Two Collars and a Free Lunch

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Abstract: Earlier this year liquidity for EUR ITM cash-settled swaptions virtually disappeared due to uncertainty about the proper pricing of those products. We take a fresh look at the no-arbitrage violations of the standard market formula and show how to construct a simple arbitrage strategy that doesn't involve trading CMS-type products. We also consider alternative pricing approaches.

Keywords: Cash-settled swaptions, arbitrage, zero-wide collars, cash-adjusted forwards, cash-physical basis, terminal swap-rate models, LTSRM, ETSRM.

Since 2008, the interest-rate derivatives industry has undergone some significant changes. Not only did the market focus again more on vanilla-like products, but also much attention has been drawn on how to correctly discount cash flows based on the type (or absence) of collateral and on the implications that warehousing derivatives has on the balance sheet. Some of the basic assumptions for pricing non-linear interest rate derivatives, however, have never been properly reassessed. As a result, the market standard for pricing cash-settled swaptions – the most liquidly traded non-linear interest rate products in Europe – is still based on a rather simplistic formula, which is not guaranteed to be arbitrage-free, see Mercurio [2008].

In some sense, this market formula rests on the assumption that physically- and cash-settled swaps[1] share the same convexity effects – an assumption that particularly dominates the pricing of deep in the money (ITM) swaptions. When EUR interest rates fell to historical lows earlier this year, and swaption books of market participants consequently became dominated by deep ITM receiver swaptions, liquidity for those options practically vanished due to the involved modeling uncertainties. In some instances, bid-offer spreads for ITM cash-settled receivers exceeded their naive time value.

In this article we will revisit the no-arbitrage violations of the naive market formula and show how to construct a simple arbitrage strategy. In contrast to the arbitrage portfolio outlined in Mercurio [2008], our strategy doesn't require trading CMS-type products, which makes it much easier to exploit any pricing inconsistencies. In the second part, we will have a closer look at possible alternative approaches for pricing cash-settled swaptions and also analyse how those approaches perform with respect to coherently pricing cash- and physically-settled swaptions.

Zero-wide Collar Arbitrage

Let us denote by S_t the time-t value of a forward swap rate associated with an N-period swap with fixed-leg schedule $0 < T_0 < \ldots < T_N$. For a strike price $K \in \mathbb{R}$ and expiry $T = T_0$, the payoff of a cash-settled (payer) swaption at time T is given by

$$C(S_T)(S_T - K)^+$$

with the so-called cash-annuity $C(S) = \sum_{i=1}^{N} \frac{\tau}{(1+\tau S)^i}$ and where for simplicity all day count fractions are assumed to be all equal to $\tau > 0$. It is market practice to value cash-settled swaptions via the following Black-like formula

$$\begin{aligned} CS(0,K,S_0,T) &= P(0,T)\mathbb{E}^T[C(S_T)(S_T-K)^+] \\ &\approx P(0,T)C(S_0)\mathbb{E}[(S_T-K)^+] \\ &= P(0,T)C(S_0)\text{Black}(0,K,S_0,T,\sigma(K)) \end{aligned} \tag{1}$$

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[1] A "cash-settled swap" being a synthetic swap consisting of long a cash-settled payer swaption and short a cash-settled receiver swaption or vice versa. In the market these are commonly known as "zero-wide collars".

where $P(0,T)$ denotes the time-0 value of a discount bond with maturity T and $\sigma(K)$ is the market-implied (cash) volatility for strike K. Mercurio [2008] derives no-arbitrage conditions that pricing functions for cash-settled swaptions must satisfy and shows that the above formula generally violates the so-called normalisation condition. He also demonstrates how to set up an arbitrage strategy if this condition is not met. The problem is that the violations need to be checked on a case by case basis via numerically integrating the implied density. Moreover, the arbitrage strategy is based on using CMS swaplets, floorlets and caplets, which need to be replicated via cash-settled swaptions. This replicating procedure may require trading swaptions for a large range of strikes and can therefore render executing the arbitrage strategy either outright impossible or at least too expensive to monetize any potential gains from the no-arbitrage violations. This may be one of the reasons why using the naive Black-like formula has by and large remained the market standard. In the following we derive a rather simple static arbitrage strategy, which only involves trading four regular cash-settled swaptions and is hence much easier to execute than a strategy based on CMS-type products.

