Advanced EONIA Curve Calibration

Avoiding unwanted shape oscillation in EUR overnight curve

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Empirical Evidence in EUR Market

- Bootstrapping quality is usually measured by the smoothness of forward rates
- For even the best interpolation scheme to be effective, market jumps must be removed before calibration, then added back at the end of the process
- The most relevant rate jump is related to the *Turn-Of-Year* (TOY)
- A rate jump is usually related to increased liquidity demand because of end-of-month or end-of-year requirements.

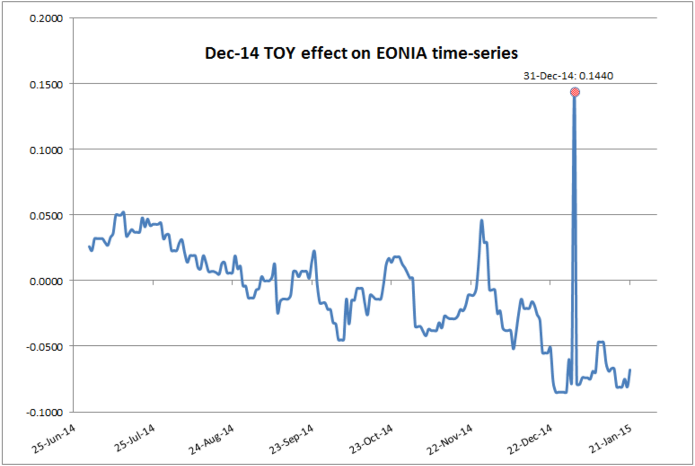

Figure: December 2014 EONIA Index turn-of-year

The U.S. market case

However, previous definition does not work for the negative jumps observed for the USD Fed Funds rate

Figure: Fed Funds fixing, source: Bloomberg Terminal.

Jump estimation methodology

In order to estimate jump sizes, Ametrano-Mazzocchi[1] propose a 4-step approach inspired by Burghardt [1]:

1. Build an overnight curve using a linear/flat interpolation, including all liquid market instruments
2. The first segment out of line with the preceding and following ones can be put back in line dumping the difference into a jump effect. For positive sizes:

$$[F^{original}(t_1, t_2) - F^{interp}(t_1, t_2)] \cdot \tau(t_1, t_2) = J^{Size} * \tau^J$$

3. Handle the jump as exogenous multiplicative coefficient for all discount factors after the jump date
4. Iterate ad libitum 2 and 3 for subsequent jump dates.

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Negative Sizes

Reviewed formula for negative sizes

The preceding formula is good for estimating positive jumps, but it needs a fix for negative jumps:

$$[F^{Interp}(t_1,t_2) - F^{Original}(t_1,t_2)] \cdot \tau(t_1,t_2) = J^{Size} * \tau^J$$

$$J^{Size} = [F^{Interp}(t_1,t_2) - F^{Original}(t_1,t_2)] \cdot \frac{\tau(t_1,t_2)}{\tau^J}$$

EONIA and USD Overnight Curves Including Jumps

The resulting EONIA and USD overnight curves including jumps estimated through the preceding approach are shown in Figure 3 and 4

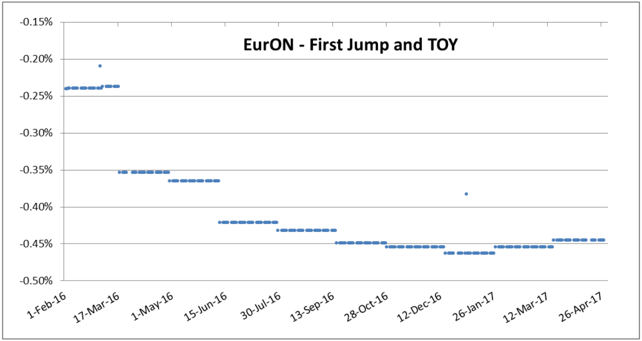

Figure: Eonia curve short end with estimated positive jumps.

