Advanced EONIA Curve Calibration

Avoiding unwanted shape oscillation in EUR overnight curve

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BANCA IMI QuantLib Deloitte.

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Empirical Evidence in EUR Market

- The key point for a "state-of-art" curve bootstrapping is to obtain smooth forward rates
- For even the best interpolation scheme to be effective, any market jump must be removed before the curve calibration and than added back at the end of the process
- The most relevant rate jump is the so called *Turn-Of-Year* (TOY)
- A rate jump can be seen, in a financial point of view, as a higher index fixing due to increased search of liquidity of market participants caused by end-of-month or end-of-year capital requirements.

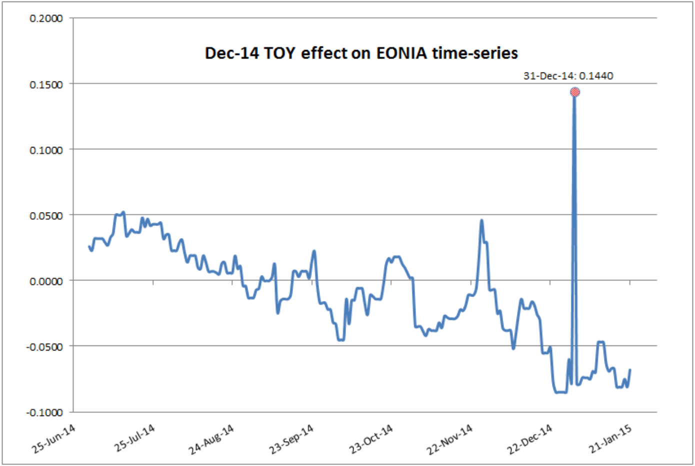

Figure: December 2014 EONIA Index turn-of-year

The U.S. market case

However, previous definition is not consistent for the U.S. case since the Fed Funds rate shows negative jumps

Figure: Last 7 months Fed Funds rate time series seen on Bloomberg Terminal.

Jump estimation methodology

In order to estimate jumps sizes, Ametrano-Mazzocchi[1] propose a 4-step approach inspired by Burghardt [1]:

1. The 1^{st} step is to built an overnight curve using a linear/flat interpolation and including all liquid market quotes available
2. After that it is possible to estimate the 1^{st} jump assuming that a segment out of line with preceding and following segments can be put back in line dumping the difference into the jump. For positive sizes:

$$[F^{original}(t_1, t_2) - F^{interp}(t_1, t_2)] \cdot \tau(t_1, t_2) = J^{Size} * \tau^J$$

3. Once the 1^{st} jump as been estimated it is possible to remove it from the curve
4. Iterate ad libitum 2 and 3 on the next jump date.

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Negative Sizes

Reviewed formula for negative sizes

The preceding formula is good for estimating positive jumps. Otherwise, in order to calculate negative sizes, the 2nd point formula must be reviewed in:

$$[F^{Interp}(t_1, t_2) - F^{Original}(t_1, t_2)] \cdot \tau(t_1, t_2) = J^{Size} * \tau^J$$

$$J^{Size} = [F^{Interp}(t_1, t_2) - F^{Original}(t_1, t_2)] \cdot \frac{\tau(t_1, t_2)}{\tau^J}$$

EONIA and USD overnight Curves including Jumps

The resulting EONIA and USD overnight curves including jumps estimated through the preceding approach are shown in Figure 3 and 4

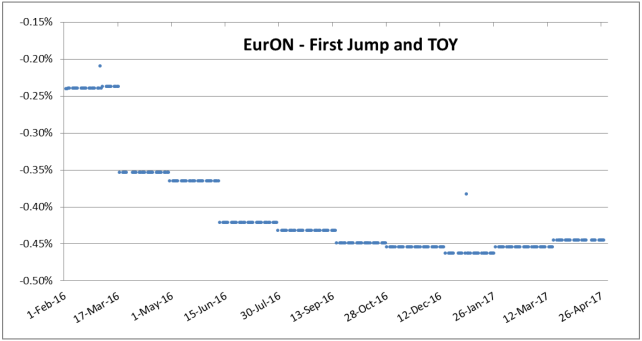

Figure: First Eonia curve's section with estimated positive jumps.