Let us fix a strike $K > S_0$ (the case $K < S_0$ works analogously). Using the naive market formula to determine the time-0 price of a zero-wide cash collar $CC(0, K, S_0)$, struck at K and with maturity $T > 0$ yields

$$CC(0, K, S_0, T) = P(0,T)C(S_0)(S_0 - K)$$

Delta-hedging (with respect to S) this zero-wide collar with an at-the-money (ATM) zero-wide collar and subtracting the (naive) premium of the strike-K collar yields for the time-0 value of the whole package

$$P(0,T)C(S_0)(S_0 - K) - \Delta P(0,T)C(S_0)(S_0 - S_0) - P(0,T)C(S_0)(S_0 - K) = 0$$

where the hedge-ratio α is given by

$$\Delta = \left.\frac{\frac{\partial}{\partial S}CC(0,K,S)}{\frac{\partial}{\partial S}CC(0,S_0,S)}\right|_{S=S_0} = 1 + \frac{C'(S_0)}{C(S_0)}(S_0 - K)$$

Proposition: For the time-T payoff of this package it holds[2]

$$C(S_T)(S_T - K) - \Delta C(S_T)(S_T - S0) - C(S_0)(S_0 - K) \geq 0,\ \forall S_T > -\frac{1}{\tau} \tag{2}$$

where the inequality is strict for $S_T \neq S_0$. So in total we have constructed a static portfolio involving two zero-wide collars (or four cash-settled swaptions) and a deterministic fee, which is worth 0 at time 0 and greater than or equal to zero at expiry T – a clear arbitrage opportunity.

Proof: Without loss of generality assume $\tau = 1$. With $K > S_0$ and $C(S) > 0$, Eqn. (2) holds if and only if

$$f(S_T) \coloneqq \frac{C(S_0)}{C(S_T)} \geq 1 - \frac{C'(S0)}{C(S0)}(S_T - S0) =: g(S_T),\ \forall S_T > -1 \tag{3}$$

For $S_T = S_0$, we trivially have $f(S_0) = g(S_0)$. Furthermore it is easy to verify that

$$f'(S_0) = -\frac{C'(S_0)}{C(S_0)} = g'(S_0)$$

Therefore, it suffices to show that $f(S)$, or equivalently $1/C(S)$, is convex in order to show that (3) holds. The second derivative of $1/C(S)$ is given by

$$\left(\frac{1}{C(S)}\right)'' = \frac{Nh(S)}{(1+S)^{2N}S^3},$$

where

$$h(S) = (N-1)(1+S)^{N+1} - (N+1)(1+S)^N + (N+1)(1+S) - (N-1).$$

Hence, in order to show that the second derivative is positive it suffices to show that

$$h(S)\begin{cases} \leq 0, & S \in [-1, 0]; \\ \geq 0, & S \in [0, \infty). \end{cases} \tag{4}$$

[2]Note that the cash-annuity formula has a singularity at $S_T = -1/\tau$.

Define

$$\tilde{h}(S) := h(S-1) = (N-1)S^{N+1} - (N+1)S^N + (N+1)S - (N-1)$$

The coefficients (in descending order) of the polynomial $\tilde{h}$ exhibit three sign changes and thus $\tilde{h}$ can have at most three positive roots by Descarte's rule of signs. Next, it is easy to verify that

$$\tilde{h}(1) = \tilde{h}'(1) = \tilde{h}''(1) = 0,$$

that is, $S = 1$ is a root of multiplicity three and therefore it is the only positive root of $\tilde{h}$. Lastly, since $\tilde{h}(0) = -(N-1) < 0$ $(N > 1)$ and $\tilde{h}$ is continuous, we must have that $\tilde{h}(S) \leq 0$ for $S \in [0, 1]$ and $h(S) \geq 0$ for $S \in [0, \infty)$, which is equivalent to (4). $\square$

As an example, Figure 1 shows the payoff profile of such an arbitrage portfolio. It is worth emphasizing that the payoff profile only depends on the tenor of the underlying swaptions and not on the time to expiry. Longer times to expiry though will generally allow S_T to move further away from S_0, resulting in higher chances of receiving larger payouts at time T.