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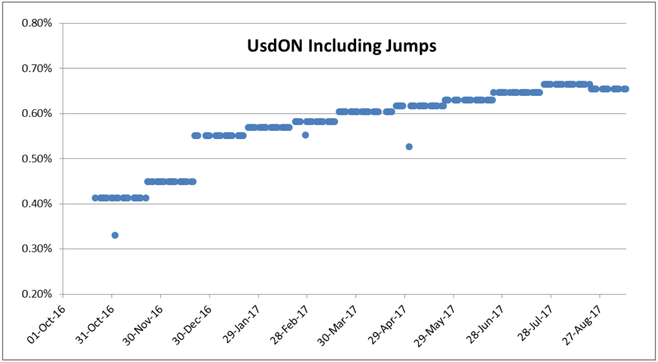

Figure: USDON curve short end with estimated negative jumps.

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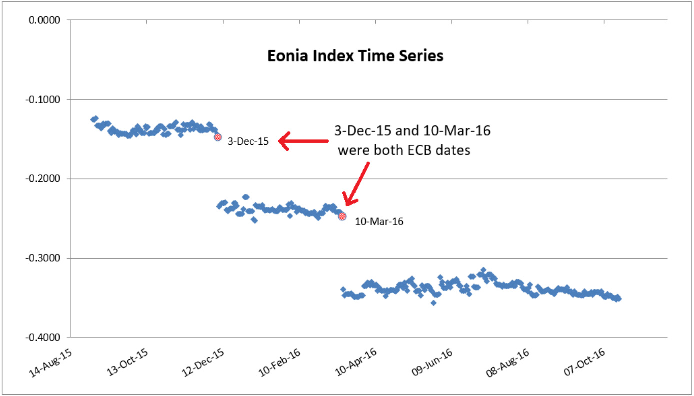

Figure: Piece-wise constant behaviour shown by EONIA fixings

Spot and Forward OIS Overlap

When mixing spot starting OIS (*Overnight Indexed Swaps*) and forward starting ECB OIS (*European Central Bank OIS*), the ECB OIS are preferred because of their greater liquidity.

Imperfect Concatenation

In the bootstrapping of EONIA curve the sequential inclusion of a spot starting instrument is performed without knowledge of the forthcoming forward starting instrument, whose information content is more relevant for the overlapping section, as visible in Figure 6.

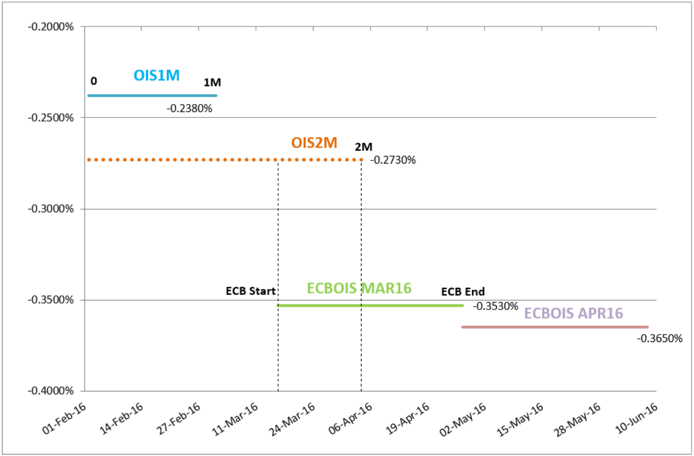

Figure: Overlapping EONIA instruments levels; dataset as of *January 29, 2016*

The calibration algorithm derives the average rate for:

- the interval $(0; 1M)$ from OIS1M
- the interval $(1M; 2M)$ from OIS2M
- the interval $(2M; ECB_{end})$ from 1^{st} ECB OIS

Distortion

As a consequence, the bootstrapping does not use the ECB OIS relevant information for the interval $(ECB_{start}; 2M)$.

