Rate Curves Framework

Banca IMI - Quantitative Structuring

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Contents

Introduction

This work has the purpose to provide a wide description about multiple IR curves modelling and their calibration using QuantLibXL.
Chapter 1 is devoted to introduce what a IR curve is, why it is important to model a interest rate term structure and to describe the methodology behind the rate curves construction. Therefore, many important points will be treated like:

- Exogenous Vs. Endogenous Discounting
- Curve Parametrization
- Best Fit Vs. Exact Fit Algorithm
- Interpolation Techniques
- Instruments Selection Criteria
- Synthetic Instruments
- First Order Sensitivities

Chapter 2 is focused on curve calibration using QuantLibXL with a particular emphasis on object building and bootstrap functions using some practical Excel examples in order to give to the user a step-by-step guideline for curves calibration in QuantLibXL.
Chapter 3 presents the so called Rate Curve Framework explaining how access it and its structure; here the reader can find a user guide for each workbook making up the framework which was born after 2007 crisis in order to integrate official systems with new functionalities (i.e. multi-curve and exogenous ois discounting) which at that time were not supported yet. Today the implementation of new curve construction models inside front office systems is not straightforward and the Excel framework, being more flexible and allowing faster changhes, is still a very useful tool.
Finally, **Chapter 4** is devoted to the Banca IMI's Framework implementation in which is described the intern practice in terms of:

- Data Provider and Front Office System;
- Framework infrastructure;
- Reference date;

- Instruments selection;
- Curve parametrization;
- Interpolation technique;
- Synthetic deposits use.

Chapter 1

Yield Curve Methodology

This chapter aims to expose some of the fundamental methodologies which represent the interest rate curve modelling base. For more details you can refer to the bibliography.

1.1 What is an Interest Rate (IR) Curve and why we need it

A IR Curve, or Term Structure of Interest Rates, could be defined as the relationship between Zero Rates and times but we will see that this is only one of the possible definitions. Therefore, a IR curve is the graph of the function mapping maturities into zero rates at time t. The plotted line begins with the spot zero rate, which is the rate for the shortest maturity, and extends out in time, typically to 60 years. A n-year zero coupon rate (or zero rate) is the interest rate related to an investment that starts today and ends n-year after without paying intermediate coupons.
Rate curve can have different kind of shapes, sometimes they are upward sloping (normal IR curve), sometimes are downward sloping (inverted IR curve) and other times they change their behaviour across maturities becoming partly upward and partly downward sloping (humped IR curve) as visible in Figure 1.1. To explain this phenomenon different theories have been proposed. one of this is called *liquidity preference theory* and its underlying basic assumption is that investors prefer to preserve their liquidity and invest funds for short period of time. Borrowers, on the other hand, usually prefer to borrow at fixed rates for long periods of time and this leads to a situation in which forward rates are greater than expected future zero rates.
Another doctrine named *market segmentation theory* conjectures that there is no relationship between different tenor interest rates. Under this theory, a major investor like a large pension fund invests in certain maturity instruments and does not readily switch from one maturity to another; for this reason the short-term part of the interest term structure is determined by supply and demand in the short-term instruments market, medium-term part by supply demand in the medium-term market and so on.
Finally, the most simplest and appealing theory is *expectations theory* which describes

long-term rates as the expected future value of the short-term rates. In particular, this theory argue that a forward interest rate corresponding to a certain future period is equal to the expected future zero rate for that period; obviously today's expected value may diverge from future interest rate fixing due to market volatility.
Typically IR curve's modelling is based on choosing a parametric form in conjunction with an interpolation scheme. The parameters will be chosen such as to reprice a set of liquid market quotes with a sufficient good precision. A very often used calibration algorithm builds rate curves cycling in iterative fashions in order to reach an almost perfect repricing for the whole set of input market quotes where by quote we mean any number that is used by the market to indicate the level of a financial instrument: rate, price and so on. This particular kind of calibration process is called bootstrap. Even if there are a lot of liquid instruments, we don't have an instrument for each maturity date and his is the reason way an "interpolation" method has to be used to obtain the entire term structure.
Pricing all interest rate derivatives and any other market instrument which value depends on interest rates requires modelling the future dynamics of the entire IR curve and so it is very important to find the best way to construct it, since an incorrect term structure will fail to produce good prices. It is reasonable to say that calibrating multiple curves is the backbone of interest rate derivatives relative pricing where relative pricing means valuing non-quoted market instruments.

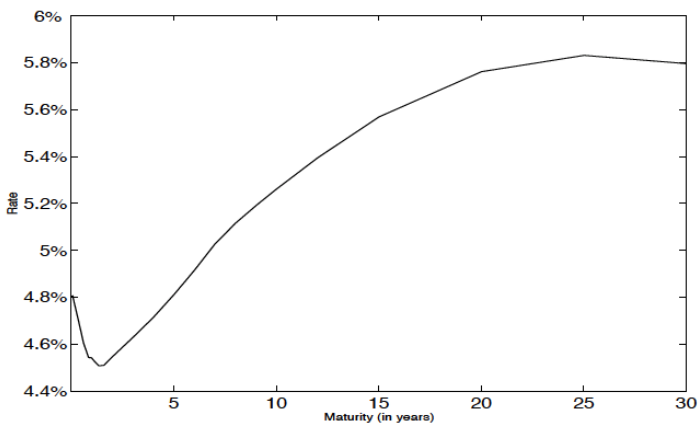


Figure 1.1: An Example of humped IR curve on EUR market.

1.2 Multiple Forwarding Curves and One Discounting Curve

Since the 2007 crisis and the resulting rates spreads discrepancy it becomes necessary to calibrate,for each currency, multiple Forwarding Curves linked to a specific underlying index of a given tenor. For this reason the Rate Curve Framework, as we will see in **Chapter 3**, is a Multi-Curve Framework. For example, in the EUR market case, we have five curves C_{ON},C_{1M},C_{3M},C_{6M},C_{12M} with underlying indexes Eonia, Euribor 1M, Euribor 3M, Euribor 6M, Euribor 12M respectively. Curves reference date is the start date of the spot calibrating instruments (forwarding has no meaning before this date). Discount factors entering into the pricing formulas of calibrating instruments could be taken

- from a predetermined discounting curve (*exogenous discounting*): discount factors are an input of the calibration process as well as market quotes;

- from the forwarding curve itself (*endogenous discounting*): discount factors are an output of the calibration process.

Discounting curve reference date is today because we want present values referring to today. At present, all relevant calibrating instruments traded on regulated markets (like Futures) or collateralized with daily margining (like FRAs, Swaps and Basis Swaps) settle their positions on collateral accounts which earn the overnight rate. This is why the ON Curve usually plays the role of Discounting Curve (as well as the role of Forwarding Curve for its specific tenor). Once calibrated, all these curves are used to price and hedge collateralized products. Before the basis explosion in 2007 summer and the obligation to clear many classes of products, all derivatives was discounted using a single curve, named Standard curve, which was built using liquid instruments of different tenors. The old Standard curve still plays the role of Discounting Curve for non-collateralized products and this is way inside our framework is defined also one Standard Curve for each currency. Obviously this is an approximation because non-collateralized products should be priced taking into account counterparty credit risk and funding costs/benefits based on the investor Funding Rate which can't be modeled and is very difficult to estimate. Even in this case all the Forwarding Curves are calibrated with exogenous OIS discounting (and not Standard Discounting)[1].

1.3 Curves Description

1.3.1 Discounts, Zero Rates, Forward Rates

In order to present the available descriptions for our curves we have to take a step back to a simplified framework where we only consider one curve related to a "risk-free". The general idea of the one-curve framework is that all interest rate derivatives depend on only one curve, usually described in terms of zero-coupon bond prices P(t,s). If t is our reference date, $P(t;s)$ for a maturity $s \geq t$ is the price in t of an instrument which pays 1 unit of currency in s. Starting from discount factors you can define simple zero rates, m-times-per-year compounded zero rates (with frequency m) and continuously

[1] for more details about multi-curves foundations you can refer, (for example), to [2] and [4].

compounded zero rates via equations

$$P(t;s) = \frac{1}{1 + z_s(t;s)\tau(t,s)} \tag{1.1}$$

$$P(t;s) = \left(\frac{1}{1 + z_c(t;s)/m}\right)^{m\tau(t,s)} \tag{1.2}$$

$$P(t;s) = e^{-z(t;s)\tau(t,s)} \tag{1.3}$$

Typical market instruments and products are defined on simple compounded rates and m-times-per-year compounded rates; they are quoted by the market (directly or indirectly) and the corresponding day count convention τ is also defined by the market. Continuously compounded rates, instead, are only a mathematical abstraction (notice that they can be defined as the limit of compounded rates when $m \to \infty$); usually in this case τ is a specific strictly monotone day count convention (for example $Act/365$) in order to ensure that every date s is mapped to a unique time $\tau(t,s)$. We now define different types of forward rates. In order to do this, we have to fix two futures dates s_1, s_2 with $t < s_1 < s_2$ and a day count convention τ.

Continuous Forward Rates

A *continuous forward rate* $F_c(t;s_1,s_2)$ is defined as the future zero rate implied by today (unique) term structure of zero rates assuming continuous compounding. Taking a deterministic example we have:

$$e^{z(t;s_1)\tau(t,s_1)} e^{F_c(t;s_1,s_2)\tau(s_1,s_2)} = e^{z(t;s_2)\tau(t,s_2)} \tag{1.4}$$

Equivalently:

$$\begin{aligned} F_c(t;s_1,s_2) &= \frac{z(t;s_2)\tau(t,s_2) - z(t;s_1)\tau(t,s_1)}{\tau(s_1,s_2)} \\ &\overset{1.3}{=} \frac{1}{\tau(s_1,s_2)} \ln\left(\frac{P(t;s_1)}{P(t;s_2)}\right) \end{aligned} \tag{1.5}$$

that defines F_c in terms of continuous compounded zero rates and discount factors.

Simple Forward Rates

Similarly, a *simple forward rate* $F(t;s_1,s_2)$ is defined by

$$(1 + z_s(t;s_1)\tau(t,s_1))(1 + F(t;s_1,s_2)\tau(s_1,s_2)) = 1 + z_s(t;s_2)\tau(t,s_2) \tag{1.6}$$

Using equations 1.1 and 1.3 we have

$$F(t;s_1,s_2) = \frac{1}{\tau(s_1,s_2)}\left(\frac{P(t;s_1)}{P(t;s_2)} - 1\right) = \frac{1}{\tau(s_1,s_2)}\left(e^{z(t;s_2)\tau(t,s_2) - z(t;s_1)\tau(t,s_1)} - 1\right) \tag{1.7}$$

that defines F in terms of discount factors and continuous compounded zero rates.

Instantaneous Forward Rates

An *instantaneous forward rate* is the continuous forward rate that applies for an infinitesimal period. It is defined by the following equation

$$f(t,s) := \lim_{s_2 \to s_1} F_c(t; s_1, s_2) \overset{1.5}{=} \frac{d}{ds}\left(z(t;s)\tau(t,s)\right) \tag{1.8}$$

By integrating

$$\int_t^s f(t;u)du = z(t;s)\tau(t,s) \tag{1.9}$$

and also

$$\int_{s_1}^{s_2} f(t;u)du = z(t;s_2)\tau(t,s_2) - z(t;s_1)\tau(t,s_1) \tag{1.10}$$

From equations 1.5 and 1.10 we have

$$F_c(t; s_1, s_2) = \frac{1}{\tau(s_1, s_2)}\left(\int_{s_1}^{s_2} f(t;u)du\right) \tag{1.11}$$

which shows that the average of the instantaneous forward rate over any interval $[s_1, s_2]$ is equal to the continuous forward rate for that interval. From equations 1.7 and 1.10 we have

$$F(t; s_1, s_2) = \frac{1}{\tau(s_1, s_2)}\left(e^{\int_{s_1}^{s_2} f(t;u)du} - 1\right) \tag{1.12}$$

which shows that simple forward rates are a continuous function of integrated instantaneous forward rates. We have finally obtained the following important relationship between discount factors, continuously compounded zero rates and instantaneous forward rates

$$P(t;s) = e^{-z(t;s)\tau(t,s)} = e^{-\int_t^s f(t;u)du} \tag{1.13}$$

or equivalently

$$-\ln P(t;s) = z(t;s)\tau(t,s) = \int_t^s f(t;u)du \tag{1.14}$$

So we have at least three possible ways to describe our unique curve: through discount factors, through continuous compounded zero rates and through instantaneous forward rates. In formulas

$$\begin{aligned} C(t) &= \{s \to P(t;s) | s \geq t\} \\ C(t) &= \{s \to z(t;s) | s > t\} \\ C(t) &= \{s \to f(t;s) | s > t\} \end{aligned}$$

The three descriptions are almost equivalent; the only difference is that only discount factors are well defined when $s = t$ and we have $P(t;t) = 1$. We now come back to our multi-curve framework.

1.3.2 Reasonable description through forward rates

Each rate curve C_x of the framework is a forwarding curve linked to a specific tenor x. The most natural description should be the one that models directly forward rates which, in most cases, are directly quoted by the market (unlike, for example, of discount factors). Forward rates directly quoted by the market through FRA, Futures and indirectly through Interest Rates Swaps and Basis Swaps, are simple forward rates corresponding to a triplet $(s, s+x, \tau_x)$ where $s \geq t$ and τ_x is the specific day count convention related to the underlying index of each instrument (example: Euribor xM). So our curve C_x should be described by

$$C_x(t) = \{s \to F_x(t; s, s+x) | s \geq t\} \tag{1.15}$$

Unfortunately, this is not the commonly used approach.

1.3.3 Classical description through pseudo-discount factors

Definitions of equations 1.7, 1.12 and 1.13 can be extended to multi-curve Framework as follows:

$$F_x(t; s_1, s_2) = \frac{1}{\tau(s_1, s_2)} \left(\frac{P_x(t; s_1)}{P_x(t; s_2)} - 1 \right) = \frac{1}{\tau(s_1, s_2)} \left(e^{\int_{s_1}^{s_2} f_x(t;u)du} - 1 \right) \tag{1.16}$$

$$P_x(t; s) = e^{-z_x(t;s)\tau(t,s)} = e^{-\int_t^s f_x(t;u)du} \tag{1.17}$$

Otherwise the only real quantities quoted directly or indirectly by the market are, as said before, the simple forward rates $F_x(t; s, s+x)$ which thus satisfies

$$\begin{aligned} F_x(t; s, s+x) &= \frac{1}{\tau_x(s, s+x)} \left(\frac{P_x(t; s)}{P_x(t; s+x)} - 1 \right) \\ &= \frac{1}{\tau_x(s, s+x)} \left(e^{\int_s^{s+x} f_x(t;u)du} - 1 \right) \end{aligned} \tag{1.18}$$

Last equation is really similar to equation 1.7 but the substance of these new relation is totally different. More precisely

- single-curve equation is the result obtained from no arbitrage condition;
- multi-curve equation is merely the definition of a *pseudo-discount factors* function $P_x(t; s)$.

To clarify this concept we can make the following example. In the old single-curve world, before 2007 crisis, in the EUR market we could calculate the forward rate $F(t; t+3M, t+6M)$ starting from the values $P(t; t+3M)$ and $P(t; t+6M)$ deduced from Euribor $3M$ and Euribor $6M$ fixings with a no arbitrage condition; the resulting value was in line with the market quote of the $3X6$ FRA. With the large Basis Swap spreads presently quoted on the market this relation is no more valid: if we calculate the $3X6$ FRA implicit in Euribor 6M and Euribor 3M fixings (as explained above) we do not obtain the $3X6$

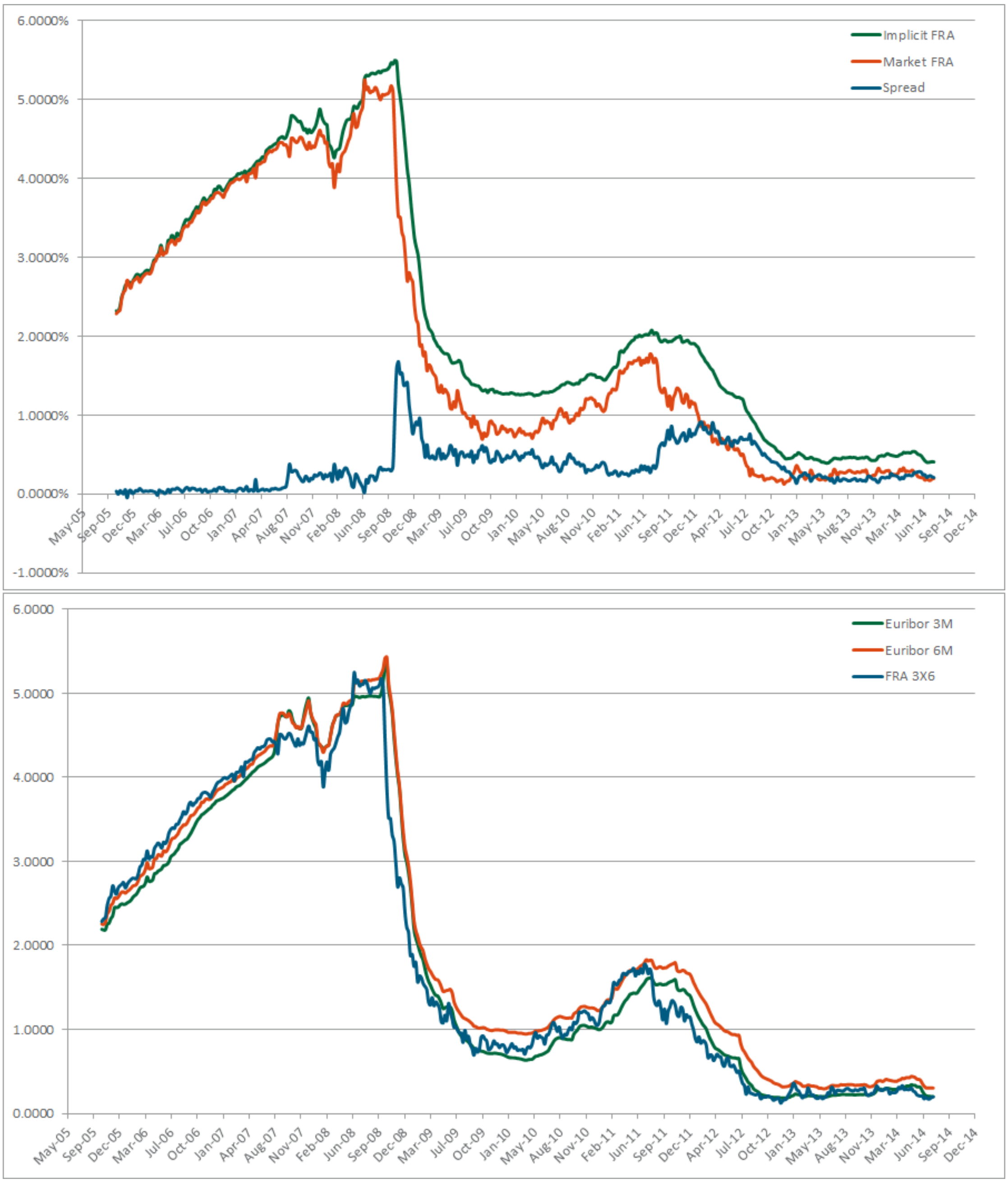


Figure 1.2: After summer 2007 the $3X6$ FRA implicit in Euribor 6M and Euribor 3M fixings it was no longer consistent with the $3X6$ FRA quoted by the market. The large Basis Swap spread observed were an evidence of the need of multiple curves, one for each tenor.

FRA quoted by the market (see Figure 1.2). We have to define two synthetic quantities $P_{3M}(t; t+3M)$ and $P_{3M}(t; t+6M)$ to make the old relationship still valid.

Looking at equation 1.18 we notice that it is always possible to calculate a simple forward rates curve $\{s \to F_x(t; s, s+x)|s \geq t\}$ if a pseudo-discount factors curve $\{s \to P_x(t; s)|s \geq t\}$ is given but this is not our case. Conversely, it is not possible to deduce a unique pseudo-discount factors curve starting form a simple forward rates curve and this is due to the arbitrariness of the equation itself: a simple forward rate defines the ratio of two pseudo-discount factors and not a unique pseudo-discount factor. In this case, to define uniquely pseudo-discount factors we have to add other conditions, for example fixing their shape on the first x-interval $[t, Spot(today)+x)$ (remember that the reference date t is today or the spot date referring to today). If we know the function $\{s \to P_x(t; s)|t \leq s \leq Spot(today)+x\}$ we can deduce the whole structure via equation 1.18.

The discussion above has proved that for each forward curve C_x we have the following alternative (almost equivalent) descriptions:

$$C_x(t) = \{s \to P_x(t; s)|s \geq t\} \tag{1.19}$$

$$C_x(t) = \{s \to z_x(t; s)|s > t\} \tag{1.20}$$

$$C_x(t) = \{s \to f_x(t; s)|s > t\} \tag{1.21}$$

The advantage of using one of the descriptions based on P, z or f (compared to a direct forward rates description) is that it can be used to calibrate forward curves inside legacy systems, which are mainly based on pseudo-discount curves. On the other hand the drawbacks are obvious. First of all, you are not modelling directly the market quantities, the ones on which we have some intuition (the forward rates). Secondly, as said before, market quotes give informations about Forward rates, whereas to determine the function $P_x(t, s)$ we need to arbitrarily fix a part of length x. When $x = ON$ this arbitrary part is really small (few days) and it can be fixed with some market quotes. So in this case we can use the pseudo-discount factors description without problems at all; also this description is consistent with the role of C_{ON} as discount curve of the framework (in this case pseudo-discount factors are real discount factors!). For the other tenors we will have to introduce synthetic deposits in order to manage this arbitrariness.

1.4 Algorithms: Best Fit vs Exact Fit

There are two main classes of curves construction algorithms. The common feature is that both classes require a set of N pre-selected market instruments.

A *best-fit algorithm* assumes a functional form for the curve C_x and calibrates its parameters using the pre-selected instruments. It is very popular due to the smoothness of the curve, calibration easiness and intuitive financial interpretation of functional form parameters but it has a big drawback: usually there are more instruments than parameters so that the result of the calibration is not a perfect reprice of the whole set of instruments but a minimization of the repricing error. This is way the quality of a best-fit is not good enough for trading purposes in liquid markets, where a basis point quarter can make the difference.

Exact-fit algorithms, instead, fix the IR curve on a time grid of N points (or *pillars*) in order to exactly reprice the selected instruments. An interpolation method is required to determine curve values between pillars. Usually the algorithm is incremental, building the curve step-by-step with the increasing maturity of the ordered instruments (*bootstrap* approach).

The interpolation method is intimately connected to the bootstrap and plays a fundamental role also during bootstrapping, not just after that. The calibration proceeds with incomplete information: in order to determine each pillar value, the already bootstrapped part of the curve has to be used, not only pillar values but also intermediate interpolated ones (see [8] for a practical example of this interaction between interpolation and bootstrap).

Usually the set of bootstrapping instruments defines the time grid used for bootstrapping: each pillar is the maturity date or the last relevant date of the corresponding instrument. But this is not the only available choice: one can define a time grid of N *custom dates* providing that each of these dates is earlier than the last relevant date of the corresponding instrument.

Henceforward we restrict ourselves to the exact curve calibration problem.

1.5 Interpolation

When we speak about interpolation we mean two distinct objects:

- the quantity to be interpolated;
- the interpolation scheme.

The quantity to be interpolated is chosen according to curve's description. If we use a description through forward rates, the most reasonable quantity to be interpolated is the forward rate itself: in this way interpolation schemes and constraints can be imposed directly on market quantities. If we have to use a description through pseudo-discount factors we have the following main choices: discount factors, zero rates, instantaneous forward rates. Another choice could be the logarithm of discount factor, that is the product between zero rate and time (see equation 1.17).

Once the quantity to be interpolated is chosen, an interpolation scheme must be used to estimate curve values between pillars. There are two big classes of schemes:

- local schemes: each intermediate point depends only on its bracketing pillars;
- non-local (or global) schemes: each intermediate point depends on points outside the interval defined by its neighborhoods pillars.

A typical example of local scheme is the linear one; cubic splines are examples of global interpolations. Notice that when bootstrapping with a local interpolation the curve's shape between two calibrated pillars can no longer change. If the interpolation method is global, the curve changes continuously until the end of the procedure because the shape of the part of the curve already bootstrapped is altered by the addition of further pillars. In this case, after a first bootstrap which might even use a local interpolation scheme, the resulting complete grid is altered one pillar at time until convergence is reached with a given precision. The first cycle can be even replaced by a good grid guess, for example the grid previous state. Also, with a global interpolation, the computational performance of bootstrapping algorithm is slowed down, with the most important effects observed during sensitivities calculation (for more information about the Quantlib optimization process refers to [9]).

The criteria used to analyse the interpolation techniques usually fall into one of the following two categories:

- the quality of the forward rates: in this case we are looking at the curve as an "accounting" tool and we are trying to answer the question "How good the forward rates look?";
- the quality of the implied hedging strategies: in this case we are looking at the curve as a "risk management" tool and we are trying to answer the question "Are the implied hedging quantities reasonable and stable?"

Sometimes an interpolation method is good to build forward rates but not good from an hedging point of view. It is not impossible to use different interpolation schemes, one for each function, but this creates inconsistencies and is not advisable. The only way is to find a compromise.

We now describe some interpolation techniques (that is, quantity to be interpolated plus an interpolation scheme) trying to analyse the first aspect, that is the quality of discrete forward rates; hedging strategies will be discussed in a separate section. Since each simple forward rate is approximately the integral of instantaneous forward rates (remember equation 1.16), we will analyze the smoothness of simple forward rates computing instantaneous forward rates via equation 1.8. The starting point is a set of known curve's points $\{(t_i, y_i = y(t_i))\}_{i=1,\dots,N}$ where y represent the quantity to be interpolated as defined above. t_0 will be the reference date.

