Advanced Eonia Curve Calibration

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Abstract

This work analyzes and proposes solutions for subtle, but relevant, problems related to the EONIA curve calibration. The first issue examined is how to deal with jumps and turn-of-year effects. The second point is related to the problem caused by imperfect concatenation between spot starting OIS and forward starting ECB dated OIS: in order to avoid distortion, a meta-instrument called "Forward Stub" shoud cover the section between the maturity of the last spot starting OIS and the settlement of the first ECB OIS. Its implied value can be derived assuming a no-arbitrage conditions. The final issue is the empirical evidence that the forward overnight rates are generally constant between ECB monetary policy board meeting dates: because of this, a log-linear discount interpolation is a good fit. Anyway, flat forward rates are hardly realistic on the long end. This is the rationale to suggest the use of a "Mixed Interpolation" which merges two different interpolation regimes. All the algorithms used to perform the analysis are implemented in the open-source QuantLib project.[1]

To perform the analysis a EUR market dataset as of January 29, 2016 (spot date February 2, 2016) shown in table 1 has been used. However, only the most liquid subset of quotes is included in the calibration in order to avoid distortive overfitting. While instruments included are perfectly re-priced by construction, the rest will show re-pricing errors. The validation of this work's analysis will mostly consist in the minimization of the excluded instruments re-pricing errors, while obtaining good properties for the overall curve.

1 Jump estimation

The key point in a "state-of-the-art" curve bootstrapping is to obtain smooth forward rates. In order to do that, for even the best interpolation scheme to be

[1] QuantLib is a C++ free/open-source library for quantitative finance, for more information see [1]

EURON Market Dataset						
Instrument	Tenor	Fixing Date	Value Date	Maturity Date	Quote Value	Included
EON	SW	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 09-Feb-2016	−0.2390%	TRUE
EON	2W	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 16-Feb-2016	−0.2390%	TRUE
EON	3W	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 23-Feb-2016	−0.2390%	TRUE
EON	1M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Mar-2016	−0.2380%	TRUE
EON	2M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 04-Apr-2016	−0.2730%	TRUE
ECBOIS	MAR16	Mon, 14-Mar-2016	Wed, 16-Mar-2016	Wed, 27-Apr-2016	−0.3530%	TRUE
EON	3M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 02-May-2016	−0.2980%	FALSE
EON	4M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Thu, 02-Jun-2016	−0.3160%	FALSE
ECBOIS	APR16	Mon, 25-Apr-2016	Wed, 27-Apr-2016	Wed, 08-Jun-2016	−0.3650%	TRUE
EON	5M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 04-Jul-2016	−0.3350%	FALSE
ECBOIS	JUN16	Mon, 06-Jun-2016	Wed, 08-Jun-2016	Wed, 27-Jul-2016	−0.4210%	TRUE
EON	6M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 02-Aug-2016	−0.3460%	FALSE
EON	7M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Fri, 02-Sep-2016	−0.3610%	FALSE
ECBOIS	JUL16	Mon, 25-Jul-2016	Wed, 27-Jul-2016	Wed, 14-Sep-2016	−0.4320%	TRUE
EON	8M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 03-Oct-2016	−0.3710%	FALSE
ECBOIS	SEP16	Mon, 12-Sep-2016	Wed, 14-Sep-2016	Wed, 26-Oct-2016	−0.4490%	TRUE
EON	9M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Nov-2016	−0.3800%	FALSE
EON	10M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Fri, 02-Dec-2016	−0.3870%	FALSE
ECBOIS	OCT16	Mon, 24-Oct-2016	Wed, 26-Oct-2016	Wed, 14-Dec-2016	−0.4540%	TRUE
EON	11M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 02-Jan-2017	−0.3930%	FALSE
ECBOIS	DEC16	Mon, 12-Dec-2016	Wed, 14-Dec-2016	Wed, 25-Jan-2017	−0.4570%	TRUE
EON	12M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Thu, 02-Feb-2017	−0.3950%	FALSE
ECBOIS	JAN17	Mon, 23-Jan-2017	Wed, 25-Jan-2017	Wed, 15-Mar-2017	−0.4540%	TRUE
EON	15M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 02-May-2017	−0.4080%	FALSE
ECBOIS	MAR17	Mon, 13-Mar-2017	Wed, 15-Mar-2017	Wed, 03-May-2017	−0.4450%	TRUE
EON	18M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Aug-2017	−0.4150%	TRUE
EON	21M	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Thu, 02-Nov-2017	−0.4160%	TRUE
EON	2Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Fri, 02-Feb-2018	−0.4130%	TRUE
EON	3Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 04-Feb-2019	−0.3730%	TRUE
EON	4Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 03-Feb-2020	−0.2920%	TRUE
EON	5Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 02-Feb-2021	−0.1870%	TRUE
EON	6Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Feb-2022	−0.0620%	TRUE
EON	7Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Thu, 02-Feb-2023	0.0690%	TRUE
EON	8Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Fri, 02-Feb-2024	0.1990%	TRUE
EON	9Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 03-Feb-2025	0.3230%	TRUE
EON	10Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 02-Feb-2026	0.4370%	TRUE
EON	11Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 02-Feb-2027	0.5390%	TRUE
EON	12Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Feb-2028	0.6310%	TRUE
EON	15Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 03-Feb-2031	0.8410%	TRUE
EON	20Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 04-Feb-2036	1.0280%	TRUE
EON	25Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 04-Feb-2041	1.0920%	TRUE
EON	30Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Fri, 02-Feb-2046	1.1160%	TRUE
EON	40Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Wed, 02-Feb-2056	1.1380%	TRUE
EON	50Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Tue, 02-Feb-2066	1.1090%	TRUE
EON	60Y	Fri, 29-Jan-2016	Tue, 02-Feb-2016	Mon, 03-Feb-2076	1.1050%	TRUE