EONIA and USD overnight Curves including Jumps

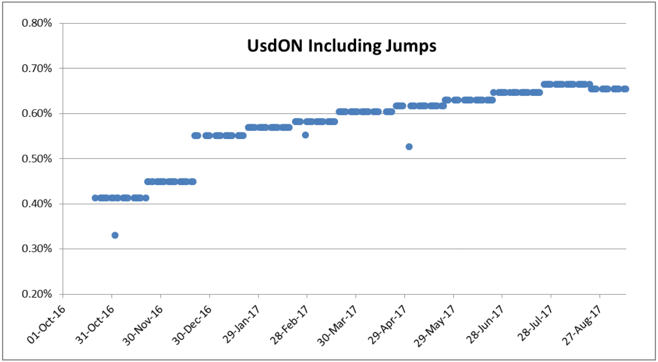

Figure: First USDON curve's section with estimated negative jumps.

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Spot and Forward OIS imperfect concatenation

The problem is related to the composition of the EONIA curve which mixes spot starting instruments (*Overnight Indexed Swaps*) and forward starting instruments (*European Central Bank OIS)*

- Otherwise, ECB OIS are preferred by traders because of their liquidity
- Best practices tend to set calibration algorithms in order to include ECB OIS instead of spot OIS which cover the same period.

Imperfect Concatenation

The EONIA curve will show an overlapping section caused by the imperfect concatenation between spot and forward OIS as visible in Figure 6.

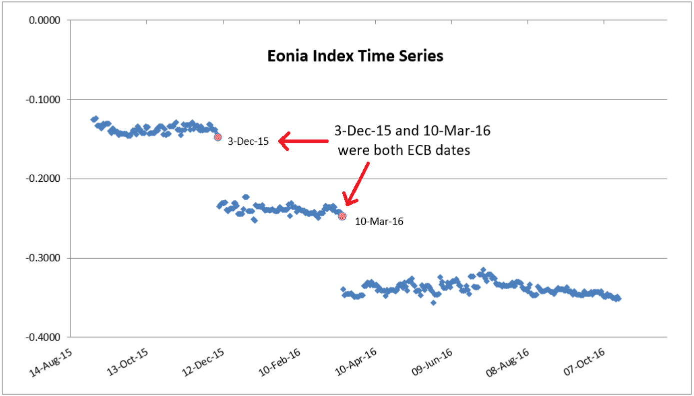

Figure: Piecewise constant behaviour shown by last year EONIA Index fixings

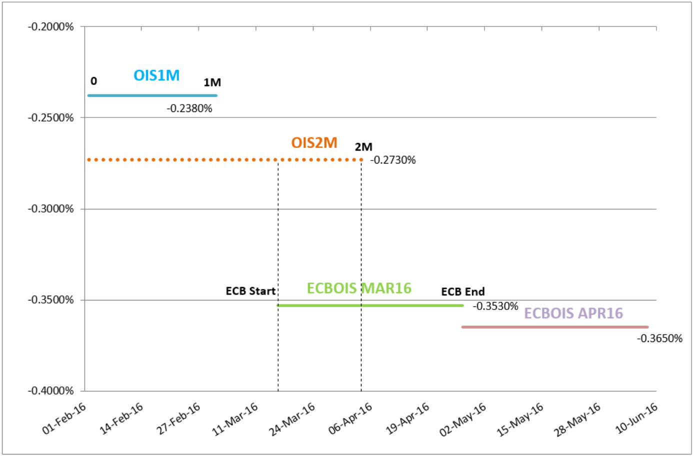

Figure: Overlapping EONIA instruments levels; dataset as of *January 29, 2016*

The calibration algorithm derives the following information:

- The average rate for the interval $(0; 1M)$ from OIS1M
- The average rate for the interval $(1M; 2M)$ from OIS2M
- The average rate for the interval $(2M; ECB_{end})$ from 1^{st} ECB OIS

Distortion

As a consequence, the calibrator can not use the information related to the interval $(ECB_{start}; 2M)$ set by the most liquid instrument on the market.

Solution: Forward Stub

This distortion could be negligible only if the information given by both instruments are almost equal. What about if the first ECB OIS is accounting a rates cut/rise expectation?