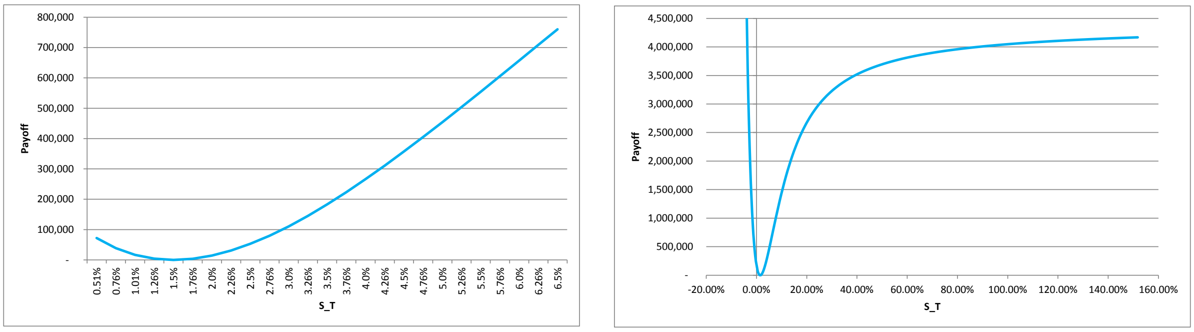

Figure 1: *Time-T payoff profile of the arbitrage portfolio for a notional value of 10m, $S_0 = 1.51\%$, $K = 6\%$, $N = 30$. Both graphs show the same payoff but for different scales of the horizontal axis.*

Also note that the above arbitrage strategy only rests on the assumption that zero-wide cash collars can be sold or bought at the naive premiums. It doesn't require any further modeling assumptions. Quantifying the fair value of this strategy however – or more importantly avoiding such kind of arbitrage opportunities in the first place – requires a more sophisticated approach for pricing cash-settled swaptions and we shall look at a few possible candidates in the following.

Alternative Pricing Approaches

In order to price cash-settled swaptions in an arbitrage-free way, one might first turn to standard term-structure models. These are not only arbitrage-free by construction, but should in addition be able to provide a realistic link between prices of cash- and physically-settled swaptions. Using full-blown term-structure models, however, is not necessarily very practical, at least not from a pure vanilla desk's point of view. Given the relative simplicity of the final products to be priced, the involved calibration routines are often overly complex and the pricing and risking is in general computationally costly. Moreover, most term-structure models are rather restrictive with respect to the possible smile shapes that they can accurately fit, further limiting the usefulness of these models for swaptions market making and risk managing.

We shall therefore focus our attention only on vanilla-like modeling approaches. In this case we can generally follow two different routes, depending on whether the main modeling work is carried out under the T-forward measure or the annuity measure. Looking at Eqn. (1) one might initially be inclined to directly model the main driving swap rate under the T-forward measure. While theoretically possible, this meant having CMS-forward quotes as input parameters. Those forwards however, generally can not be accurately observed in the market and are normally considered an output rather than an input of swaptions modeling. Furthermore, one had to deal with further constraints in the sense that it needed to be ensured that the final model-implied swap-rate distributions are consistent with the initially imposed CMS-forward quotes.

The other alternative, which we shall follow, is to choose a standard vanilla model for the distribution of the swap-rate under the annuity measure, and then resort to so-called terminal swap-rate models (TSRM) for pricing cash-settled swaptions. TSRM's aim to provide a functional link between the value of the main driving swap-rate at time T, determining the payoff, and the various discount bonds (and hence the annuity) introduced via the change of measure

$$\mathbb{E}^T[\phi(S_T)] = A_0 \mathbb{E}^A\left[\frac{1}{A_T}\phi(S_T)\right] \tag{5}$$

where $\phi(S_T)$ is some time-T payoff[3] and where the physical annuity A_t is defined in terms of forward discount bonds[4]

$$A_t = \sum_{i=1}^{N} \tau_i P(t, T_i)/P(t, T)$$

More concretely, TSRM's specify functional forms for the so-called annuity mapping function

$$M(x) = \mathbb{E}^A\left[\frac{A_0}{A_T}|S_T = x\right] \tag{6}$$

such that Eqn. (5) can be written as

$$\mathbb{E}^T[\phi(S_T)] = \mathbb{E}^A[M(S_T)\phi(S_T)] \tag{7}$$

i.e. everything inside the expectation becomes a function of S_T only. These models are widely used in practice for pricing CMS-linked products or more generally for dealing with simple forms of payment delays. We refer the reader to Andersen and Piterbarg [2010], Section 16.3 for a general introduction and an overview of the different varieties.