Table 1: Market dataset used to perform the EUR analysis.

effective, any market rate jump must be removed before the curve calibration and than added back at the end of the process. A rate jump can be seen, in a financial point of view, as a higher index fixing due to increased search of liquidity of market participants caused by end-of-month or end-of-year capital requirements. This definition is good for the EUR case in which market evidence shows positive jumps as visible in Figure 1. However, this definition is not consistent for the U.S. case in which market evidence shows negative jumps as it will be shown in A. The most relevant rate jump is the so called *Turn-Of-Year* (TOY)

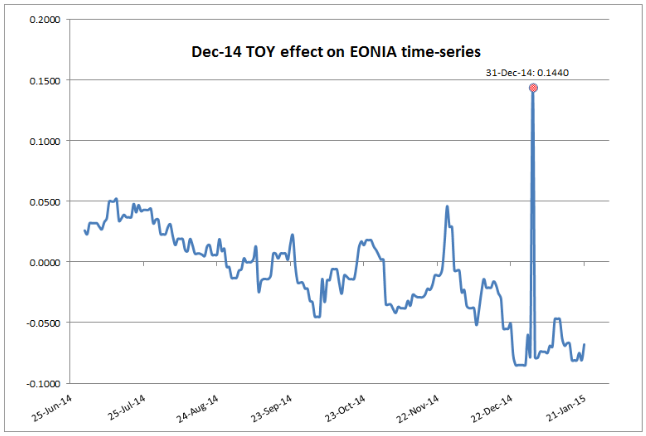

Figure 1: December 2014 Eonia turn of year effect.

effect observable in the last working day in market quotations spanning across the end of the year. Other Euribor indexes with longer tenor display smaller jumps when their maturity crosses the same border; in fact jumps amplitude decrease as the rate tenor increase.
To estimate jumps sizes Ametrano and Mazzocchi ([4]) propose a four-step approach inspired by Burghardt ([5]):

1. The first step is to built an overnight curve calibrating by means of a linear/flat interpolation and including all liquid market quotes available (high pillar density is strictly recommended);

2. After that it is possible to estimate the first jump assuming that a segment out of line with preceding and following segments can be put back in line dumping the difference into the jump:

$$[F^{original}(t_1,t_2) - F^{interp}(t_1,t_2)] \cdot \tau(t_1,t_2) = J^{Size} * \tau^J \tag{1}$$

$$J^{Size} = [F^{original}(t_1,t_2) - F^{interp}(t_1,t_2)] \cdot \frac{\tau(t_1,t_2)}{\tau^J} \tag{2}$$

3. Ones the first jumps as been estimated it is possible to remove it from the curve;

4. Iterate ad libitum 2 and 3 on the next jump date.

EONIA Index fixings show that it occurs a jump at least at every end of month. To obtain a smooth forwarding curve, this particular behaviour must be addressed in the calibration taking care to clear the entire curve from jumps before starting the bootstrap. Figures 2 and 3 shows the impact of first and second jump inclusion in the curves shape where jumps sizes have been estimated taking advantage of the approach proposed above. As visible, the short term section of the curve is altered due the fact that jumps have been accounted. To preserve the value of the instantaneous forward rate integral in that section, the calibration algorithm pushes down all the segment changing the curve shape.

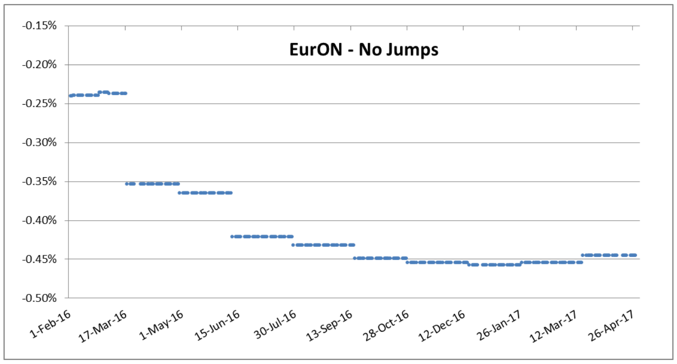

Figure 2: First two year section of the EUR overnight curve without jumps.