Solution: Forward Stub

The error in $(ECB_{start}; 2M)$ is especially relevant if the ECB OIS is accounting a rates cut/rise expectation

To solve this problem the suggestion is to build a "Forward Stub" meta-quote:

- Start date equal to the maturity of the last spot starting OIS non-overlapping with ECB OIS ($1M$ in our case)
- Maturity equal to ECB_{start}, the settlement date of the first ECB OIS

This new meta-quote handle the transition between spot and forward starting instruments without overlap

The Forward Stub value is implied by market rates, ensuring the re-pricing of the discarded overlapping spot instrument:

Condition

$$\int_0^{1M} f(s)ds + \int_{1M}^{ECB_{start}} f(s)ds + \int_{ECB_{start}}^{2M} f(s)ds = \int_0^{2M} f(s)ds$$

- $\int_0^{1M} f(s)ds = OIS1M$ value
- $\int_{1M}^{ECB_{start}} f(s)ds =$ Forward Stub value *(unknown)*
- $\int_{ECB_{start}}^{2M} f(s)ds =$ it is not a quoted market instrument
- $\int_0^{2M} f(s)ds = OIS2M$ value

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Average rate in $(ECB_{start}; 2M)$

Which rate level in $(ECB_{start}; 2M)$? There is no market instrument for this period.

Proposal

Set the rate in $(ECB_{start}; 2M)$ at the $(ECB_{start}; ECB_{end})$ level, as this is supported by the empirical evidence of mostly flat rate between ECB meetings

- where the average rate in $(ECB_{start}; ECB_{end})$ is known and equal to the 1^{st} ECB OIS

Since the instantaneous forward rates integral in the interval $(ECB_{start}; 2M)$ is known, the Forward Stub is the only unknown value, leading to:

Forward Stub value

$$\int_{1M}^{ECB_{start}} f(s)ds = \frac{\int_0^{2M} f(s)ds}{\int_0^{1M} f(s)ds + \int_{ECB_{start}}^{2M} f(s)ds}$$

Assuming continuous compounding

$$Forward\ Stub = \frac{\left[\frac{e^{F(0,2M)\cdot\tau(0,2M)}}{e^{F(0,1M)\cdot\tau(0,1M)}\cdot e^{F(ECB_{start},2M)\cdot\tau(ECB_{start},2M)}} - 1\right]}{\tau(1M, ECB_{start})}$$

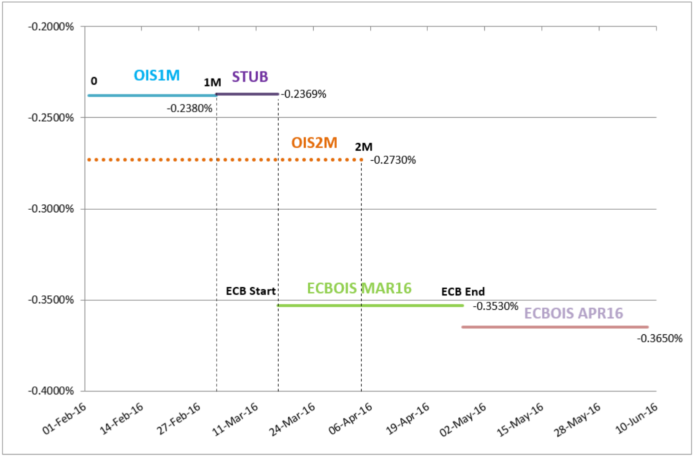

Figure: EONIA levels including the Forward Stub.

The Forward Stub algorithm is stable also in the limiting case of becoming a Spot Stub.

Spot Stub

In particular calendar conditions, the discarded spot instrument might be the 1W OIS, making the Forward Stub actually spot starting: $\tau(0, ECB_{start})$ (where ECB_{start} is the 1^{st} ECB OIS fixing date)

Spot Stub value

The Spot Stub value can be derived using the following formula:

$$Spot\ Stub = \frac{\left[\frac{e^{F(0,ECB_{end})\cdot\tau(0,ECB_{end})}}{e^{F(ECB_{start},ECB_{end})\cdot\tau(ECB_{start},ECB_{end})}} - 1\right]}{\tau(0, ECB_{start})}$$

Repricing Errors analysis

Repricing Errors in Basis Points (Bps)			
Instrumets		Overlapping	Forward Stub
OIS	2M	0.00	0.00
OIS	3M	-1.09	-0.05
OIS	4M	-0.72	0.05
OIS	5M	-0.64	-0.03
OIS	6M	-0.83	-0.32
OIS	7M	-0.45	-0.02
OIS	8M	-0.42	-0.04
OIS	9M	-0.33	0.01
OIS	10M	-0.31	-0.01
OIS	11M	-0.27	0.01
OIS	12M	-0.60	-0.35
OIS	15M	-0.24	-0.04
	RMSE	0.57	0.14
	ME	-1.09	-0.35

Figure: Repricing errors for instruments **not included** in the calibration.