In order to actually use the TSRM approach for pricing products, we first need to specify the distribution of the swap rate under the physical measure, or equivalently, fix the implied volatility smile of physically-settled swaptions. Typically, only one type of swaption is liquidly traded in a given market. In case of EUR or GBP for example, physically-settled swaptions are rarely traded and hence the corresponding volatilities are hard to come by. Simply substituting physical volatilities with cash volatilities is not a viable option, as we would generally not be able to reproduce the market prices of cash-settled swaptions. Please see also Henrard [2011] for further results regarding the common market practice of using cash and physical volatilities interchangeably.

Thus, before pricing arbitrary cash-settled swaptions, we will first have to carry out a calibration step, where we back out physical volatilities by calibrating a (physical) volatility surface via a TSRM to premiums of liquidly traded OTM cash-settled swaptions. By going this route, we first make sure that the produced prices of cash-settled swaptions (including the non-observable prices of ITM swaptions) are arbitrage-free[5]. Second, as a positive side effect, we also obtain physical volatilities which can later be used to calibrate LMM's or short-rate models for pricing more exotic products[6].

In practice, the most commonly used model is the so-called linear TSRM (LTSRM), where the inverse of the physical annuity is modeled as a linear function $M(S_T) = aS_T + b$ of the swap rate. The coefficients of the linear function are normally obtained by drawing an analogy to a one-factor Gaussian short-rate model, which results in having a mean-reversion level as a free parameter. The linearity assumption of this model comes with great numerical tractability and may explain its popularity among practitioners. It also comes with certain drawbacks though, for instance the fact that the annuity ratio can become negative, potentially leading to negative option premiums. Furthermore, as we shall see later, the implied physical volatility levels for longer expiries do not necessarily need to be very realistic.

The fact that the original LTSRM is based on mean-reversion levels as input parameters is not always very convenient or practical from a risk-management perspective. Not only does it mean that we have to mark or calibrate additional parameters. It also provides us only with a rather indirect link to proper vega risks, which are generally more intuitive risk measures and can be hedged more easily than mean-reversion risks. Cedervall and Piterbarg [2012] therefore present a modeling framework, which directly takes swap-rate volatilities and correlations of all relevant tenors as inputs. In the article, the authors analyse two concrete representations, a

[3]For clarity of exposition we assume here and in the following that the payment time, the swap-rate fixing time and the swap start date all collapse to T.

[4]The annuity measure $\mathbb{P}^A$ however is of course defined in terms of the usual annuity $P(t,T)A_t$

[5]Provided the chosen TSRM respects the fundamental no-arbitrage constraints to a certain degree.

[6]Note that virtually all swaptions approximations used in the calibration routines of term-structure models are for physically- rather than cash-settled swaptions.

linear and a non-linear model, mainly in the context of pricing CMS-related payoffs. In the following we will have a closer look at the non-linear version[7], which we shall call Exponential TSRM[8] (ETSRM), in the context of pricing cash-settled swaptions. For clarity, we will briefly restate the main formulas defining the ETSR model in the simplest case.

First, observe that Eqn. (8) can be written as[9]

$$M(s) = \frac{A_0}{\mathbb{E}^T[A_T|S_T = s]} \tag{8}$$

Next, consider a sequence of co-initial swap-rates $S_1, \ldots, S_N = S$, all with the same start date T but with end dates corresponding to the fixed leg payment dates of the main driving rate S. All those swap-rates will be linked to a common stochastic driver $X \sim N(0,1)$ under the T-forward measure via

$$1 + \tau_i S_i(T) = (1 + \tau_i S_i(0))e^{\mu_i + v_i \sqrt{T} X} \tag{9}$$

The "drifts" μ_i are given by no-arbitrage considerations, while the factor loadings v_i are linked to swap-rate correlations $\rho_{i,N}$ and normal swap-rate volatilities σ_i via

$$v_i = \frac{\tau_i}{1 + \tau_i S_i(0)} \rho_{i,N} \sigma_i$$

Using swap-rate volatilities (from the underlying physical volatility surface) rather than mean-reversion parameters means that we will get proper (bucketed) vega risks. In particular, a time-T expiry cash-settled swaption or a CMS-type payoff will generally exhibit (physical) vegas in the entire T-expiry row of the vega grid, up to the underlying tenor of product.