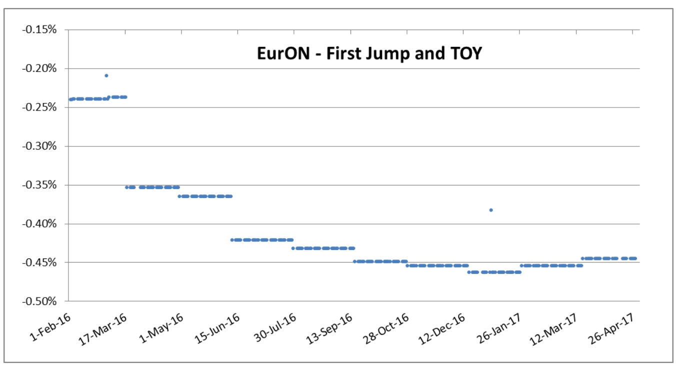

Figure 3: The same curve in Figure 2 including first jump and TOY effect.

Since the piecewise flat steps of the curve are given by market quotes which are the integral of instantaneous forward rates between an interval (t_1, t_2), if this integral must account a deep above level point, the residual part is shifted down in order to maintain the integral value equal.

2 Forward Stub

The problem concerns the composition of the EUR overnight curve which instruments' selection best practice is to mix spot starting derivatives, the Overnight Indexed Swaps *(OIS)* and forward starting derivatives, the European Central Bank Overnight Indexed Swaps *(ECB OIS)*. Table 1 summarize all the instruments available on the market but only the most liquid subset will be used to perform the calibration in order to avoid distortive overfitting. The instruments' selection criteria is based on liquidity. As a consequence, whenever an year fraction is covered by many market quotes, only the most liquid one will be included in the calibration. For this reason, since ECB OIS are preferred by traders because are the most liquid overnight indexed instruments, the calibration algorithm is set in order to give them priority despite of spot starting OIS which cover the same period[2]. Otherwise, since ECB OIS are forward starting instruments, the concatenation between spot OIS and forward ECB OIS could not be perfect leading to an overlapping section as the one shown in Figure 4. Figure 4 shows an example of imperfect concatenation where OIS2M

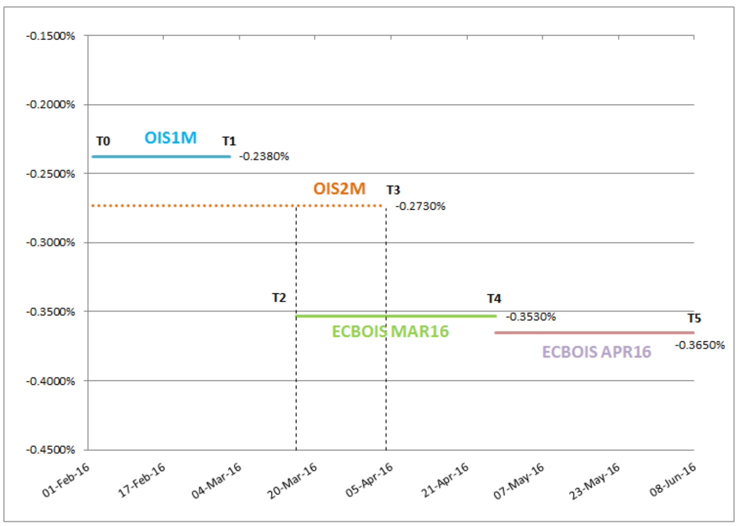

Figure 4: Overlapping EONIA instruments levels; dataset as of *January 29, 2016*.

with maturity $t_3 = April 4, 2016$ overlaps the first ECB OIS with settlement $t_2 = March 16, 2016$.

The calibration algorithm derives information about the average rate for the interval $(t_0; t_1)$ from the OIS1M. Similarly, OIS2M fixes the average rate in the interval $(t_1; t_3)$ and the information about $(t_3; t_4)$ is given by the first ECB OIS. Since the bootstrap algorithm uses this information to perform a perfect reprice of all the included market quotes, it is not able to derive the information set by the first ECB OIS for the interval $(t_2; t_3)$ because it uses the level fixed by OIS2M. This means that the calibrator can not use the information related to the interval $(t_2; t_3)$ set by the most liquid instrument on the market. This distortion could be negligible only if the information given by both instruments are almost equal, like when the curve has a flat behaviour in that region. Otherwise, there are situations where the level fixed by these two instruments is

[2]For more information about curve calibration best practices see [2] and [3]

very different as, for example, when first ECB OIS market quote is accounting a rates cut/rise expectation. The distortion resulting from the overlapping section is visible in Figure 5 where the calibrator, which is not using the the most reliable information for the interval $(t_2; t_3)$, pushes down the curve till -0.40% in order to reprice perfectly the first ECB OIS. It is straightforward that this strange behaviour is due to calibration's discrepancy resulting from the overlapping section. In order to fix this problem the suggestion is to create

