EONIA Jumps and Proper Euribor Forwarding

The Case of Synthetic Deposits in Legacy Discount-Based Systems

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QuantLib User Meeting, Düsseldorf, 5 December 2014
https://speakerdeck.com/nando1970/eonia-jumps-and-proper-euribor-forwarding

Outline

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To obtain proper forward rate curves in legacy discount-based systems it is very important to construct in a good way the first section, the one for maturities shorter than the first market pillar.

FRA equation

It is useful to write down the relation between the value of a generic *FRA* contract, on x tenor rate, and pseudo-discount factors ($P(d)$, $d =$ date):

$$1 + F_x(d, d+x) \cdot \tau = \frac{P(d)}{P(d+x)} = e^{\int_d^{d+x} f_x(s)ds} \tag{1}$$

Therefore we have that this contract depends on the values of two pseudo-discount factors: $P(d)$ and $P(d+x)$.

Example

We consider the Euribor $6M$ curve. The first instrument, in order of increasing maturities, that we can find on market is the 0x6 *FRA* (e.g. *FRA* over today, *FRA* over tomorrow, index fixing[a])
Since $P(0) = 1$, using equation (1), we have:

$$P(6M) = \frac{1}{1 + F_{6M}(0, 6M) \cdot \tau} = e^{-\int_0^{6M} f_{6M}(s)ds} = e^{-z_{6M}(6M)\tau} \qquad (2)$$

With $z_{6M}(6M)$ we indicate the zero rate at $6M$.

[a] Euribor fixing = 0x6 *FRA*, therefore $r^{fix}(6M) = F_{6M}(0, 6M)$

Example

Imagine that we now want to insert the 1x7 *FRA*; using equation (1), we have:

$$P(7M) = \frac{P(1M)}{1 + F_{6M}(1M, 7M) \cdot \tau} = \frac{e^{-\int_0^{1M} f_{6M}(s)ds}}{1 + F_{6M}(1M, 7M) \cdot \tau} \quad (3)$$

We need to know $\int_0^{1M} f_{6M}(s)$

We could interpolate $P(1M)$ between $P(0) = 1$ and $P(6M)$.

This produces very bad result: without any other information we easily underestimate or overestimate the proper values of the $1M$ pseudo-discount factor.

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Example

These kind of errors lead to an incorrect calculation of forward rates: when market quotes become less dense (i.e. where the FRA's strip ends) the curve shows an hump:

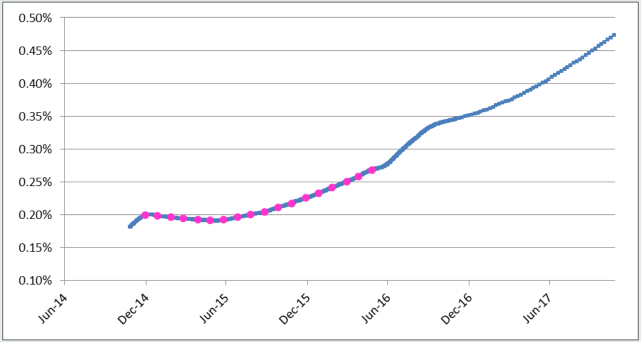

Figure: Forward Euribor 6M Curve

Example

It is helpful to take a look at the *instantaneous forward rates*, since everything else is obtained through their integration. In the first 6 months, they have been obtained in an arbitrary way, leading to oscillations:

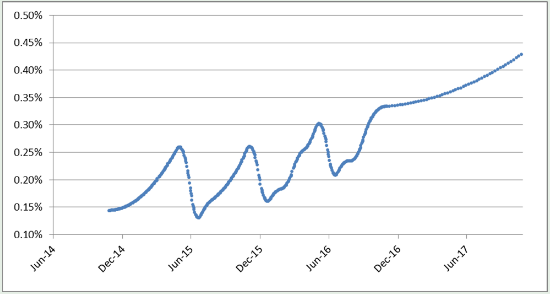

Figure: Instantaneous Forward Rate[a] on Euribor 6M Curve (3 years)

[a] we calculate them as *ON* forward rate on 6 months curve

Outline

Synthetic Deposits construction

The model

To solve this problem we need a reliable way to prescribe the shape of the instantaneous forward on the short end. We can use *ON* instantaneous forward rate plus a spread, $\delta_x(t)$:

$$f_x(t) = f_{on}(t) + \delta_x(t) \qquad \forall\ 0 \leq t \leq x \tag{4}$$

The model

In general[a], even with t_2 not equal to $t_1 + x$:

$$
\begin{aligned}
1 + F_x(t_1, t_2)\tau_x &= e^{\int_{t_1}^{t_2} f_x(s)ds} \\
&= e^{\int_{t_1}^{t_2} (f_{on}(s)+\delta_x(s))ds} = e^{\int_{t_1}^{t_2} f_{on}(s)ds} \cdot e^{\int_{t_1}^{t_2} \delta_x(s)ds} \\
&= [1 + F_{on}(t_1, t_2)\tau_x] \cdot e^{\Delta_x(t_1,t_2)}
\end{aligned}
$$

Where:

$$
\Delta_x(t_1, t_2) = \int_{t_1}^{t_2} \delta_x(s)ds
$$

$$
F_{on}(t_1, t_2) = \frac{1}{\tau_x}\left[\frac{P_{on}(t_1)}{P_{on}(t_2)} - 1\right]
$$

[a] τ_x = x-months discrete forward rate year fraction

The model

Isolating $\Delta_x(t_1, t_2)$, we have:

$$\begin{aligned}\Delta_x(t_1, t_2) &= \ln\left[\frac{1 + F_x(t_1, t_2)\tau_x}{1 + F_{on}(t_1, t_2)\tau_x}\right] \\ &= \ln\left[1 + F_x(t_1, t_2)\tau_x\right] - \ln\left[1 + F_{on}(t_1, t_2)\tau_x\right]\end{aligned}$$

We can approximate the previous equation neglecting higher order terms:

$$\Delta_x(t_1, t_2) \approx F_x(t_1, t_2)\tau_x - F_{on}(t_1, t_2)\tau_x \approx B_x(t_1, t_2)\tau_x \tag{5}$$

Where:

$$B_x(t_1, t_2) = F_x(t_1, t_2) - F_{on}(t_1, t_2) \tag{6}$$

is the simply compounded basis.