1.5.1 Linear Interpolation

For $t \in [t_{i-1}, t_i]$ the interpolation formula is

$$y(t) = \frac{t - t_{i-1}}{t_i - t_{i-1}} y(t_i) + \frac{t_i - t}{t_i - t_{i-1}} y(t_{i-1})$$

On Zero Rates

Let $y(t) = z(t_0; t) \equiv z(t)$. Since $z(t)$ is piecewise linear also $f(t_0, t) \equiv f(t)$ is piecewise linear because it is the derivative of the piecewise quadratic function $z(t)t$. In formulas

$$z(t)t = \frac{t - t_{i-1}}{t_i - t_{i-1}} z(t_i)t + \frac{t_i - t}{t_i - t_{i-1}} z(t_{i-1})t$$

and

$$f(t) = \frac{d}{dt}z(t)t = \frac{2t - t_{i-1}}{t_i - t_{i-1}}z(t_i) + \frac{t_i - 2t}{t_i - t_{i-1}}z(t_{i-1})$$

But there is a difference. While $z(t)$ is C^0 on $[t_0, t_N]$, this is not the case for $f(t)$ that presents a jump at each node; in fact values

$$f(t_i^+) = \lim_{t \to t_i}\left(\frac{2t - t_{i-1}}{t_i - t_{i-1}}z(t_i) + \frac{t_i - 2t}{t_i - t_{i-1}}z(t_{i-1})\right) = \frac{2t_i - t_{i-1}}{t_i - t_{i-1}}z(t_i) - \frac{t_i}{t_i - t_{i-1}}z(t_{i-1})$$

and

$$f(t_i^-) = \lim_{t \to t_i}\left(\frac{2t - t_i}{t_{i+1} - t_i}z(t_{i+1}) + \frac{t_{i+1} - 2t}{t_{i+1} - t_i}z(t_i)\right) = -\frac{t_i}{t_{i+1} - t_i}z(t_{i+1}) + \frac{t_{i+1} - 2t_i}{t_{i+1} - t_i}z(t_i)$$

are different. Simple forward rates are obtained integrating $f(t)$ on a "rolling" interval of length x (see equation 1.16); thus they are smoother than instantaneous forward rates. In particular, $F_x(t; s, s + x)$ is C^0 with a "sawtooth" shape (see Figure 1.3: we use daily forward rates as best proxy for instantaneous forward rates). This is clearly not the best interpolation method because the resulting forward rates have an improbable shape.

On Log of Discount Factors

If $y(t) = \ln P(t_0; t) \equiv \ln P(t)$ we have

$$z(t)t = -\ln P(t) = -\frac{t - t_{i-1}}{t_i - t_{i-1}}\ln P(t_i) - \frac{t_i - t}{t_i - t_{i-1}}\ln P(t_{i-1})$$

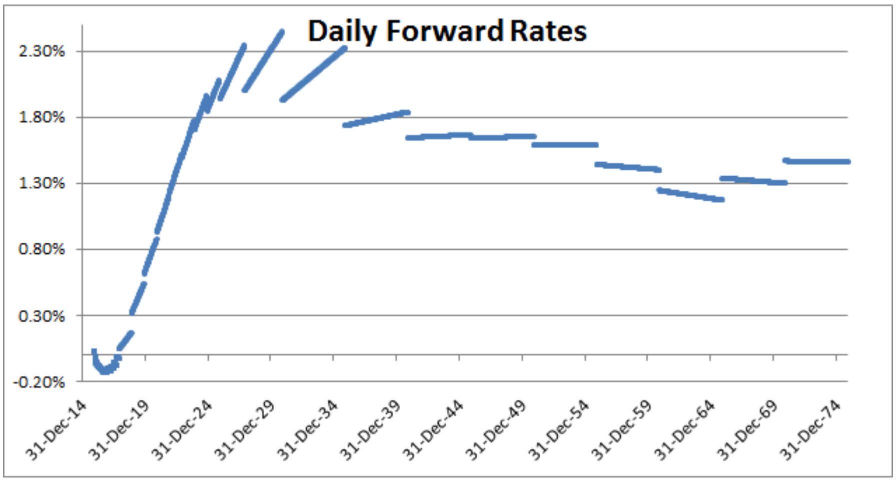
Using equation 1.8 we get

$$f(t) = \frac{d}{dt}z(t)t = \frac{-\ln P(t_i) + \ln P(t_{i-1})}{t_i - t_{i-1}} = \frac{z(t_i)t_i - z(t_{i-1})t_{i-1}}{t_i - t_{i-1}}$$

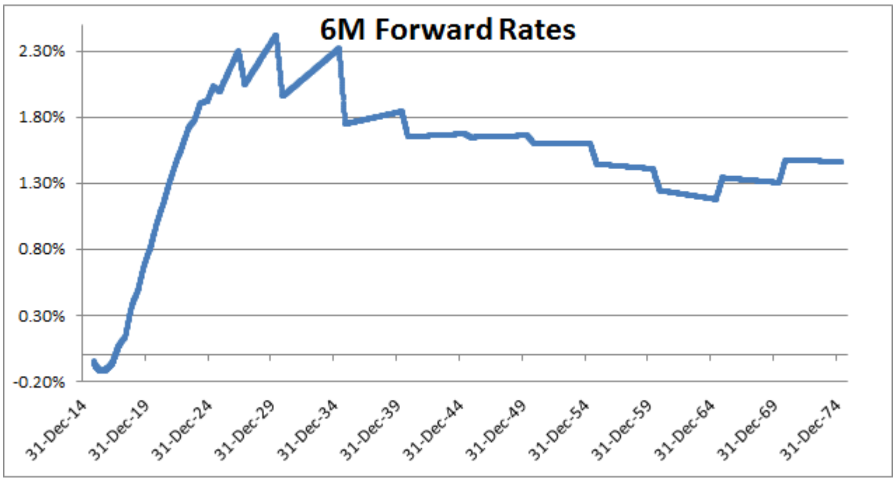
So this method has piecewise constant instantaneous forward rates (last equation doesn't depend on t) which generate "stepped" forward rates (see Figure 1.4). The result is not better than the one obtained with linear interpolation on zero rates. Nevertheless this method is more popular because it can be used to describe the first section of overnight-related curves with instantaneous forward rates jumping at specific dates. This can be a desired feature to represent policy committee meeting where central banks can decide on jumps of reference rates.

1.5.2 Constrained Cubic Spline Interpolation (or "Kruger Scheme")

Traditional cubic spline interpolation methods describe the unknown function f as a collection of $N - 1$ spline functions f_i ($i = 1, ..., N - 1$), each one defined on the interval $[t_i, t_{i+1}]$ through the following criteria:

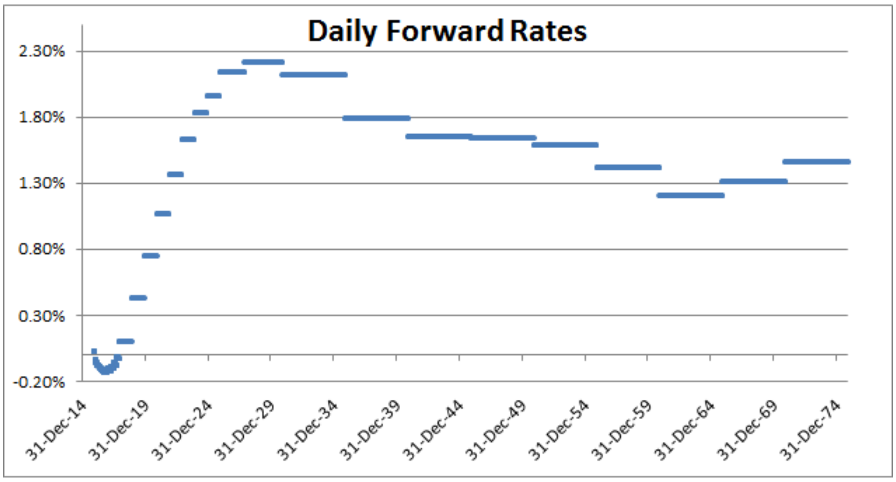


(a) Piecewise Linear Daily Forward Rates.

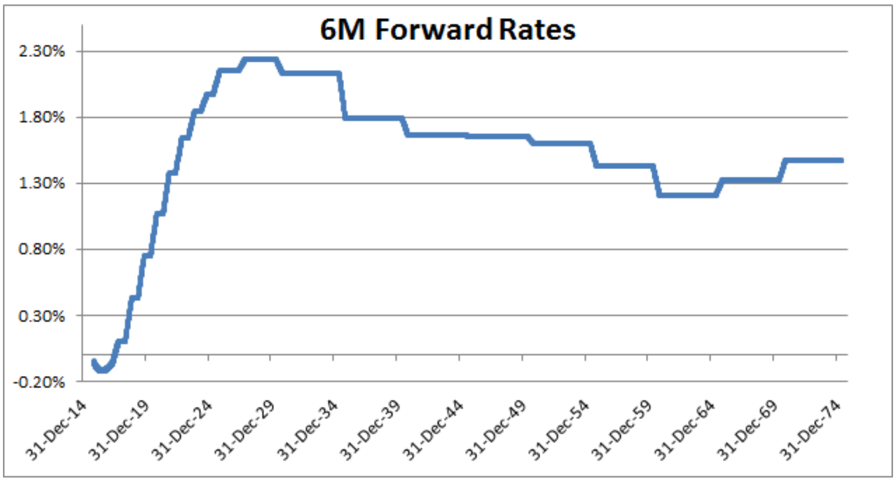


(b) Sawtooth 6M Forward Rates.

Figure 1.3: Euribor 6M Curve with Linear interpolation on Zero Rates.



(a) Piecewise Constant Daily Forward Rates.



(b) Stepped 6M Forward Rates.

Figure 1.4: Euribor 6M Curve with Linear interpolation on Log Discount Factors.

- f_i is a third order polynomial

$$f_i(t) = a_i + b_i t + c_i t^2 + d_i t^3 \tag{1.22}$$

for $i = 1, ..., N-1$

- f_i pass through all the known points

$$f_i(t_i) = y_i \ , \ f_i(t_{i+1}) = y_{i+1} \tag{1.23}$$

for $i = 1, ..., N-1$

- First order derivative is the same for both functions on either side of a point

$$f_i'(t_{i+1}) = f_{i+1}'(t_{i+1}) \tag{1.24}$$

for $i = 1, ..., N-2$

- Second order derivative is the same for both functions on either side of a point

$$f_i''(t_{i+1}) = f_{i+1}''(t_{i+1}) \tag{1.25}$$

for $i = 1, ..., N-2$

The first equation give us $4N-4$ unknown parameters; the other equations give us $2(N-1)+(N-2)+(N-2) = 4N-6$ equations. The two remaining equations are based on a border conditions for the starting and ending points. If we choose the following conditions

$$f_1''(t_1) = f_{N-1}''(t_N) = 0 \tag{1.26}$$

the resulting spline is called *Natural Spline*.

These interpolation methods suffer of well-documented problems, such as spurious inflection points, excessive convexity, lack of locality and wide oscillations (the spline only alleviates the problem of oscillation seen when fitting a single polynomial). We present now the method of Kruger ([6]) which combines the smooth curve characteristics of spline interpolation with non-overshooting behaviour of linear interpolation.

Kruger's approach consists in constructing a Constrained Cubic Spline using equations 1.23, 1.24, 1.26 and replacing 1.25 (equal second order derivative at every point) with

$$f_i'(t_{i+1}) = f_{i+1}'(t_{i+1}) = f'(t_{i+1}) \tag{1.27}$$

for $i = 1, ..., N-2$ where $f'(t_{i+1})$ is a specified first order derivative. The result is an interpolated function less smooth but with a specific slope at every point. Intuitively we know the slope of the spline will be between the slopes of the adjacent straight lines. If we define

$$S_i = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$$

a good choice is

$$f'(t_i) = \begin{cases} \frac{2}{\frac{1}{S_i}+\frac{1}{S_{i-1}}} & \text{if } S_i S_{i-1} \geq 0 \\ 0 & \text{if } S_i S_{i-1} < 0 \end{cases} \tag{1.28}$$

Note that this interpolation scheme preserves monotonicity: in regions of monotonicity of the inputs - three successive increasing or decreasing points - the interpolating function preserves this property. Maximum and minimum points are allowed only on pillars.

The effect of constrained cubic interpolation on zero rates and on the logarithm of discount factors is shown in Figures 1.5 and 1.6. We can notice that the quality of forward rates is really better than the quality achieved with linear interpolation, the best effect obtained interpolating on log discount factors. The heritage of linear interpolation is evident, mainly interpolating on zero rates: the "sawtooth" forwards have become "humps". To reduce this effect the only solution is to increase the number of pillars. Often this is achieved using interpolated quotes.

1.5.3 Monotonic Cubic Natural Spline Interpolation (or "Hyman Scheme")

The method of Hyman or *Hyman filter* ([5]) attempts to address traditional cubic spline problems in a different way. It's a method that could be applied to any cubic interpolation scheme, for example the cubic natural spline one, to preserve monotonicity. In case of C^2 interpolation schemes the Hyman filter ensures monotonicity at the expenses of the second derivative of the interpolated function which will no longer be continuous in the points where the filter has been applied.

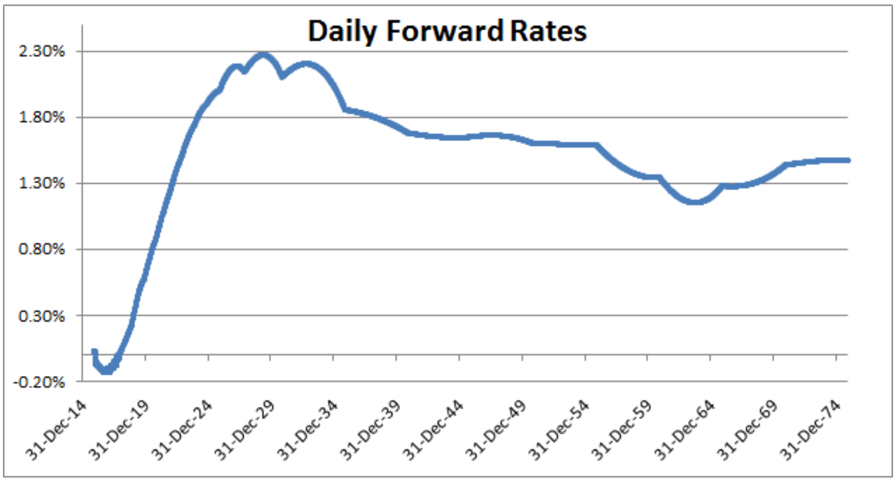
Let us briefly sketch how Hyman filter works. When input data are locally monotone (three successive increasing or decreasing points), if the chosen interpolating function is already monotonic, the Hyman filter leaves it unchanged preserving all of its original features, otherwise it changes the slopes locally in order to guarantee monotonicity. When the data are not locally monotone, instead, the interpolated function will have a maximum/minimum at the node. Maximum and minimum points are allowed also between pillars.

The effect of Hyman Filter applied to Cubic Natural Spline Interpolation on zero rates and on the logarithm of discount factors is shown in Figures 1.7 and 1.8. Looking at the smoothness of forward rates, it's clear that this is the best approach from this point of view.

1.6 Market Instruments Selection

The selection of calibrating instruments follows two fundamental criteria:

1. *Homogeneity*: for each currency, build multiple separated sets of Interest Rate Instruments according to the tenor of the underlying rate (1M, 3M, 6M or 12M tenors); instruments depending on two indexes are allowed (for instance Basis Swaps).



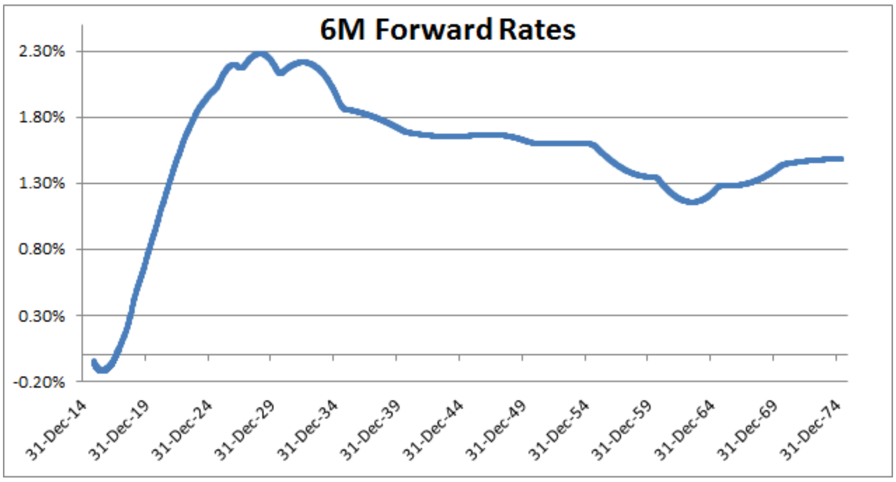
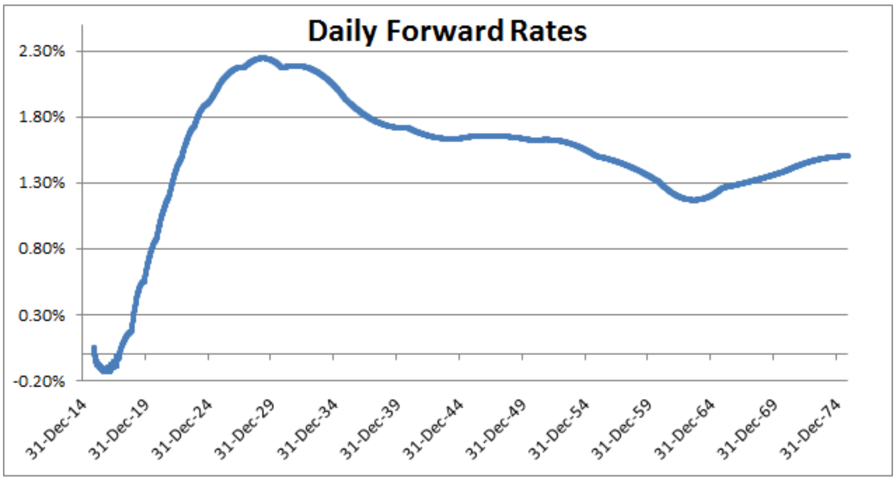


Figure 1.5: Constrained Cubic interpolation on Zero Rates for Euribor 6M Curve.



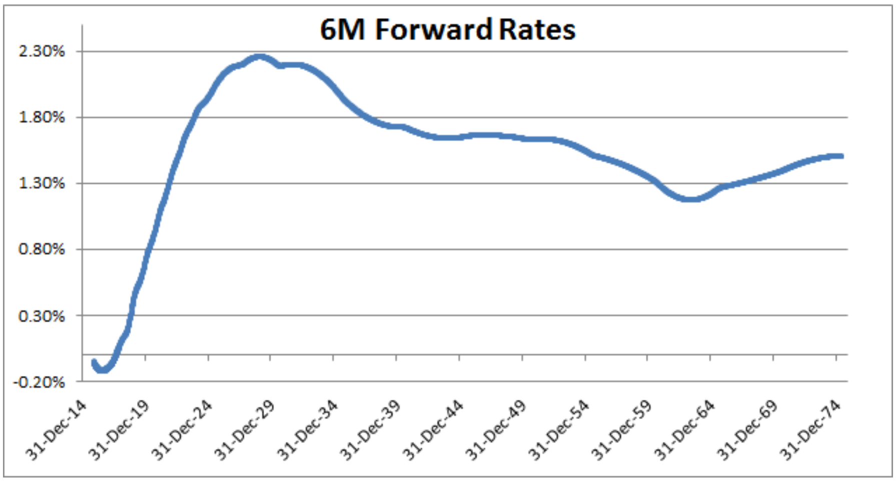


Figure 1.6: Euribor 6M Curve with Constrained Cubic interpolation on Log Discount Factors.

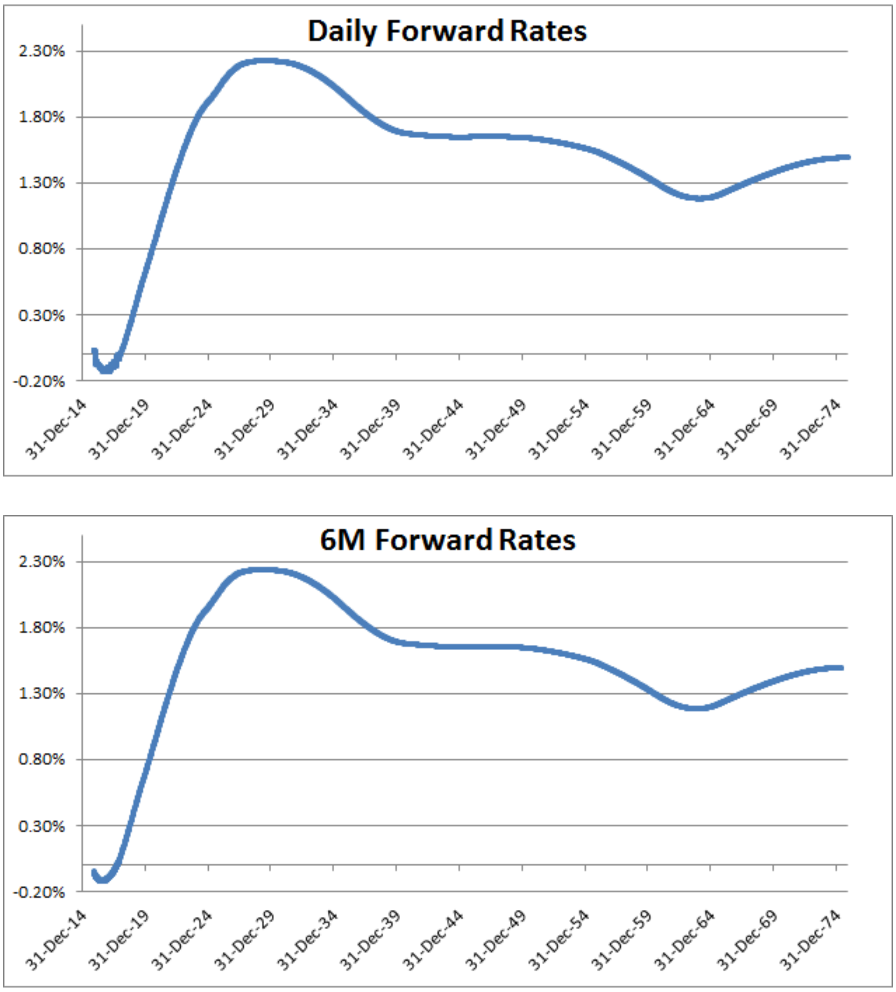


Figure 1.7: Euribor 6M Curve with Monotonic Cubic Natural Spline interpolation on Zero Rates.

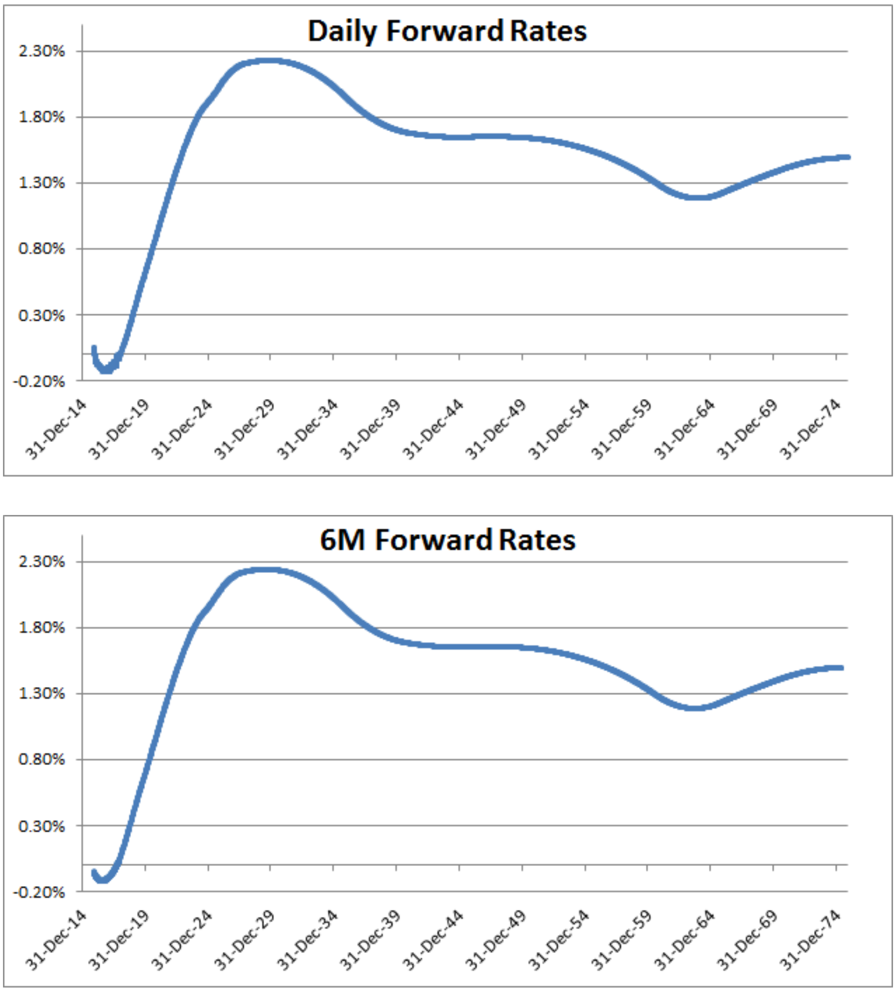


Figure 1.8: Euribor 6M Curve with Monotonic Cubic Natural Spline interpolation on Log Discount Factors.

2. *Maximum Liquidity*: for each currency-tenor pair, the chosen instruments may overlap in some sections; in order to define a subset of (mostly) non-overlapping instruments preference must be given to the more liquid ones.

You can refer to [2] or [7] for a review of most important calibrating instruments and corresponding bootstrapping equations.

1.7 Synthetic Instruments

A synthetic bootstrapping instrument is an instrument that is not directly quoted on the market but can be built starting from other quoted instruments. We can at least define two classes of synthetic instruments.

1.7.1 Synthetic Interpolated Instruments

These are instruments whose quotes can be determined directly interpolating on available market quotes. For example, if we need an IRS with maturity 27 years and it is not quoted by the market, we can choose an interpolation method and interpolate 25 years swap and 30 years swap quotes to find the missing quote. This approach is quite rough and can generate very bad forward rates.