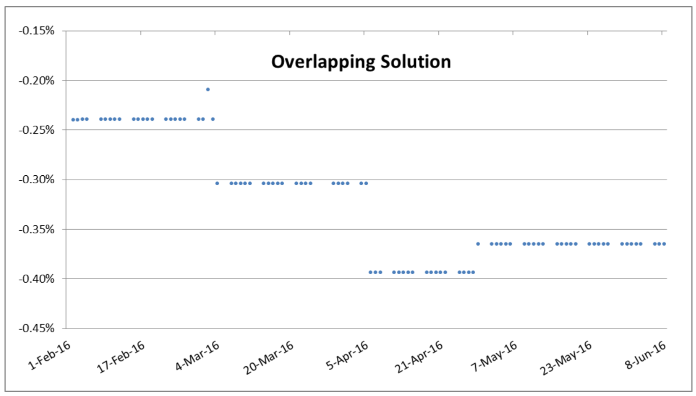

Figure 5: EONIA curve calibrated including the overlapping instrument.

a new "Meta-Instrument", from now called *Forward Stub*, that covers the interval $(t_1; t_2)$ and which value can be derived from available market quotes. The aim of the Forward Stub is to cover the section between the last not-overlapping spot starting OIS maturity and the first ECB OIS settlement date in order to link perfectly spot instruments to forward instruments as shown in Figure 6. Looking to the dataset used, the time values are:

t_0 = February 2, 2016 - Settlement date
t_1 = March 2, 2016 - OIS1M maturity
t_2 = March 16, 2016 - First ECB OIS settlement date
t_3 = April 4, 2016 - OIS2M maturity date
t_4 = April 27, 2016 - First ECB OIS maturity date
t_5 = June 8, 2016 - Second ECB OIS maturity date

From available market quotes, the following information is well known:

- The average forward rate for (t_0, t_1) is given by OIS1M market quote and it is equal to: $-0,2380\%$;

- The average forward rate for (t_0, t_3) is given by OIS2M market quote and it is equal to: $-0,2730\%$;

- The average forward rate for (t_2, t_4) is given by the 1^{st} ECB OIS market quote and it is equal to: $-0,3530\%$;

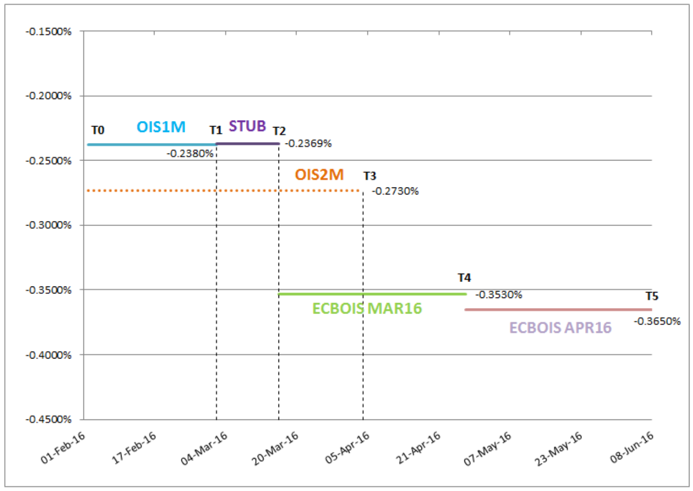

Figure 6: Overlapping EONIA instruments' levels including the Forward Stub.

- The average forward rate for (t_4, t_5) is given by the 2^{nd} ECB OIS market quote and it is equal to: $-0,3650\%$.

The Forward Stub quote is the rate which covers the year fraction (t_1, t_2). This rate is implied in the market and must be derived assuming a no-arbitrage condition which impose the OIS2M to be perfectly repriced. Otherwise, in order to obtain the unknown Meta-Quote value, it is necessary to have information about the average forward rates in the interval (t_2, t_3). This value can not be derived directly from market instruments. Since the average forward rate for (t_2, t_4) is known and equal to the 1^{st} ECB OIS market quote it is possible to derive the (t_2, t_3) value assuming that forward rates in (t_2, t_4) are constant. This assumption is consistent with the empirical market evidence which shows an almost flat behaviour between ECB monetary policy dates as the Figure 9 suggest. All the information necessary to derive the Forward Stub implied market quotes is now available and it is possible to exploit the no-arbitrage condition to set:

$$\int_{t_0}^{t_1} f(s)\mathrm{d}s + \int_{t_1}^{t_2} f(s)\mathrm{d}s + \int_{t_2}^{t_3} f(s)\mathrm{d}s = \int_{t_0}^{t_3} f(s)\mathrm{d}s \tag{3}$$