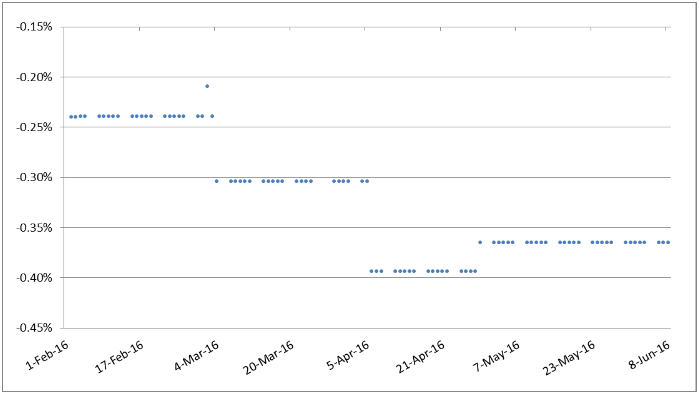

Figure: EONIA curve bootstrapped with overlapping instruments.

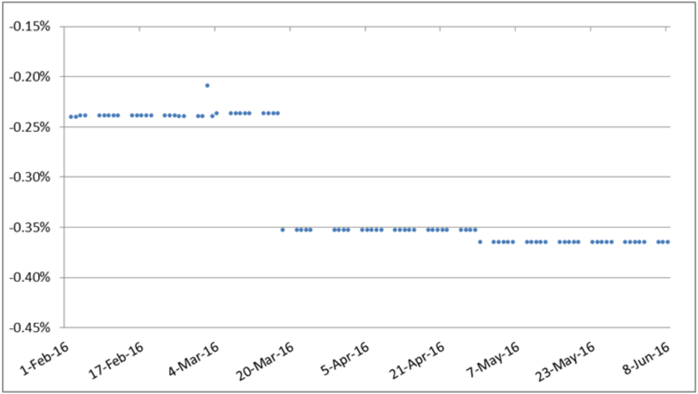

Figure: EONIA curve bootstrapped including the Forward Stub.

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Fitting EONIA Curve Functional Form

- EONIA fixings show an almost flat behaviour between ECB monetary policy meeting dates.
- On the short end we want piece-wise constant forward rates between ECB dates
- On the mid-long section of the curve to have constant forward rates between pillars spaced years apart is unrealistic; smooth interpolation is to be preferred

Interpolation Problem

We need to accommodate conflicting interpolation requirements to model the overnight curve

Solution: Mixed Interpolation technique

Solution

The solution proposed is to build a new interpolation scheme named: "Mixed-Interpolation" that gives the possibility to merge two different interpolation techniques.

Critical issues

1. At which point the interpolation scheme must be switched?
2. Which merging approach can be used?

Our suggestion

- Merge a piecewise constant interpolation on the short end (up to the end of the ECB OIS strip) with a monotone cubic Hyman[a] filtered interpolation on the mid-long end.
- Set the "Switch Pillar" equal to the maturity of the last quoted ECB OIS

[a] for more information see [2]

QuantLib implementation

The Mixed Interpolation algorithm is available in QuantLib for merging two different interpolations at the switch-pillar, using the first one for the short end and the second one for the long end. There are two merging alternatives:

1. *Share Range*: each interpolation is defined on the whole curve
2. *Split Range*: each interpolation is defined on (and restricted to) its own time period only