We can define another kind of synthetic interpolated instruments. Imagine you have already calibrated a discounting curve and a forwarding curve of a certain tenor. Now you are able to price all the instruments related to that specific tenor, included non quoted ones. For example, you have calibrated your forwarding curve using Basis Swaps because Swaps are not quoted by the market. With this curve plus an appropriate discounting curve you can calculate synthetic Swap quotes and use them to perform a second bootstrap. Clearly the product of this second calibration will be different from the product of the first one. Another example. You have to calibrate using few Swap quotes because the market doesn't quote a lot of maturities. After that you can calculate synthetic Swaps quotes for every maturity you want and use the whole set of Swap quotes to perform a second different calibration. In this case we do not interpolate directly market quotes but we interpolate the bootstrapped curve. We will see in chapter 4 why it is important to define this kind of instruments.

1.7.2 Synthetic Deposits

As anticipated in section 1.3, it may happen that we have to describe each forward curve C_x of our framework with pseudo-discount factors. When $x = ON$ this kind of description is consistent with the role of C_{ON} as discount curve of the framework. Hence we can consider to have at our disposal the discounting curve C_{ON}. Conversely when $x \neq ON$ we have to manage the arbitrariness of the function $P_x(t; s)$ imposing more conditions (as explained in section 1.3). We first show with a practical example what happens if we do not add more constraints.

Let C_{6M} be the Euribor $6M$ forwarding curve. We can try to calibrate this curve using only the available market quotes (6M FRA and Swaps) and a specific interpolation algorithm. As shown in Figure 1.9 this approach leads to bad results even using sophisticated interpolation methods (Hyman scheme on Log Discount Factors): daily forward rates have an oscillatory behavior in the first section of the curve; consequently $6M$ forward rates curve shows humps in the same section. The reason of this behavior could be that each market FRA quote determines two discount factors. For example, the $1X7$ FRA quote fixes the ratio of the values of $P_{6M}(t; t+1M)$ and $P_{6M}(t; t+7M)$: this is possible (using the iterative procedure mentioned in section 1.4) but we do not have control on the final result, which is completely determined by the interpolation method. For more details you can refer to [7].

We can try to improve the method using the underlying index fixing value as best proxy of the discrete forward $F_x(t; t, t + x)$. In our example, taking into account this information, we can fix the pseudo-discount factor $P_{6M}(t; t+1M)$ interpolating between the two nodes $P_{6M}(t; t) = 1$ and

$$P_{6M}(t; t + 6M) = \frac{1}{1 + \tau_x(t, t + 6M)F_{6M}(t; t + 6M)}$$

Then, using this value and the $1X7$ FRA quote, we can determine $P_{6M}(t; t+7M)$. First of all, notice that this method can be used only if we are interpolating on discount factors (zero rates and instantaneous forward rates are not defined in $s = t$). Secondly, as explained in [3] and [7], also this method produces bad results for at least two reasons: the fixing value used to calculate the discount factor $P_x(t; t+x)$ does not change during the day and so is not really representative of the simple forward $F_x(t; t, t+x)$ implicitly quoted by the market; most of the work is still done by the interpolation method.

For all these reasons we usually try to solve the arbitrariness issue before starting calibration. We choose to fix the shape of C_x on the first x interval $[t, t + x)$ ($t = spot(today)$ is the curve reference date) and we have to do this with a reliable method ensuring the smoothness of the curve and consistency with the available x-tenor market quotes. Notice that a quoted instrument with underlying rate of tenor x has maturity date not earlier than $t + x$. For example, if $x = 6M$ the first available instrument is the $1X7$ FRA (as said before) whose maturity date falls outside the chosen interval $[t, t+x)$. So we can't use directly market quotes to determine the short term part of C_x.

The basic idea is to build the short term part of each C_x curve by modifying the C_{ON} curve with a suitable corrector, called shift, such as the result is consistent with the available x-tenor market quotes. Our goal is to define a set of n synthetic-deposits with maturity dates that fall inside the range $[t, t + x)$ which will then be used to bootstrap the first section of the C_x curve using the equation

$$P_x(t; s_i) = \frac{1}{1 + r_x^i \tau_x(t, s_i)} \tag{1.29}$$

where r_x^i and $s_i \in [t, t + x)$ are the quote and the maturity date of the i-th instrument respectively. In practice we have to calculate the synthetic quotes $\{r_x^i\}_{i=1,\ldots,n}$ using the

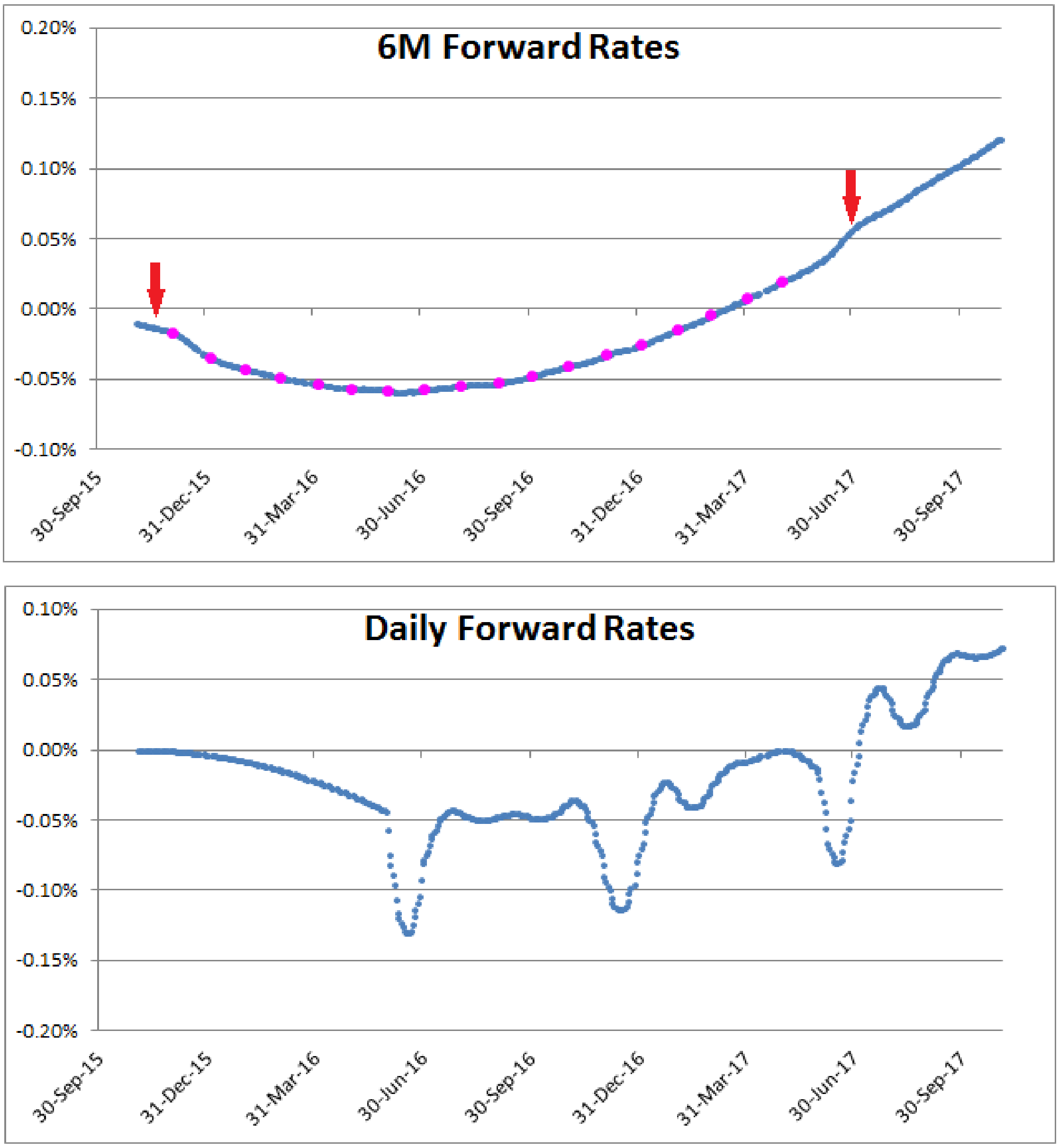


Figure 1.9: Euribor $6M$ Curve calibrated using only the available market quotes and interpolating with the Hyman scheme on Log Discount Factors (Evaluation Date: 30 October 2015). Daily forward rates curve in the first section has an oscillatory behavior; $6M$ forward rates curve shows humps in the same section (red arrows). The pink dots represent the FRA market quotes used for calibration.

available market quotes. Notice that

$$r_x^i = \frac{1}{\tau_x}\left(\frac{1}{P_x(t;s_i)} - 1\right) \overset{1.16}{=} F_x(t;t,s_i) \tag{1.30}$$

so that we can determine the whole set of synthetic quotes fixing the shape of simple forward rates on $[t, t+x)$. In order to do this we define the *continuous compounded basis* δ_x and the *integrated continuous compounded basis* Δ_x as

$$\delta_x(t;u) = f_x(t;u) - f_{ON}(t;u) \tag{1.31}$$

$$\Delta_x(t;s_1,s_2) = \int_{s_1}^{s_2} \delta_x(t;u)du \tag{1.32}$$

where $u, s_1, s_2 \geq t$, and then calculate

$$\begin{aligned} F_x(t;s_1,s_2) &= \frac{1}{\tau_x(s_1,s_2)}\left(e^{\int_{s_1}^{s_2} f_{ON}(t;u)du} e^{\int_{s_1}^{s_2} \delta_x(t;u)du} - 1\right) = \\ &= \frac{1}{\tau_x(s_1,s_2)}\left((1 + F_{ON}(t;s_1,s_2)\tau_x(s_1,s_2))\, e^{\int_{s_1}^{s_2} \delta_x(t;u)du} - 1\right) = \\ &= \frac{1}{\tau_x(s_1,s_2)}\left((1 + F_{ON}(t;s_1,s_2)\tau_x(s_1,s_2))\, e^{\Delta_x(t;s_1,s_2)} - 1\right) \end{aligned} \tag{1.33}$$

Equivalently

$$\Delta_x(t;s_1,s_2) = \ln\left(\frac{1 + F_x(t;s_1,s_2)\tau_x(s_1,s_2)}{1 + F_{ON}(t;s_1,s_2)\tau_x(s_1,s_2)}\right) \tag{1.34}$$

Now it's simple to understand how we can proceed to determine the synthetic quotes $\{r_x^i\}_{i=1,\ldots,n}$:

1. First we assume a functional form for Δ_x (or equivalently δ_x) with N parameters; usually the continuous compounded ON-x basis is approximated with a polynomial of degree $N-1$ with N parameters

   $$\delta_x(t;u) = \sum_{j=1}^{N} \alpha_j \tau_x(t;u)^{j-1}$$

   so that

   $$\Delta_x(t;s_1,s_2) = \sum_{j=1}^{N}\left[\frac{\alpha_j}{j}\left(\tau_x(t;s_2)^j - \tau_x(t;s_1)^j\right)\right]$$

2. Secondly we calculate N values $\{\Delta_x(t;t_j,t_j+x)\}_{j=1,\ldots,N}$ via equation 1.34 starting from

   - N available market quotes $F_x(t;t_j,t_j+x)$;
   - N corresponding forward rates $F_{ON}(t;t_j,t_j+x)$ calculated using C_{ON}.

3. We then calibrate Δ_x parameters using the N values $\{\Delta_x(t;t_j,t_j+x)\}_{j=1,\ldots,N}$.

4. Finally we calculate our synthetic deposits quotes $\{r_x^i \equiv F_x(t;t,s_i)\}_{i=1,\ldots,n}$ using equation 1.33 and

   - n corresponding forward rates $F_{ON}(t;t,s_i)$ calculated using C_{ON};
   - n corresponding basis values $\Delta_x(t;t,s_i)$ calculated using the calibrated basis Δ_x.

   These quotes are used as an input of the bootstrapping procedure as they were real market quotes.

The result of this approach applied to the Euribor $6M$ curve is shown in Figure 1.10. The difference with Figure 1.9 is evident.

The quotes used for Δ_x calibration must be selected carefully. For example, the underlying index fixing value must not be taken into account directly because (as said before) is not representative of the discrete forward $F_x(t;t,t+x)$ quoted by the market. Also, to improve synthetic deposits calculation, it's possible to model "jumps" (*Turn Of Year* and *End of Month* effects) as it will be discussed in 1.8.1.

1.8 Advanced Calibration Issues

This section analyzes and proposes solutions for subtle, but relevant, problems related to the EONIA curve calibration. The first issue examined is how to deal with jumps and turn-of-year effects. The second point is related to the problem caused by imperfect concatenation between spot starting OIS and forward starting ECB dated OIS: in order to avoid distortion, a meta-instrument called "Forward Stub" should cover the section between the maturity of the last spot starting OIS and the settlement of the first ECB OIS. Its implied value can be derived assuming a no-arbitrage conditions. The final issue is the empirical evidence that the forward overnight rates are generally constant between ECB monetary policy board meeting dates: because of this, a log-linear discount interpolation is a good fit. Anyway, flat forward rates are hardly realistic on the long end. This is the rationale to suggest the use of a "Mixed Interpolation" which merges two different interpolation regimes.

1.8.1 Jump estimation

The key point in a "state-of-the-art" curve bootstrapping is to obtain smooth forward rates. In order to do that, for even the best interpolation scheme to be effective, any market rate jump must be removed before the curve calibration and than added back at the end of the process. A rate jump can be seen, in a financial point of view, as a higher index fixing due to increased search of liquidity of market participants caused by end-of-month or end-of-year capital requirements. This definition is good for the EUR case in which market evidence shows positive jumps as visible in Figure 1.11. However, this definition is not consistent for the U.S. case in which market evidence shows negative

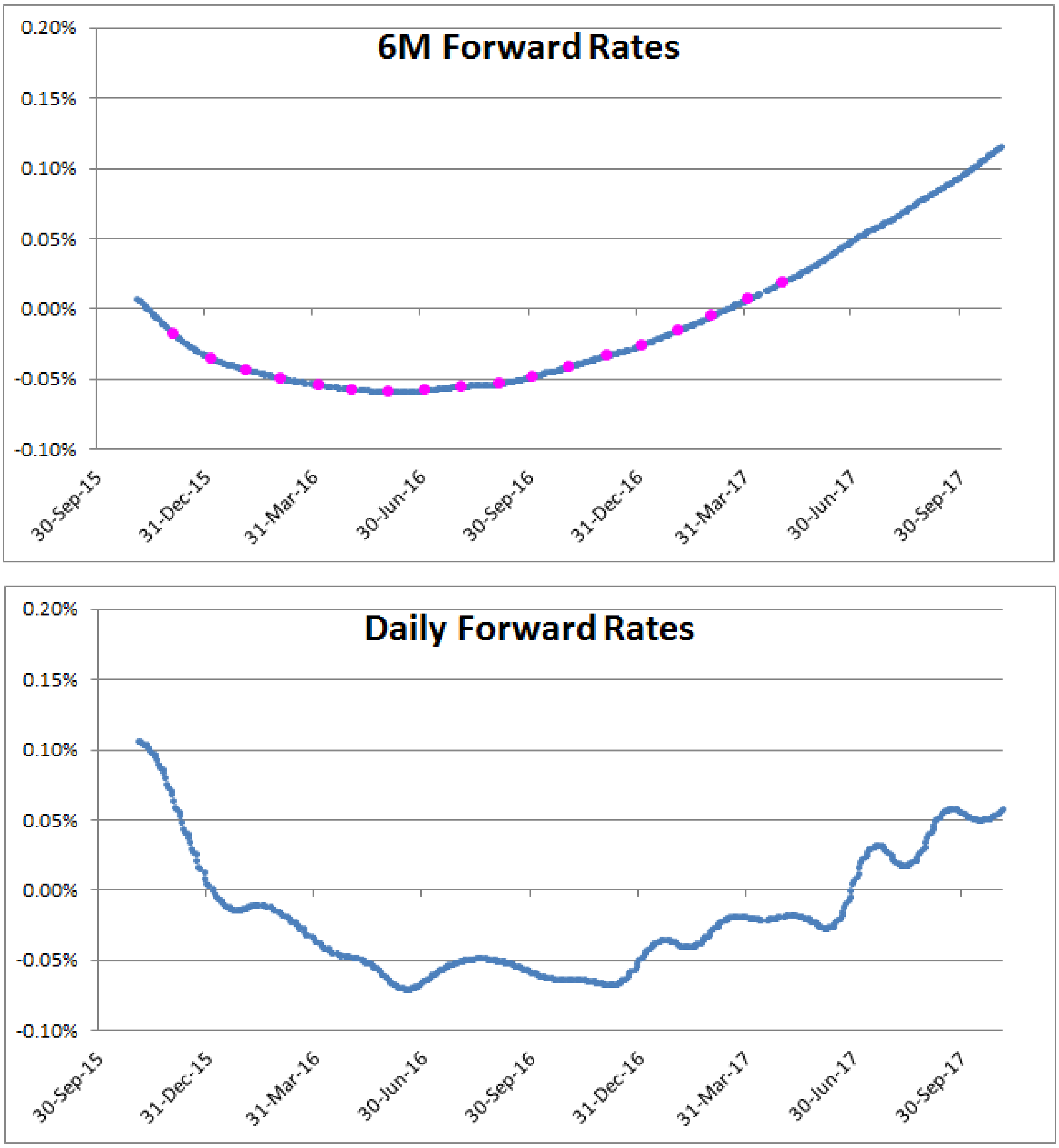


Figure 1.10: Euribor $6M$ Curve calibrated using synthetic deposits quotes, the available market quotes and interpolating with the Hyman scheme on Log Discount Factors (Evaluation Date: 30 October 2015). Daily forward rates curve in the first section has less oscillations than in Figure 1.9; $6M$ forward rates curve do not show humps. The pink dots represent the FRA market quotes used for calibration.

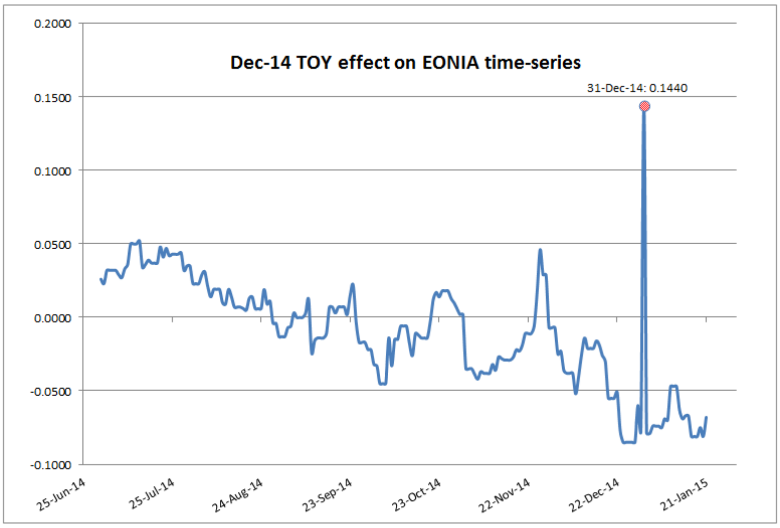


Figure 1.11: December 2014 Eonia turn of year effect.

jumps as it will be shown in 1.8.2. The most relevant rate jump is the so called *Turn-Of-Year* (TOY) effect observable in the last working day in market quotations spanning across the end of the year. Other Euribor indexes with longer tenor display smaller jumps when their maturity crosses the same border; in fact jumps amplitude decrease as the rate tenor increase.
To estimate jumps sizes Ametrano and Mazzocchi ([3]) propose a four-step approach inspired by Burghardt ([1]):

1. The first step is to built an overnight curve calibrating by means of a linear/flat interpolation and including all liquid market quotes available (high pillar density is strictly recommended);

2. After that it is possible to estimate the first jump assuming that a segment out of line with preceding and following segments can be put back in line dumping the difference into the jump:

$$[F^{original}(t_1, t_2) - F^{interp}(t_1, t_2)] \cdot \tau(t_1, t_2) = J^{Size} * \tau^J \tag{1.35}$$

$$J^{Size} = [F^{original}(t_1, t_2) - F^{interp}(t_1, t_2)] \cdot \frac{\tau(t_1, t_2)}{\tau^J} \tag{1.36}$$

3. Ones the first jumps as been estimated it is possible to remove it from the curve;

4. Iterate ad libitum 2 and 3 on the next jump date.

EONIA Index fixings show that it occurs a jump at least at every end of month. To obtain a smooth forwarding curve, this particular behaviour must be addressed in the calibration taking care to clear the entire curve from jumps before starting the bootstrap. Figures 1.12 and 1.13 shows the impact of first and second jump inclusion in the curves shape where jumps sizes have been estimated taking advantage of the approach proposed above. As visible, the short term section of the curve is altered due the fact that jumps have been accounted. To preserve the value of the instantaneous forward rate integral in that section, the calibration algorithm pushes down all the segment changing the curve shape.

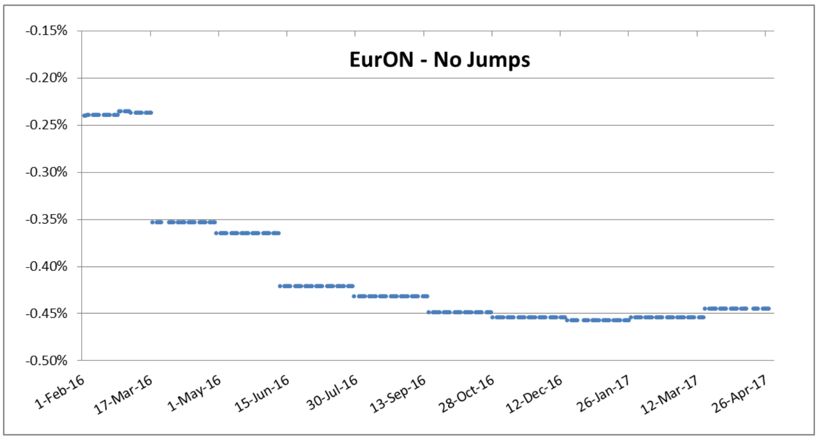


Figure 1.12: First two year section of the EUR overnight curve without jumps.

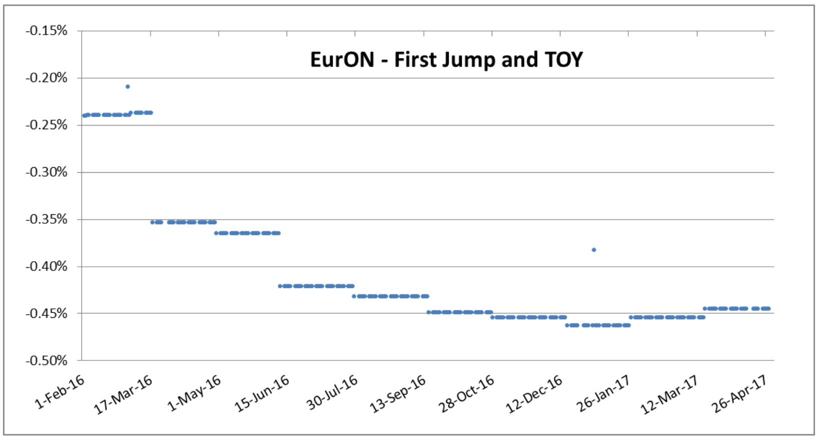


Figure 1.13: The same curve in Figure 1.12 including first jump and TOY effect.

Since the piecewise flat steps of the curve are given by market quotes which are the integral of instantaneous forward rates between an interval (t_1, t_2), if this integral must account a deep above level point, the residual part is shifted down in order to maintain the integral value equal.

1.8.2 Negative Jump for USD Market

The USD market case shows an inverted situation compared to the EUR market presented before. As visible in Figure 1.14, the Fed Funds fixings shows negative jumps every end-of-month and with a particular deep size. In a financial point of view this is due to the excess of liquidity that market participants hold and want to sell at the end of each month. As a consequence U.S market participants tend to have a decreasing necessity of liquidity every end-of-month. This behaviour tends to be constant during

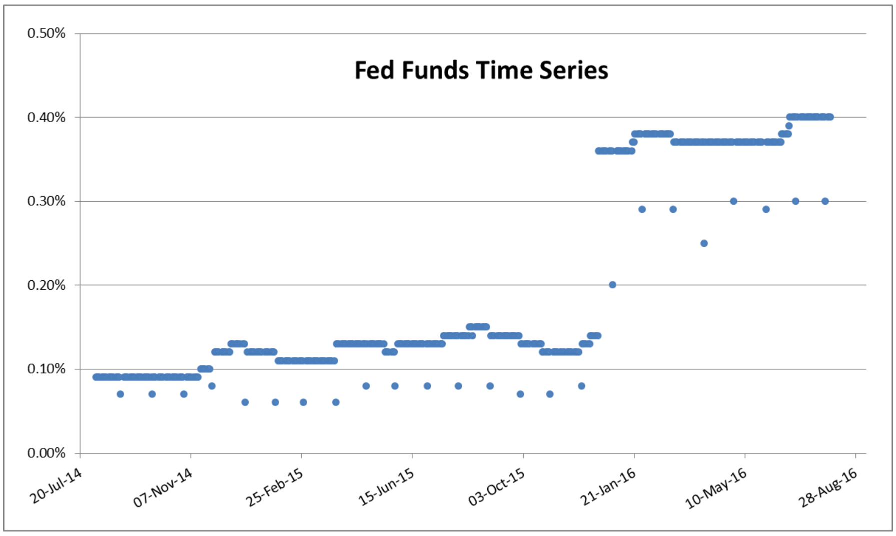


Figure 1.14: Last 2 years Fed Funds fixings time series.

time, in fact, for the last two years, the Fed Funds time series have shown negative jumps every end-of-month with no exceptions. To estimate negative sizes, however, the Ametrano-Mazzocchi approach proposed for the EUR market case must be reviewed. In particular the formula at point 2 becomes:

$$[F^{Interp}(t_1, t_2) - F^{Original}(t_1, t_2)] \cdot \tau(t_1, t_2) = J^{Size} * \tau^J \tag{1.37}$$

$$J^{Size} = [F^{Interp}(t_1, t_2) - F^{Original}(t_1, t_2)] \cdot \frac{\tau(t_1, t_2)}{\tau^J} \tag{1.38}$$

Applying the same procedure but using the reviewed formula it is now possible to perform negative jumps estimations and account them in the USD overnight curve as shown in Figure 1.16.