As visible, no-arbitrage condition implies that investing in OIS1M (from t_0 to t_1), Forward Stub (from t_1 to t_2) and in ECB OIS till the OIS2M maturity (from t_2 to t_3) must be equal to investing in OIS2M (from t_0 to t_3). Solving the equation for the Forward Stub unknown value:

$$\int_{t_1}^{t_2} f(s)\mathrm{d}s = \frac{\int_{t_0}^{t_3} f(s)\mathrm{d}s}{\int_{t_0}^{t_1} f(s)\mathrm{d}s + \int_{t_2}^{t_3} f(s)\mathrm{d}s} \tag{4}$$

Assuming continuous compounding:

$$Stub = \frac{\left[\frac{e^{F(t_0,t_3)\cdot\tau(t_0,t_3)}}{e^{F(t_0,t_1)\cdot\tau(t_0,t_1)}\cdot e^{F(t_2,t_3)\cdot\tau(t_2,t_3)}} - 1\right]}{\tau(t_1,t_2)} \tag{5}$$

Including this Meta-Instrument in the overnight curve calibration improves consistently the output quality avoiding unwanted distortion and oscillation as it will be shown later taking advantage of the repricing error analysis. Otherwise, this solution can be implemented only if the bootstrap algorithm uses a flat interpolation for the short-term section of the curve. This condition is necessary to match the assumption that forward rates in (t_2, t_4) are constant. It is also important to underlying that the Stub algorithm produces good results also in limit cases as, for example, when $\tau(t_2, t_4) \longrightarrow \frac{1}{365}$ that represents the particular case in which the Forward Stub quote duration is just 1 day. Furthermore, during a year, since the first ECB OIS fixing becomes earlier and earlier, it will happen that the discarded spot instrument is the spot week OIS (which is the one with shorter maturity). This case implies that the so called "Forward Stub" it is not forward start anymore because it becomes a spot instrument which covers the year fraction: $\tau(t_0, t_1)$ (where t_1 is the first ECB OIS fixing date). In this particular situation, the "Spot Stub" implied value is:

$$Spot\ Stub = \frac{\left[\frac{e^{F(t_0,t_2)\cdot\tau(t_0,t_2)}}{e^{F(t_1,t_2)\cdot\tau(t_1,t_2)}} - 1\right]}{\tau(t_0,t_1)} \tag{6}$$

The impact of the Forward Stub inclusion in presented in Figure 7. It is visible that this approach avoids strange curve's behaviour and is the one which minimizes all the out-of-curve instruments repricing errors ensuring also the perfect reprice of the excluded overlapping instrument.

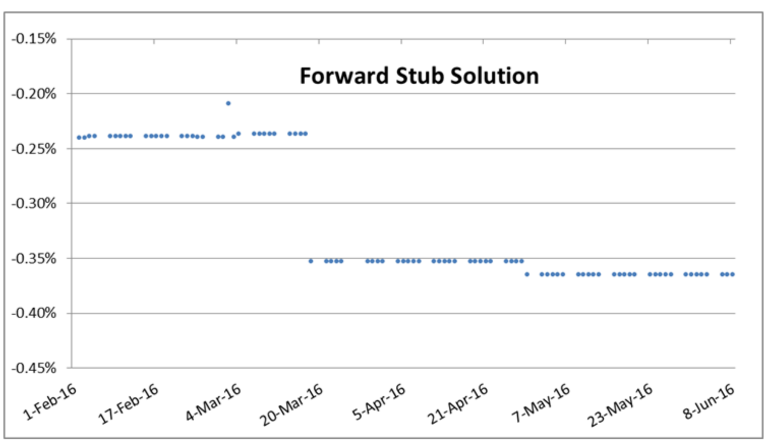

Figure 7: EONIA Curve calibrated including Forward Stub.

Finally, the Figure 8 shows the repricing error analysis that validates the Forward Stub solution as the one able to minimize errors and to improve the curve's quality. The other solution instead is able to reprice perfectly the 2-months maturity OIS because it is an included instrument but it is not able to do a good re-price for all the not included market quotes which show relevant repricing errors. Also the lower values of Root Mean Square Error (RMSE) and Max Error (ME) validate the Forward Stub solution as the one that ensures errors' minimization.

Repricing Errors in Basis Points (Bps)			
Instrumets		Overlapping	Forward Stub
OIS	2M	0.00	0.00
OIS	3M	-1.09	-0.05
OIS	4M	-0.72	0.05
OIS	5M	-0.64	-0.03
OIS	6M	-0.83	-0.32
OIS	7M	-0.45	-0.02
OIS	8M	-0.42	-0.04
OIS	9M	-0.33	0.01
OIS	10M	-0.31	-0.01
OIS	11M	-0.27	0.01
OIS	12M	-0.60	-0.35
OIS	15M	-0.24	-0.04
	RMSE	0.57	0.14
	ME	-1.09	-0.35

Figure 8: Repricing errors summary related to all available instruments not included in the calibration (included instruments don't show errors by construction).

