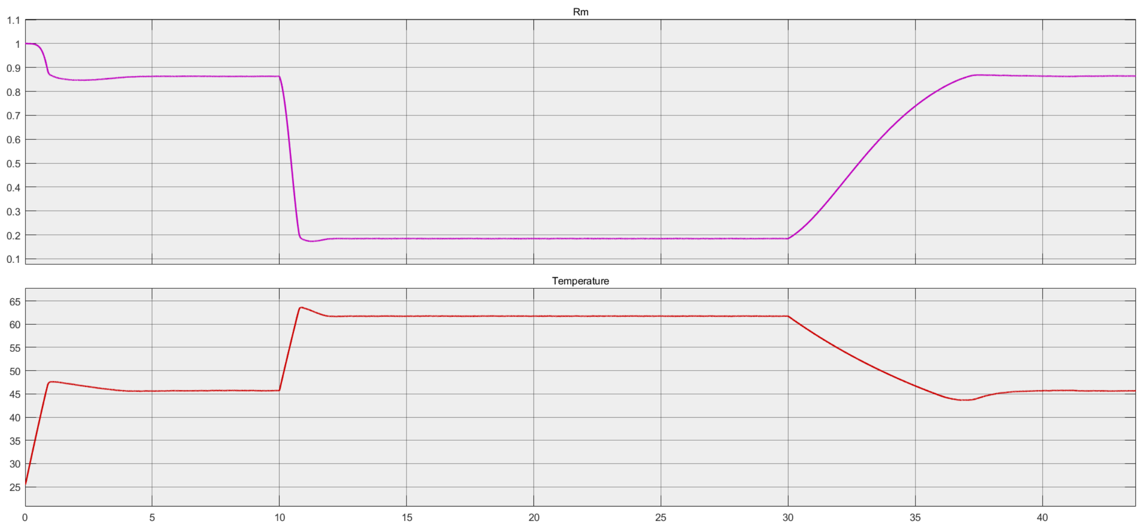
In this final section graphs with comparison of the real and simulated results are plotted.
The graphs shown show the analysis of both the model and the workbench controlled by the same PID. Two problems were encountered in obtaining valid measurements on the workbench:

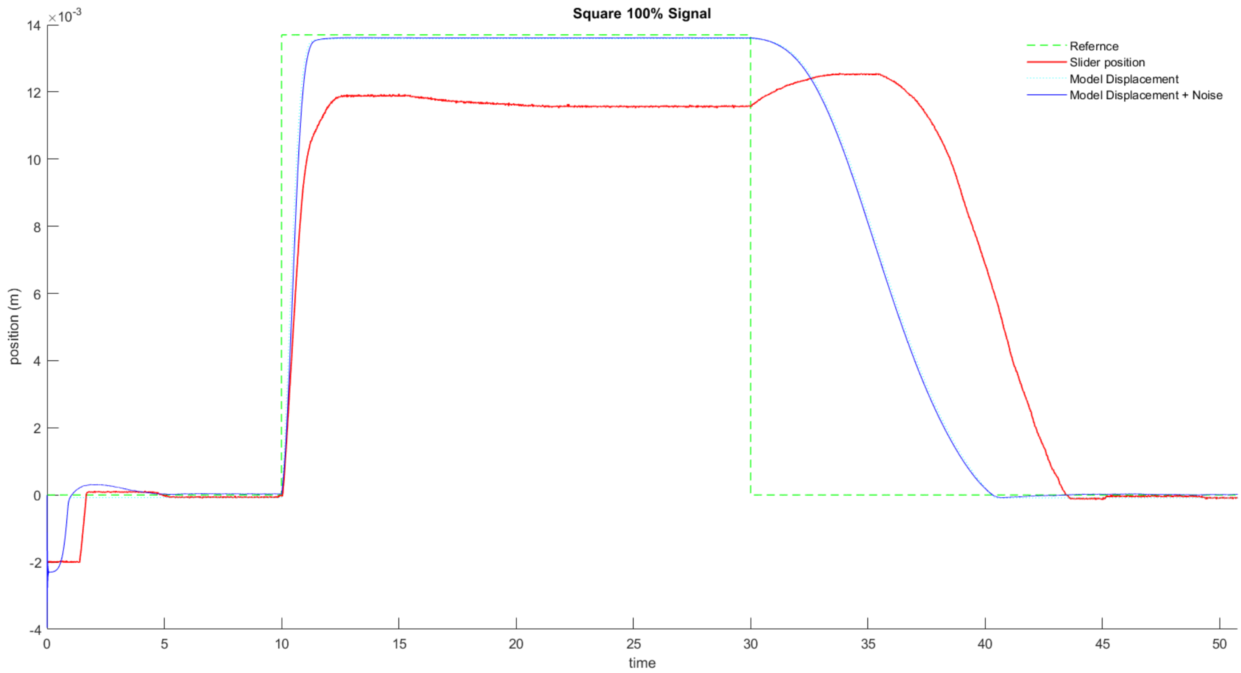
- the length in which the SMA is fully stretched was variable from day to day. Therefore, a stretch length was chosen which did not always coincide with the actual one. This implies that in the graphs the 0 of the position of the slider does not coincide with the length of complete stretching and the PID, therefore, supplies a small duty cycle at the start to keep the length of the SMA coinciding with the length to reach the 0 of the imposed reference system. Therefore, also from the graph of R_m, it can be seen that the SMA never reaches a complete state of Martensite.