1.8.3 Forward Stub

The problem concerns the composition of the EUR overnight curve which instruments' selection best practice is to mix spot starting derivatives, the Overnight Indexed Swaps

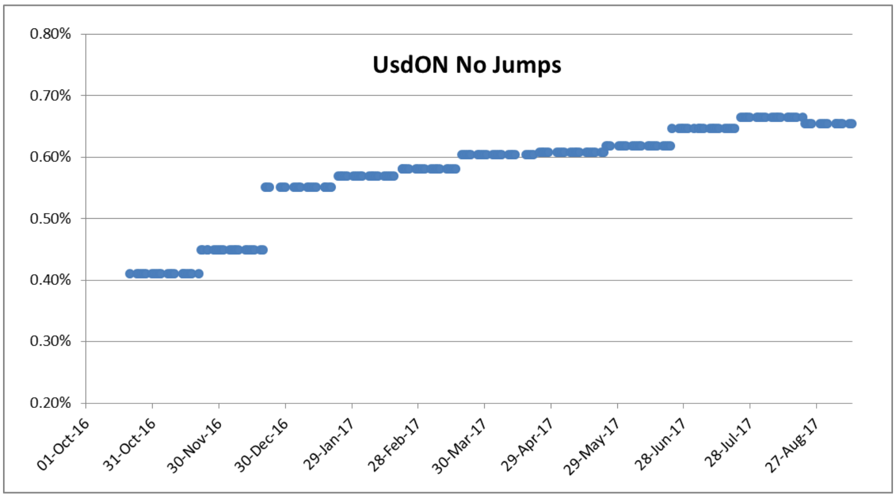


Figure 1.15: First USD overnight curve section without jumps.

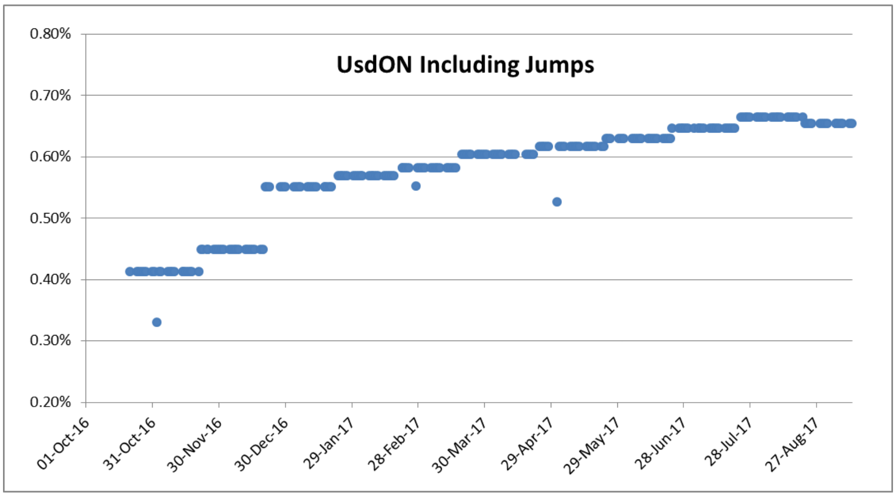


Figure 1.16: The same curve in Figure 1.15 including all estimated jumps.

(OIS) and forward starting derivatives, the European Central Bank Overnight Indexed Swaps *(ECB OIS)*. From the whole set of instruments available on the market only the most liquid subset will be used to perform the calibration in order to avoid distortive overfitting. The instruments' selection criteria is based on liquidity. As a consequence, whenever an year fraction is covered by many market quotes, only the most liquid one will be included in the calibration. For this reason, since ECB OIS are preferred by traders because are the most liquid overnight indexed instruments, the calibration algorithm is set in order to give them priority despite of spot starting OIS which cover the same period. Otherwise, since ECB OIS are forward starting instruments, the concatenation between spot OIS and forward ECB OIS could not be perfect leading to an overlapping

section as the one shown in Figure 1.17.

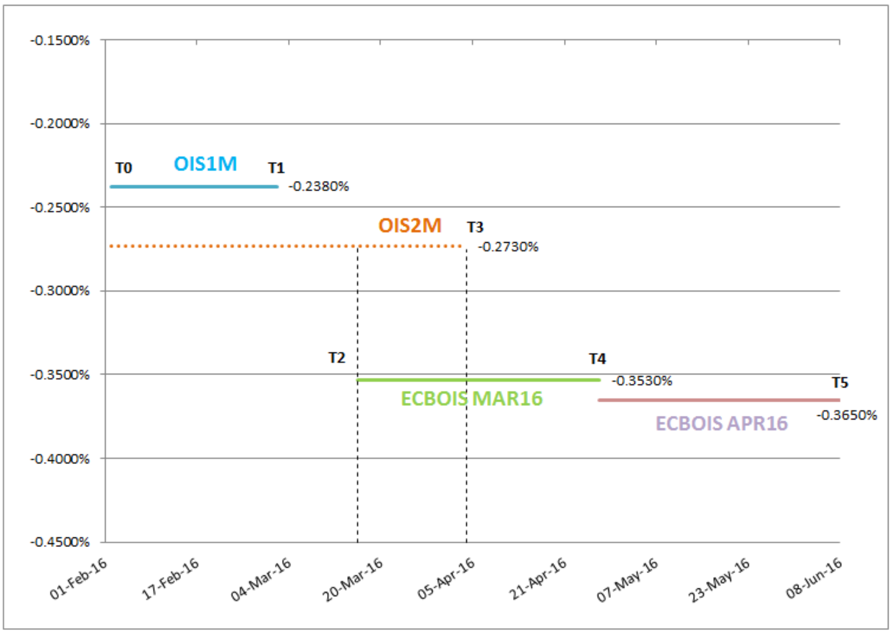


Figure 1.17: Overlapping EONIA instruments levels; dataset as of *January 29, 2016*.

Figure 1.17 shows an example of imperfect concatenation where OIS2M with maturity $t_3 = April4, 2016$ overlaps the first ECB OIS with settlement $t_2 = March16, 2016$. The calibration algorithm derives information about the average rate for the interval $(t_0; t_1)$ from the OIS1M. Similarly, OIS2M fixes the average rate in the interval $(t_1; t_3)$ and the information about $(t_3; t_4)$ is given by the first ECB OIS. Since the bootstrap algorithm uses this information to perform a perfect re-price of all the included market quotes, it is not able to derive the information set by the first ECB OIS for the interval $(t_2; t_3)$ because it uses the level fixed by OIS2M. This means that the calibrator can not use the information related to the interval $(t_2; t_3)$ set by the most liquid instrument on the market. This distortion could be negligible only if the information given by both instruments are almost equal, like when the curve has a flat behaviour in that region. Otherwise, there are situations where the level fixed by these two instruments is very different as, for example, when first ECB OIS market quote is accounting a rates cut/rise expectation. The distortion resulting from the overlapping section is visible in Figure 1.18 where the calibrator, which is not using the the most reliable information for the interval $(t_2; t_3)$, pushes down the curve till -0.40% in order to reprice perfectly the first ECB OIS. It is straightforward that this strange behaviour is due to calibration's discrepancy resulting from the overlapping section. In order to fix this problem the suggestion is to create a new "Meta-Instrument", from now called *Forward Stub*, that covers the interval $(t_1; t_2)$ and which value can be derived from available market quotes. The aim of the Forward Stub is to cover the section between the last not-overlapping spot starting OIS maturity and the first ECB OIS settlement date in order to link perfectly

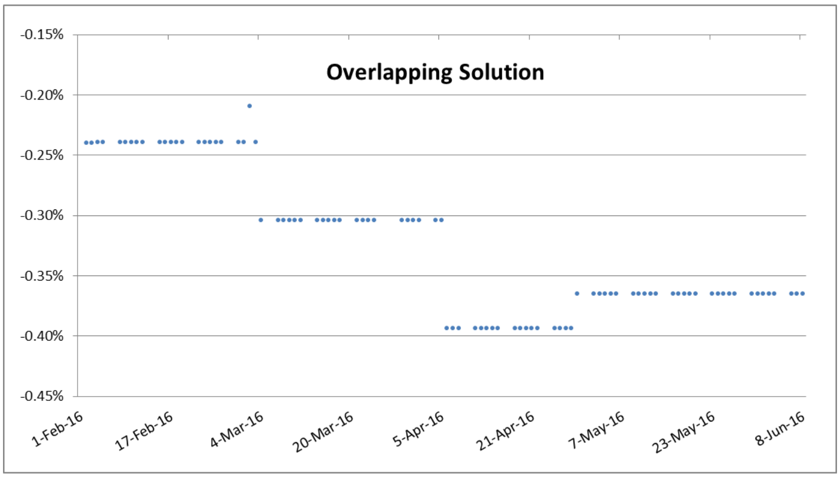


Figure 1.18: EONIA curve calibrated including the overlapping instrument.

spot instruments to forward instruments as shown in Figure 1.19. The Forward Stub

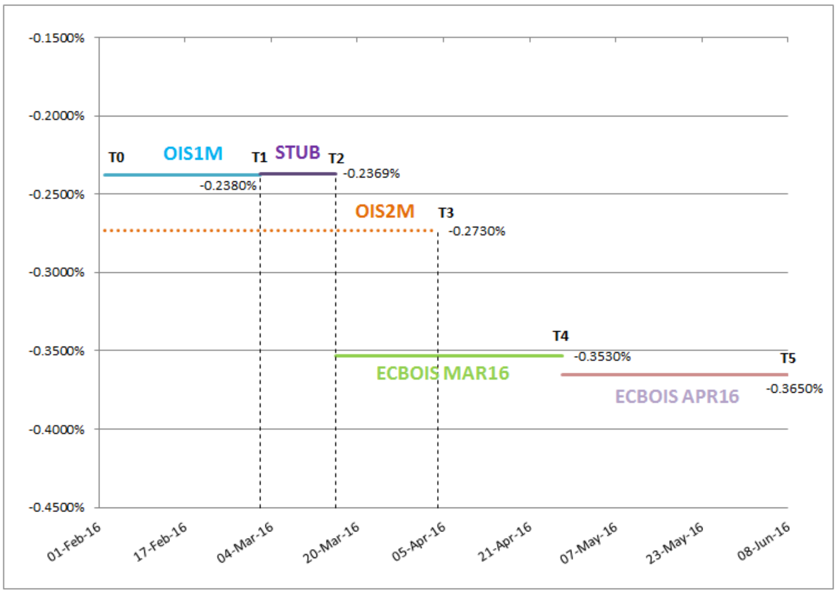


Figure 1.19: Overlapping EONIA instruments' levels including the Forward Stub.

quote is the rate which covers the year fraction (t_1, t_2). This rate is implied in the market and must be derived assuming a no-arbitrage condition which impose the OIS2M to be perfectly repriced. Otherwise, in order to obtain the unknown Meta-Quote value, it is necessary to have information about the average forward rates in the interval (t_2, t_3). This value can not be derived directly from market instruments. Since the average forward rate for (t_2, t_4) is known and equal to the 1^{st} ECB OIS market quote it is possible to derive the (t_2, t_3) value assuming that forward rates in (t_2, t_4) are constant.

This assumption is consistent with the empirical market evidence which shows an almost flat behaviour between ECB monetary policy dates as the Figure 1.22 suggest. All the information necessary to derive the Forward Stub implied market quotes is now available and it is possible to exploit the no-arbitrage condition to set:

$$\int_{t_0}^{t_1} f(s)\mathrm{d}s + \int_{t_1}^{t_2} f(s)\mathrm{d}s + \int_{t_2}^{t_3} f(s)\mathrm{d}s = \int_{t_0}^{t_3} f(s)\mathrm{d}s \tag{1.39}$$

As visible, no-arbitrage condition implies that investing in OIS1M (from t_0 to t_1), Forward Stub (from t_1 to t_2) and in ECB OIS till the OIS2M maturity (from t_2 to t_3) must be equal to investing in OIS2M (from t_0 to t_3). Solving the equation for the Forward Stub unknown value:

$$\int_{t_1}^{t_2} f(s)\mathrm{d}s = \frac{\int_{t_0}^{t_3} f(s)\mathrm{d}s}{\int_{t_0}^{t_1} f(s)\mathrm{d}s + \int_{t_2}^{t_3} f(s)\mathrm{d}s} \tag{1.40}$$

Assuming continuous compounding:

$$Stub = \frac{\left[\frac{e^{F(t_0,t_3)\cdot\tau(t_0,t_3)}}{e^{F(t_0,t_1)\cdot\tau(t_0,t_1)}\cdot e^{F(t_2,t_3)\cdot\tau(t_2,t_3)}} - 1\right]}{\tau(t_1,t_2)} \tag{1.41}$$

Including this Meta-Instrument in the overnight curve calibration improves consistently the output quality avoiding unwanted distortion and oscillation as it will be shown later taking advantage of the repricing error analysis. Otherwise, this solution can be implemented only if the bootstrap algorithm uses a flat interpolation for the short-term section of the curve. This condition is necessary to match the assumption that forward rates in (t_2, t_4) are constant. It is also important to underlying that the Stub algorithm produces good results also in limit cases as, for example, when $\tau(t_2, t_4) \longrightarrow \frac{1}{365}$ that represents the particular case in which the Forward Stub quote duration is just 1 day. Furthermore, during a year, since the first ECB OIS fixing becomes earlier and earlier, it will happen that the discarded spot instrument is the spot week OIS (which is the one with shorter maturity). This case implies that the so called "Forward Stub" it is not forward start anymore because it becomes a spot instrument which covers the year fraction: $\tau(t_0, t_1)$ (where t_1 is the first ECB OIS fixing date). In this particular situation, the "Spot Stub" implied value is:

$$Spot\ Stub = \frac{\left[\frac{e^{F(t_0,t_2)\cdot\tau(t_0,t_2)}}{e^{F(t_1,t_2)\cdot\tau(t_1,t_2)}} - 1\right]}{\tau(t_0,t_1)} \tag{1.42}$$

The impact of the Forward Stub inclusion in presented in Figure 1.20. It is visible that this approach avoids strange curve's behaviour and is the one which minimizes all the out-of-curve instruments repricing errors ensuring also the perfect reprice of the excluded overlapping instrument. Finally, the Figure 1.21 shows the repricing error analysis that validates the Forward Stub solution as the one able to minimize errors and to improve the curve's quality. The other solution instead is able to reprice perfectly the 2-months

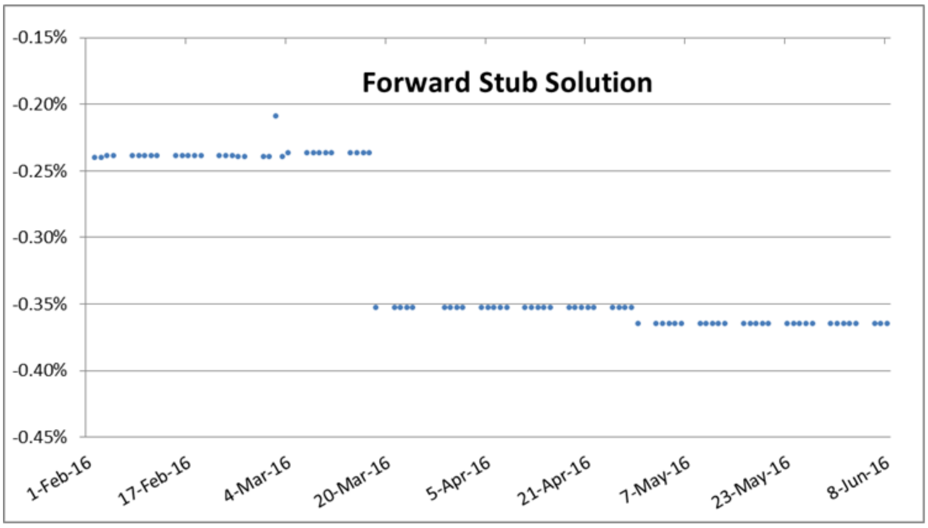


Figure 1.20: EONIA Curve calibrated including Forward Stub.

maturity OIS because it is an included instrument but it is not able to do a good re-price for all the not included market quotes which show relevant repricing errors. Also the lower values of Root Mean Square Error (RMSE) and Max Error (ME) validate the Forward Stub solution as the one that ensures errors' minimization.

Repricing Errors in Basis Points (Bps)			
Instrumets		Overlapping	Forward Stub
OIS	2M	0.00	0.00
OIS	3M	-1.09	-0.05
OIS	4M	-0.72	0.05
OIS	5M	-0.64	-0.03
OIS	6M	-0.83	-0.32
OIS	7M	-0.45	-0.02
OIS	8M	-0.42	-0.04
OIS	9M	-0.33	0.01
OIS	10M	-0.31	-0.01
OIS	11M	-0.27	0.01
OIS	12M	-0.60	-0.35
OIS	15M	-0.24	-0.04
	RMSE	0.57	0.14
	ME	-1.09	-0.35

Figure 1.21: Repricing errors summary related to all available instruments not included in the calibration (included instruments don't show errors by construction).

1.8.4 Mixed Interpolation

The *Mixed Interpolation*, as already introduced before, is a particular technique which merges two different kind of interpolation. Figure 1.22 shows that EONIA Index time series have a piecewise constant behaviour between ECB monetary policy board meeting dates. In order to replicate this particular shape, the best way is by means of a flat interpolation on discount factor which is able to provide a stepped curve's behaviour. Since instruments' granularity decrease in the mid-long term, assuming flat forward rates becomes inconsistent because it would produce flat rates period of more than a year. As a consequence, it becomes necessary to switch to another interpolation technique which is able to provide smooth forwards. In order to obtain smooth forward rates Ametrano-Bianchetti [2] propose to use an Hyman[2] filtered monotone cubic natural spline. The correct point where switching interpolation schemes can not be established consistently but the suggestion is to use a flat interpolator up to the end of the ECB dated OIS strip and then start with the cubic spline. The mixed interpolation is implemented in QuantLib 1.9 version in two different ways:

- The *Split Range* approach consists in interpolating till a pre-determined pillar, from now called the *Switch Pillar*, using the 1^{st} interpolation technique and then switch to the 2^{nd} interpolation for the rest of the curve.

- *Share Range* approach consists in interpolating two times the whole curve and then merging the obtained curves in the switch pillar. The curve calibrated taking advantage of this approach is plotted in Figure 1.23.

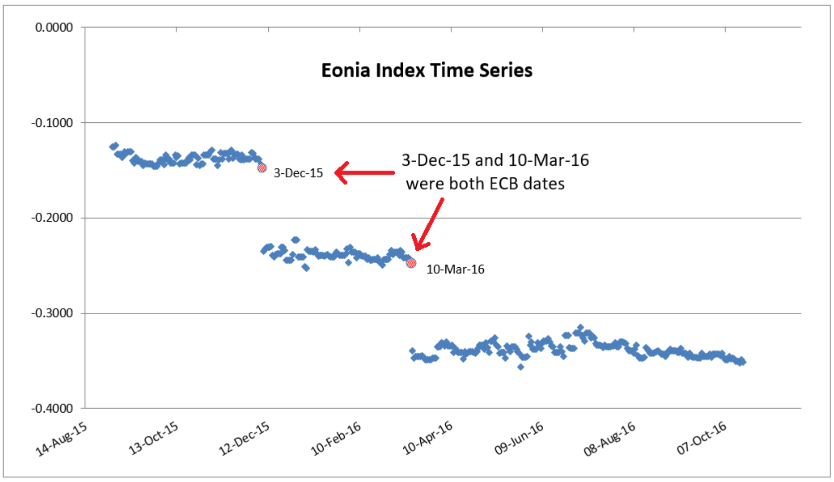


Figure 1.22: Last year EONIA Index fixings showing a piecewise flat behaviour between ECB monetary policy meetings

[2]More details on monotonicity preserving filters can be found in [5]

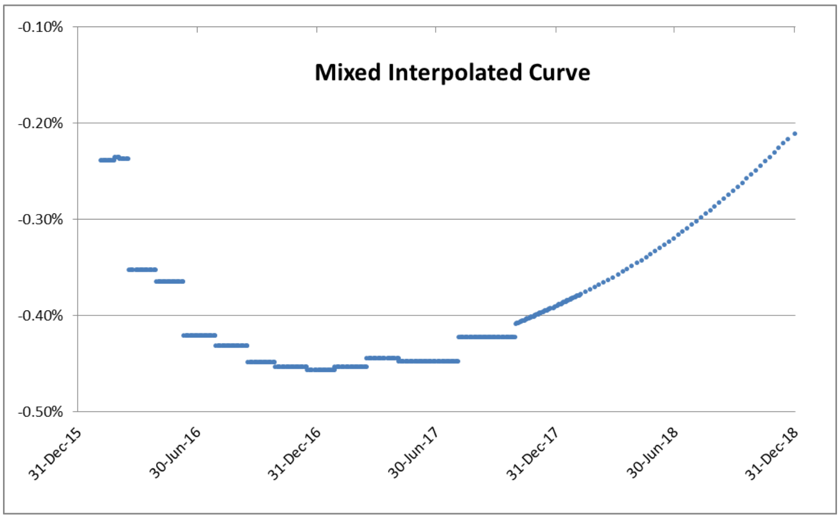


Figure 1.23: Mixed Interpolation approach merging linear and monotonic cubic natural spline on log-discounts.

As visible, this approach produces a stepped behaviour till the switch pillar and then forward rates become smooth due to the monotone cubic interpolation. To validate the Mixed "Linear-HymanCubic" interpolator as the best technique to model the EONIA curve, Figure 1.24 summarize the same repricing error analysis used for the preceding section. Analyzing the repricing table it is possible to draw some conclusions. The comparison is made between a log-linear interpolated curve which includes the Forward Stub; a Hyman filtered monotone cubic spline curve which includes an "*Ad Hoc*" Stub obtained through a root finding approach and the mixed interpolated curves following both the possibles methods. Obviously, it is visible that the mixed approaches and the log-linear one lead to the same errors since the mixed technique uses a log-linear interpolation up to the switch pillar (which is set on the last quoted ECB OIS maturity). The monotone cubic curve, instead, even if bootstrapped including the Stub, produces higher errors for each instrument not included in the calibration. The conclusion is that the EONIA curve's short term section is fitted better by a piecewise flat interpolation rather than a cubic one. Otherwise, on the mid/long term, assuming constant forwards can not be consistent anymore. That's why this work suggests to mix the log-linear interpolation up to the end of the ECB OIS strip and then switch to a monotone cubic spline interpolator which produces smooth forward rates for the mid/long term section. To confirm the analysis the Root Mean Square Error (RMSE) and Max Error (ME) showing higher values related to the monotone cubic approach.

1.9 First order sensitivities (or Deltas)

Curves are not only accounting tools but also risk management tools to analyse the risks. We concentrate on first order risks called *deltas*. Let's consider a portfolio of interest

Repricing Errors in Basis Points (Bps)					
Instrumets		Linear + Stub	MonotoneCubic + Stub	SplitRange	ShareRange
OIS	2M	0.00	0.00	0.00	0.00
OIS	3M	-0.05	-0.52	-0.05	-0.05
OIS	4M	0.05	-0.21	0.05	0.05
OIS	5M	-0.03	-0.12	-0.03	-0.03
OIS	6M	-0.32	-0.55	-0.32	-0.32
OIS	7M	-0.02	-0.19	-0.02	-0.02
OIS	8M	-0.04	-0.18	-0.04	-0.04
OIS	9M	0.01	-0.13	0.01	0.01
OIS	10M	-0.01	-0.13	-0.01	-0.01
OIS	11M	-0.03	-0.11	-0.03	-0.03
OIS	12M	-0.35	-0.47	-0.35	-0.35
OIS	15M	-0.04	-0.13	-0.04	-0.04
RMSE		0.14	0.29	0.14	0.14
ME		-0.35	-0.55	-0.35	-0.35

Figure 1.24: Repricing error analysis using a log-linear interpolation + Stub for the whole curve (1^{st} column), a monotone cubic spline + Stub for the whole curve (2^{nd} column), a share range mixed interpolation (3^{rd} column) and a split range mixed interpolation (4^{th} column).

rate derivatives depending on our set of calibrated curves $\{C_i\}_{i=1,\dots,n}$ each characterised by a time grid $\{T_{ij}\}_{j=1,\dots,k_i}$ and a set of bootstrapping instruments with market quotes $\{Q_{ij}\}_{j=1,\dots,k_i}$. Defining $Q = \{Q_{ij}\}_{i,j}$ as the entire set of bootstrapping market quotes, the price of our portfolio at time t will be denoted by $\Pi(t;Q)$.

The portfolio's delta is the first order estimate of the price change for a change of the quotes of the instruments in the bootstrapping basket. We now have to make an assumption on possible changes of quotes.

Usually by quote we mean a rate or a price: an interest rate swap quote is always a rate; futures, instead, are quoted in terms of price. The simplest change we can imagine is a rate shift δ, which for example correspond to a future price shift of $-100\cdot\delta$. Usually $\delta = 1bps$, which at present is the choice of our front office system.

We assume that all the possible quote changes are the ones just described; we will indicate with δ_{ij} the shift corresponding to the quote Q_{ij} ($\delta_{ij} = \delta$ in case of a rate, $\delta_{ij} = -100\cdot\delta$ in case of price and so on).

When one single quote Q_{ij} is shifted, the first order estimation of the price change is

$$\Delta^{\Pi}_{ij}(t;Q) = \frac{\partial \Pi}{\partial Q_{ij}}\delta_{ij} \tag{1.43}$$

that is called *bucketed delta for pillar* T_{ij}. We can define also the *partial delta for curve* C_i as

$$\Delta^{\Pi}_{i}(t;Q) = \sum_{j=1}^{k_i} \Delta^{\Pi}_{ij}(t;Q) \tag{1.44}$$

which corresponds to a parallel movement of the set of quotes associated to the curve C_i (every quote related to curve C_i is shifted as explained before). Finally the *total delta* is defined by

$$\Delta^{\Pi}(t;Q) = \sum_{i=1}^{n} \Delta_i^{\Pi}(t;Q) \tag{1.45}$$

which corresponds to a parallel movement of the whole set of quotes Q. Usually the derivatives $\frac{\partial \Pi}{\partial Q_{ij}}$ are calculated using a finite differences method. The shift h used to perform the calculation must be selected carefully: a shift too big or too small could negatively affect calculation.