3 Mixed Interpolation

The *Mixed Interpolation*, as already introduced before, is a particular technique which merges two different kind of interpolation. Figure 9 shows that EONIA Index time series have a piecewise constant behaviour between ECB monetary policy board meeting dates. In order to replicate this particular shape, the best way is by means of a flat interpolation on discount factor which is able to provide a stepped curve's behaviour. Since instruments' granularity decrease in the mid-long term, assuming flat forward rates becomes inconsistent because it would produce flat rates period of more than a year. As a consequence, it becomes necessary to switch to another interpolation technique which is able to provide smooth forwards. In order to obtain smooth forward rates Ametrano-Bianchetti [2] propose to use an Hyman[3] filtered monotone cubic natural spline. The correct point where switching interpolation schemes can not be established

[3] More details on monotonicity preserving filters can be found in [6]

consistently but the suggestion is to use a flat interpolator up to the end of the ECB dated OIS strip and then start with the cubic spline. The mixed interpolation is implemented in QuantLib 1.9 version in two different ways:

- The *Split Range* approach consists in interpolating till a pre-determined pillar, from now called the *Switch Pillar*, using the 1^{st} interpolation technique and then switch to the 2^{nd} interpolation for the rest of the curve.
- *Share Range* approach consists in interpolating two times the whole curve and then merging the obtained curves in the switch pillar. The curve calibrated taking advantage of this approach is plotted in Figure 10.

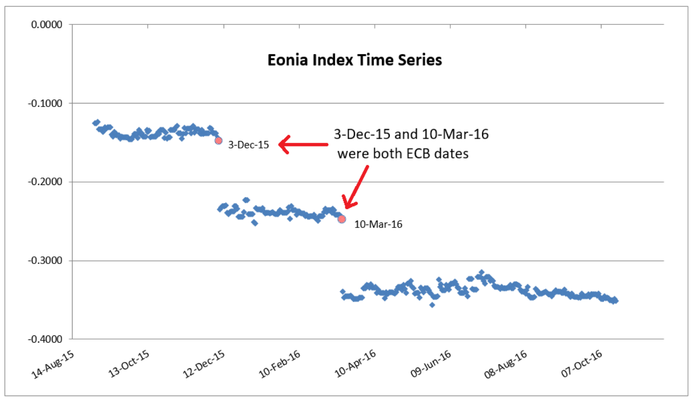

Figure 9: Last year EONIA Index fixings showing a piecewise flat behaviour between ECB monetary policy meetings

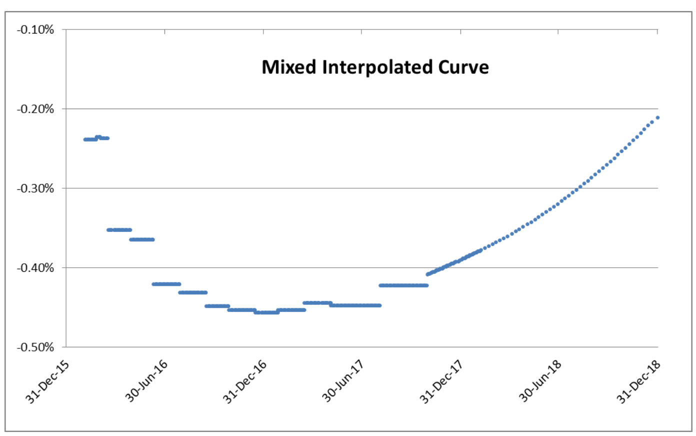

Figure 10: Mixed Interpolation approach merging linear and monotonic cubic natural spline on log-discounts.

As visible, this approach produces a stepped behaviour till the switch pillar and then forward rates become smooth due to the monotone cubic interpolation. To validate the Mixed "Linear-HymanCubic" interpolator as the best technique to model the EONIA curve, Figure 11 summarize the same repricing error analysis used for the preceding section. Analyzing the repricing table it

Repricing Errors in Basis Points (Bps)					
Instrumets		Linear + Stub	MonotoneCubic + Stub	SplitRange	ShareRange
OIS	2M	0.00	0.00	0.00	0.00
OIS	3M	-0.05	-0.52	-0.05	-0.05
OIS	4M	0.05	-0.21	0.05	0.05
OIS	5M	-0.03	-0.12	-0.03	-0.03
OIS	6M	-0.32	-0.55	-0.32	-0.32
OIS	7M	-0.02	-0.19	-0.02	-0.02
OIS	8M	-0.04	-0.18	-0.04	-0.04
OIS	9M	0.01	-0.13	0.01	0.01
OIS	10M	-0.01	-0.13	-0.01	-0.01
OIS	11M	-0.03	-0.11	-0.03	-0.03
OIS	12M	-0.35	-0.47	-0.35	-0.35
OIS	15M	-0.04	-0.13	-0.04	-0.04
	RMSE	0.14	0.29	0.14	0.14
	ME	-0.35	-0.55	-0.35	-0.35

Figure 11: Repricing error analysis using a log-linear interpolation + Stub for the whole curve (1^{st} column), a monotone cubic spline + Stub for the whole curve (2^{nd} column), a share range mixed interpolation (3^{rd} column) and a split range mixed interpolation (4^{th} column).