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Repricing Errors Analysis

Repricing Errors in Basis Points (Bps)					
Instrumets		Linear + Stub	MonotoneCubic + Stub	SplitRange	ShareRange
OIS	2M	0.00	0.00	0.00	0.00
OIS	3M	-0.05	-0.52	-0.05	-0.05
OIS	4M	0.05	-0.21	0.05	0.05
OIS	5M	-0.03	-0.12	-0.03	-0.03
OIS	6M	-0.32	-0.55	-0.32	-0.32
OIS	7M	-0.02	-0.19	-0.02	-0.02
OIS	8M	-0.04	-0.18	-0.04	-0.04
OIS	9M	0.01	-0.13	0.01	0.01
OIS	10M	-0.01	-0.13	-0.01	-0.01
OIS	11M	-0.03	-0.11	-0.03	-0.03
OIS	12M	-0.35	-0.47	-0.35	-0.35
OIS	15M	-0.04	-0.13	-0.04	-0.04
	RMSE	0.14	0.29	0.14	0.14
	ME	-0.35	-0.55	-0.35	-0.35

Figure: Repricing errors for instruments **not included** in the calibration.

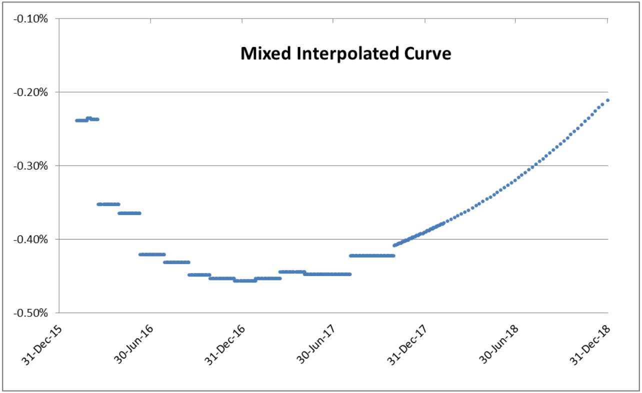

Figure: A mixed interpolated EONIA curve linearly interpolating log-discounts up to the last ECB OIS and then switching to a monotone log-cubic Hyman filtered interpolation.

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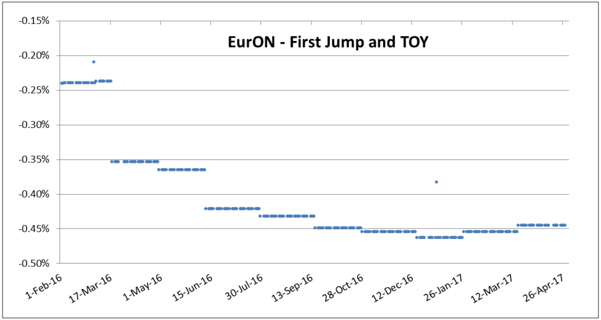
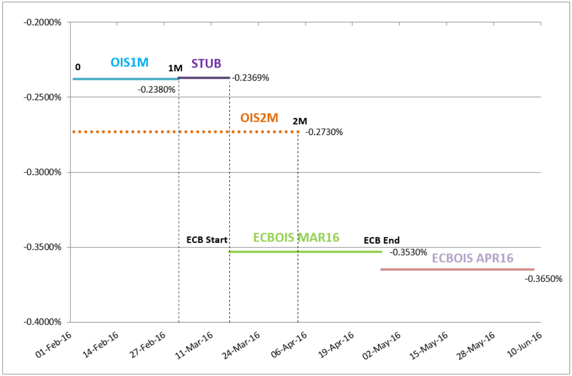
Conclusions

- 1) Estimate TOYs and other jumps, account them before calibration



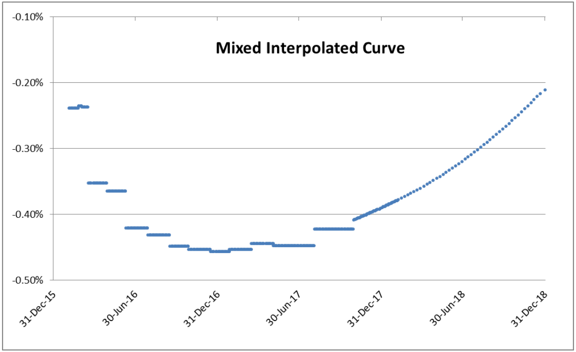
Conclusions

- 2) Link spot instruments to forward instruments using the Forward Stub in order to avoid error in the overlapping section



Conclusions

- 3) Use a mixed (log) linear-cubic (discount factor) interpolator to account for different requirements on the short and long end of the curve



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