Finally, making use of some bootstrapping arguments, we can write the conditional expectation in Eqn. (8) as

$$\mathbb{E}^T[A_T|S_T = s] = \sum_{a=1}^{N} \tau_a \exp\{-\sum_{b=a}^{N}(\mu_b - \frac{v_b}{v_N})\} \prod_{b=a}^{N} \left(\frac{1 + \tau_N S(0)}{1 + \tau_N s}\right)^{\frac{v_b}{v_n}} \prod_{b=a}^{N} \frac{1}{a + \tau_b S_b(0)} \tag{10}$$

The formulas derived above are for the simplest case, where the payment time is equal to the swap start date and where we work in a single curve (self-discounting) framework. The model can be easily extended though to work in a multi-curve framework and to also handle payment delays, see Cedervall and Piterbarg [2012] for further details.

Results for Cash-settled Swaptions

Even though the ETSR model is not a proper term-structure model, it respects many of the standard no-arbitrage constraints by construction. In particular, it returns a strictly positive premium for the arbitrage portfolio that we constructed earlier: The price of the package with $N = 30$, $T = 10$, $S_0 = 1.51\%$ and $K = 6\%$ and with EUR market data as of 06/05/2015 would be 227k EUR rather than zero.

Another consequence of using a more realistic model for pricing cash-settled swaptions is that put-call parity will not necessarily hold anymore at the naive (physical) forward S_0. That is, the prices of receiver and payer swaptions struck at $K = S_0$ may be different, contrary to what is implied by the market formula[10]. For future reference, let us call the strike prices at which put-call parity does hold "cash-adjusted" forwards. The differences between cash-adjusted forwards and naive (physical) forwards shall be called "cash-forward adjustments". In the following we will compare cash-adjusted forwards implied by various models. In order to avoid any discrepancies due to differences in the implied volatility smiles, we will focus in this brief analysis only on models which are based on normal-like distributions. More specifically, we will compare the LTSR/ETSR models (based on normal physical surfaces) to a normal-diffusion type one-factor Libor market model (LMM) and to a one-factor Gaussian short-rate model[11] (SR), both calibrated to the same physical surfaces. Note that term-structure models

[7] Please note that although the linear version does provide vega risks, it comes with many of the drawbacks of the original LTSRM. Hence, we shall not consider this model here any further.

[8] This should not be confused with the exponential TSR model introduced in Andersen and Piterbarg [2010], p. 714.

[9] Note the measure the change.

[10] Please note though that having cash-adjusted forwards being equal to the naive forwards, doesn't necessarily by itself represent a direct arbitrage opportunity.

[11] For each expiry the model was calibrated to all the swaption volatilities in the given expiry row.

generally can't be efficiently calibrated to premiums of cash-settled swaptions due a lack of accurate analytical approximations. Hence, for the purpose of this analysis, we used a given physical surface as benchmark to calibrate the term-structure models. The same surfaces were used when pricing with ETSRM/LTSRM. Properly calibrated, all models will therefore return identical physical swaption premiums, while cash swaption premiums would vary. In practice, it would be opposite: premiums for cash-settled swaptions were given by the market and served as the fixed benchmark, while premiums of physically-settled swaptions would have to be implied via a model of choice.

Figure 2 shows cash-forward adjustments for 30Y tenor swaptions implied by the aforementioned models when using different volatility surfaces. Volatilities for the 30Y tenor column were always kept fixed at 60bp, whereas volatilities in the remaining columns were linearly decreasing/increasing in tenor direction, with volatilities in the 1Y tenor column being set to either 75bp, 65bp or 55bp. The interest-rate curve in all cases was linearly upward sloping from 2% to 4% in 6M forward space. As can be seen from the graphs in Figure 2, increasing/decreasing the volatilities for the shorter tails while keeping the 30Y tenor volatilities fixed, yields higher/lower cash-forward adjustments. We can also observe that ETSRM implies cash-forward adjustments very similar to the ones from the two term-structure models. LTSRM on the other hand is usually a bit more off, especially for longer expiries, although that depends on the chosen mean-reversion levels[12].