is possible to draw some conclusions. The comparison is made between a log-linear interpolated curve which includes the Forward Stub; a Hyman filtered monotone cubic spline curve which includes an "*Ad Hoc*" Stub obtained through a root finding approach and the mixed interpolated curves following both the possibles methods. Obviously, it is visible that the mixed approaches and the log-linear one lead to the same errors since the mixed technique uses a log-linear interpolation up to the switch pillar (which is set on the last quoted ECB OIS maturity). The monotone cubic curve, instead, even if bootstrapped including the Stub, produces higher errors for each instrument not included in the calibration. The conclusion is that the EONIA curve's short term section is fitted better by a piecewise flat interpolation rather than a cubic one. Otherwise, on the mid/long term, assuming constant forwards can not be consistent anymore. That's why this work suggests to mix the log-linear interpolation up to the end of the ECB OIS strip and then switch to a monotone cubic spline interpolator which produces smooth forward rates for the mid/long term section. To confirm the analysis the Root Mean Square Error (RMSE) and Max Error (ME) showing higher values related to the monotone cubic approach.

4 Conclusions

This paper faces 3 advanced EONIA curve calibration problems and presents some possible solutions. First of all, the jumps problem has been shown. Following the best practice proposed by Ametrano-Mazzocchi ([4]) and ispired by Burghardt ([5]), this work proposed a simple way to estimate jumps and turn-of-year effects as the ones visible in the EONIA Index time series and shows how the shape is changed after the jumps inclusion.

After that, it proposes to build a Meta-Quote called Forward Stub in order to fix the distortion caused by the not perfect concatenation between spot starting OIS and forward starting ECB OIS. The results have been presented with a repricing errors analysis which shows that including the Forward Stub despite of the overlapping instrument produces a more accurate curve.

Finally, the mixed interpolation has been presented as a particular interpolator which merge two different techniques. In order to reach a good fit of the overnight curve's shape, this work proposes to mix a linear interpolation on log-discounts up to the end of the ECB OIS strip and then switch to a monotone cubic spline for the mid-long term section. Using the same repricing approach proposed in the previous issue it has been shown that this configuration leads to a very good curve's fit which is better than the one obtained through a log-linear interpolation or a monotone cubic spline interpolation for the whole curve.

Appendix A USD market jumps estimation

In this appendix, the same analysis presented in has been applied to the USD maarket case. To perform the analysis a dataset as of October 5, 2016 (spot date October 7, 2016), has been used which is shown hereunder.

USDON Market Dataset						
Instrument	Tenor	Fixing Date	Value Date	Maturity Date	Quote Value	Flag
OIS	SW	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 28-Oct-2016	0.4070%	TRUE
OIS	2W	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 04-Nov-2016	0.4030%	TRUE
OIS	3W	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 14-Nov-2016	0.4080%	TRUE
OIS	1M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 21-Nov-2016	0.4100%	TRUE
OIS	2M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Wed, 21-Dec-2016	0.4290%	TRUE
OIS	3M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Jan-2017	0.4720%	TRUE
OIS	4M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Tue, 21-Feb-2017	0.4950%	TRUE
OIS	5M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Tue, 21-Mar-2017	0.5110%	TRUE
OIS	6M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 21-Apr-2017	0.5270%	TRUE
OIS	7M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 22-May-2017	0.5390%	TRUE
OIS	8M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Wed, 21-Jun-2017	0.5490%	TRUE
OIS	9M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 21-Jul-2017	0.5600%	TRUE
OIS	10M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 21-Aug-2017	0.5710%	TRUE
OIS	11M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Thu, 21-Sep-2017	0.5790%	TRUE
OIS	1Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Oct-2017	0.5880%	TRUE
OIS	15M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 22-Jan-2018	0.6140%	TRUE
OIS	18M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Apr-2018	0.6370%	TRUE
OIS	21M	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Jul-2018	0.6580%	TRUE
OIS	2Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 22-Oct-2018	0.6780%	TRUE
OIS	3Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 21-Oct-2019	0.7500%	TRUE
OIS	4Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Wed, 21-Oct-2020	0.8220%	TRUE
OIS	5Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Thu, 21-Oct-2021	0.8960%	TRUE
OIS	6Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 21-Oct-2022	0.9700%	TRUE
OIS	7Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Oct-2023	1.0410%	TRUE
OIS	8Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 21-Oct-2024	1.1050%	TRUE
OIS	9Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Tue, 21-Oct-2025	1.1620%	TRUE
OIS	10Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Wed, 21-Oct-2026	1.2140%	TRUE
OIS	12Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Oct-2028	1.3030%	TRUE
OIS	15Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Tue, 21-Oct-2031	1.3990%	TRUE
OIS	20Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Tue, 21-Oct-2036	1.4950%	TRUE
OIS	25Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 21-Oct-2041	1.5370%	TRUE
OIS	30Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 22-Oct-2046	1.5560%	TRUE
OIS	40Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Mon, 23-Oct-2056	1.5640%	TRUE
OIS	45Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Fri, 21-Oct-2061	1.5550%	TRUE
OIS	50Y	Wed, 19-Oct-2016	Fri, 21-Oct-2016	Thu, 21-Oct-2066	1.5490%	TRUE

Figure 12: The dataset used for the USD market analysis.