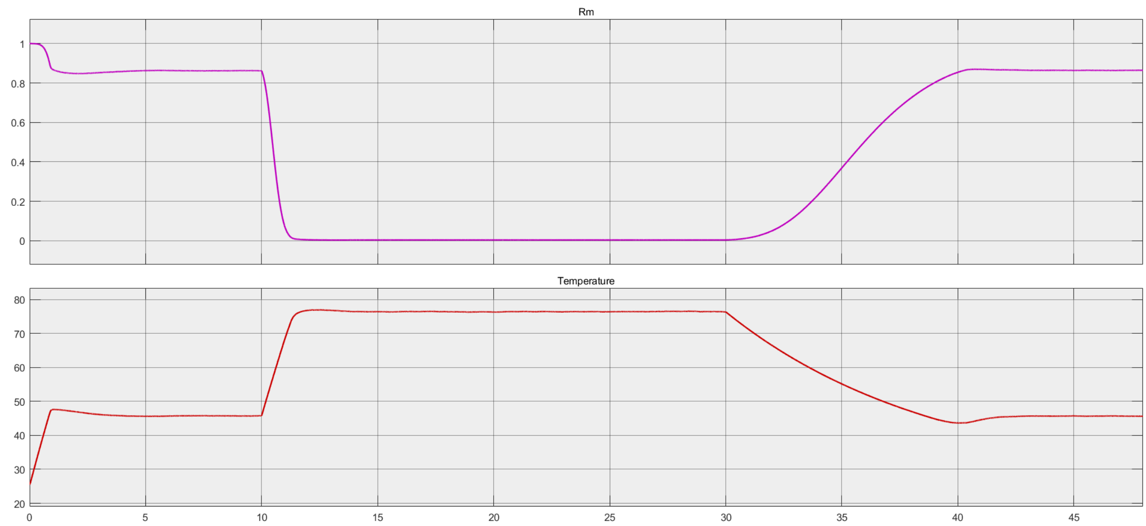
- The maximum shortening length is a length that varies a lot, depending on the conditions of the SMA. Therefore very often also the maximum effective shortening length did not coincide with that measured in the past.
  Furthermore, the SMA in complete shortening presents strongly non-linear and difficult to model behaviors. As if shortened to the maximum, therefore in full Austenite, once that the power supply is removed, and therefore it is in the cooling phase, the SMA presents a small further shortening before starting to stretch (this phenomenon can be observed in the graph of the square signal at 100%, in the time interval between 30-45 sec). Therefore, to avoid this behavior, most of the charts analyzed were made by reaching the SMA at a shortening to 80% of the maximum.