$\Delta_x(t_1, t_1 + x)$ and $B_x(t_1, t_1 + x)$ are observable from market quotes

Synthetic Deposits

We can construct synthetic spot instruments, $F_x^{Synth}(0, t)$ with $t \leq x$, for the bootstrapping of the forward curve of tenor x between 0 and x.

$$P(t) = \frac{1}{1 + F_x^{Synth}(0, t) \cdot \tau_x} = e^{-\int_0^t f_x(s)ds}$$

Therefore in equations (5) and (6) we consider the case:

- $t_1 = 0$
- $t_2 = t \leq t_1 + x = x$

Synthetic Deposits equation

With these hypothesis:

$$F_x^{Synth}(0,t) = \frac{[1 + F_{on}(0,t)\tau_x] \cdot e^{\Delta_x(0,t)} - 1}{\tau_x} \tag{7}$$

Remark:

- $\Delta_x(0,t)$, with $t \leq x$, is not observable on the market;
- $\Delta_x(t_i, t_i + x)$, with $t_i = 0M, 1M, 2M, \ldots$ and/or $t_i = 0M, IMM1, IMM2, \ldots$ is observable from $F_x(t_i, t_i + x)$ and $F_{on}(t_i, t_i + x)$;

Basis calculation

$$\Delta_x(0,t) = \int_0^t \delta_x(s)ds$$

We parametrize $\delta_x(s)$ using an n degree polynomial calibrated to market available quotes, $F_x(t_i, t_i + x)$, and $F_{on}(t_i, t_i + x)$ calculated from the *ON* curve.

1 - Flat parametrization

$$f_x(t) = f_{on}(t) + \delta_x(t) \qquad \forall\ 0 \le t \le x$$

$$\delta_x(t) = \alpha_x$$

To fix the value of α_x we need:

$$\begin{aligned}\int_0^x \delta_x(s)ds = \alpha_x \tau_x &= \Delta_x(0,x) \\ &= \ln[1 + F_x(0,x)\tau_x] - \ln[1 + F_{on}(0,x)\tau_x]\end{aligned}$$

We have the following equation:

$$\Delta_x(0,x) = \alpha_x \tau_x$$

2 - Linear parametrization

$$\delta_x(t) = \alpha_x + \beta_x \tau_x(0, t)$$

To determine the value of α_x, β_x we integrate the previous equation between a generic t_i and $t_i + x$:[a]

$$\int_{t_i}^{t_i+x} \delta_x(s) ds = \alpha_x(\tau_{t_i+x} - \tau_{t_i}) + \frac{1}{2}\beta_x(\tau^2_{t_i+x} - \tau^2_{t_i}) = \Delta_x(t_i, t_i + x)$$

[a] $\tau_t = \tau_x(0, t)$

$\Delta_x(t_i, t_i + x)$ is observable for $t_i = 0M, 1M, \ldots$ and/or $t_i = IMM1, IMM2, \ldots$

2 - Linear parametrization

We need 2 equations to calibrate α_x and β_x:

$$\begin{cases} \Delta_x(0, 0+x) = & \ln[1 + F_x(0, 0+x)\tau_x(0, x)] \\ & - \ln[1 + F_{on}(0, 0+x)\tau_x(0, x)] \\ \Delta_x(t_1, t_1+x) = & \ln[1 + F_x(t_1, t_1+x)\tau_x(t_1, t_1+x)] \\ & - \ln[1 + F_{on}(t_1, t_1+x)\tau_x(t_1, t_1+x)] \end{cases}$$

We have the following system of 2 equations:

$$\begin{cases} \Delta_x(0, 0+x) & = \alpha_x \tau_x + \frac{1}{2}\beta_x \tau_x^2 \\ \Delta_x(t_1, t_1+x) & = \alpha_x(\tau_{t_1+x} - \tau_{t_1}) + \frac{1}{2}\beta_x(\tau_{t_1+x}^2 - \tau_{t_1}^2) \end{cases}$$

3 - Quadratic parametrization

$$\delta_x(t) = \alpha_x + \beta_x \tau_x(0,t) + \gamma_x \tau_x(0,t)^2$$

To determine the value of α_x,β_x,γ_x as before:

$$\int_{t_i}^{t_i+x} \delta_x(s)ds = \alpha_x(\tau_{t_i+x} - \tau_{t_i}) + \frac{1}{2}\beta_x(\tau^2_{t_i+x} - \tau^2_{t_i}) + \frac{1}{3}\gamma_x(\tau^3_{t_i+x} - \tau^3_{t_i}) = \Delta(t_i, t_i + x)$$

$\Delta_x(t_i, t_i + x)$ is observable for $t_i = 0M$,$1M$,... and/or $t_i = IMM1$,$IMM2$,...