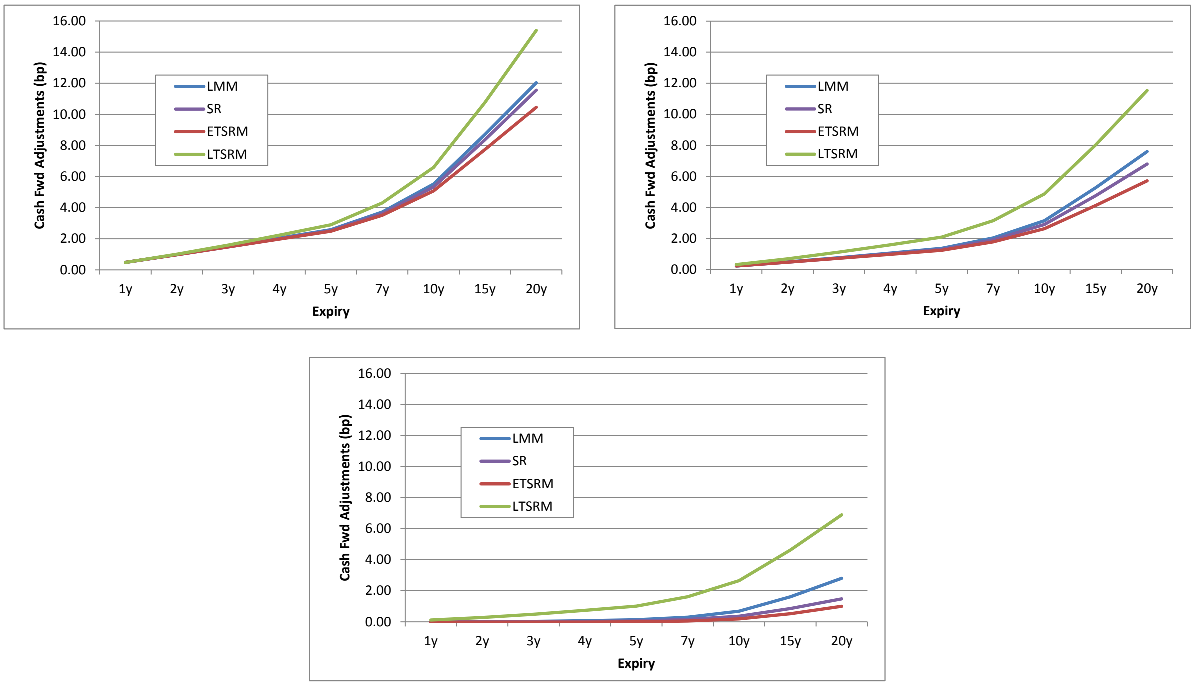

Figure 2: *Cash-forward adjustments of 30Y tenor swaptions for different volatility term-structures in tenor direction. 1Y tenor swaption BPVols decreasing/increasing linearly from 75bp (top left), 65bp (top right) or 55bp (bottom) to 60bp for the 30Y vol column.*

Next, we consider real EUR market data as of 06/05/2015. We calibrated a physical surface (going through ETSRM) to market prices of cash-settled ATM straddles and OTM payer/receiver swaptions. Table 1 shows the corresponding ETSRM implied cash-adjusted forwards. The negative values especially for the shorter expiries are mainly a consequence of the inverted volatility term-structure in tenor direction, with volatilities of the shorter tenors being lower than longer tenor volatilities. Using the same market data, Figure 3 shows the implied cash BPVol smile[13] for EUR 10Y30Y cash-settled swaptions. Note that we obtain two separate volatility branches, one for payer and one for receiver swaptions. That is, for a given strike we need to plug in two different volatilities into the market formula in order to retrieve the ETSRM premiums, depending on whether we want to price either a payer or a receiver swaption. Unsurprisingly, OTM volatilities from ETSRM match the market volatilities fairly well, as those served as calibration targets. Volatilities for in-the-money (ITM) swaptions, however, can

[12] For this analysis we used mean-reversion parameters consistent with the used volatility term structures. The parameters may be tweaked though to obtain results more in line with the other models, although this would have to be done manually on a case by case basis.

[13] Where we define the implied cash BP vols (for a given strike) to be the normal annual basis point volatility, that we need to plug into the market formula in order to retrieve a given swaption premium.

differ significantly from the naive market volatilities. In particular, for strikes further away from ATM, the BPVol inversion for ITM swaptions can even completely break down, as the ETSRM implied premiums drop below the naive intrinsic value $P(0,T)C(S_0)(wS_0 - wK)^+,\ w = \pm 1$ of the (Black-like) market formula, see Figure 4. This raises the question whether using cash BPVols as a way of quoting prices of cash-settled swaptions is the right concept, at least when it comes to quoting prices for ITM swaptions.