Within a curve-based pricing framework, the value of Π doesn't depend directly on market rates Q but indirectly through discount factors and forward rates appearing in the corresponding pricing formulas. Since forward rates could be written in terms of their associated discount factors and since discount factors could be written in terms of their corresponding zero rates, we may think that Π depends directly on a set of zero rates $\{z_{ij}\}$. Note that a single zero rate z_{ij} may depend on more than one single market quote due to non-local effects in bootstrapping. For the sake of simplicity, let us neglects effects due to exogenous discounting so that each zero rate depends only on market quotes related to its specific curve. In this case we can write

$$\frac{\partial \Pi}{\partial Q_{ij}} = \sum_{h=1}^{k_i} \frac{\partial z_{ih}}{\partial Q_{ij}} \frac{\partial \Pi}{\partial z_{ih}} \tag{1.46}$$

So in matrix notation we have

$$\Delta_i^{\Pi}(t;Q) = \delta_i \cdot J_i \cdot \nabla_i \Pi \tag{1.47}$$

where

$$J_i = \left[\frac{\partial z_{ih}}{\partial Q_{ij}} \right]_{jh} \tag{1.48}$$

is the *Jacobian Matrix* for curve C_i,

$$\nabla_i = \left[\frac{\partial}{\partial z_{ih}} \right]_{h} \tag{1.49}$$

is the *gradient operator* for curve C_i and

$$\delta_i = [\delta ij]_j \tag{1.50}$$

Finally

$$\Delta^{\Pi}(t;Q) = \sum_{i=1}^{n} \delta_i \cdot J_i \cdot \nabla_i \Pi$$

This is the formula used by our front office systems to perform the calculation. Obviously formulas which consider exogenous discounting are more sophisticated but it is not our main purpose presenting them here.

Once the partial and total deltas has been computed, we want to hedge our portfolio by trading appropriate amounts of hedging instruments (each one with unit nominal amount). Typically the set of hedging instruments for curve C_i is a subset of the most liquid bootstrapping instruments of C_i but their selection is subjective and is part of an interest rate trader work. Note that for each instruments used in the curve construction only a delta with respect to the instrument itself will appear because its quote is not affected by other instruments in the basket. Hedging amounts are chosen in such a way that the total portfolio (consisting of the original portfolio plus the appropriate amount of each hedging instrument) satisfies the zero delta condition.

Different interpolation methods may lead to huge differences in bucketed deltas. If a method implies that you have to use ten year swap to hedge a seven month swap, probably is not the right method. We will now analyze the impact of interpolation choice on deltas calculation calibrating a discount curve C_{ON} using the same data but different interpolation techniques. In particular we compare

- Linear interpolation on zero rates
- Linear interpolation on logarithm of discount factors
- Constrained Cubic interpolation on zero rates
- Constrained Cubic interpolation on logarithm of discount factors
- Monotonic Cubic Natural Spline Interpolation on zero rates
- Monotonic Cubic Natural Spline on logarithm of discount factors

Then, for each interpolation method we calculate deltas for a 5Y OIS with 2M forward start (notional 100000000) and look at the differences. Results are shown in figure 1.25.

It's clear from our example that local methods are the best choice from an hedging point of view because they imply an intuitive and simple hedging strategy (deltas are concentrated around pillars which correspond to the start and end dates of the instrument). Unfortunately we know from section 1.5 that these methods don't produce good forward rates. In the same section we claimed that Hyman scheme produces the best forward rates but this new test shows that its non-locality level is too high for hedging purposes. The best compromise seems to be the Kruger scheme; we will apply it to zero rates and not on to the logarithm of discount factors because this last method has shown in some market conditions a lot of instability concerning deltas calculation.

Quotes	Interpolation Method					
	Linear on Zero Rates	Linear on Log Discounts	Kruger on Zero Rates	Kruger on Log Discounts	Hyman on Zero Rates	Hyman on Log Discounts
EURONDPON=	0.39	- 0.02	- 0.14			
EURONDPTN=	0.13	- 0.01	- 0.05			
EURONDPSN=						
EURONOISSW=						
EURONOIS2W=						
EURONOIS3W=						
EURONOIS1M=						
EURONOIS2M=	- 1,723.26	- 1,723.26	- 1,723.26	- 1,723.26	- 1,723.26	- 1,723.26
EURONOIS3M=						
EURONOIS4M=						
EURONOIS5M=						
EURONOIS6M=						
EURONOIS7M=					0.01	
EURONOIS8M=					- 0.03	- 0.01
EURONOIS9M=					0.12	0.04
EURONOIS10M=					- 0.57	- 0.20
EURONOIS11M=			- 0.02		2.33	0.85
EURONOIS1Y=	- 5.01	- 5.32	- 5.25	- 5.24	- 7.31	- 5.97
EURONOIS13M=			- 0.19	- 0.11	- 0.18	- 0.17
EURONOIS14M=	6.76	6.75	7.08	6.98	6.97	6.97
EURONOIS15M=	0.23	0.24	0.07	0.13	0.26	0.22
EURONOIS16M=			- 0.01		- 0.28	- 0.12
EURONOIS17M=					0.83	0.24
EURONOIS18M=					- 2.78	- 0.80
EURONOIS19M=					12.00	3.65
EURONOIS20M=					- 42.58	- 13.56
EURONOIS21M=					148.39	49.64
EURONOIS22M=					- 607.79	- 213.42
EURONOIS23M=			- 15.92	- 2.30	2,190.05	802.05
EURONOIS2Y=	0.75	- 0.70	15.57	1.49	- 2,054.28	- 787.82
EURONOIS3Y=	2.84	1.95	2.35	2.64	1,475.62	859.54
EURONOIS4Y=	3.83	2.59	- 3,460.05	- 3,690.91	- 5,221.10	- 4,042.30
EURONOIS5Y=	43,637.48	42,199.03	49,978.98	50,720.37	49,557.84	47,934.17
EURONOIS6Y=	8,977.30	10,412.29	6,708.55	6,269.59	9,031.50	10,477.54
EURONOIS7Y=			- 605.69	- 609.56	- 2,367.74	- 3,203.47
EURONOIS8Y=					639.07	987.88
EURONOIS9Y=					- 172.52	- 300.13
EURONOIS10Y=					47.10	91.01
EURONOIS11Y=					- 12.73	- 27.05
EURONOIS12Y=					3.45	7.99
EURONOIS13Y=					- 0.93	- 2.34
EURONOIS14Y=					0.25	0.68
EURONOIS15Y=					- 0.07	- 0.20
EURONOIS16Y=					0.02	0.06
EURONOIS17Y=					- 0.01	- 0.02
EURONOIS18Y=						
EURONOIS19Y=						
EURONOIS20Y=						
EURONOIS21Y=						
EURONOIS22Y=						
EURONOIS23Y=						
EURONOIS24Y=						
EURONOIS25Y=						
EURONOIS26Y=						
EURONOIS27Y=						
EURONOIS28Y=						
EURONOIS29Y=						
EURONOIS30Y=						
EURONOIS35Y=						
EURONOIS40Y=						
EURONOIS50Y=						
EURONOIS60Y=						

Figure 1.25: Deltas for a 2M Forward Start 5Y OIS with different interpolation methods.

Chapter 2

Calibrating Interest Rate Curves in QuantiLibXL

This chapter aims to present QuantLibXL with its object based logic and to explain step-by-step how to calibrate a rate curve taking advantage of its functionalities. In order to do that, a practical approach will be followed showing Excel figures and analyzing each function needed to perform a good curve bootstrap.

2.1 Curve Calibration in QuantLibXL

QuantLib is a C++ free/open-source library for quantitative finance based on an object-oriented programming (OOP). OOP is a programming paradigm based on concept of "objects" which may contain data, often known as attributes, and functions often known as methods. QuantLib library implements all the basic classes and methods used for calibration and these functionalities are exported to Microsoft Excel through QuantLibXL Add-IN (for more information please visit `http://quantlib.org`).
The Banca IMI's Framework implementation techniques will be discussed into detail in **Chapter 4** but here we present a simple static case of Eonia Curve calibration in order to give to the reader a basic know-how for self implement his own model.
Before starting, the first step is to summarize curve general settings like: evaluation date, calendar, currency, day counter and so on. This passage isn't necessary but can simplify the process. A really important role is played by the Trigger; this cell (yellow coloured in Figure 2.4) force the sheet recalculation and it is a backbone of real time calibrations in which market data change in continuous time and recalculation is strictly required. If you want your workbook to recalculate each time that you trigger, it is necessary to trace cell dependency filling each trigger field for all QuantLibXL functions.
First of all we have to set an evaluation date using: *"qlSettingsSetEvaluationDate"*, which needs only a date as input parameter: a specific date. After that you can calculate the relative settlement date using: *"qlCalendarAvance"*, which needs more input to be set like: "calendar, day counter, settlement days (usually 2) and many more".

The calibration starting point is the creation of a set of financial instruments (like

Trigger	
EvaluationDate	11-Jul-16
SetEvaluationDate	TRUE
Calendar	TARGET
SettlementDays	2
SettlementDay	13-lug-16
Currency	EUR
Compounding	Continuous
Frequency	Semiannual
DayCountConvention	Act/360
BusinessDayConvention	Following

Figure 2.1: A general settings example for curve calibration.

swaps, FRAs, Futures or Deposits) corresponding to meaningful market quotes that depends on the same underlying (eg a ON or a 3M rate). For each of these instruments you need to create a Quote Object using the *"qlSimpleQuote"* function that is an easy and straightforward feature which creates an instrument object and does not need any input parameter (otherwise it is always recommended to pass it a *"QuoteID"* in the *"ObjectID"* field). After that we have to pass to all Quote Objects the respective market quotes using: *"qlSimpleQuoteSetValue"*. This function requires 2 input parameter: the SimpleQuotes built in the previous step and the respective MarketValue. We also need to build an Eonia Index object through the *"qlEonia"* function which does not need required inputs (note that for all other tenor indexes must be used a different function called *"qlEuribor"*).
We are now ready to build the main calibration objects, namely Rate Helpers. The aim of QuantLibXL Rate Helpers is to link a market quote to a specific instrument with all the related conventions necessary to let the bootstrap algorithm know how it has to re-price that specific market value during the calibration process. As a consequence, there are many types of Rate Helpers, one for each kind of market instruments (for example you'll find OIS Rate Helpers, for overnight indexed swaps, or FRA Rate Helpers, for forward rate agreements and so on). QuantLib uses directly this kind of objects in addition to priority and selection criteria for bootstrapping any curve. This criteria aren't necessary for the bootstrap algorithm but can widely improve the output.
There are 3 different kind of criteria:

- **Selection Criteria**: implemented with a TRUE/FALSE array that must be passed to the *"ohFilter"* function. This function permits to select which instruments must be included in the calibration and which not. In practice, when the user FALSE flag an instrument is forcing the calibration to not include it in any case.

- **Priority and Minimum Distance Criteria**: implemented as 2 lists of numbers. In particular the *"Min Dist"* list represents the neighborhood of distance days required by an instrument from the previous and following pillar. If two pillars are near each other in less than the minimum distance, just one of them will be included in the calibration, namely the one that has the higher priority expressed in the *"Priority"* list. In practice, if you have an high liquid instrument concentration in a time frame you can express your inclusion criteria linking each quote to a couple of number (priority and minimum distance); so, for example, if you want to be sure that Futures will be included at the expense of FRAs you can set a higher Priority to Futures.

- **Futures and Deposits selection criteria**: that are 4 different criteria:
    1. nImmFutures: requests an integer representing the maximum number of IMM Futures that can be included in the calibration;
    2. nSerialFutures: requests an integer representing the maximum number of Serial Futures that can be included in the calibration;
    3. FutureRollDays: requests an integer representing how many days before its expiry the Front Futures must be discarded (zero implies the use of the Front Futures during its expiry day);
    4. DepoInclusion: sets up the deposits inclusion criterion (if missing, default = AllDepos).

qlEonia	Eonia#0000

Instrument	Maturity	MarketValue
OIS	ON	0.97%
OIS	TN	0.98%
OIS	1W	0.99%
OIS	2W	0.98%
OIS	3W	1.00%
OIS	1M	0.99%
OIS	3M	0.99%
OIS	6M	0.97%
OIS	1Y	0.96%
OIS	2Y	0.97%
OIS	3Y	1.00%
OIS	4Y	1.04%
OIS	5Y	1.11%
OIS	10Y	1.30%
OIS	20Y	1.43%
OIS	30Y	1.68%
OIS	60Y	1.89%

QuoteID	qlSimpleQuote	qlSimpleQuoteSetValue	qlOISRateHelper	Selection	Priority	Min Dist
OisOn	OisOn#0000	0.00%	obj_0001e#0000	FALSE	50	1
OisTn	OisTn#0000	0.00%	obj_00014#0000	FALSE	50	1
Ois1wk	Ois1wk#0000	0.00%	obj_0001d#0000	TRUE	50	1
Ois2wk	Ois2wk#0000	0.00%	obj_0001a#0000	TRUE	50	1
Ois3wk	Ois3wk#0000	0.00%	obj_00019#0000	TRUE	50	1
Ois1mt	Ois1mt#0000	0.00%	obj_0000f#0000	TRUE	50	1
Ois3mt	Ois3mt#0000	0.00%	obj_00017#0000	TRUE	50	1
Ois6mt	Ois6mt#0000	0.00%	obj_00010#0000	TRUE	50	1
Ois1yr	Ois1yr#0000	0.00%	obj_0001c#0000	TRUE	50	1
Ois2yr	Ois2yr#0000	0.00%	obj_0001f#0000	TRUE	50	1
Ois3yr	Ois3yr#0000	0.00%	obj_00021#0000	TRUE	50	1
Ois4yr	Ois4yr#0000	0.00%	obj_00020#0000	TRUE	50	1
Ois5yr	Ois5yr#0000	0.00%	obj_00018#0000	TRUE	50	1
Ois10yr	Ois10yr#0000	0.00%	obj_00022#0000	TRUE	50	1
Ois20yr	Ois20yr#0000	0.00%	obj_0001b#0000	TRUE	50	1
Ois30yr	Ois30yr#0000	0.00%	obj_00016#0000	TRUE	50	1
Ois60yr	Ois60yr#0000	0.00%	obj_00011#0000	TRUE	50	1

Figure 2.2: Rate helpers building using QuantLibXL.

Once rate helpers have been built, before starting the curve calibration, it is necessary to execute a pre-processing procedure over the whole set of helpers in order to filter them according to the selection, priority, minimum distance, futures and deposit criteria. The result is the subset of instruments which will be included in the bootstrap process. This can be achieved by means of the *"qlRateHelperSelection"* function used in conjunction with the *"ohFilter"* function. ohFilter passes to QuantLib the information concerning what instruments have been selected with the TRUE/FALSE flag. It requires two vectors as input whose one must be a sequence of TRUE/FALSE. An example of how using these functions is given by Figure 2.3;

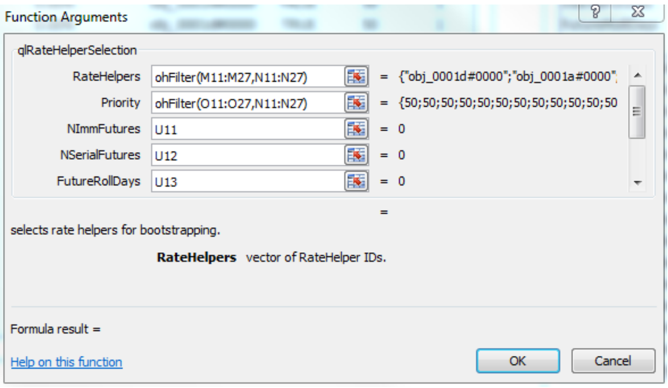


Figure 2.3: An example of how using ohFilter in qlRateHelperSelection for considering selection, priority, futures and deposit criteria.

When rate helpers have been selected, a good practice is to check the list of instruments used for bootstrapping as well as the main curve characteristics like: day counter, parametrization and interpolation technique as reported in at the bottom of Figure 2.4. After that, we have all the input needed to perform the calibration function *"qlPiecewiseYieldCurve"* which request a wide input range including the pre-selected rate helpers.

We have finally created the object which contains the calibrated overnight curve bootstrapped from selected instrument's market values and consistent with all conventions and characteristics chosen previously. This object can be passed to other QuantLibXL function to extrapolate some information as done in Figure 2.5 in which, for example, we use the *"qlPiecewiseYieldCurveDates"* and *"qlPiecewiseYieldCurveData"* functions to obtain a set of dates and the corresponding discount factors from curve object. Furthermore we can always request the corresponding zero and forward rates using *"qlYieldTSZeroRate"* and *"qlYieldTSForwardRate"* although we parametrize the curve in terms of discount factors (remember that the 3 parametrization are equivalent, as explained before in **Chapter 2**).

Select Rate Helpers	
nIMMFutures	0
nSerialFutures	0
FutureRollDays	0
DepoInclusion	AllDepos
qlRateHelperSelection	obj_0001d
qlRateHelperSelection	obj_0001a
qlRateHelperSelection	obj_00019
qlRateHelperSelection	obj_0000f
qlRateHelperSelection	obj_00017
qlRateHelperSelection	obj_00010
qlRateHelperSelection	obj_0001c
qlRateHelperSelection	obj_0001f
qlRateHelperSelection	obj_00021
qlRateHelperSelection	obj_00020
qlRateHelperSelection	obj_00018
qlRateHelperSelection	obj_00022
qlRateHelperSelection	obj_0001b
qlRateHelperSelection	obj_00016
qlRateHelperSelection	obj_00011

Curve Definition	
DayCountConvention	Act/360
Traits	Discount
Interpolator	MonotonicCubicNaturalSpline
qlPiecewiseYieldCurve	obj_00023#0001

Figure 2.4: Rate helper selection and curve calibration.

qlPiecewiseYieldCurveDates	qlPiecewiseYieldCurveData
Monday, July 11, 2016	1
Wednesday, July 20, 2016	0.999750944
Wednesday, July 27, 2016	0.999564589
Wednesday, August 03, 2016	0.999359465
Monday, August 15, 2016	0.99903605
Thursday, October 13, 2016	0.997417575
Friday, January 13, 2017	0.994990308
Thursday, July 13, 2017	0.990334458
Friday, July 13, 2018	0.980659333
Monday, July 15, 2019	0.970188864
Monday, July 13, 2020	0.958713854
Tuesday, July 13, 2021	0.945480945
Monday, July 13, 2026	0.87635115
Monday, July 14, 2036	0.747909379
Friday, July 13, 2046	0.590509256
Monday, July 13, 2076	0.297644394

Figure 2.5: Data extrapolation from curve object.

Chapter 3

Banca IMI's Rate Curve Framework

3.1 Software/Addin needed

The Rate Curve Framework is a set of Excel workbooks which allows the users to perform curves calibration. It permits to bootstrap yield curves by means of adavanced interpolation techniques, synthetic deposits and including Turn-of-Year effects. Actually this workbooks are built using Quantlib analytics exported in Excel through the QuantLibAddin and are meant to be feed with real time datas. At present, the chosen provider is Thomson Reuters[1] and to properly use the framework the users need:

- Thomson Reuters Eikon License;
- Thomson Reuters Eikon platform and Thomson Reuters Eikon Microsoft Excel installed on his workstation.

Thomson Reuters Eikon Microsoft Excel is an Excel Add-In that allows you to download real time data from Thomson Reuters Eikon platform to Microsoft Excel: in this way all market quotes that we need for calibration are available directly on Excel interface where they can interact with QuantLibXL functions. Furthermore, Banca IMI integrates the set of open-source workbooks with the Murex Contributor. The contributor allows to feed the intern front office system (in our case Murex) with a set of bootstrapped "meta-quote" which will be involved in second Murex bootstrap. Of course it is possible to switch to another front office system but an analytics change will be necessary.

[1] It is also possible to use another data provider to get real time quotes but, in this case, it will be necessary to change all import data functions

3.2 How to access the framework

3.2.1 Access via .bat Files

In order to access the Rate Curve Framework, a possible way is to download from **`http://quantlib.org`** the QuantLibXL-xxx.zip [2] file. Once you do it, in the unzipped folder you will find a sequence of *.bat* and *.xml* files.
For each currency (CCY), you can find two batch (.bat) and their related (i.e. with the same name) xml files. The batch files are used to launch the Excel Rate Curve Framework session: **session_file.CCY-s-live.bat** is used for a live data feed session by means of Thomson Reuters Eikon, **session_file.CCY-s-static.bat** is used to load an historical data session. The .xml files contains a list of start-up parameters and options needed by the Add-in (for example: currency, evaluation date, xll path etc...).

3.2.2 Access via XL-Launcher Application

Another easy way to access to the Framework is by mean of XL-Launcher application that works as a front end to Excel and allows various Excel session configurations, such as Add-ins to be loaded and start-up parameters to be passed to the Add-ins.

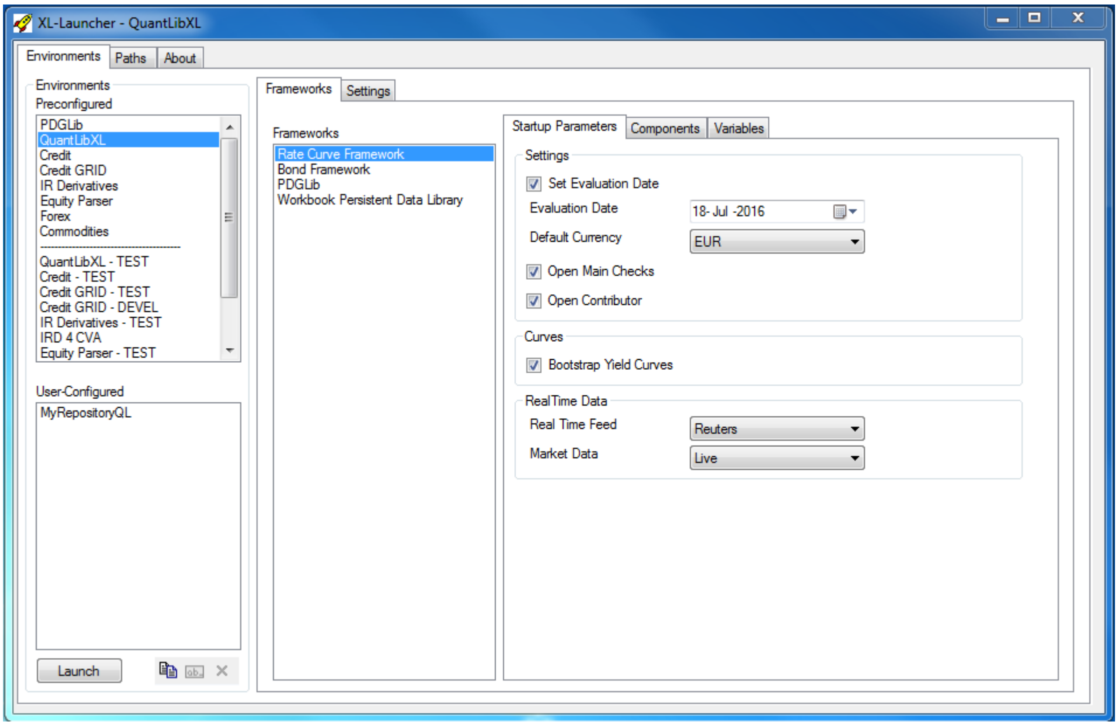


Figure 3.1: XL-Launcher Application interface.

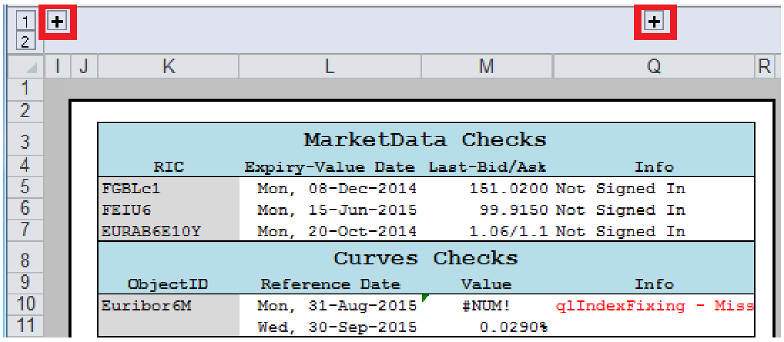
As visible, by means of XL-Launcher it is possible to configure a Framework session

[2]xxx represents the version number

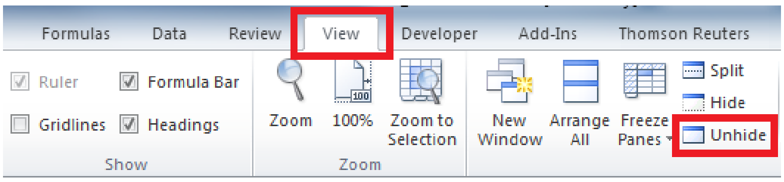
setting up many parameters like: currency, evaluation date, data provider; but also choose whether or not open the MainChecks, open the Contributor, bootstrap Yield Curves and choose Live/Static data.

3.3 Rate Curve Framework Structure

Once you have opened the Framework, you will be able to see on the board, after a few minutes of loading, the MainChecks workbook (next section is devoted to a user guide of the whole set of workbooks which compose the Framework). First of all, at least all worksheets have some hidden part that can be viewed simply clicking on plus buttons under the Excel quick access toolbar (see Figure **??**.



The other workbooks are hidden and, to unhide them, you must go to *"View"*, on Excel ribbon, and then click *"Unhide"*.



After that, you will see on the board the table in Figure 3.2 and you will be able to access to all workbooks. The *MainChecks*, as the name suggests, is devoted to a quick connection and curve control in order to be sure that curve bootstrapping has been successful and to check the real time data feed. The *Market* workbook creates the simple quote objects for a wide number and type of market instruments setting the respective market values imported from Thomson Reuters. The *JumpsQuotesFeedON* has the target of finding and calculating jumps while *SynthQuotesFeed* is dedicated to synthetic deposits calculation. Finally the *CurveBootstrapping* workbook is the one intended for creating the rate helper objects, selecting the preferred instruments and performing the rate curves calibration.