3 - Quadratic parametrization

We need 3 equations to calibrate α_x,β_x,γ_x:

$$\begin{cases} \Delta_x(0,0+x) = & \ln[1+F_x(0,0+x)\tau_x(0,x)] \\ & -\ln[1+F_{on}(0,0+x)\tau_x(0,x)] \\ \Delta_x(t_1,t_1+x) = & \ln[1+F_x(t_1,t_1+x)\tau_x(t_1,t_1+x)] \\ & -\ln[1+F_{on}(t_1,t_1+x)\tau_x(t_1,t_1+x)] \\ \Delta_x(t_1,t_1+x) = & \ln[1+F_x(t_2,t_2+x)\tau_x(t_2,t_2+x)] \\ & -\ln[1+F_{on}(t_2,t_2+x)\tau_x(t_2,t_2+x)] \end{cases}$$

We have the following system of 3 equations:

$$\begin{cases} \Delta_x(0,0+x) & = \alpha_x\tau_x + \frac{1}{2}\beta_x\tau_x^2 + \frac{1}{3}\gamma_x\tau_x^3 \\ \Delta_x(t_1,t_1+x) & = \alpha_x(\tau_{t_1+x} - \tau_{t_1}) + \frac{1}{2}\beta_x(\tau_{t_1+x}^2 - \tau_{t_1}^2) + \frac{1}{3}\gamma_x(\tau_{t_1+x}^3 - \tau_{t_1}^3) \\ \Delta_x(t_2,t_1+x) & = \alpha_x(\tau_{t_2+x} - \tau_{t_2}) + \frac{1}{2}\beta_x(\tau_{t_2+x}^2 - \tau_{t_2}^2) + \frac{1}{3}\gamma_x(\tau_{t_2+x}^3 - \tau_{t_2}^3) \end{cases}$$

To find $\Delta_x(t_i, t_i + x) = \int_{t_i}^{t_i+x} \delta_x(s)ds$, we use:

- market available quotes on x-months tenor Euribor, like: index fixing, Futures, FRA, IRS.
- equivalent *ON* discrete forward rates, built using the *ON* curve (in the Euro market they are equivalent to forward *OIS* contract).

Basis Calculation

Example

Let's consider the 6 Months Euribor curve.
Until 2 years ago, to calculate $\Delta_{6M}(0, 6M)$ we had:

- FRA over today and FRA over tomorrow;

Today they are not quoted anymore, therefore we need to find a different way to calculate $\Delta_{6M}(0, 6M)$. The only contract that we can use is the index fixing.
After that instruments, to calculate $\Delta_{6M}(t_i, t_i + x)$, we have:

- FRA up to 2 years.

We calculate *ON* discrete forward rates insisting on the same set of dates as the $6M$ market quotes and we calculate the difference between these two values.

Basis Calculation

Example

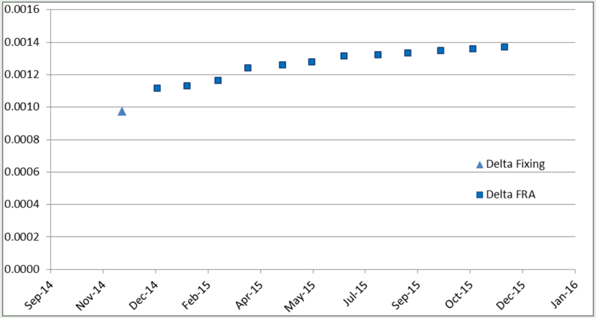

Figure: $\Delta_{6M}(t_i, t_i + 6M) = \ln[1 + F_{6M}(t_i, t_i + 6M)] - \ln[F_{on}(t_i, t_i + 6M)]$

Basis Calculation

Example

$$\Delta_{6M}(t_i, t_i + 6M) = \int_{t_i}^{t_i+6M} \delta_x(s)ds$$

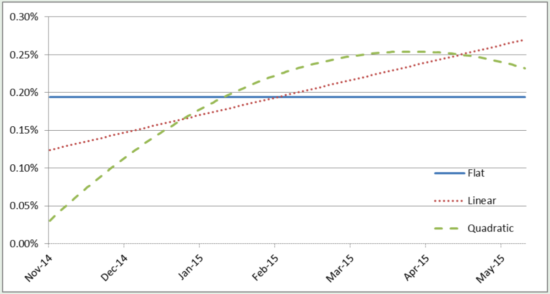

Figure: Continuously compounded basis

Synthetic Deposits Calculation

Example

It is important to note that the integral of $\delta_{6M}(s)$ in the first 6 months it is the same for all of the parametrization used, but with different shape:

$$\Delta_x(0,6M) = \int_0^{6M} \delta_x(s)ds = \begin{cases} \int_0^{6M} \alpha_{6M} ds & \text{flat} \\ \int_0^{6M} (\hat{\alpha}_{6M} + \hat{\beta}_{6M}\tau_s)ds & \text{linear} \\ \int_0^{6M} (\tilde{\alpha}_{6M} + \tilde{\beta}_{6M}\tau_s + \tilde{\gamma}_{6M}\tau_s^2)ds & \text{quad} \end{cases}$$

Synthetic Deposits Calculation

Example

We can build *Synthetic Deposits* for every $0 \leq t \leq 6M$:

$$F_{6M}^{Synth}(0,t) = \frac{[1 + F_{on}(0,t)\tau_{6M}] \cdot e^{\Delta_{6M}(0,t)} - 1}{\tau_{6M}}$$

Usually we construct *Synthetic Deposits* to match the start date of market *FRAs* or Futures.

Example

Not using $1M, 2M, 3M, 4M, 5M, 6M$ *Synthetic Deposits* for the $6M$ curve.

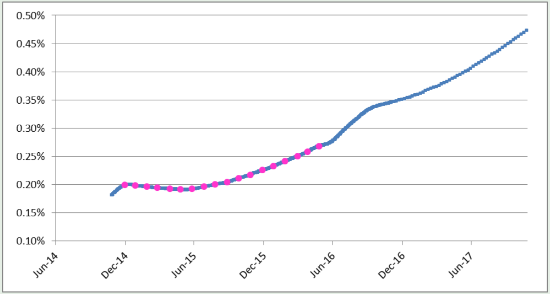

Figure: Forward Euribor 6M Curve without Synthetic Deposits

Example

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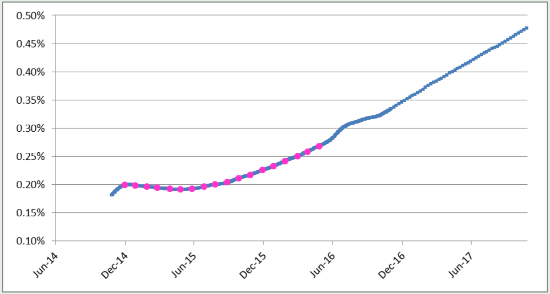

Figure: Forward Euribor 6M Curve with Synthetic Deposits

Outline

Problem

Until about two years ago this approach was satisfactory: it produced smooth forward rates.