Expiry\Tenor	2Y	5Y	10Y	30Y
2Y	-0.01	-0.16	-0.65	-1.00
5Y	-0.02	-0.10	-0.49	-0.12
7Y	-0.02	-0.11	-0.19	0.82
10Y	-0.07	0.03	0.17	2.86
20Y	-0.05	0.19	0.64	5.31
30Y	-0.12	0.56	1.27	4.17

Table 1: *ETSRM-implied cash-forward adjustments in basis points for EUR market data as of 06/05/2015.*

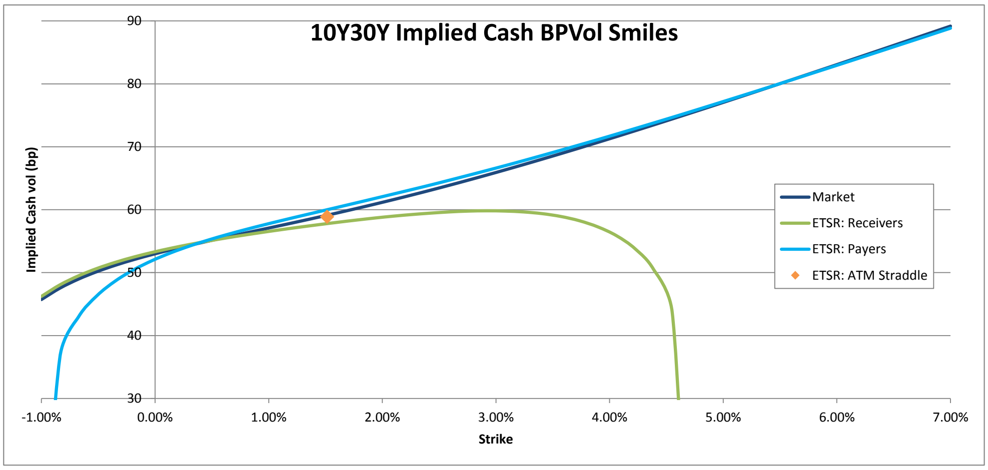

Figure 3: *10Y30Y implied cash BPVols for EUR market data as of 06/05/2015.*

The Cash-Physical Basis

Although physically-settled swaptions are not liquidly traded in the European markets, their premiums or volatilities usually form the basis for calibrating term-structure models, and thus for determining the price levels of Bermudan swaptions and other more exotic interest-rate derivatives. As cash-settled swaptions represent the natural hedging instruments for those products, it is clearly desirable that both physically- and cash-settled swaptions are priced consistently with each other. As a consequence, the models that we use to back out physical volatility surfaces should be able to capture the "cash-physical basis" in a realistic way.

Despite the problems of the market formula with no-arbitrage violations, one may naturally ask whether it can nevertheless be used as a tool for backing out physical volatilities from cash-settled swaption premiums[14]. And it is indeed common market practice to use cash and physical volatilities interchangeably, i.e. to use the very same Black-implied volatility (for a given strike) to either price a cash-settled swaption (using the naive market formula) or a physically-settled swaption (using the standard physical formula). The results in Henrard [2011], however, suggest that this practice is in general not justified, as it doesn't yield price levels consistent with what is obtained by using proper term-structure models. Since Henrard [2011] doesn't provide us with any other alternatives for efficiently backing out physical volatilities, let us have a brief look at how LTSRM/ETSRM perform in this regard. As a measure for the cash-physical basis, we show in Figure 5 differences between forward premiums of ATM[15]

[14] The market formula can be derived in a heuristic way which involves the physical annuity measure. This suggests that both cash and physical volatilities should indeed be the same.

[15] Where "ATM" refers the naive physical forward for both types of instruments.