The USD market case shows an inverted situation compared to the EUR market presented before. As visible in Figure 13, the Fed Funds fixings shows negative jumps every end-of-month and with a particular deep size. In a financial point of view this is due to the excess of liquidity that market participants hold and want to sell at the end of each month. As a consequence U.S market participants tend to have a decreasing necessity of liquidity every end-of-month. This behaviour tends to be constant during time, in fact, for the last two years, the Fed Funds time series have shown negative jumps every end-of-month with no exceptions. To estimate negative sizes, however, the Ametrano-Mazzocchi approach proposed for the EUR market case must be reviewed. In particular

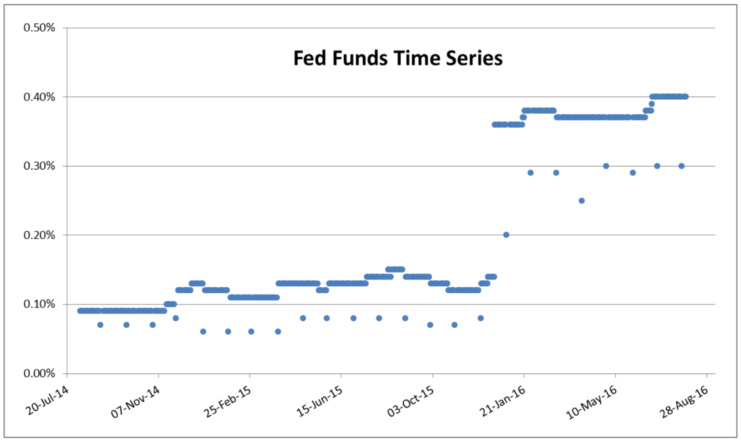

Figure 13: Last 2 years Fed Funds fixings time series.

the formula at point 2 becomes:

$$[F^{Interp}(t_1,t_2) - F^{Original}(t_1,t_2)] \cdot \tau(t_1,t_2) = J^{Size} * \tau^J \tag{7}$$

$$J^{Size} = [F^{Interp}(t_1,t_2) - F^{Original}(t_1,t_2)] \cdot \frac{\tau(t_1,t_2)}{\tau^J} \tag{8}$$

Applying the same procedure but using the reviewed formula it is now possible to perform negative jumps estimations and account them in the USD overnight curve as shown in Figure 15.

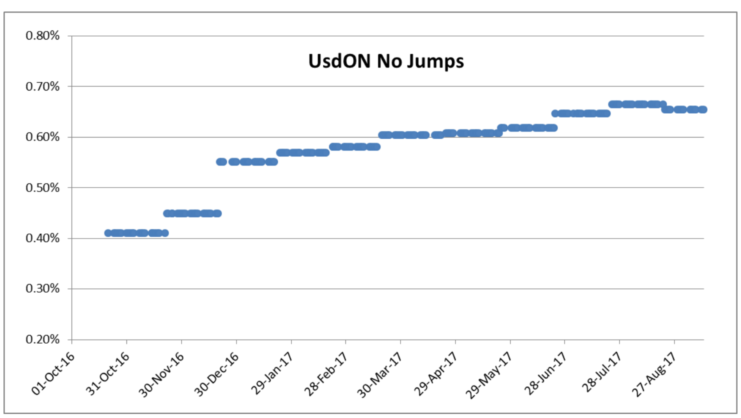

Figure 14: First USD overnight curve section without jumps.

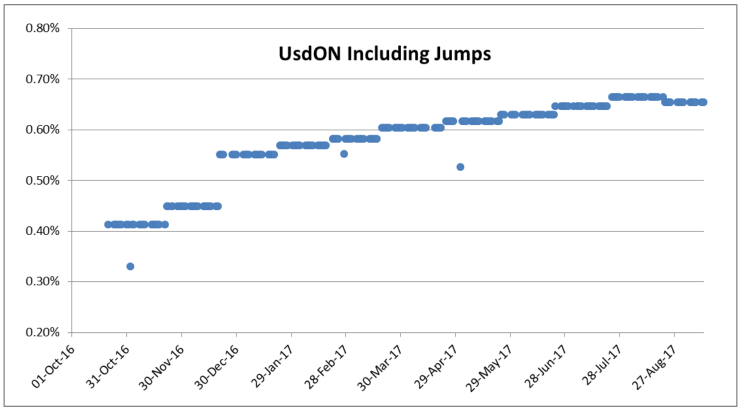

Figure 15: The same curve in Figure 14 including all estimated jumps.

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