Recently it began to show some problems, because of two crucial issues:

- instruments available to calculate the basis, $F_x(t, t+x)$[a]
- shape of the *ON* term structure, $F_{on}(t, t+x)$

[a] *FRA* over today/tomorrow are not available anymore

$$\Delta_x(t, t+x) = \ln[1 + F_x(t, t+x)\tau_x] - \ln[1 + F_{on}(t, t+x)\tau_x]$$

First Problem

The first problem is the correct selection of market instruments used to determine the values of the basis: an illiquid product will produce an incorrect estimation.
E.g.: the index fixing is not an optimal choice. Its value remains constant during the day, insensitive to market changes.

Solution

A solution is to interpolate/extrapolate only on liquid values of $B_x(t_i, t_i + x)$[a], the simply compounded basis; then we calculate the interpolated/extrapolated values of F_x, for the illiquid ones and we use them to determine Δ_x.

[a] $\Delta_x(t_i, t_i + x) \approx B_x(t_i, t_i + x)\tau$ cannot be directly interpolated because τ is not exactly constant (business days adjustment)

Basis Calculation

Example

$$B_{6M}(t_i, t_i + 6M) = F_{6M}(t_i, t_i + 6M) - F_{on}(t_i, t_i + 6M)$$

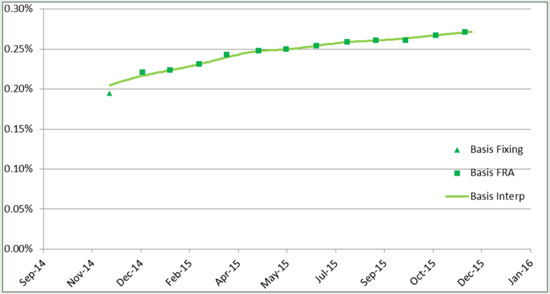

Figure: Interpolated Simply Compounded Basis

Second Problem

The second critical point is the *ON* term structure: to obtain a good approximation of the basis, it is fundamental to build a good *ON* curve.

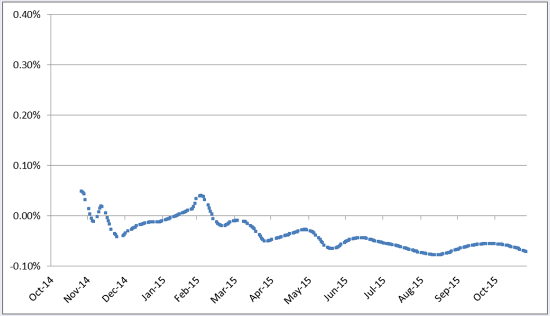

Figure: ON interest rate curve with log-cubic interpolation on Discount factor

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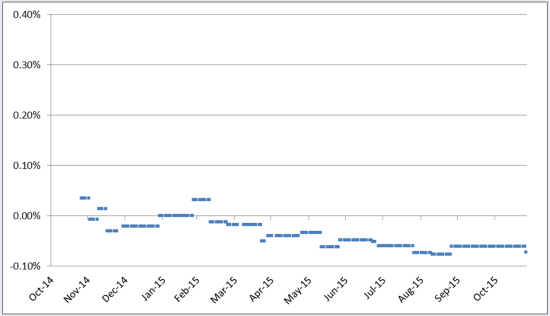

Figure: ON interest rate curve with log-linear interpolation on Discount factor

Outline

How many Jumps?

The *Eonia* fixing has a jump at least at every end of month.

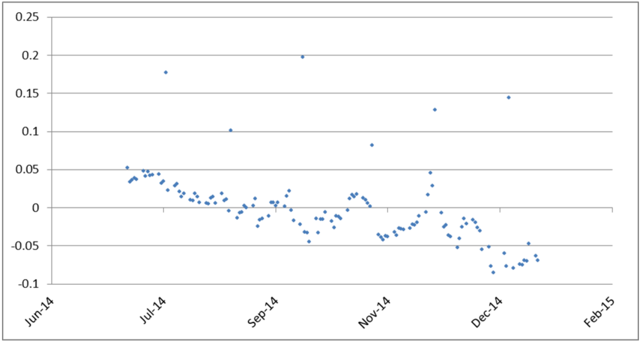

Figure: Eonia fixings, last 6 months

Jump Size estimation

We show how the jump estimation improves the quality of the curve with a concrete example:

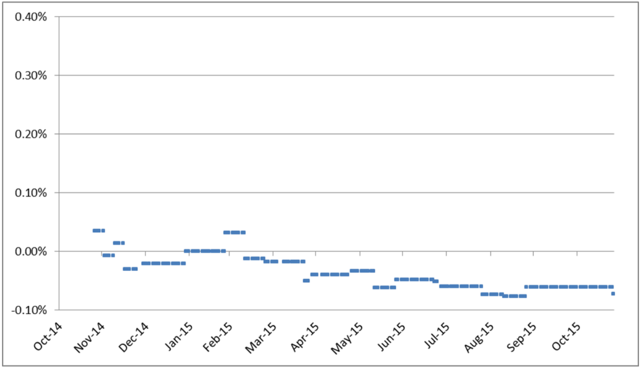

Figure: ON interest rate curve: starting point

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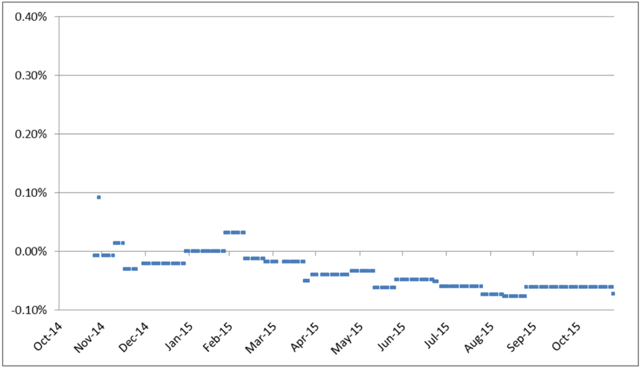

Figure: ON interest rate curve with the first jump

Jumps size estimation

To calculate Jumps' size we can follow an approach similar to the one used by Burghardt [3] to estimate turn of year jumps:

1. Construct an *ON* curve using all liquid market quotes using a flat interpolation on forward rate
2. Estimate the first jump assuming that a segment out of line with preceding and following segments can be put back in line dumping the difference into the jump[1]:

$$[F^{original}(t_1, t_2) - F^{interp}(t_1, t_2)] \cdot \tau(t_1, t_2) = Jump \cdot \tau_{Jump}$$

3. Clean the curve from the jump at point 2

Iterate ad libitum 2 and 3 on next jump date

[1] τ_{Jump} is the year fraction between jump business day and the next business day

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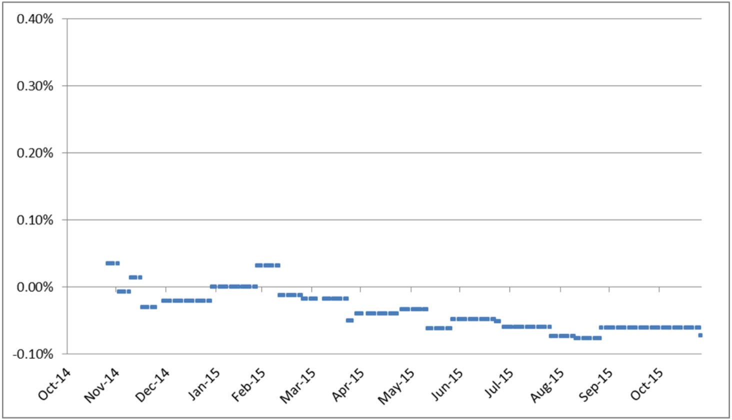

Figure: ON interest rate curve: starting point

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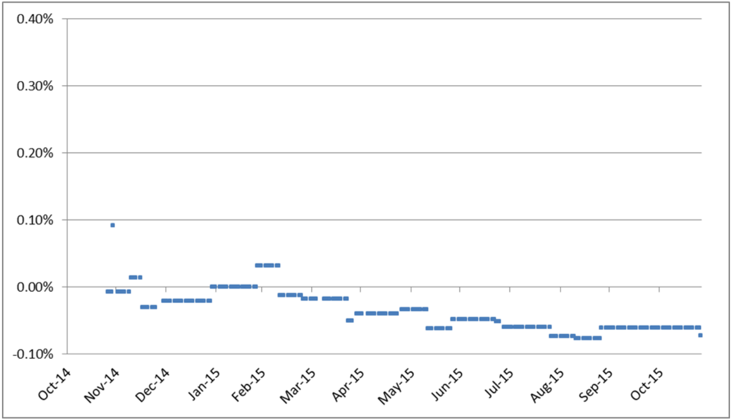