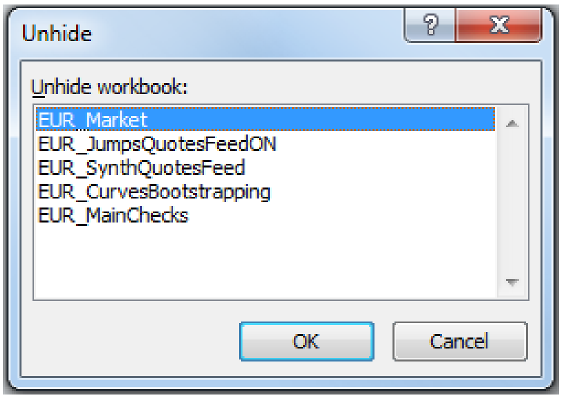


Figure 3.2: A complete list of the Rate Curve Framework workbooks.

3.4 Workbooks User Guide

This section wants to be a Rate Curve Framework guideline which could help an end-user to understand the rationale behind each workbook. In particular it wants to focus where the user can interact with the Framework, changing parameters or switching settings and where he can't.

3.4.1 MainChecks

As already sad, this is the first workbook visible on the board once the Framework session as been launched. It is dedicated to a fast check on live data connection and to each curve calibration. An example of MainChecks layout is given by Figure 3.3.

We can summarize all controls in 3 steps:

- Check the loading: before start it is necessary to see the *"Ready"* message in the Excel status bar (lower left corner). If not, the workbook is still loading and this passage may require a few minutes.

- Check the Real Time Feed: in cells **Q5-Q6-Q7** you must see the message *"Updated at [current time]"* as you can see in the underlying figure.

  If not, there are several different messages that can appear. One of them is *"Paused at..."*, it means that the data downloading is paused and can be adjusted simply clicking on *Resume Updates* using Thomson Reuters tab. Another one is *"Offline at..."*, it can occurs when you are not online with Eikon and can be adjusted simply forcing the Reuters Login. Finally there is the *"Access denied"* message which occurs when a page has become private and the data provider should be contacted for support.

- Check Curve Calibrations: first of all go to cell **U2** *("Trigger")* and delete the content (this passage is fundamental to eliminate false errors which may occur if

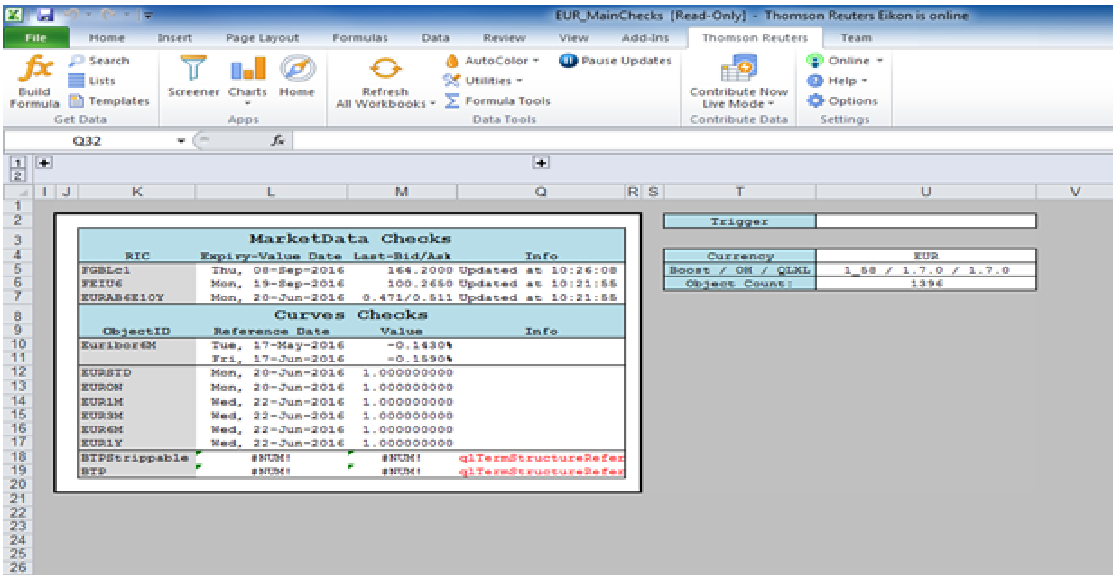
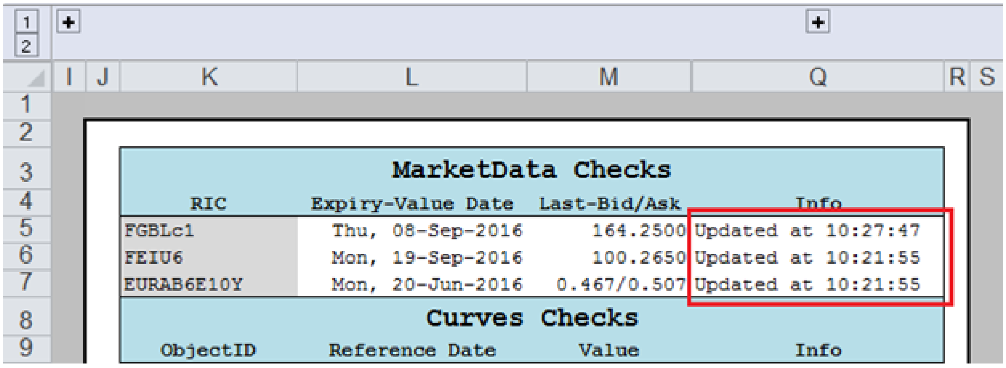
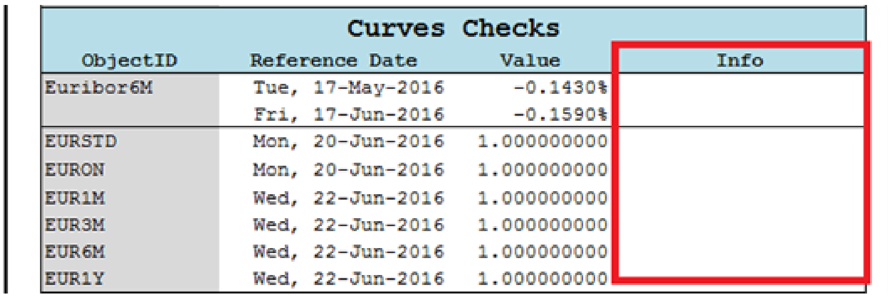


Figure 3.3: Eur_MainChecks workbook.



MarketData Checks			
RIC	Expiry-Value Date	Last-Bid/Ask	Info
FGBLc1	Thu, 08-Sep-2016	164.2500	Updated at 10:27:47
FEIU6	Mon, 19-Sep-2016	100.2650	Updated at 10:21:55
EURAB6E10Y	Mon, 20-Jun-2016	0.467/0.507	Updated at 10:21:55
Curves Checks			
ObjectID	Reference Date	Value	Info

the workbook is not triggered). If everything works, you will see no error message in *"Info"* like shown in next figure.



Curves Checks			
ObjectID	Reference Date	Value	Info
Euribor6M	Tue, 17-May-2016	-0.1430%	
	Fri, 17-Jun-2016	-0.1590%	
EURSTD	Mon, 20-Jun-2016	1.000000000	
EURON	Mon, 20-Jun-2016	1.000000000	
EUR1M	Wed, 22-Jun-2016	1.000000000	
EUR3M	Wed, 22-Jun-2016	1.000000000	
EUR6M	Wed, 22-Jun-2016	1.000000000	
EUR1Y	Wed, 22-Jun-2016	1.000000000	

If you still have error messages in Info Column try to Refresh All Workbook (Thomson Reuters Tab) and cancel again the Trigger.

At the end, it is good practice to check if the Thomson Reuters values in **Q** column *("Last-Bid/Ask")* for instruments in **K** column *("RIC")* are correct. In particular this Reuters RICs refers to the most liquid instruments quoted in the reference market. Logically, this are well known values for traders that can immediately check if Reuters is really downloading live quotes. It is also good practice to check if:

1. For each curve, column **L** *("Reference Date")* is equal to [curve's spot date] and **M** *("Value")* is equal to 1.000000000.

2. For Ibor Indexes (for example Euribor6M for EUR), Index Reference Date must be [yesterday] if you launch the framework before fixings publication, otherwise [today] and Index Value must be the last published index fixing.

Curves Checks			
ObjectID	Reference Date	Value	Info
Euribor6M	Tue, 17-May-2016	-0.1430%	
	Fri, 17-Jun-2016	-0.1590%	
EURSTD	Mon, 20-Jun-2016	1.000000000	
EURON	Mon, 20-Jun-2016	1.000000000	
EUR1M	Wed, 22-Jun-2016	1.000000000	
EUR3M	Wed, 22-Jun-2016	1.000000000	
EUR6M	Wed, 22-Jun-2016	1.000000000	
EUR1Y	Wed, 22-Jun-2016	1.000000000	

3.4.2 Market

This workbook has the task to create an object for each market quote, retrieve from real time Data provider the relative market values and then associate this values to the corresponding quotes. In the *GeneralSettings* sheet you can find the workbook's main setting and, in particular, it is possible to chose from which broker import market quotes as visible in Figure 3.4 in which we are taking FRA, OIS and IMM OIS from ICAP and ECB OIS from Tullets. As usual it is also possible to chose currency and other general settings. The Live Feed table **(G2:J7)**, summarize the type of session showing TRUE when you have launched a live session, otherwise FALSE. Under that you can see the date of the last update that must be [current time], [current date] in live session and a historical date in static sessions. Furthermore, in Triggers table **(B2:E10)**, is important that the *TriggerCounter* cell, in live session, continue triggering during time meaning that the Framework is really updating market quotes in real time; to check this, it is sufficient that trigger counter rises over time.

Moving to the next sheet, *Euribor*, you can find the construction of the objects which represent all Euribor Indexes. As soon as the framework have performed the curves bootstrap, this objects will be directly linked to the curves using the QuantLibXL *qlRelinkableHandleYieldTermStructure* function. This process might appear useless but it's a fundamental step especially for live sessions. In fact, doing this, index objects won't be re-created each time that curves changes due to the updating of market values.

Triggers

Trigger	Mon, 30-May-2016, 09:01:24
EvaluationDate	Mon, 30-May-2016
TriggerCounter	2
LastFixingsTrigger	0

Contributors

Reuters	
KLIEM	CKCC
ICAP	ICAP
Tullets	TTKL
Tradition	TRIT
Prebon	PREL
COSMOREX	COSZ

INSTRUMENT	EUROPE
Deposits	KLIEM
FRA	ICAP
OIS	ICAP
ECB OIS	TTKL
IMM OIS	ICAP

Figure 3.4: Market contributor table.

In fact, when a user want to create a generical framework using QuantLibXL, must keep in mind that the best practice is to build each object just one time because, every time an object is created, Excel allocates memory and this can overload the entire framework.

Tenor	*Relinkable Handle*	*Index*	*LastFixing_Quote*
ON	EURON#0000	Eonia#0000	EoniaLastFixing_Quote#0000
SW		EuriborSW#0000	EuriborSWLastFixing_Quote#0000
2W		Euribor2W#0000	Euribor2WLastFixing_Quote#0000
1M	EUR1M#0000	Euribor1M#0000	Euribor1MLastFixing_Quote#0000
2M		Euribor2M#0000	Euribor2MLastFixing_Quote#0000
3M	EUR3M#0000	Euribor3M#0000	Euribor3MLastFixing_Quote#0000
6M	EUR6M#0000	Euribor6M#0000	Euribor6MLastFixing_Quote#0000
9M		Euribor9M#0000	Euribor9MLastFixing_Quote#0000
1Y	EUR1Y#0000	Euribor1Y#0000	Euribor1YLastFixing_Quote#0000
STD	EURSTD#0000		

Each of the remaining sheets serve the purpose of creating simple quotes related to particular kind of market instruments (e.g. FRA, OIS, Futures...).
In the *Mid* column you find the Live Thomson Reuters quotes and, in the *Static* column, the historical ones referred to the old date (see the general settings Last Update cell). If you have launched a live session, the *Effective* column takes *Mid* column data, otherwise it takes the *Static* column data. Finally the *Change* column calculate the percentage

change in market quotes.

Mid	Bid/Ask Spread	Static	Effective	Change
-0.2590	0.0500	-0.2590	-0.2590	0.0000%
-0.2620	0.0500	-0.2620	-0.2620	0.0000%
-0.2660	0.0500	-0.2660	-0.2660	0.0000%
-0.2680	0.0500	-0.2680	-0.2680	0.0000%
-0.2740	0.0500	-0.2740	-0.2740	0.0000%
-0.2800	0.0500	-0.2800	-0.2800	0.0000%
-0.2820	0.0500	-0.2820	-0.2820	0.0000%
-0.2840	0.0500	-0.2840	-0.2840	0.0000%
-0.2870	0.0500	-0.2870	-0.2870	0.0000%
-0.2910	0.0500	-0.2910	-0.2910	0.0000%
-0.2950	0.0500	-0.2950	-0.2950	0.0000%
-0.2990	0.0500	-0.2990	-0.2990	0.0000%

3.4.3 JumpsQuotesFeedON

As usual the starting point is the general settings sheet. The first step in the jump analysis is to bootstrap an overnight curve till 2Y using a flat interpolation on forward rates and where jumps are not considered. To check if the calibration goes right you can have a look to **D18-D19** cells.

Wed, 1-Jun-2016		1.000000000	
Fri, 1-Jun-2018		1.003476556	

If you see and error message there, please try to trigger or refresh the session. The error message can also be generated by Thomson Reuters connection, so, have a look also to **D23-D24** cells which, in live sessions, must contain: "TRUE" near *LiveDataFeed* and [current-date], [current-time] near *LastUpdate*.

LastUpdate	Wed, 22-Oct-2014, 19:52:02
LiveDataFeed	FALSE

In general setting sheet you also find the Final Curve settings which is the workbook's outcome; in fact, cell **I14** represents the new "jump-corrected" curve. As before you can check if this passage have worked looking to cells **I18-I19**

Final Curve	
PiecewiseYieldCurve Error	obj_0043b#0000
Yield Curve Index	obj_0043c#0000
Wed, 1-Jun-2016	1.000000000
Fri, 1-Jun-2018	1.003476556

Moving to the Rate Helpers sheet you can find the whole set of instruments used to bootstrap the overnight curve till 2Y. This passage will be discussed more accurately in section 3.4.5. Next sheet is the one which performs the jump size estimation. The objective is to check where, in the pre-calibrated overnight curve, a jump can occur and calculate its size. If the analysis find a jump, you will see a value different from zero (which is the jump size) in the red-written cell, otherwise you will find: 0.00

	Jump	99.9996%
Thu, 30-Jun-2016	tau	0.0028
obj_0042e#0000	overwrite	Jump (bps)
TRUE		0.13%

In the last sheet you can find the jumps sizes and dates contribution. Here the pre-calculated jumps are fixed in quotes in order to be accounted in the final curve calibration.

Jump ID		Effective
EURJump1_SYNTHON_Quote	EURJump1_SYNTHON_Quote#0000	99.9998%
EURJump2_SYNTHON_Quote	EURJump2_SYNTHON_Quote#0000	100.0000%
EURJump3_SYNTHON_Quote	EURJump3_SYNTHON_Quote#0000	99.9991%
EURJump4_SYNTHON_Quote	EURJump4_SYNTHON_Quote#0000	100.0000%
EURJump5_SYNTHON_Quote	EURJump5_SYNTHON_Quote#0000	100.0000%
EURJump6_SYNTHON_Quote	EURJump6_SYNTHON_Quote#0000	100.0000%
EURJump7_SYNTHON_Quote	EURJump7_SYNTHON_Quote#0000	100.0000%
EURJump8_SYNTHON_Quote	EURJump8_SYNTHON_Quote#0000	100.0000%
EURJump9_SYNTHON_Quote	EURJump9_SYNTHON_Quote#0000	100.0000%
EURJump10_SYNTHON_Quote	EURJump10_SYNTHON_Quote#0000	100.0000%
EURJump11_SYNTHON_Quote	EURJump11_SYNTHON_Quote#0000	100.0000%
EURJump12_SYNTHON_Quote	EURJump12_SYNTHON_Quote#0000	100.0000%
EURJump13_SYNTHON_Quote	EURJump13_SYNTHON_Quote#0000	99.9993%

In the example shown in previous figure, as visible, the analysis found 3 positive jumps.

3.4.4 SynthQuotesFeed

The General settings sheet contains the usual features and checks so let's skip directly to the other sheets. The xMSynthDepo sheets has the objective to build synthetic deposits for the related xM curve. In the *"Selected"* column it is possible to decide which Delta's[3] must be included in the α, β, γ calibration.

coef α	coef β	coef γ	selected
0.0861	-0.0549	0.0350	FALSE
0.0861	0.0037	0.0002	TRUE
0.0861	0.0111	0.0015	TRUE
0.0917	0.0197	0.0043	TRUE
0.0833	0.0252	0.0077	FALSE

Deltas can be selected simply writing: "TRUE" in the corresponding cell. Remember that the Quadratic parametrization needs 3 deltas to be calibrated so, if you flag just 2 deltas, you will be able to use just the Linear and Flat parametrization.
This deltas are calculated in the xM Delta sheets for each of the xM Curve. Even in this case you can decide which basis must be computed directly and which one must be interpolated writing: "TRUE" or "FALSE" in the *"Selected"* columns.

3M Market quote	F ON	Basis	Selected
-0.0400%	-0.1466%	0.1066%	FALSE
-0.0460%	-0.1558%	0.1098%	TRUE
-0.0490%	-0.1618%	0.1128%	TRUE
-0.0530%	-0.1712%	0.1182%	TRUE
-0.0550%	-0.1754%	0.1204%	TRUE
-0.0570%	-0.1812%	0.1242%	TRUE

Finally all synthetic deposits created are fixed in quotes in the *Contribution* sheet. As you can see, there is a different column for *Live*, *Static* and *Effective* values. In particular, values in the *Effective* column change in according to the fact that you are using or not a live session (as already explained before).

[3] We already discuss Delta's calculation in Section 1.7.1

	Live
SND	-0.1097%
SWD	-0.1152%
2WD	-0.1155%
3WD	-0.1158%

Static
-0.1060%
-0.1122%
-0.1135%
-0.1147%

Effective
-0.1060%
-0.1122%
-0.1135%
-0.1147%

For a final check remember to control the *Error* column; there must be no error messages.

Quote Object	Error
EURSND_SYNTH1M_Quote#0001	
EURSWD_SYNTH1M_Quote#0001	
EUR2WD_SYNTH1M_Quote#0001	
EUR3WD_SYNTH1M_Quote#0001	
EUR1MD_SYNTH1M_Quote#0001	

3.4.5 CurveBootstrapping

Let's consider the case of EUR curves bootstrapping. The General Setting sheet summarize all bootstrapped curve objects near the *PiecewiseYieldCurve* cells. As usual you can switch settings (like Calendar, Discounting Curve, Currency, Pillar Date etc...) simply clicking on the right cell and selecting the parameter you need from the drop down menu. Always remember to trigger after changing parameters.

Currency	EUR
Permanent	TRUE
Trigger	Fri, 27-May-2016, 11:26:47
ObjectOverwrite	TRUE
Discounting	EURON
Pillar Date	MaturityDate

Evaluation Date	Fri, 27-May-2016
SettlementDate	Tue, 31-May-2016
Calendar	TARGET

Family Name	ibor
QuoteSuffix	_Quote
JumpSuffix	_SYNTHON

In the same ways you can change curves settings like:

1. **Settlement days** *"NDays"* - Usually it' set to 0 or 2 and it fixes the day which discount factor = 1;
2. **Curve parametrization** *"TraitsID"* - If is missing, default = Discount;
3. **Interpolation scheme** *"InterpolationID"* - If is missing, default = LogLinear;

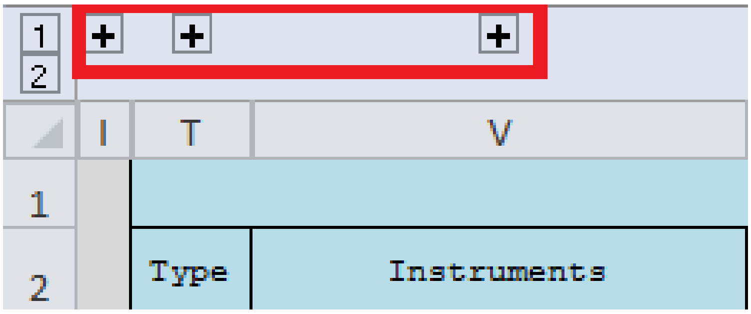
To check if all curves have been correctly calibrated, please control cells **D21-D22** (for Overnight Curve): the first discount (**D21**) is set on the reference date for the relative term structure and so must be 1.000000000; for the second rate (**D22**) it's sufficient

PiecewiseYieldCurve	_EURON#0005
Error	
ObjectID	_EURON
NDays	0
Accuracy	
TraitsID	ZeroYield
InterpolatorID	MonotonicCubicNaturalSpline

that you don't see an error message. The same rules are applied for all other Curves. Furthermore, if you write *TRUE* near the *Extrapolation* cell the framework will be able to provide rates for dates after the last curve pillar using a flat extrapolator.

Mon, 30-May-2016	1.000000000
Mon, 1-Jun-2076	0.438156678
EURON#0000	TRUE
Extrapolation	FALSE

Moving to the EURON sheet it's is important to know that there are some hidden cells that you can unhide clicking on plus buttons visible over **I, T** and **V** columns. The hidden parts show the construction of *Rate Helpers* for the whole set of instruments indexed on overnight rate.



A very important task is played by columns *Include Flag*, *Priority* and *Minimum Distance*; in fact they define the inclusion and priority criteria for all instruments.

Include Flag	Priority	Min Dist
TRUE	70	1
TRUE	70	1
FALSE	70	1

For example, you can chose whether or not to exclude *OIS60Y* writing: *FALSE* in the right cell or change inclusion priority (for less priority digit a lower number). In particular the *Min Dist* column represents the minimum distance (in days) required by an instrument from another pillar. If two pillars are near each other in less than the minimum distance, just one of them will be included in the calibration, namely the one with the higher priority.

OIS	obj_00514#0000	EUREON60Y_Quote	1.3470%	TRUE	50	1	31-May-16	29-May-76

The column *Instruments* summarize the set of market instruments which are included in the bootstrap algorithm and will contribute to the curve calibration.
Cells **AH3-AH4-AH5-AH6**, instead, define the futures and deposits selection criteria already discussed in 3.

Rate Helpers Selection	
nIMMFutures	0
nSerialFutures	0
FrontFuturesRollingDays	0
DepoFuturesPriority	AllDepos

Finally from column **AC** to column **AJ** you find the jumps summary table calculated in the JumpQuotesFeedON sheet. In particular, in column *overwrite*, you can force the calibration to **NOT** consider a jump simply writing: 0 in the jump's corresponding cell or set another jump value overwriting the estimated jump size.

#	Dates	Size	overwrite	used
1	Thu, 30-Jun-2016	0.07%	0.00%	100.00000%
2	Fri, 29-Jul-2016	0.00%		EURJump2_SYNTHON_Quote
3	Wed, 31-Aug-2016	0.32%		EURJump3_SYNTHON_Quote
4	Fri, 30-Sep-2016	0.00%		EURJump4_SYNTHON_Quote
5	Mon, 31-Oct-2016	0.00%		EURJump5_SYNTHON_Quote
6	Wed, 30-Nov-2016	0.00%		EURJump6_SYNTHON_Quote
7	Fri, 30-Dec-2016	0.00%		EURJump7_SYNTHON_Quote
8	Tue, 31-Jan-2017	0.00%		EURJump8_SYNTHON_Quote
9	Tue, 28-Feb-2017	0.00%		EURJump9_SYNTHON_Quote
10	Fri, 31-Mar-2017	0.00%		EURJump10_SYNTHON_Quote
11	Fri, 28-Apr-2017	0.00%		EURJump11_SYNTHON_Quote
12	Wed, 31-May-2017	0.00%		EURJump12_SYNTHON_Quote
13	Fri, 29-Dec-2017	0.06%		EURJump13_SYNTHON_Quote

Notice that a similar table can be founded in the JumpQuotesFeedON workbook. However, the jump overwrite in the two tables is slightly different. In particular overwriting in JumpQuotesFeedON sheet implies overwriting in the whole set of curves while doing the same in CurveBootsapping workbook means change jumps size in relation to the single curve.

	Jump Date	Jump Value	Jump size	Overwrite JumpSize
Jump1	Thu, 30-Jun-2016	99.99988%	0.04%	
Jump2	Fri, 29-Jul-2016	100.00000%	0.00%	0.00%
Jump3	Wed, 31-Aug-2016	100.00000%	0.00%	0.00%
Jump4	Fri, 30-Sep-2016	100.00000%	0.00%	0.00%
Jump5	Mon, 31-Oct-2016	100.00000%	0.00%	0.00%
Jump6	Wed, 30-Nov-2016	100.00000%	0.00%	0.00%
Jump7	Fri, 30-Dec-2016	100.00000%	0.00%	0.00%
Jump8	Tue, 31-Jan-2017	100.00000%	0.00%	0.00%
Jump9	Tue, 28-Feb-2017	100.00000%	0.00%	0.00%
Jump10	Fri, 31-Mar-2017	100.00000%	0.00%	0.00%
Jump11	Fri, 28-Apr-2017	100.00000%	0.00%	0.00%
Jump12	Wed, 31-May-2017	100.00000%	0.00%	0.00%
Jump13	Fri, 29-Dec-2017	99.99911%	0.08%	0.08%

For all other sheets: EUR1M, EUR3M, EUR6M and EUR12M the same methodology described above can be applied with no significant differences.

Chapter 4

Banca IMI's Implementation

4.1 Rate Curve Framework Infrastructure

4.2 Introduction

In this chapter we apply the methodology illustrated in the previous chapter to the concrete EUR market case in order to produce a new proposal for the EUR Rate Curve Framework. Obviously we do not claim that our choices are the best solution to any problems, being related to many factors as the particular market situation we have experienced during last years.

We will calibrate five curves: $C_{ON}, C_{1M}, C_{3M}, C_{6M}, C_{12M}$ whose underlying indexes are Eonia, Euribor 1M, Euribor 3M, Euribor 6M, Euribor 12M respectively. Actually for each curve we will have two calibrations: first an Excel calibration, performed with market quotes and synthetic deposits quotes needed to fix the curve over the interval $[t, spot(today) + x]$; secondly a front office system calibration, performed with a set of contribution instruments which includes part or all of the instruments used for the first calibration plus other synthetic interpolated instruments (as explained in section 1.7.1). Also, the second calibration could be performed with different settings (reference date, interpolation, ...). Let us explain why this double calibration is still used in the actual Banca IMI framework.