Square Reference



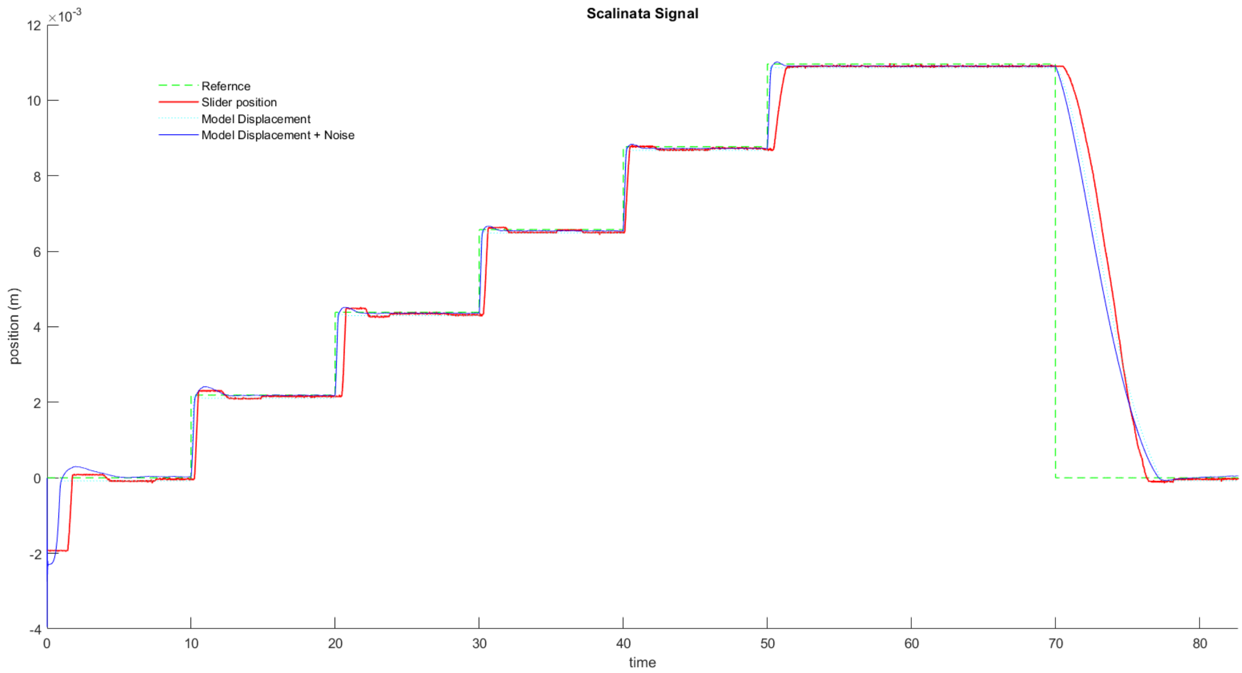


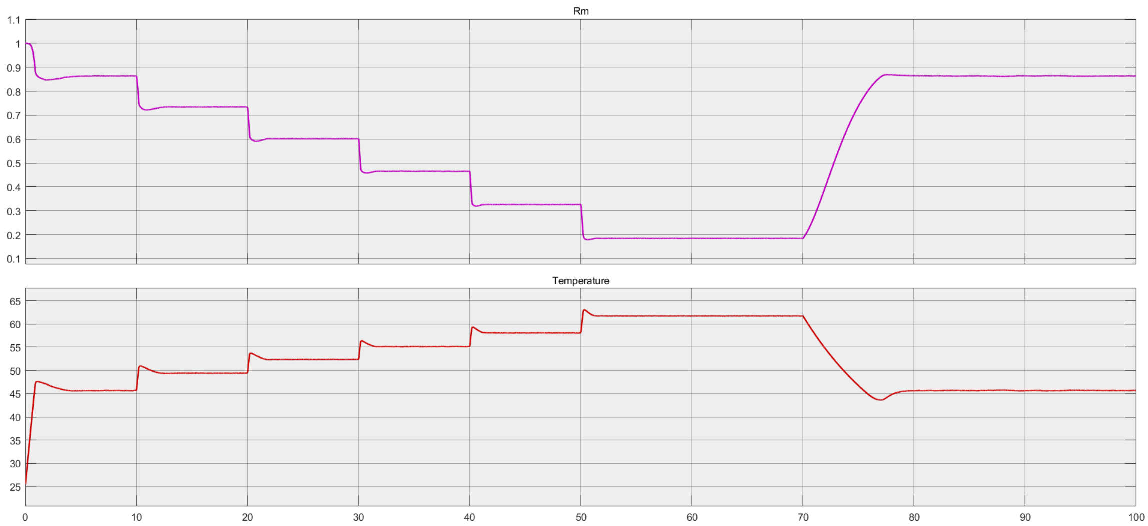




The performance of both the model and the real system controlled with a double step reference can be acceptable. As for the heating step the SMA manages to have dynamics as fast as possible and the PID manages to maintain the value of the reference position with a margin of error that is negligible. Regarding the cooling phase, the SMA has a slightly slower behavior. And in the 100% displacement graph, therefore by forcing the SMA wire to perform its maximum shortening, it is possible to notice a behavior that is not modeled in the cooling phase as the SMA, before assuming a plastic behavior, presents a further contraction that exceeds the maximum reached from its warm-up.