To solve this problem the suggestion is to build a Meta-Quote called: "Forward Stub" with this characteristics:

- Start date equal to the maturity of the last not-overlapping OIS
- Maturity equal to the settlement date of the first forward ECB OIS

The result is a new quote that links spot to forward market instruments perfectly

The Forward Stub value is implied in the market and can be derived imposing a no-arbitrage condition which always ensures the perfect re-price of the overlapping spot instrument:

Condition

$$\int_0^{1M} f(s)ds + \int_{1M}^{ECB_{start}} f(s)ds + \int_{ECB_{start}}^{2M} f(s)ds = \int_0^{2M} f(s)ds$$

- $\int_0^{1M} f(s)ds = OIS1M$ value
- $\int_{1M}^{ECB_{start}} f(s)ds =$ Forward Stub value *(unknown)*
- $\int_{ECB_{start}}^{2M} f(s)ds =$ it is not a quoted market instrument
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Average rate in $(ECB_{start}; 2M)$

How to define the average rate in $(ECB_{start}; 2M)$ there is no instrument on the market covering the same period?

Assumption

To have information about the average rate in $(ECB_{start}; 2M)$ it is necessary to assume flat forwards in the interval $(ECB_{start}; ECB_{end})$. This means that the average rate in the interval $(ECB_{start}; 2M)$ would be equal to the average rate in the interval $(ECB_{start}; ECB_{end})$

- where the average rate in $(ECB_{start}; ECB_{end})$ is known and equal to the 1^{st} ECB OIS

Since the instantaneous forward rates integral in the interval $(ECB_{start}; 2M)$ is known, the problem is reduced to a simple equation where the only unknown value is the Forward Stub quote which is equal to:

Forward Stub value

$$\int_{1M}^{ECB_{start}} f(s)ds = \frac{\int_0^{2M} f(s)ds}{\int_0^{1M} f(s)ds + \int_{ECB_{start}}^{2M} f(s)ds}$$

Assuming continuous compounding

$$Forward\ Stub = \frac{\left[\frac{e^{F(0,2M)\cdot\tau(0,2M)}}{e^{F(0,1M)\cdot\tau(0,1M)}\cdot e^{F(ECB_{start},2M)\cdot\tau(ECB_{start},2M)}} - 1\right]}{\tau(1M, ECB_{start})}$$

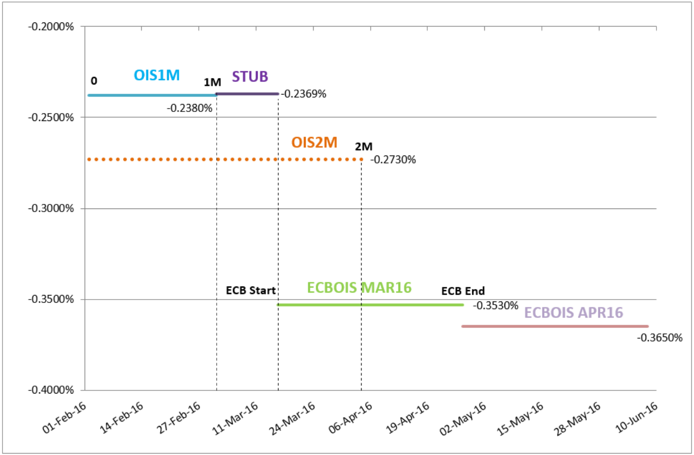

Figure: Overlapping EONIA instruments levels including the Forward Stub.

The Forward Stub algorithm is stable also in limit cases.

Spot Stub

In particular calendar conditions, the discarded spot instrument is the one-week maturity OIS *(OIS1W)*. This case implies that the so called Forward Stub it is not forward start anymore because it covers the year fraction: $\tau(0, ECB_{start})$ (where ECB_{start} is the 1^{st} ECB OIS fixing date)

Spot Stub value

In this limit case, the "Spot Stub" implied value can be derived using the following formula:

$$Spot\ Stub = \frac{\left[\frac{e^{F(0,ECB_{end})\cdot\tau(0,ECB_{end})}}{e^{F(ECB_{start},ECB_{end})\cdot\tau(ECB_{start},ECB_{end})}} - 1\right]}{\tau(0, ECB_{start})}$$

Repricing Errors analysis

Repricing Errors in Basis Points (Bps)			
Instrumets		Overlapping	Forward Stub
OIS	2M	0.00	0.00
OIS	3M	-1.09	-0.05
OIS	4M	-0.72	0.05
OIS	5M	-0.64	-0.03
OIS	6M	-0.83	-0.32
OIS	7M	-0.45	-0.02
OIS	8M	-0.42	-0.04
OIS	9M	-0.33	0.01
OIS	10M	-0.31	-0.01
OIS	11M	-0.27	0.01
OIS	12M	-0.60	-0.35
OIS	15M	-0.24	-0.04
RMSE		0.57	0.14
ME		-1.09	-0.35