Figure: ON interest rate curve with the first jump

Jump Size estimation

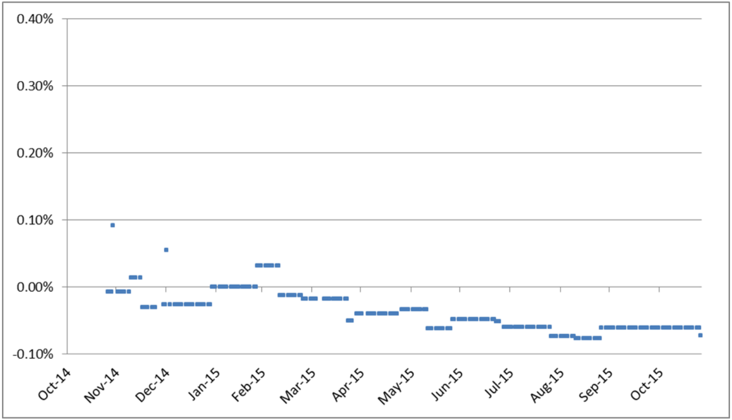


Figure: ON interest rate curve with the second jump

Jump Size estimation

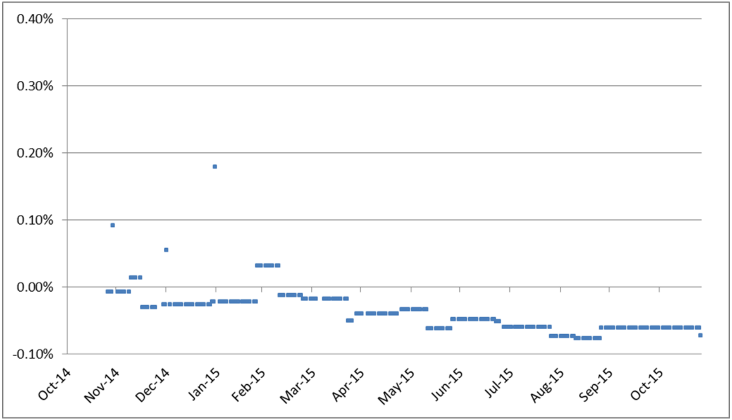


Figure: ON interest rate curve with the third jump

Jump Size estimation

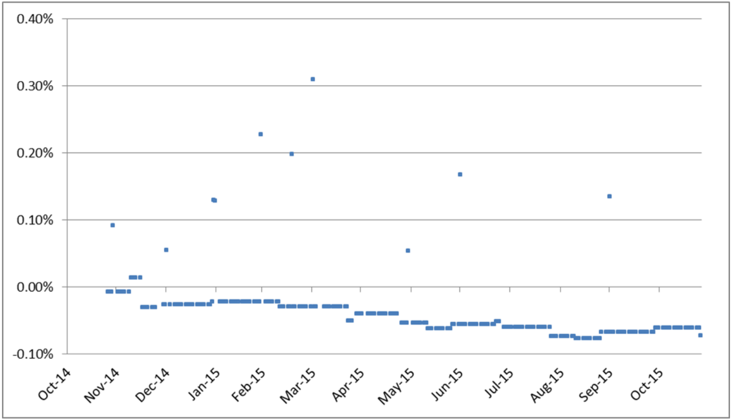


Figure: ON interest rate curve with jumps

Jump Size estimation

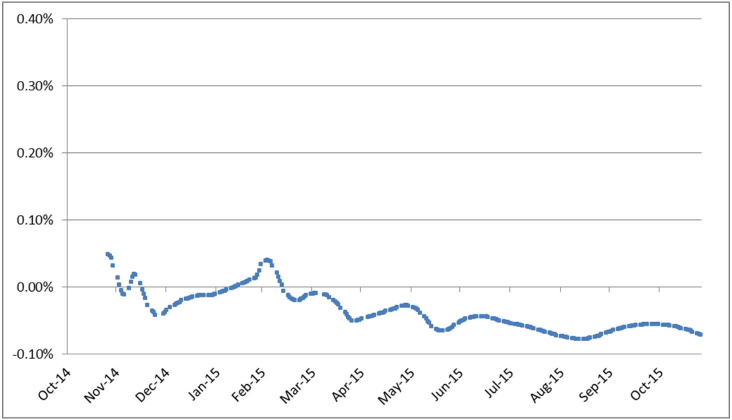


Figure: ON interest rate curve with log-cubic interpolation on Discount factor

Outline

Synthetic Deposits Construction

We had 2 residual problems for construction of *Synthetic Deposits*:

- sub-optimal instruments selection
- wrong *ON* curve calculation

Modelling *ON* jumps we fixed the *ON* curve and obtained reliable $B(0, x)$ to be interpolated/extrapolated.

6-Months Euribor Curve

Example

Interpolating only liquid $B_{6M}(t_i, t_i + 6M)$, i.e. fixing not included, we match the fixing level nonetheless!

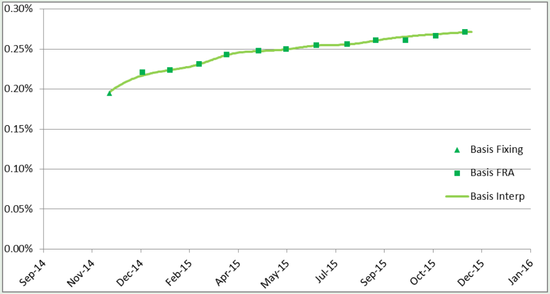


Figure: Interpolated Simply Compounded Basis with jumps

6-Months Euribor Curve

Example

Interpolating only liquid $B_{6M}(t_i, t_i + 6M)$, i.e. fixing not included, we match the fixing level nonetheless!

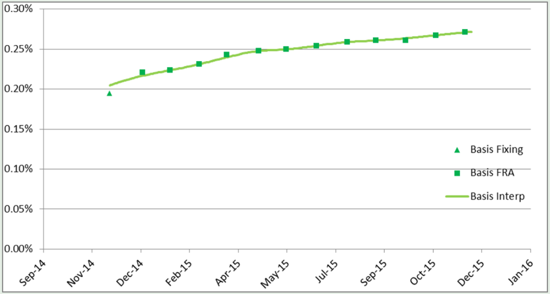


Figure: Interpolated Simply Compounded Basis without jumps

6-Months Euribor Curve

Example

In order to estimate $\delta_{6M}(t)$ we use:

- flat parametrization:
  $\Delta_{6M}(0,6) = \ln[1 + F_{6M}^{extrap}(0,6)\cdot\tau] - \ln[1 + F_{ON}(0,6)\cdot\tau]$
- linear parametrization:

$$\begin{cases} \Delta_{6M}(0,6) = \ln[1 + F_{6M}^{extrap}(0,6)\cdot\tau] - \ln[1 + F_{ON}(0,6)\cdot\tau] \\ \Delta_{6M}(1,7) = \ln[1 + F_{6M}^{interp}(1,7)\cdot\tau] - \ln[1 + F_{ON}(1,7)\cdot\tau] \end{cases}$$

- quadratic parametrization:

$$\begin{cases} \Delta_{6M}(0,6) = \ln[1 + F_{6M}^{extrap}(0,6)\cdot\tau] - \ln[1 + F_{ON}(0,6)\cdot\tau] \\ \Delta_{6M}(1,7) = \ln[1 + F_{6M}^{interp}(1,7)\cdot\tau] - \ln[1 + F_{ON}(1,7)\cdot\tau] \\ \Delta_{6M}(2,8) = \ln[1 + F_{6M}^{interp}(2,8)\cdot\tau] - \ln[1 + F_{ON}(2,8)\cdot\tau] \end{cases}$$

6-Months Euribor Curve

Example

$\delta_{6M}(t)$ using index fixing:

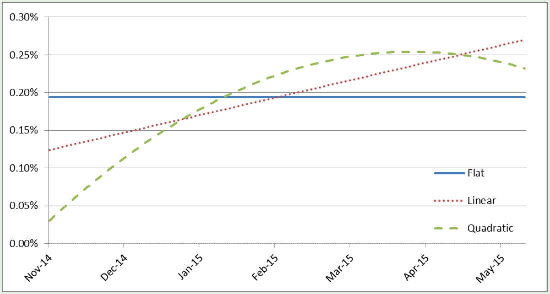


Figure: Continuously Compounded Basis $\delta_{6M}(t)$ for $0 \leq t \leq 6M$

6-Months Euribor Curve

Example

$\delta_{6M}(t)$ extrapolating B_{6M}, instead of index fixing:

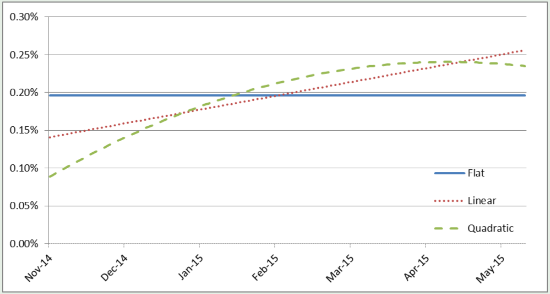


Figure: Continuously Compounded Basis $\delta_{6M}(t)$ for $0 \leq t \leq 6M$

3-Months Euribor Curve

Example

Interpolating only liquid $B_{3M}(t_i, t_i + 3M)$, i.e. fixing not included, we match the fixing level nonetheless!

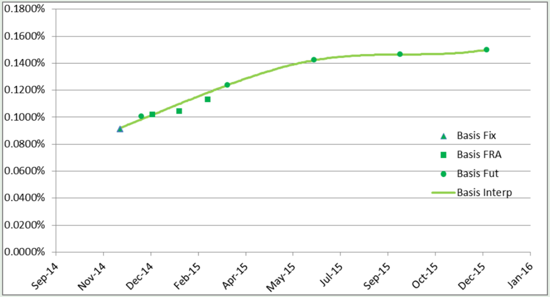


Figure: Interpolated Simply Compounded Basis

3-Months Euribor Curve

Example

In order to estimate $\delta_{3M}(t)$ we use:

- flat parametrization:
  $\Delta_{3M}(0,3) = \ln[1 + F_{3M}^{extrap}(0,3) \cdot \tau] - \ln[1 + F_{ON}(0,3) \cdot \tau]$
- linear parametrization:

$$\begin{cases} \Delta_{3M}(0,3) = \ln[1 + F_{3M}^{extrap}(0,3) \cdot \tau] - \ln[1 + F_{ON}(0,3) \cdot \tau] \\ \Delta_{3M}(1,4) = \ln[1 + F_{3M}^{interp}(1,4) \cdot \tau] - \ln[1 + F_{ON}(1,4) \cdot \tau] \end{cases}$$

- quadratic parametrization:

$$\begin{cases} \Delta_{3M}(0,3) = \ln[1 + F_{3M}^{extrap}(0,3) \cdot \tau] - \ln[1 + F_{ON}(0,3) \cdot \tau] \\ \Delta_{3M}(1,4) = \ln[1 + F_{3M}^{interp}(1,4) \cdot \tau] - \ln[1 + F_{ON}(1,4) \cdot \tau] \\ \Delta_{3M}(2,5) = \ln[1 + F_{3M}^{interp}(2,5) \cdot \tau] - \ln[1 + F_{ON}(2,5) \cdot \tau] \end{cases}$$

3-Months Euribor Curve

Example

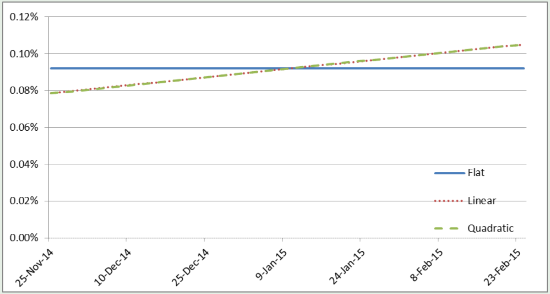


Figure: Continuously Compounded Basis $\delta_{3M}(t)$ for $0 \leq t \leq 3M$

6-Months Euribor Curve

Example

Forward Euribor 6M without synthetic deposits[a]:

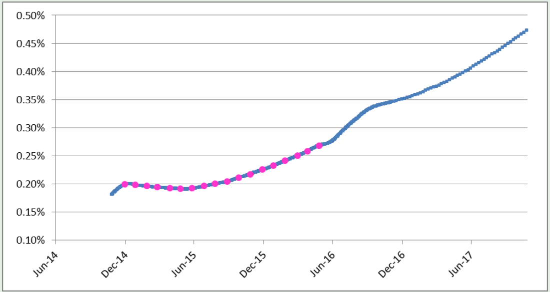


Figure: Forward Euribor 6M Curve

[a] using index fixing

6-Months Euribor Curve

Example

Forward Euribor 6M with synthetic deposits[a]:

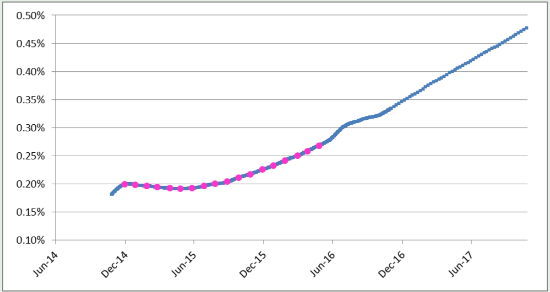


Figure: Forward Euribor 6M Curve

[a] using index fixing and without ON jumps

6-Months Euribor Curve

Example

Forward Euribor 6M with synthetic deposits[a]:

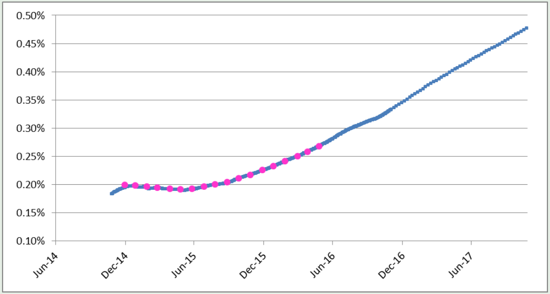


Figure: Forward Euribor 6M Curve

[a] without index fixing, with correct instruments selection and ON jumps

6-Months Euribor Curve

Example

Instantaneous Forward Euribor 6M without synthetic deposits[a]:

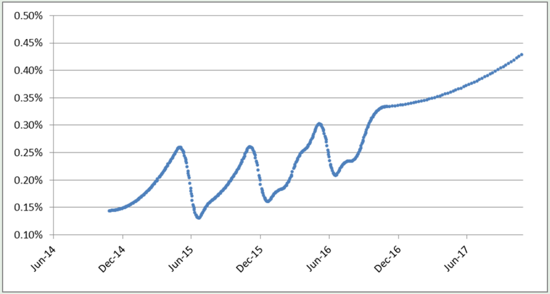


Figure: Instantaneous Forward Rate on Euribor 6M Curve

[a] using index fixing

6-Months Euribor Curve

Example

Instantaneous Forward Euribor 6M with synthetic deposits[a]:

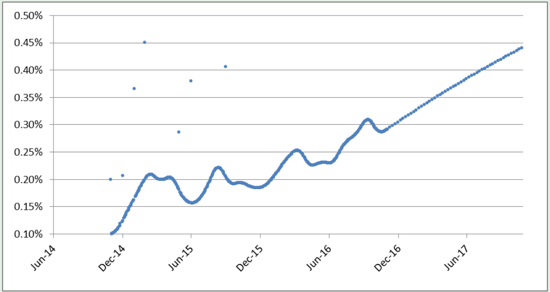


Figure: Instantaneous Forward Rate on Euribor 6M Curve

[a] without index fixing, with correct instruments selection and ON jumps

3-Months Euribor Curve

Example

Forward Euribor 3M without synthetic deposits[a]:

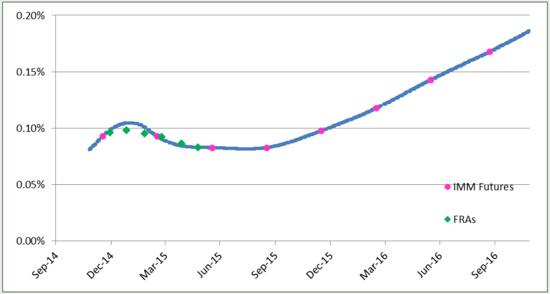


Figure: Forward Euribor 3M Curve

[a] using index fixing

3-Months Euribor Curve

Example

Forward Euribor 3M with synthetic deposits[a]:

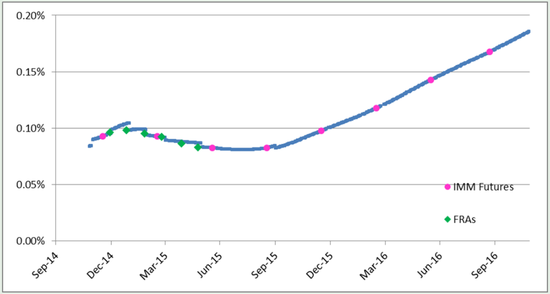


Figure: Forward Euribor 3M Curve

[a] without index fixing, with correct instruments selection and ON jumps

3-Months Euribor Curve

Example

Instantaneous Forward Euribor 3M without synthetic deposits[a]:

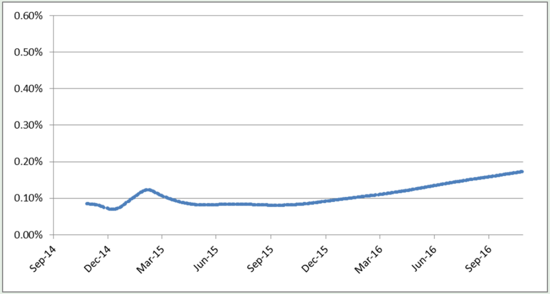


Figure: Instantaneous Forward Rate on Euribor 3M Curve

[a] using index fixing

3-Months Euribor Curve

Example

Instantaneous Forward Euribor 3M with synthetic deposits[a]:

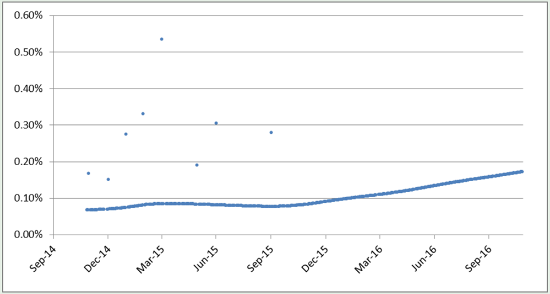
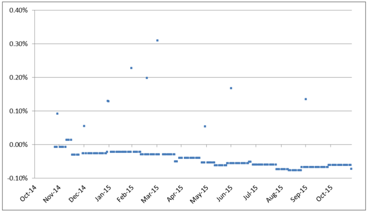


Figure: Instantaneous Forward Rate on Euribor 3M Curve

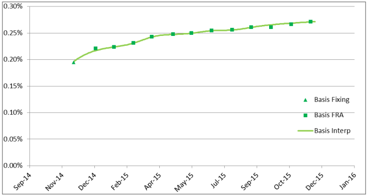
[a] without index fixing, with correct instruments selection and ON jumps

Conclusion

- *ON* curve jumps must be taken into account



- the smoothness of the basis, not of the forward rates, is the relevant factor



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