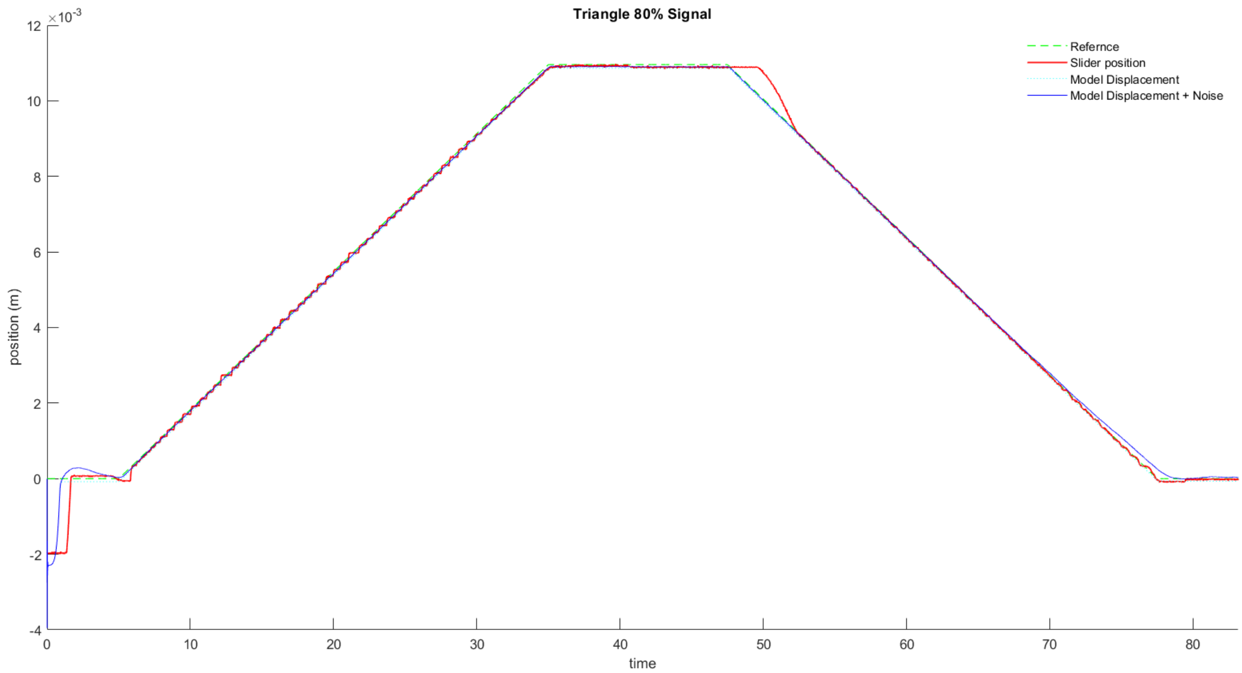
Scalinata Reference

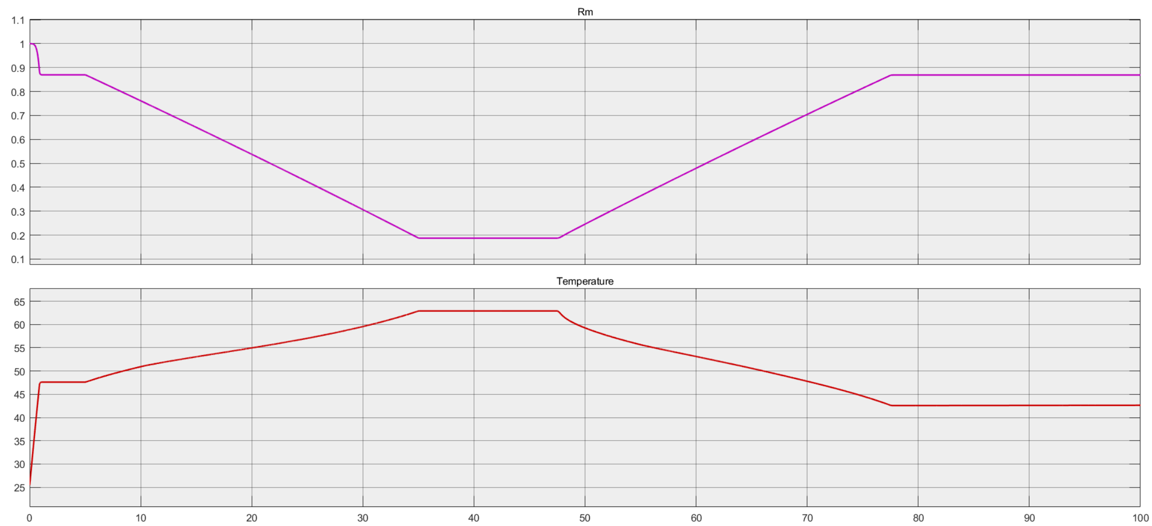




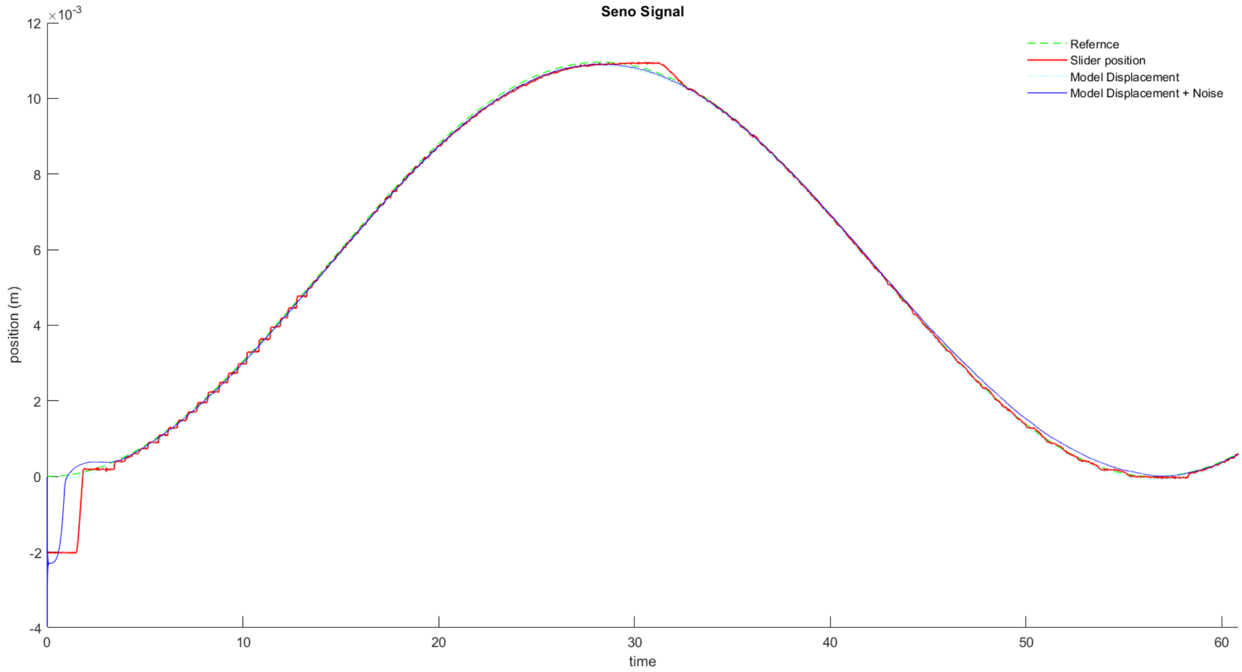
Using this reference signal, we tested the minor cycle of the change hysteresis of the crystal lattice. As can be seen from the graphs above, the action of the PID controller is able to stop the shortening of the SMA wire in particular required position values, whatever it is between its minimum and maximum length, with almost maximum accuracy. Performing a small period of adjustment which includes a small overshoot of no more than 5% of the displacement.