With the actual infrastructure, it is not possible to completely avoid the double calibration process (for example, the front office system can't calculate synthetic deposits). Our proposal, instead, is to align as much as possible the two calibrations, in order to have both on Excel and inside the system a set of curves representative of the real market. These obviously imply a change in hedging practices that is still ongoing. Unfortunately some differences between first and second calibration still remains. We illustrate here the main ones.

Curves reference date

As discussed in section 1.7.2, we have to set discounting curve reference date to today (in order to be able to calculate today's value of cash flows). For forwarding curves, instead, we prefer to set reference date to the spot date of calibrating instruments. This is possible on Excel but not inside the front office system where the whole set of curves has reference date equal to today. This is a small inconsistency that still remains.

Interpolation scheme and instruments selection

As discussed in section 1.9 the best compromise between having good forward rates and stable/reasonable deltas calculation seems a bootstrap performed with a Kruger scheme applied on zero rates even if the corresponding forwards rates are not the best ones. In order to weaken their humped behavior, we follow these steps:

- we use Hyman scheme on Zero Rates for Excel Calibration in order to have a good curve from a forward rates point of view; this choice produces non-local deltas but we can tolerate it because the main purpose of this calibration is contribution (not sensitivities calculation); the only synthetic instruments used here are deposits.

- we create the set of contribution instruments including all the calibrating instruments plus a set of synthetic interpolated instruments to obtain a ticker time grid (mostly for the long term part of the curve);

- finally we use Kruger scheme on zero rates for the second calibration; both synthetic deposits and synthetic interpolated instruments are used here.

The result of this process for Euribor 6M Curve (Evaluation Date: 23 June 2016) is shown in Figures 4.1 and 4.2.

Synthetic Instruments

As said before, the Excel calibration is performed using only the necessary synthetic deposits (needed to fix the curve over the interval $[t, spot(today) + x]$) and no other synthetic instruments. Instead the system calibration makes use of synthetic interpolated instruments of two kinds: instruments corresponding to non quoted maturities (as just illustrated) or instruments not quoted at all (example: the market quotes basis swaps but system calibration is performed with swaps, which are preferred for hedging).

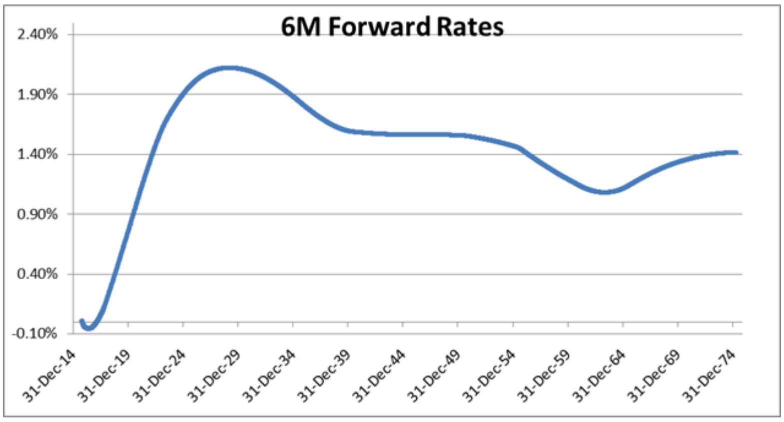
4.3 ON Curve

4.3.1 Excel Calibration

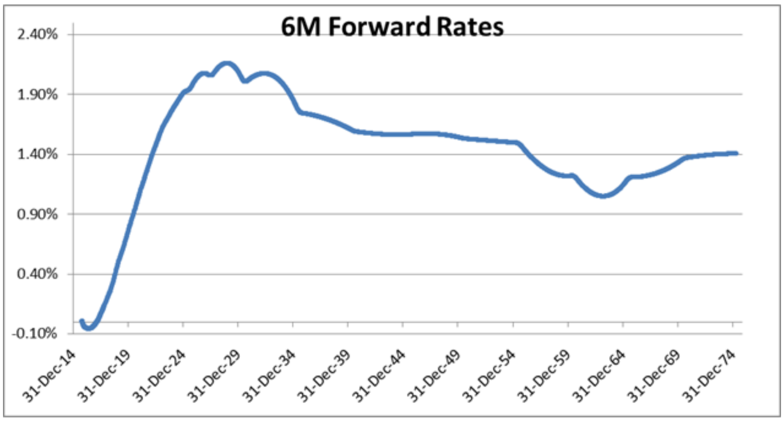
Eonia curve is bootstrapped using the following instruments:

Euribor 6M System Calibration Vs Excel Calibration					
Pillar	Instruments	Earliest Date	Pillar Date	XL Calibration	Mx Calibration
S/N	Deposit	Mon, 27-Jun-2016	Tue, 28-Jun-2016		x
1W	Deposit	Mon, 27-Jun-2016	Thu, 7-Apr-2016		x
2W	Deposit	Mon, 27-Jun-2016	Mon, 7-Nov-2016		x
3W	Deposit	Mon, 27-Jun-2016	Mon, 18-Jul-2016		x
1M	Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	x	x
2M	Deposit	Mon, 27-Jun-2016	Mon, 29-Aug-2016	x	x
3M	Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	x	x
4M	Deposit	Mon, 27-Jun-2016	Thu, 27-Oct-2016	x	x
5M	Deposit	Mon, 27-Jun-2016	Mon, 28-Nov-2016	x	x
6M	Deposit	Mon, 27-Jun-2016	Tue, 27-Dec-2016	x	x
1X7	FRA	Wed, 27-Jul-2016	Fri, 27-Jan-2017	x	x
2X8	FRA	Mon, 29-Aug-2016	Mon, 27-Feb-2017	x	x
3X9	FRA	Tue, 27-Sep-2016	Mon, 27-Mar-2017	x	x
4X10	FRA	Thu, 27-Oct-2016	Thu, 27-Apr-2017	x	x
5X11	FRA	Mon, 28-Nov-2016	Mon, 29-May-2017	x	x
6X12	FRA	Tue, 27-Dec-2016	Tue, 27-Jun-2017	x	x
7X13	FRA	Fri, 27-Jan-2017	Thu, 27-Jul-2017	x	x
8X14	FRA	Mon, 27-Feb-2017	Mon, 28-Aug-2017	x	x
9X15	FRA	Mon, 27-Mar-2017	Wed, 27-Sep-2017	x	x
10X16	FRA	Thu, 27-Apr-2017	Fri, 27-Oct-2017	x	x
11X17	FRA	Mon, 29-May-2017	Mon, 27-Nov-2017	x	x
12X18	FRA	Tue, 27-Jun-2017	Wed, 27-Dec-2017	x	x
13X19	FRA	Thu, 27-Jul-2017	Mon, 29-Jan-2018	x	x
14X20	FRA	Mon, 28-Aug-2017	Tue, 27-Feb-2018	x	x
15X21	FRA	Wed, 27-Sep-2017	Tue, 27-Mar-2018	x	x
16X22	FRA	Fri, 27-Oct-2017	Fri, 27-Apr-2018	x	x
17X23	FRA	Mon, 27-Nov-2017	Mon, 28-May-2018	x	x
18X24	FRA	Wed, 27-Dec-2017	Wed, 27-Jun-2018	x	x
24X30	FRA	Wed, 27-Jun-2018	Thu, 27-Dec-2018		x
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	x	x
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	x	x
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	x	x
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	x	x
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	x	x
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	x	x
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	x	x
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	x	x
11Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2027		x
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	x	x
13Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2029		x
14Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-1930		x
15Y	Swap	Mon, 27-Jun-2016	Sat, 27-Jun-1931	x	x
16Y	Swap	Mon, 27-Jun-2016	Tue, 28-Jun-1932		x
17Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-1933		x
18Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-1934		x
19Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-1935		x
20Y	Swap	Mon, 27-Jun-2016	Sat, 27-Jun-1936	x	x
21Y	Swap	Mon, 27-Jun-2016	Tue, 29-Jun-1937		x
22Y	Swap	Mon, 27-Jun-2016	Tue, 28-Jun-1938		x
23Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-1939		x
24Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-1940		x
25Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-1941	x	x
26Y	Swap	Mon, 27-Jun-2016	Sat, 27-Jun-1942		x
27Y	Swap	Mon, 27-Jun-2016	Tue, 29-Jun-1943		x
28Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-1944		x
29Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-1945		x
30Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-1946	x	x
31Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-1947		x
32Y	Swap	Mon, 27-Jun-2016	Tue, 29-Jun-1948		x
33Y	Swap	Mon, 27-Jun-2016	Tue, 28-Jun-1949		x
34Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-1950		x
35Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-1951	x	x
36Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-1952		x
37Y	Swap	Mon, 27-Jun-2016	Sat, 27-Jun-1953		x
38Y	Swap	Mon, 27-Jun-2016	Tue, 29-Jun-1954		x
39Y	Swap	Mon, 27-Jun-2016	Tue, 28-Jun-1955		x
40Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-1956	x	x
15YX30Y	Forward Swap	Fri, 27-Jun-2031	Mon, 27-Jun-2061	x	
50Y	Swap	Mon, 27-Jun-2016	Tue, 28-Jun-1966	x	x
25YX30Y	Forward Swap	Thu, 27-Jun-2041	Mon, 29-Jun-2071	x	
60Y	Swap	Tue, 27-Jun-2017	Tue, 29-Jun-1976	x	x

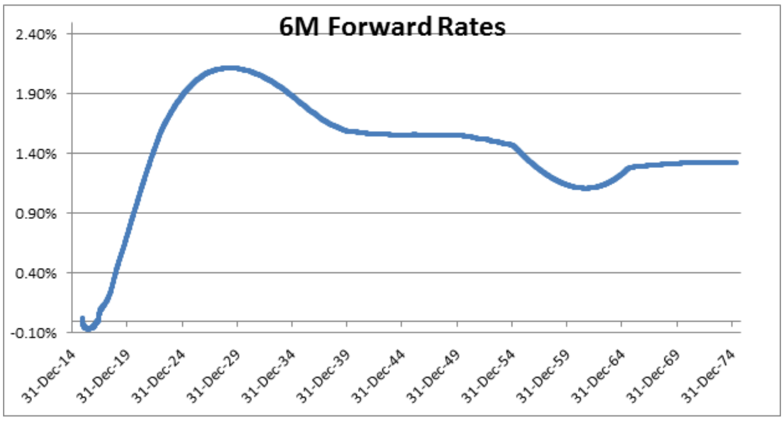
Figure 4.1: Excel Calibration vs System Calibration for Euribor 6M Curve (Evaluation Date 23 June 2016)



(a) Hyman on Zero Rates - Synthetic Deposits and Market Instruments (Excel Calibration)



(b) Kruger on Zero Rates - Synthetic Deposits and Market Instruments



(c) Kruger on Zero Rates - Synthetic Deposits, Synthetic Interpolated Instruments and Market Instruments (System Calibration)

Figure 4.2: Excel Calibration vs System Calibration for Euribor 6M Curve

- ON and TN Deposits in order to set the curve reference date to today's date (these instruments are not properly based on Eonia - the underlying index is the one-day tenor Euribor - thus we are introducing a very small inconsistency);
- all the available forward starting OIS on ECB dates;
- spot starting OIS up to 60Y.

The ECB OIS are more liquid than spot starting OIS and this is way we always use them when available. There are no synthetic instruments.

The curve structure with corresponding Market RICs is shown in Figure 4.3.

Eonia Curve - Excel Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Market RIC
ON	Deposit	Thu, 23-Jun-2016	Fri, 24-Jun-2016	EUROND=CKCC
TN	Deposit	Fri, 24-Jun-2016	Mon, 27-Jun-2016	EURTND=CKCC
SW	OIS	Mon, 27-Jun-2016	Mon, 04-Jul-2016	EUREONSW=ICAP
2W	OIS	Mon, 27-Jun-2016	Mon, 11-Jul-2016	EUREON2W=ICAP
3W	OIS	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EUREON3W=ICAP
1M	OIS	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EUREON1M=ICAP
2M	OIS	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUREON2M=ICAP
Jul-16	ECB OIS	Wed, 27-Jul-2016	Wed, 14-Sep-2016	EURECBOISM1=ICAP
Sep-16	ECB OIS	Wed, 14-Sep-2016	Wed, 26-Oct-2016	EURECBOISM2=ICAP
Oct-16	ECB OIS	Wed, 26-Oct-2016	Wed, 14-Dec-2016	EURECBOISM3=ICAP
Dec-16	ECB OIS	Wed, 14-Dec-2016	Wed, 25-Jan-2017	EURECBOISM4=ICAP
8M	OIS	Mon, 27-Jun-2016	Mon, 27-Feb-2017	EUREON8Y=ICAP
9M	OIS	Mon, 27-Jun-2016	Mon, 27-Mar-2017	EUREON9Y=ICAP
10M	OIS	Mon, 27-Jun-2016	Thu, 27-Apr-2017	EUREON10Y=ICAP
11M	OIS	Mon, 27-Jun-2016	Mon, 29-May-2017	EUREON11Y=ICAP
1Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2017	EUREON12Y=ICAP
15M	OIS	Mon, 27-Jun-2016	Wed, 27-Sep-2017	EUREON15M=ICAP
18M	OIS	Mon, 27-Jun-2016	Wed, 27-Dec-2017	EUREON18M=ICAP
21M	OIS	Mon, 27-Jun-2016	Tue, 27-Mar-2018	EUREON21M=ICAP
2Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2018	EUREON2Y=ICAP
3Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUREON3Y=ICAP
4Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUREON4Y=ICAP
5Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUREON5Y=ICAP
6Y	OIS	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUREON6Y=ICAP
7Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUREON7Y=ICAP
8Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUREON8Y=ICAP
9Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUREON9Y=ICAP
10Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUREON10Y=ICAP
11Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EUREON11Y=ICAP
12Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUREON12Y=ICAP
15Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUREON15Y=ICAP
20Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUREON20Y=ICAP
25Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUREON25Y=ICAP
30Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUREON30Y=ICAP
40Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUREON40Y=ICAP
50Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUREON50Y=ICAP
60Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUREON60Y=ICAP

Figure 4.3: Bootstrapping instruments selected for Excel calibration of Eonia Curve and corresponding Market RICs (Evaluation Date 23 June 2016)

4.3.2 System Calibration

The curve structure with corresponding Internal RICs is shown in Figure 4.4. The contribution instruments (i.e. instruments priced using the Excel curve and then used to perform the system calibration) are listed below:

- ON, TN, SN Deposits;

- spot starting OIS up to 60Y.

Notice the presence of synthetic interpolated instruments.

Eonia Curve - System Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Internal RIC
O/N	Deposits	Thu, 23-Jun-2016	Fri, 24-Jun-2016	EURONDPON=
T/N	Deposits	Fri, 24-Jun-2016	Mon, 27-Jun-2016	EURONDPTN=
S/N	Deposits	Mon, 27-Jun-2016	Tue, 28-Jun-2016	EURONDPSN=
1W	OIS	Mon, 27-Jun-2016	Thu, 7-Apr-2016	EURONOISSW=
2W	OIS	Mon, 27-Jun-2016	Mon, 7-Nov-2016	EURONOIS2W=
3W	OIS	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EURONOIS3W=
1M	OIS	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EURONOIS1M=
2M	OIS	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EURONOIS2M=
3M	OIS	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EURONOIS3M=
4M	OIS	Mon, 27-Jun-2016	Thu, 27-Oct-2016	EURONOIS4M=
5M	OIS	Mon, 27-Jun-2016	Mon, 28-Nov-2016	EURONOIS5M=
6M	OIS	Mon, 27-Jun-2016	Tue, 27-Dec-2016	EURONOIS6M=
7M	OIS	Mon, 27-Jun-2016	Fri, 27-Jan-2017	EURONOIS7M=
8M	OIS	Mon, 27-Jun-2016	Mon, 27-Feb-2017	EURONOIS8M=
9M	OIS	Mon, 27-Jun-2016	Mon, 27-Mar-2017	EURONOIS9M=
10M	OIS	Mon, 27-Jun-2016	Thu, 27-Apr-2017	EURONOIS10M=
11M	OIS	Mon, 27-Jun-2016	Mon, 29-May-2017	EURONOIS11M=
1Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2017	EURONOIS1Y=
13M	OIS	Mon, 27-Jun-2016	Thu, 27-Jul-2017	EURONOIS13M=
14M	OIS	Mon, 27-Jun-2016	Mon, 28-Aug-2017	EURONOIS14M=
15M	OIS	Mon, 27-Jun-2016	Wed, 27-Sep-2017	EURONOIS15M=
16M	OIS	Mon, 27-Jun-2016	Fri, 27-Oct-2017	EURONOIS16M=
17M	OIS	Mon, 27-Jun-2016	Mon, 27-Nov-2017	EURONOIS17M=
18M	OIS	Mon, 27-Jun-2016	Wed, 27-Dec-2017	EURONOIS18M=
19M	OIS	Mon, 27-Jun-2016	Mon, 29-Jan-2018	EURONOIS19M=
20M	OIS	Mon, 27-Jun-2016	Tue, 27-Feb-2018	EURONOIS20M=
21M	OIS	Mon, 27-Jun-2016	Tue, 27-Mar-2018	EURONOIS21M=
22M	OIS	Mon, 27-Jun-2016	Fri, 27-Apr-2018	EURONOIS22M=
23M	OIS	Mon, 27-Jun-2016	Mon, 28-May-2018	EURONOIS23M=
2Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2018	EURONOIS2Y=
3Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EURONOIS3Y=
4Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EURONOIS4Y=
5Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EURONOIS5Y=
6Y	OIS	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EURONOIS6Y=
7Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EURONOIS7Y=
8Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EURONOIS8Y=
9Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EURONOIS9Y=
10Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EURONOIS10Y=
11Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EURONOIS11Y=
12Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EURONOIS12Y=
13Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2029	EURONOIS13Y=
14Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2030	EURONOIS14Y=
15Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EURONOIS15Y=
16Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2032	EURONOIS16Y=
17Y	OIS	Mon, 27-Jun-2016	Mon, 27-Jun-2033	EURONOIS17Y=
18Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2034	EURONOIS18Y=
19Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2035	EURONOIS19Y=
20Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EURONOIS20Y=
21Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2037	EURONOIS21Y=
22Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2038	EURONOIS22Y=
23Y	OIS	Mon, 27-Jun-2016	Mon, 27-Jun-2039	EURONOIS23Y=
24Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2040	EURONOIS24Y=
25Y	OIS	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EURONOIS25Y=
26Y	OIS	Mon, 27-Jun-2016	Fri, 27-Jun-2042	EURONOIS26Y=
27Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2043	EURONOIS27Y=
28Y	OIS	Mon, 27-Jun-2016	Mon, 27-Jun-2044	EURONOIS28Y=
29Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2045	EURONOIS29Y=
30Y	OIS	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EURONOIS30Y=
35Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EURONOIS35Y=
40Y	OIS	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EURONOIS40Y=
50Y	OIS	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EURONOIS50Y=
60Y	OIS	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EURONOIS60Y=

Figure 4.4: Bootstrapping instruments selected for System calibration of Eonia Curve and corresponding Internal RICs (Evaluation Date: 23 June 2016)

4.4 1M Curve

4.4.1 Excel Calibration

1M Euribor curve is bootstrapped using the following instruments:

- SW, 2W, 3W, 1M Synthetic Deposits;
- 1M Swap with *Act*/360 convention up to 12M;
- 1M Swap with 30/360 convention up to 60Y.

The curve structure with corresponding Market RICs is shown in Figure 4.5.

Euribor 1M Curve - Excel Calibration				
Pillar	**Instruments**	**Earliest Date**	**Pillar Date**	**Market RIC**
SW	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 04-Jul-2016	
2W	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 11-Jul-2016	
3W	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 18-Jul-2016	
1M	Synthetic Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	
2M	Swap	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUR2X1S=ICAP
3M	Swap	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EUR3X1S=ICAP
4M	Swap	Mon, 27-Jun-2016	Thu, 27-Oct-2016	EUR4X1S=ICAP
5M	Swap	Mon, 27-Jun-2016	Mon, 28-Nov-2016	EUR5X1S=ICAP
6M	Swap	Mon, 27-Jun-2016	Tue, 27-Dec-2016	EUR6X1S=ICAP
7M	Swap	Mon, 27-Jun-2016	Fri, 27-Jan-2017	EUR7X1S=ICAP
8M	Swap	Mon, 27-Jun-2016	Mon, 27-Feb-2017	EUR8X1S=ICAP
9M	Swap	Mon, 27-Jun-2016	Mon, 27-Mar-2017	EUR9X1S=ICAP
10M	Swap	Mon, 27-Jun-2016	Thu, 27-Apr-2017	EUR10X1S=ICAP
11M	Swap	Mon, 27-Jun-2016	Mon, 29-May-2017	EUR11X1S=ICAP
12M	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2017	EUR12X1S=ICAP
2Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2018	EUR1E6E2Y=ICAP
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR1E6E3Y=ICAP
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR1E6E4Y=ICAP
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR1E6E5Y=ICAP
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR1E6E6Y=ICAP
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR1E6E7Y=ICAP
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR1E6E8Y=ICAP
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR1E6E9Y=ICAP
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR1E6E10Y=ICAP
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR1E6E12Y=ICAP
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR1E6E15Y=ICAP
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR1E6E20Y=ICAP
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR1E6E25Y=ICAP
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR1E6E30Y=ICAP
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR1E6E40Y=ICAP
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR1E6E50Y=ICAP
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUR1E6E60Y=ICAP

Figure 4.5: Bootstrapping instruments selected for Excel calibration of Euribor 1M Curve and corresponding Market RICs (Evaluation Date: 23 June 2016)

4.4.2 System Calibration

The curve structure with corresponding Internal RICs is shown in Figure 4.6. The contribution instruments are listed below:

- SN, SW, 2W, 3W, 1M Deposits;
- 1M Swap with *Act*/360 convention up to 12M;
- 1M Swap with 30/360 convention up to 60Y.

Once again we notice the presence of many synthetic interpolated instruments added to have one pillar every three months from 1Y to 3Y maturities and one pillar every year from 3Y to 40Y maturities.

4.5 3M Curve

4.5.1 Excel Calibration

3M Euribor curve is bootstrapped using the following instruments:

- 1M, 2M, 3M Synthetic Deposits;
- 3M tenor FRAs up to 15M
- 10 IMM Futures;
- 3M Swaps up to 60Y.

The curve structure with corresponding Market RICs is shown in Figure 4.7.

4.5.2 System Calibration

The curve structure with corresponding Internal RICs is shown in Figure 4.8. The contribution instruments are listed below:

- Deposits from SN to 3M maturity;
- 12 FRAs
- 9 IMM Futures;
- 3M Swaps up to 60Y.

4.6 6M Curve

4.6.1 Excel Calibration

6M Euribor curve is bootstrapped using the following instruments:

- Synthetic Deposits from 1M to 6M;
- FRAs up to 2Y;
- 6M Swaps up to 60Y;
- two Forward Swaps with tenor 30Y to cover 45Y and 55Y maturities.

The curve structure with corresponding Market RICs is shown in Figure 4.9.