Figure: Repricing errors summary related on instruments **not included** in the calibration

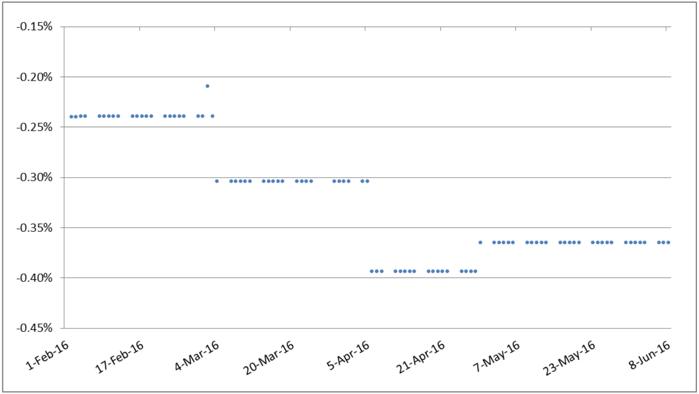

Figure: EONIA curve bootstrapped with overlapping instruments.

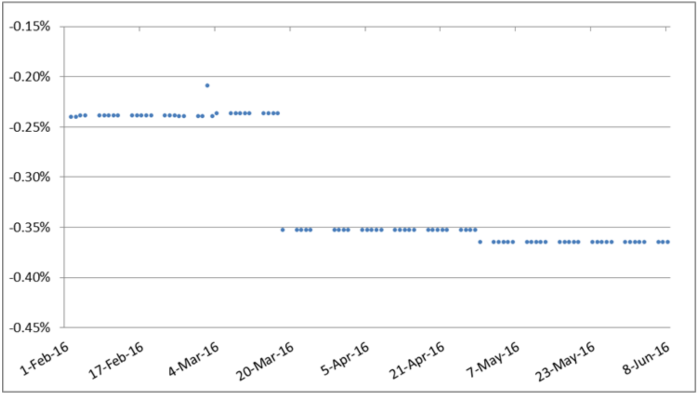

Figure: EONIA curve bootstrapped including the Forward Stub.

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Fitting EONIA Curve Functional Form

- EONIA Index fixings show an almost flat behaviour between ECB monetary policy meeting dates. Otherwise, assuming piecewise constant forward rates in the mid-long term leads to big inconsistency
- Modelling the EONIA curve requires all shape's specific features to be accounted

Interpolation Problem

The problem is related to the choice of the best interpolation scheme able to reproduce the overnight shape behaviour

Since interpolating log-linearly the whole curve is not advisable, the best practise converges towards cubic interpolation schemes. Otherwise, this kind of techniques leads to two problems:

1. They don’t have a good fit with the short-term curve’s section.
2. The Forward Stub algorithm presented before would not be consistent anymore since it assumes piecewise flat forward rates between ECB dates.
    - However the implied "Meta-Quote" could be calculated anyway by means of root finding methods

Solution: Mixed Interpolation technique

Solution

The solution proposed is to build a new interpolation scheme named: "Mixed-Interpolation" that gives the possibility to merge two different interpolation techniques.

Critical issues

1. At which point the interpolation scheme must be switched?
2. Which merging approach can be used?

Our suggestion

- Merge a log-linear interpolation up to the end of the ECB OIS strip and then switch to a monotone cubic Hyman[a] filtered interpolation till the end of the curve.
- Set the "Switch Pillar" equal to the maturity of the last quoted ECB OIS

[a] for more information see [2]

QuantLib implementation

The Mixed Interpolation algorithm can be found in QuantLib 1.9 version in two different ways:

1. The *Split Range* consists in interpolating till the switch pillar using the 1^{st} interpolation technique and then switch to the 2^{nd} interpolation for the rest of the curve.
2. *Share Range* consists in interpolating two times the whole curve and then merging the obtained curves in the switch pillar.