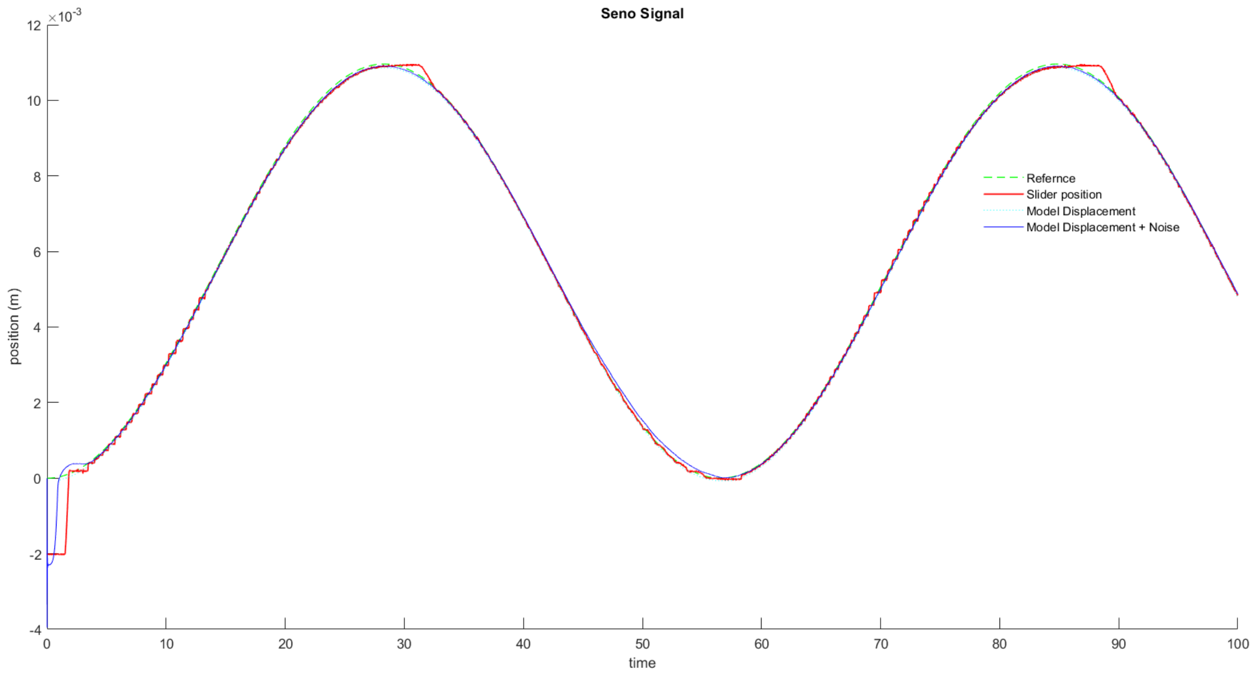
Trapezoidal Reference

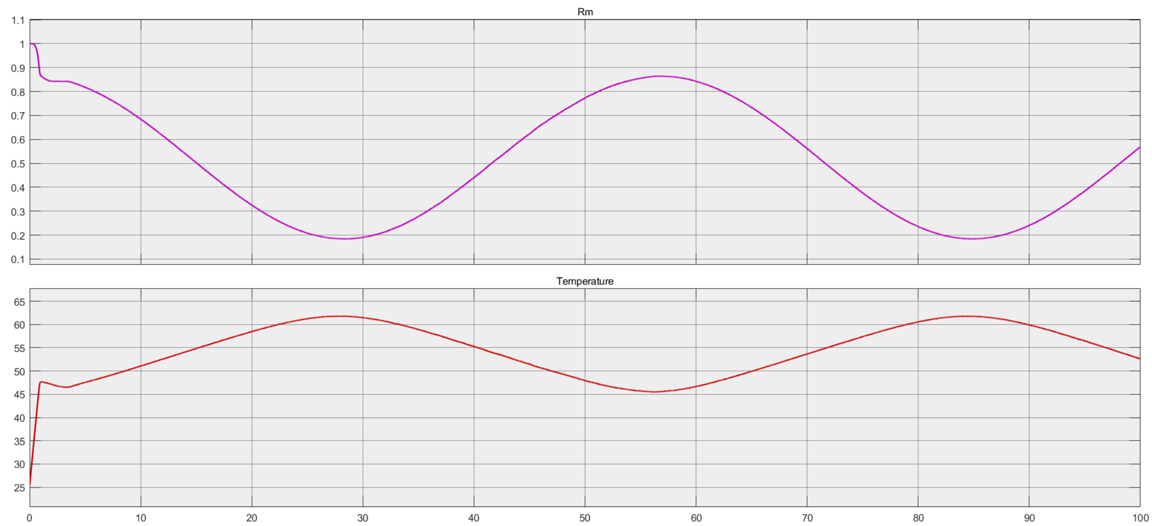




Seno Reference







By looking at the last two reference signal responses, representing the controlled system performance following a trapezoidal and a sine wave, it can be noticed that the system can be controlled also in the cooling dynamics, so in the dynamics that lead the system from an Austenite state back to a Martensite one. However, this is only possible when the reference signal dynamic is slower than the system dynamic.

Bibliography

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[2] Fathi H. Ghorbel Sushant M. Dutta. *Differential Hysteresis Modeling of a Shape Memory Alloy Wire Actuator*. IEEE/ASME Transactions on Mechatronics. 10(2): 189-97, 2005 Apr.