Euribor 1M Curve - System Calibration				
Pillar	**Instruments**	**Earliest Date**	**Pillar Date**	**Internal RIC**
S/N	Deposits	Mon, 27-Jun-2016	Tue, 28-Jun-2016	EUR1MDPSN=
1W	Deposits	Mon, 27-Jun-2016	Thu, 7-Apr-2016	EUR1MDPSW=
2W	Deposits	Mon, 27-Jun-2016	Mon, 7-Nov-2016	EUR1MDP2W=
3W	Deposits	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EUR1MDP3W=
1M	Deposits	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EUR1MDP1M=
2M	Swap	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUR1MSW2M=
3M	Swap	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EUR1MSW3M=
4M	Swap	Mon, 27-Jun-2016	Thu, 27-Oct-2016	EUR1MSW4M=
5M	Swap	Mon, 27-Jun-2016	Mon, 28-Nov-2016	EUR1MSW5M=
6M	Swap	Mon, 27-Jun-2016	Tue, 27-Dec-2016	EUR1MSW6M=
7M	Swap	Mon, 27-Jun-2016	Fri, 27-Jan-2017	EUR1MSW7M=
8M	Swap	Mon, 27-Jun-2016	Mon, 27-Feb-2017	EUR1MSW8M=
9M	Swap	Mon, 27-Jun-2016	Mon, 27-Mar-2017	EUR1MSW9M=
10M	Swap	Mon, 27-Jun-2016	Thu, 27-Apr-2017	EUR1MSW10M=
11M	Swap	Mon, 27-Jun-2016	Mon, 29-May-2017	EUR1MSW11M=
12M	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2017	EUR1MSW12M=
15M	Swap	Mon, 27-Jun-2016	Wed, 27-Sep-2017	EUR1MSW15M=
18M	Swap	Mon, 27-Jun-2016	Wed, 27-Dec-2017	EUR1MSW18M=
21M	Swap	Mon, 27-Jun-2016	Tue, 27-Mar-2018	EUR1MSW21M=
2Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2018	EUR1MSW2Y=
2Y3M	Swap	Mon, 27-Jun-2016	Thu, 27-Sep-2018	EUR1MSW2Y3M=
2Y6M	Swap	Mon, 27-Jun-2016	Thu, 27-Dec-2018	EUR1MSW2Y6M=
2Y9M	Swap	Mon, 27-Jun-2016	Wed, 27-Mar-2019	EUR1MSW2Y9M=
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR1MSW3Y=
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR1MSW4Y=
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR1MSW5Y=
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR1MSW6Y=
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR1MSW7Y=
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR1MSW8Y=
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR1MSW9Y=
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR1MSW10Y=
11Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EUR1MSW11Y=
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR1MSW12Y=
13Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2029	EUR1MSW13Y=
14Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2030	EUR1MSW14Y=
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR1MSW15Y=
16Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2032	EUR1MSW16Y=
17Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2033	EUR1MSW17Y=
18Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2034	EUR1MSW18Y=
19Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2035	EUR1MSW19Y=
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR1MSW20Y=
21Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2037	EUR1MSW21Y=
22Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2038	EUR1MSW22Y=
23Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2039	EUR1MSW23Y=
24Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2040	EUR1MSW24Y=
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR1MSW25Y=
26Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2042	EUR1MSW26Y=
27Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2043	EUR1MSW27Y=
28Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2044	EUR1MSW28Y=
29Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2045	EUR1MSW29Y=
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR1MSW30Y=
31Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2047	EUR1MSW31Y=
32Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2048	EUR1MSW32Y=
33Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2049	EUR1MSW33Y=
34Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2050	EUR1MSW34Y=
35Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EUR1MSW35Y=
36Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2052	EUR1MSW36Y=
37Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2053	EUR1MSW37Y=
38Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2054	EUR1MSW38Y=
39Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2055	EUR1MSW39Y=
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR1MSW40Y=
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR1MSW50Y=
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUR1MSW60Y=

Figure 4.6: Bootstrapping instruments selected for System calibration of Euribor 1M Curve and corresponding Internal RICs (Evaluation Date 23 June 2016)

Euribor 3M Curve - Excel Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Market RIC
1M	Synthetic Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	
2M	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 29-Aug-2016	
3M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	
1X4	FRA	Wed, 27-Jul-2016	Thu, 27-Oct-2016	EUR1X4F=ICAP
2X5	FRA	Mon, 29-Aug-2016	Mon, 28-Nov-2016	EUR2X5F=ICAP
3X6	FRA	Tue, 27-Sep-2016	Tue, 27-Dec-2016	EUR3X6F=ICAP
4X7	FRA	Thu, 27-Oct-2016	Fri, 27-Jan-2017	EUR4X7F=ICAP
5X8	FRA	Mon, 28-Nov-2016	Mon, 27-Feb-2017	EUR5X8F=ICAP
6X9	FRA	Tue, 27-Dec-2016	Mon, 27-Mar-2017	EUR6X9F=ICAP
7X10	FRA	Fri, 27-Jan-2017	Thu, 27-Apr-2017	EUR7X10F=ICAP
8X11	FRA	Mon, 27-Feb-2017	Mon, 29-May-2017	EUR8X11F=ICAP
9X12	FRA	Mon, 27-Mar-2017	Tue, 27-Jun-2017	EUR9X12F=ICAP
10X13	FRA	Thu, 27-Apr-2017	Thu, 27-Jul-2017	EUR10X13F=ICAP
11X14	FRA	Mon, 29-May-2017	Mon, 28-Aug-2017	EUR11X14F=ICAP
12X15	FRA	Tue, 27-Jun-2017	Wed, 27-Sep-2017	EUR12X15F=ICAP
17-Sep	Futures	Wed, 20-Sep-2017	Wed, 20-Dec-2017	FEIU6
17-Dec	Futures	Wed, 20-Dec-2017	Tue, 20-Mar-2018	FEIZ6
18-Mar	Futures	Wed, 21-Mar-2018	Thu, 21-Jun-2018	FEIH7
18-Jun	Futures	Wed, 20-Jun-2018	Thu, 20-Sep-2018	FEIM7
18-Sep	Futures	Wed, 19-Sep-2018	Wed, 19-Dec-2018	FEIU7
18-Dec	Futures	Wed, 19-Dec-2018	Tue, 19-Mar-2019	FEIZ7
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EURAB3E3Y=ICAP
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EURAB3E4Y=ICAP
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EURAB3E5Y=ICAP
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EURAB3E6Y=ICAP
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EURAB3E7Y=ICAP
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EURAB3E8Y=ICAP
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EURAB3E9Y=ICAP
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EURAB3E10Y=ICAP
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EURAB3E12Y=ICAP
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EURAB3E15Y=ICAP
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EURAB3E20Y=ICAP
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EURAB3E25Y=ICAP
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EURAB3E30Y=ICAP
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EURAB3E40Y=ICAP
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EURAB3E50Y=ICAP
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EURAB3E60Y=ICAP

Figure 4.7: Bootstrapping instruments selected for Excel calibration of Euribor 3M Curve and corresponding Market RICs (Evaluation Date: 23 June 2016)

4.6.2 System Calibration

The curve structure with corresponding Internal RICs is shown in Figure 4.10. The contribution instruments are listed below:

- Deposits from SN to 6M maturity;
- FRAs up to 30M;
- 6M Swaps up to 60Y.

Notice the presence of one additional synthetic interpolated FRA (in addition to interpolated deposits and swaps): as said before, we have to lock as many pillars as possible to be able to use Kruger interpolation, which guarantees stable deltas, and obtain good forward rates.

Euribor 3M Curve - System Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Internal RIC
S/N	Deposits	Mon, 27-Jun-2016	Tue, 28-Jun-2016	EUR3MDPSN=
1W	Deposits	Mon, 27-Jun-2016	Thu, 7-Apr-2016	EUR3MDPSW=
2W	Deposits	Mon, 27-Jun-2016	Mon, 7-Nov-2016	EUR3MDP2W=
3W	Deposits	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EUR3MDP3W=
1M	Deposits	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EUR3MDP1M=
2M	Deposits	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUR3MDP2M=
3M	Deposits	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EUR3MDP3M=
1X4	FRA	Wed, 27-Jul-2016	Thu, 27-Oct-2016	EUR3MFRA1X4F=
2X5	FRA	Mon, 29-Aug-2016	Mon, 28-Nov-2016	EUR3MFRA2X5F=
3X6	FRA	Tue, 27-Sep-2016	Tue, 27-Dec-2016	EUR3MFRA3X6F=
4X7	FRA	Thu, 27-Oct-2016	Fri, 27-Jan-2017	EUR3MFRA4X7F=
5X8	FRA	Mon, 28-Nov-2016	Mon, 27-Feb-2017	EUR3MFRA5X8F=
16-Dec	Futures	Wed, 21-Dec-2016	Tue, 21-Mar-2017	EUR3MFUTZ6
6X9	FRA	Tue, 27-Dec-2016	Mon, 27-Mar-2017	EUR3MFRA6X9F=
7X10	FRA	Fri, 27-Jan-2017	Thu, 27-Apr-2017	EUR3MFRA7X10F=
8X11	FRA	Mon, 27-Feb-2017	Mon, 29-May-2017	EUR3MFRA8X11F=
17-Mar	Futures	Wed, 15-Mar-2017	Thu, 15-Jun-2017	EUR3MFUTH7
9X12	FRA	Mon, 27-Mar-2017	Tue, 27-Jun-2017	EUR3MFRA9X12F=
10X13	FRA	Thu, 27-Apr-2017	Thu, 27-Jul-2017	EUR3MFRA10X13F=
11X14	FRA	Mon, 29-May-2017	Mon, 28-Aug-2017	EUR3MFRA11X14F=
17-Jun	Futures	Wed, 21-Jun-2017	Thu, 21-Sep-2017	EUR3MFUTM7
12X15	FRA	Tue, 27-Jun-2017	Wed, 27-Sep-2017	EUR3MFRA12X15F=
17-Sep	Futures	Wed, 20-Sep-2017	Wed, 20-Dec-2017	EUR3MFUTU7
17-Dec	Futures	Wed, 20-Dec-2017	Tue, 20-Mar-2018	EUR3MFUTZ7
18-Mar	Futures	Wed, 21-Mar-2018	Thu, 21-Jun-2018	EUR3MFUTH8
18-Jun	Futures	Wed, 20-Jun-2018	Thu, 20-Sep-2018	EUR3MFUTM8
18-Sep	Futures	Wed, 19-Sep-2018	Wed, 19-Dec-2018	EUR3MFUTU8
18-Dec	Futures	Wed, 19-Dec-2018	Tue, 19-Mar-2019	EUR3MFUTZ8
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR3MSW3Y=
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR3MSW4Y=
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR3MSW5Y=
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR3MSW6Y=
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR3MSW7Y=
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR3MSW8Y=
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR3MSW9Y=
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR3MSW10Y=
11Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EUR3MSW11Y=
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR3MSW12Y=
13Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2029	EUR3MSW13Y=
14Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2030	EUR3MSW14Y=
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR3MSW15Y=
16Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2032	EUR3MSW16Y=
17Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2033	EUR3MSW17Y=
18Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2034	EUR3MSW18Y=
19Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2035	EUR3MSW19Y=
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR3MSW20Y=
21Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2037	EUR3MSW21Y=
22Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2038	EUR3MSW22Y=
23Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2039	EUR3MSW23Y=
24Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2040	EUR3MSW24Y=
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR3MSW25Y=
26Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2042	EUR3MSW26Y=
27Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2043	EUR3MSW27Y=
28Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2044	EUR3MSW28Y=
29Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2045	EUR3MSW29Y=
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR3MSW30Y=
31Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2047	EUR3MSW31Y=
32Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2048	EUR3MSW32Y=
33Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2049	EUR3MSW33Y=
34Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2050	EUR3MSW34Y=
35Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EUR3MSW35Y=
36Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2052	EUR3MSW36Y=
37Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2053	EUR3MSW37Y=
38Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2054	EUR3MSW38Y=
39Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2055	EUR3MSW39Y=
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR3MSW40Y=
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR3MSW50Y=
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUR3MSW60Y=

Figure 4.8: Bootstrapping instruments selected for System calibration of Euribor 3M Curve and corresponding Internal RICs (Evaluation Date: 23 June 2016)

Euribor 6M Curve - Excel Calibration				
Pillar	**Instruments**	**Earliest Date**	**Pillar Date**	**Market RIC**
1M	Synthetic Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	
2M	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 29-Aug-2016	
3M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	
4M	Synthetic Deposit	Mon, 27-Jun-2016	Thu, 27-Oct-2016	
5M	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 28-Nov-2016	
6M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Dec-2016	
1X7	FRA	Wed, 27-Jul-2016	Fri, 27-Jan-2017	EUR1X7F=ICAP
2X8	FRA	Mon, 29-Aug-2016	Mon, 27-Feb-2017	EUR2X8F=ICAP
3X9	FRA	Tue, 27-Sep-2016	Mon, 27-Mar-2017	EUR3X9F=ICAP
4X10	FRA	Thu, 27-Oct-2016	Thu, 27-Apr-2017	EUR4X10F=ICAP
5X11	FRA	Mon, 28-Nov-2016	Mon, 29-May-2017	EUR5X11F=ICAP
6X12	FRA	Tue, 27-Dec-2016	Tue, 27-Jun-2017	EUR6X12F=ICAP
7X13	FRA	Fri, 27-Jan-2017	Thu, 27-Jul-2017	EUR7X13F=ICAP
8X14	FRA	Mon, 27-Feb-2017	Mon, 28-Aug-2017	EUR8X14F=ICAP
9X15	FRA	Mon, 27-Mar-2017	Wed, 27-Sep-2017	EUR9X15F=ICAP
10X16	FRA	Thu, 27-Apr-2017	Fri, 27-Oct-2017	EUR10X16F=ICAP
11X17	FRA	Mon, 29-May-2017	Mon, 27-Nov-2017	EUR11X17F=ICAP
12X18	FRA	Tue, 27-Jun-2017	Wed, 27-Dec-2017	EUR12X18F=ICAP
13X19	FRA	Thu, 27-Jul-2017	Mon, 29-Jan-2018	EUR13X19F=ICAP
14X20	FRA	Mon, 28-Aug-2017	Tue, 27-Feb-2018	EUR14X20F=ICAP
15X21	FRA	Wed, 27-Sep-2017	Tue, 27-Mar-2018	EUR15X21F=ICAP
16X22	FRA	Fri, 27-Oct-2017	Fri, 27-Apr-2018	EUR16X22F=ICAP
17X23	FRA	Mon, 27-Nov-2017	Mon, 28-May-2018	EUR17X23F=ICAP
18X24	FRA	Wed, 27-Dec-2017	Wed, 27-Jun-2018	EUR18X24F=ICAP
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EURAB6E3Y=ICAP
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EURAB6E4Y=ICAP
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EURAB6E5Y=ICAP
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EURAB6E6Y=ICAP
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EURAB6E7Y=ICAP
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EURAB6E8Y=ICAP
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EURAB6E9Y=ICAP
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EURAB6E10Y=ICAP
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EURAB6E12Y=ICAP
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EURAB6E15Y=ICAP
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EURAB6E20Y=ICAP
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EURAB6E25Y=ICAP
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EURAB6E30Y=ICAP
35Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EURAB6E35Y=ICAP
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EURAB6E40Y=ICAP
15X30	Forward Swap	Fri, 27-Jun-2031	Mon, 27-Jun-2061	EUR15F30Y=ICAP
50	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EURAB6E50Y=ICAP
25X30	Forward Swap	Thu, 27-Jun-2041	Mon, 29-Jun-2071	EUR25F30Y=ICAP
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EURAB6E60Y=ICAP

Figure 4.9: Bootstrapping instruments selected for Excel calibration of Euribor 6M Curve and corresponding Market RICs (Evaluation Date: 23 June 2016)

4.7 12M Curve

4.7.1 Excel Calibration

12M Euribor curve is bootstrapped using the following instruments:

- 3M, 6M, 9M, 12 Synthetic Deposits;
- FRAs up to 2Y;
- 6M-12M Basis Swaps up to 60Y.

The curve structure with corresponding Market RICs is shown in Figure 4.11.

4.7.2 System Calibration

The curve structure with corresponding Internal RICs is shown in Figure 4.12. The contribution instruments are listed below:

- Deposits from SN to 12M maturity;
- FRAs up to 2Y;
- 12M Synthetic Swaps up to 60Y.

Euribor 6M Curve - System Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Internal RIC
S/N	Deposit	Mon, 27-Jun-2016	Tue, 28-Jun-2016	EUR6MDPSN=
1W	Deposit	Mon, 27-Jun-2016	Thu, 7-Apr-2016	EUR6MDPSW=
2W	Deposit	Mon, 27-Jun-2016	Mon, 7-Nov-2016	EUR6MDP2W=
3W	Deposit	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EUR6MDP3W=
1M	Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EUR6MDP1M=
2M	Deposit	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUR6MDP2M=
3M	Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EUR6MDP3M=
4M	Deposit	Mon, 27-Jun-2016	Thu, 27-Oct-2016	EUR6MDP4M=
5M	Deposit	Mon, 27-Jun-2016	Mon, 28-Nov-2016	EUR6MDP5M=
6M	Deposit	Mon, 27-Jun-2016	Tue, 27-Dec-2016	EUR6MDP6M=
1X7	FRA	Wed, 27-Jul-2016	Fri, 27-Jan-2017	EUR6MFRA1X7F=
2X8	FRA	Mon, 29-Aug-2016	Mon, 27-Feb-2017	EUR6MFRA2X8F=
3X9	FRA	Tue, 27-Sep-2016	Mon, 27-Mar-2017	EUR6MFRA3X9F=
4X10	FRA	Thu, 27-Oct-2016	Thu, 27-Apr-2017	EUR6MFRA4X10F=
5X11	FRA	Mon, 28-Nov-2016	Mon, 29-May-2017	EUR6MFRA5X11F=
6X12	FRA	Tue, 27-Dec-2016	Tue, 27-Jun-2017	EUR6MFRA6X12F=
7X13	FRA	Fri, 27-Jan-2017	Thu, 27-Jul-2017	EUR6MFRA7X13F=
8X14	FRA	Mon, 27-Feb-2017	Mon, 28-Aug-2017	EUR6MFRA8X14F=
9X15	FRA	Mon, 27-Mar-2017	Wed, 27-Sep-2017	EUR6MFRA9X15F=
10X16	FRA	Thu, 27-Apr-2017	Fri, 27-Oct-2017	EUR6MFRA10X16F=
11X17	FRA	Mon, 29-May-2017	Mon, 27-Nov-2017	EUR6MFRA11X17F=
12X18	FRA	Tue, 27-Jun-2017	Wed, 27-Dec-2017	EUR6MFRA12X18F=
13X19	FRA	Thu, 27-Jul-2017	Mon, 29-Jan-2018	EUR6MFRA13X19F=
14X20	FRA	Mon, 28-Aug-2017	Tue, 27-Feb-2018	EUR6MFRA14X20F=
15X21	FRA	Wed, 27-Sep-2017	Tue, 27-Mar-2018	EUR6MFRA15X21F=
16X22	FRA	Fri, 27-Oct-2017	Fri, 27-Apr-2018	EUR6MFRA16X22F=
17X23	FRA	Mon, 27-Nov-2017	Mon, 28-May-2018	EUR6MFRA17X23F=
18X24	FRA	Wed, 27-Dec-2017	Wed, 27-Jun-2018	EUR6MFRA18X24F=
24X30	FRA	Wed, 27-Jun-2018	Thu, 27-Dec-2018	EUR6MFRA24X30F=
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR6MSW3Y=
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR6MSW4Y=
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR6MSW5Y=
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR6MSW6Y=
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR6MSW7Y=
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR6MSW8Y=
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR6MSW9Y=
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR6MSW10Y=
11Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EUR6MSW11Y=
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR6MSW12Y=
13Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2029	EUR6MSW13Y=
14Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2030	EUR6MSW14Y=
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR6MSW15Y=
16Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2032	EUR6MSW16Y=
17Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2033	EUR6MSW17Y=
18Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2034	EUR6MSW18Y=
19Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2035	EUR6MSW19Y=
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR6MSW20Y=
21Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2037	EUR6MSW21Y=
22Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2038	EUR6MSW22Y=
23Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2039	EUR6MSW23Y=
24Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2040	EUR6MSW24Y=
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR6MSW25Y=
26Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2042	EUR6MSW26Y=
27Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2043	EUR6MSW27Y=
28Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2044	EUR6MSW28Y=
29Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2045	EUR6MSW29Y=
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR6MSW30Y=
31Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2047	EUR6MSW31Y=
32Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2048	EUR6MSW32Y=
33Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2049	EUR6MSW33Y=
34Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2050	EUR6MSW34Y=
35Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EUR6MSW35Y=
36Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2052	EUR6MSW36Y=
37Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2053	EUR6MSW37Y=
38Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2054	EUR6MSW38Y=
39Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2055	EUR6MSW39Y=
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR6MSW40Y=
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR6MSW50Y=
60Y	Swap	Tue, 27-Jun-2017	Mon, 29-Jun-2076	EUR6MSW60Y=

Figure 4.10: Bootstrapping instruments selected for System calibration of Euribor 6M Curve and corresponding Internal RICs (Evaluation Date: 23 June 2016)

Euribor 12M Curve - Excel Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Market RIC
3M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	
6M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Dec-2016	
9M	Synthetic Deposit	Mon, 27-Jun-2016	Mon, 27-Mar-2017	
12M	Synthetic Deposit	Mon, 27-Jun-2016	Tue, 27-Jun-2017	
1X13	FRA	Wed, 27-Jul-2016	Thu, 27-Jul-2017	EUR1X13F=ICAP
2X14	FRA	Mon, 29-Aug-2016	Mon, 28-Aug-2017	EUR2X14F=ICAP
3X15	FRA	Tue, 27-Sep-2016	Wed, 27-Sep-2017	EUR3X15F=ICAP
4X16	FRA	Thu, 27-Oct-2016	Fri, 27-Oct-2017	EUR4X16F=ICAP
5X17	FRA	Mon, 28-Nov-2016	Mon, 27-Nov-2017	EUR5X17F=ICAP
6X18	FRA	Tue, 27-Dec-2016	Wed, 27-Dec-2017	EUR6X18F=ICAP
7X19	FRA	Fri, 27-Jan-2017	Mon, 29-Jan-2018	EUR7X19F=ICAP
8X20	FRA	Mon, 27-Feb-2017	Tue, 27-Feb-2018	EUR8X20F=ICAP
9X21	FRA	Mon, 27-Mar-2017	Tue, 27-Mar-2018	EUR9X21F=ICAP
10X22	FRA	Thu, 27-Apr-2017	Fri, 27-Apr-2018	EUR10X22F=ICAP
11X23	FRA	Mon, 29-May-2017	Mon, 28-May-2018	EUR11X23F=ICAP
12X24	FRA	Tue, 27-Jun-2017	Wed, 27-Jun-2018	EUR12X24F=ICAP
3Y	Basis Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR6E12E3Y=ICAP
4Y	Basis Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR6E12E4Y=ICAP
5Y	Basis Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR6E12E5Y=ICAP
6Y	Basis Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR6E12E6Y=ICAP
7Y	Basis Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR6E12E7Y=ICAP
8Y	Basis Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR6E12E8Y=ICAP
9Y	Basis Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR6E12E9Y=ICAP
10Y	Basis Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR6E12E10Y=ICAP
12Y	Basis Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR6E12E12Y=ICAP
15Y	Basis Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR6E12E15Y=ICAP
20Y	Basis Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR6E12E20Y=ICAP
25Y	Basis Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR6E12E25Y=ICAP
30Y	Basis Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR6E12E30Y=ICAP
40Y	Basis Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR6E12E40Y=ICAP
50Y	Basis Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR6E12E50Y=ICAP
60Y	Basis Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUR6E12E60Y=ICAP

Figure 4.11: Bootstrapping instruments selected for Excel calibration of Euribor 12M Curve and corresponding Market RICs (Evaluation Date: 23 June 2016)

Euribor 12M Curve - System Calibration				
Pillar	Instruments	Earliest Date	Pillar Date	Internal RIC
S/N	Deposit	Mon, 27-Jun-2016	Tue, 28-Jun-2016	EUR1YDPSN=
1W	Deposit	Mon, 27-Jun-2016	Thu, 7-Apr-2016	EUR1YDPSW=
2W	Deposit	Mon, 27-Jun-2016	Mon, 7-Nov-2016	EUR1YDP2W=
3W	Deposit	Mon, 27-Jun-2016	Mon, 18-Jul-2016	EUR1YDP3W=
1M	Deposit	Mon, 27-Jun-2016	Wed, 27-Jul-2016	EUR1YDP1M=
2M	Deposit	Mon, 27-Jun-2016	Mon, 29-Aug-2016	EUR1YDP2M=
3M	Deposit	Mon, 27-Jun-2016	Tue, 27-Sep-2016	EUR1YDP3M=
4M	Deposit	Mon, 27-Jun-2016	Thu, 27-Oct-2016	EUR1YDP4M=
5M	Deposit	Mon, 27-Jun-2016	Mon, 28-Nov-2016	EUR1YDP5M=
6M	Deposit	Mon, 27-Jun-2016	Tue, 27-Dec-2016	EUR1YDP6M=
7M	Deposit	Mon, 27-Jun-2016	Fri, 27-Jan-2017	EUR1YDP7M=
8M	Deposit	Mon, 27-Jun-2016	Mon, 27-Feb-2017	EUR1YDP8M=
9M	Deposit	Mon, 27-Jun-2016	Mon, 27-Mar-2017	EUR1YDP9M=
10M	Deposit	Mon, 27-Jun-2016	Thu, 27-Apr-2017	EUR1YDP10M=
11M	Deposit	Mon, 27-Jun-2016	Mon, 29-May-2017	EUR1YDP11M=
12M	Deposit	Mon, 27-Jun-2016	Tue, 27-Jun-2017	EUR1YDP12M=
1X13	FRA	Wed, 27-Jul-2016	Thu, 27-Jul-2017	EUR1YFRA1X13F=
2X14	FRA	Mon, 29-Aug-2016	Mon, 28-Aug-2017	EUR1YFRA2X14F=
3X15	FRA	Tue, 27-Sep-2016	Wed, 27-Sep-2017	EUR1YFRA3X15F=
4X16	FRA	Thu, 27-Oct-2016	Fri, 27-Oct-2017	EUR1YFRA4X16F=
5X17	FRA	Mon, 28-Nov-2016	Mon, 27-Nov-2017	EUR1YFRA5X17F=
6X18	FRA	Tue, 27-Dec-2016	Wed, 27-Dec-2017	EUR1YFRA6X18F=
7X19	FRA	Fri, 27-Jan-2017	Mon, 29-Jan-2018	EUR1YFRA7X19F=
8X20	FRA	Mon, 27-Feb-2017	Tue, 27-Feb-2018	EUR1YFRA8X20F=
9X21	FRA	Mon, 27-Mar-2017	Tue, 27-Mar-2018	EUR1YFRA9X21F=
10X22	FRA	Thu, 27-Apr-2017	Fri, 27-Apr-2018	EUR1YFRA10X22F=
11X23	FRA	Mon, 29-May-2017	Mon, 28-May-2018	EUR1YFRA11X23F=
12X24	FRA	Tue, 27-Jun-2017	Wed, 27-Jun-2018	EUR1YFRA12X24F=
3Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2019	EUR1YSW3Y=
4Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2020	EUR1YSW4Y=
5Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2021	EUR1YSW5Y=
6Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2022	EUR1YSW6Y=
7Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2023	EUR1YSW7Y=
8Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2024	EUR1YSW8Y=
9Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2025	EUR1YSW9Y=
10Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2026	EUR1YSW10Y=
11Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2027	EUR1YSW11Y=
12Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2028	EUR1YSW12Y=
13Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2029	EUR1YSW13Y=
14Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2030	EUR1YSW14Y=
15Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2031	EUR1YSW15Y=
16Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2032	EUR1YSW16Y=
17Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2033	EUR1YSW17Y=
18Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2034	EUR1YSW18Y=
19Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2035	EUR1YSW19Y=
20Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2036	EUR1YSW20Y=
21Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2037	EUR1YSW21Y=
22Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2038	EUR1YSW22Y=
23Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2039	EUR1YSW23Y=
24Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2040	EUR1YSW24Y=
25Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2041	EUR1YSW25Y=
26Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2042	EUR1YSW26Y=
27Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2043	EUR1YSW27Y=
28Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2044	EUR1YSW28Y=
29Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2045	EUR1YSW29Y=
30Y	Swap	Mon, 27-Jun-2016	Wed, 27-Jun-2046	EUR1YSW30Y=
31Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2047	EUR1YSW31Y=
32Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2048	EUR1YSW32Y=
33Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2049	EUR1YSW33Y=
34Y	Swap	Mon, 27-Jun-2016	Mon, 27-Jun-2050	EUR1YSW34Y=
35Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2051	EUR1YSW35Y=
36Y	Swap	Mon, 27-Jun-2016	Thu, 27-Jun-2052	EUR1YSW36Y=
37Y	Swap	Mon, 27-Jun-2016	Fri, 27-Jun-2053	EUR1YSW37Y=
38Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2054	EUR1YSW38Y=
39Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2055	EUR1YSW39Y=
40Y	Swap	Mon, 27-Jun-2016	Tue, 27-Jun-2056	EUR1YSW40Y=
50Y	Swap	Mon, 27-Jun-2016	Mon, 28-Jun-2066	EUR1YSW50Y=
60Y	Swap	Mon, 27-Jun-2016	Mon, 29-Jun-2076	EUR1YSW60Y=

Figure 4.12: Bootstrapping instruments selected for System calibration of Euribor 12M Curve and corresponding Internal RICs (Evaluation Date: 23 June 2016)

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