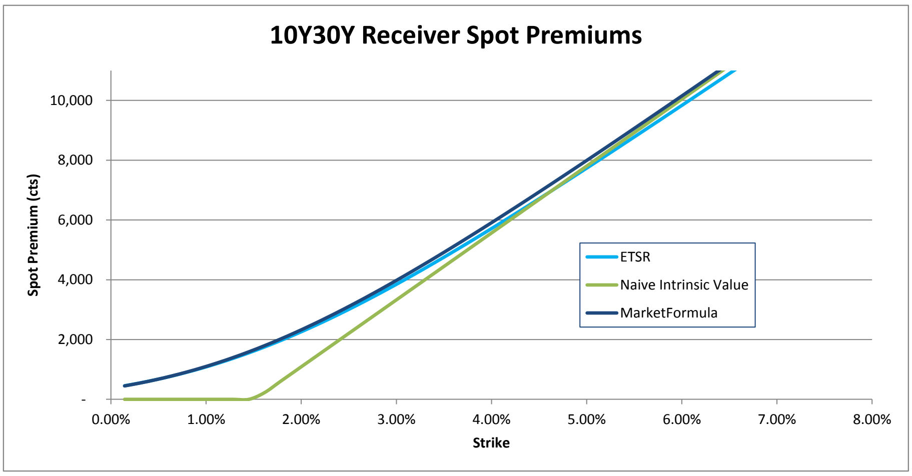

Figure 4: *10Y30Y receiver spot premiums (cents) for EUR market data as of 06/05/2015.*

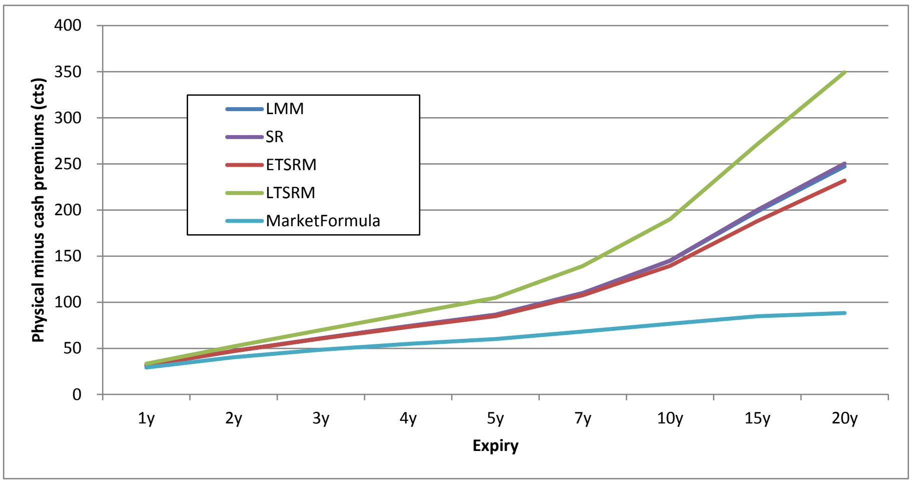

Figure 5: *Differences (in cents) between forward premiums of physically- and cash-settled ATM straddles.*

physically- and cash-settled straddles. Here, we only show results for the surface with 75bp vols in the 1Y column, as in the other two cases the pictures would look nearly identical. Firstly, note that the market formula implies premium differences significantly lower than the ones implied by the other models. Secondly, ETSRM is again very close to the two term-structure models while the cash-physical basis implied by LTSRM is generally far too big. If one is only concerned with pricing cash-settled swaptions, then the LTSRM mean-reversion parameters can generally be chosen so as to yield ITM swaption prices or cash-adjusted forward similar to those from ETSRM or proper term-structure models[16]. When it comes to backing out physical volatilities though, it is generally difficult to bring LTSRM back "in the pack", no matter what kind of (reasonable) mean-reversion levels are chosen, and the implied cash-physical bases will usually be too big. This is mainly a consequence of the linearity assumption that underlies LTSRM.

Please note that in order to generate the results in this article, we used an ETSR model where all the swap-rate/swap-rate correlations that are internally used are assumed to be one. Slightly more realistic results may be obtained by allowing for non-perfectly correlated swap rates, although analysing this in detail would be beyond the scope of this paper.

[16] Although those mean-reversion levels do not necessarily need to be consistent with mean-reversion levels implied by calibrated Gaussian short-rate models.

Conclusion

It has been widely acknowledged that the naive market formula for pricing cash-settled swaptions may produce prices which are not completely arbitrage-free. Yet, using this formula for pricing and risk-managing these products has by and large remained the market standard. One reason is certainly the convenience that comes with being able to use a simple closed formula. Another reason, however, may be the fact that the arbitrage strategy outlined in Mercurio [2008] requires trading or at least replicating CMS-type products, which makes it difficult to exploit possible arbitrage opportunities. In this article we have presented an arbitrage strategy, which only involves buying and selling four standard cash-settled swaptions, and can thus be executed more easily and cost efficiently. We have also briefly looked at other possible valuation approaches and found that the ETSR model not only prices cash-settled swaptions in an arbitrage-free way, but that it also yields realistic results for cash-adjusted forwards and cash-physical bases, in line with proper term-structure models.

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