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Repricing Errors Analysis

Repricing Errors in Basis Points (Bps)					
Instrumets		Linear + Stub	MonotoneCubic + Stub	SplitRange	ShareRange
OIS	2M	0.00	0.00	0.00	0.00
OIS	3M	-0.05	-0.52	-0.05	-0.05
OIS	4M	0.05	-0.21	0.05	0.05
OIS	5M	-0.03	-0.12	-0.03	-0.03
OIS	6M	-0.32	-0.55	-0.32	-0.32
OIS	7M	-0.02	-0.19	-0.02	-0.02
OIS	8M	-0.04	-0.18	-0.04	-0.04
OIS	9M	0.01	-0.13	0.01	0.01
OIS	10M	-0.01	-0.13	-0.01	-0.01
OIS	11M	-0.03	-0.11	-0.03	-0.03
OIS	12M	-0.35	-0.47	-0.35	-0.35
OIS	15M	-0.04	-0.13	-0.04	-0.04
	RMSE	0.14	0.29	0.14	0.14
	ME	-0.35	-0.55	-0.35	-0.35

Figure: Repricing errors summary related on instruments **not included** in the calibration for all analyzed solutions.

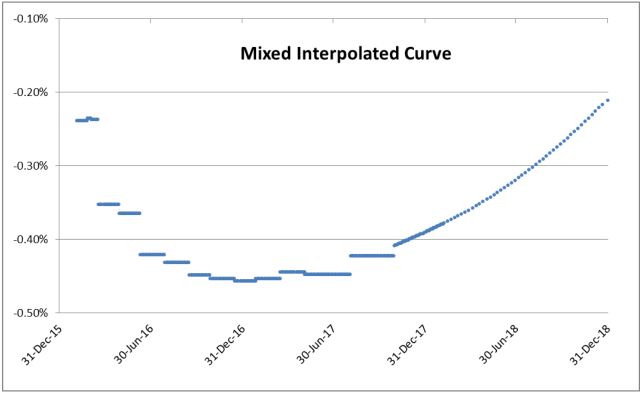

Figure: A mixed interpolated EONIA curve obtained through a linear interpolation on log-discounts up to the last ECB OIS and a monotone cubic Hyman filtered interpolation till the end curve.

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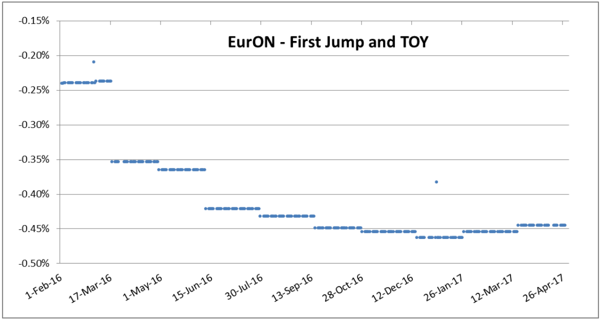
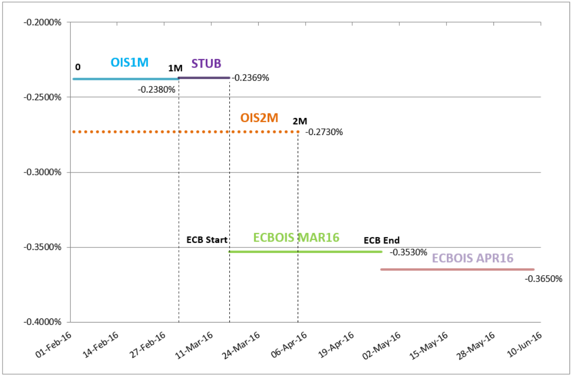
Conclusions

- 1) Estimate jumps and TOYs and account them before calibration



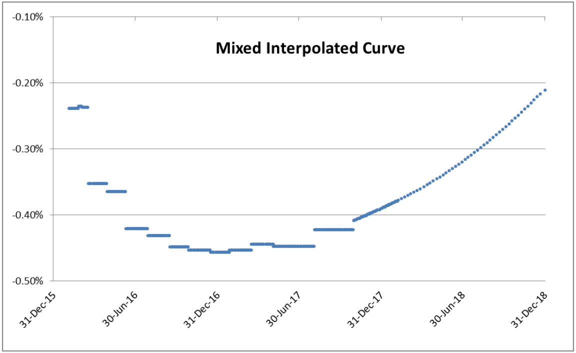
Conclusions

- 2) Link spot instruments to forward instruments using the Forward Stub in order to avoid shape oscillation



Conclusions

- 3) Use a Mixed linear-cubic interpolator in order to reach a curve's behaviour better fit



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