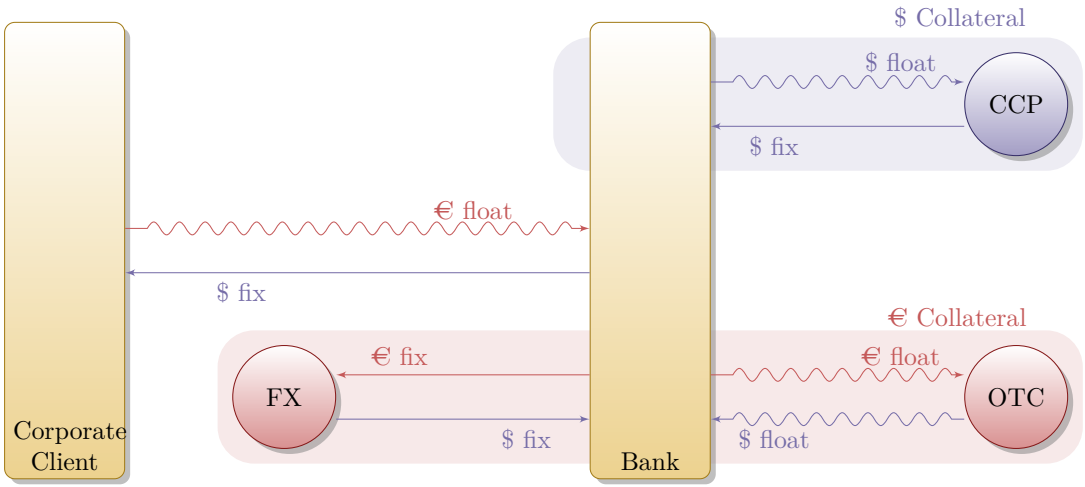
FX Modelling in Collateralized Markets: foreign measures, basis curves, practical approximations



Andrea Pallavicini
Banca IMI & Imperial College

joint work with

Nicola Moreni
Banca IMI

New Themes in Finance, Insurance and Energy Markets
Novara, 22 September 2016

Talk Outline

1. Frictions and Dislocations in the FX Market
2. Collateralization and Funding Costs in Derivative Pricing
3. Numerical Investigations of Practical Approximations

Disclaimer

The opinions expressed in this work are solely those of the authors and do not represent in any way those of their current and past employers.

Reference Papers

- Shabani, M., Stenfors, A. and Toporowski, J. (2016)
  $\longrightarrow$ "Bank Risk Premia and Abenomics: The Return of the Japan Premium in the Cross-Currency Swap Market". *LSE Financial Markets Group Paper Series*.
- Fujii, M., Takahashi, A. (2015)
  $\longrightarrow$ "Choice of Collateral Currency Updated: A market model for the benchmark pricing". Working paper on arXiv and SSRN.
- Moreni, N., Pallavicini, A. (2015)
  $\longrightarrow$ "FX Modelling in Collateralized Markets: foreign measures, basis curves, and pricing formulae". Working paper on arXiv and SSRN.
- Baba, N. and Packer, F. and Nagano, T. (2008)
  $\longrightarrow$ "The Spillover of Money Market Turbulence to FX Swap and Cross-Currency Swap Markets". *BIS Quarterly Review* 1, 2008.

Talk Outline

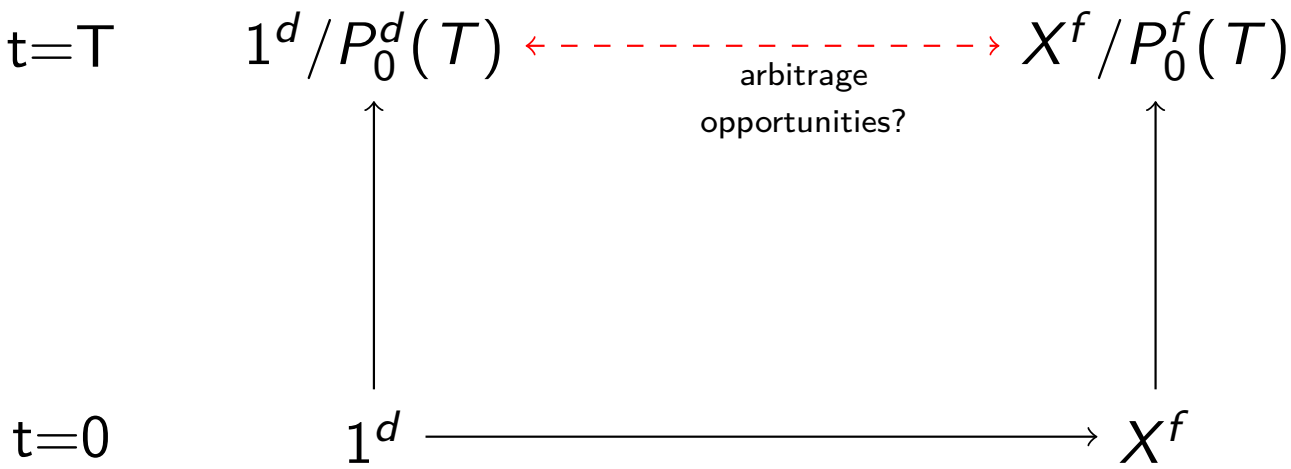
Covered Interest Parity – I

- Interest-rate differentials between two currencies should be perfectly reflected in the foreign-exchange (FX) swap prices, otherwise arbitrages would be possible.
- Investors cannot earn profits by
  - $\longrightarrow$ borrowing in a country with a lower interest rate,
  - $\longrightarrow$ exchanging for foreign currency, and
  - $\longrightarrow$ investing in a foreign country with a higher interest rate.

  due to gains or losses from exchanging back to their domestic currency at maturity.
- The equivalence in this two strategies is known as covered interest parity (CIP).

Covered Interest Parity – II

- The CIP may fail during period of financial crisis reflecting funding difficulties of financial institutions.
  - $\longrightarrow$ The Japanese banking crisis of late '90s.
  - $\longrightarrow$ The credit crunch occurred since 2007.

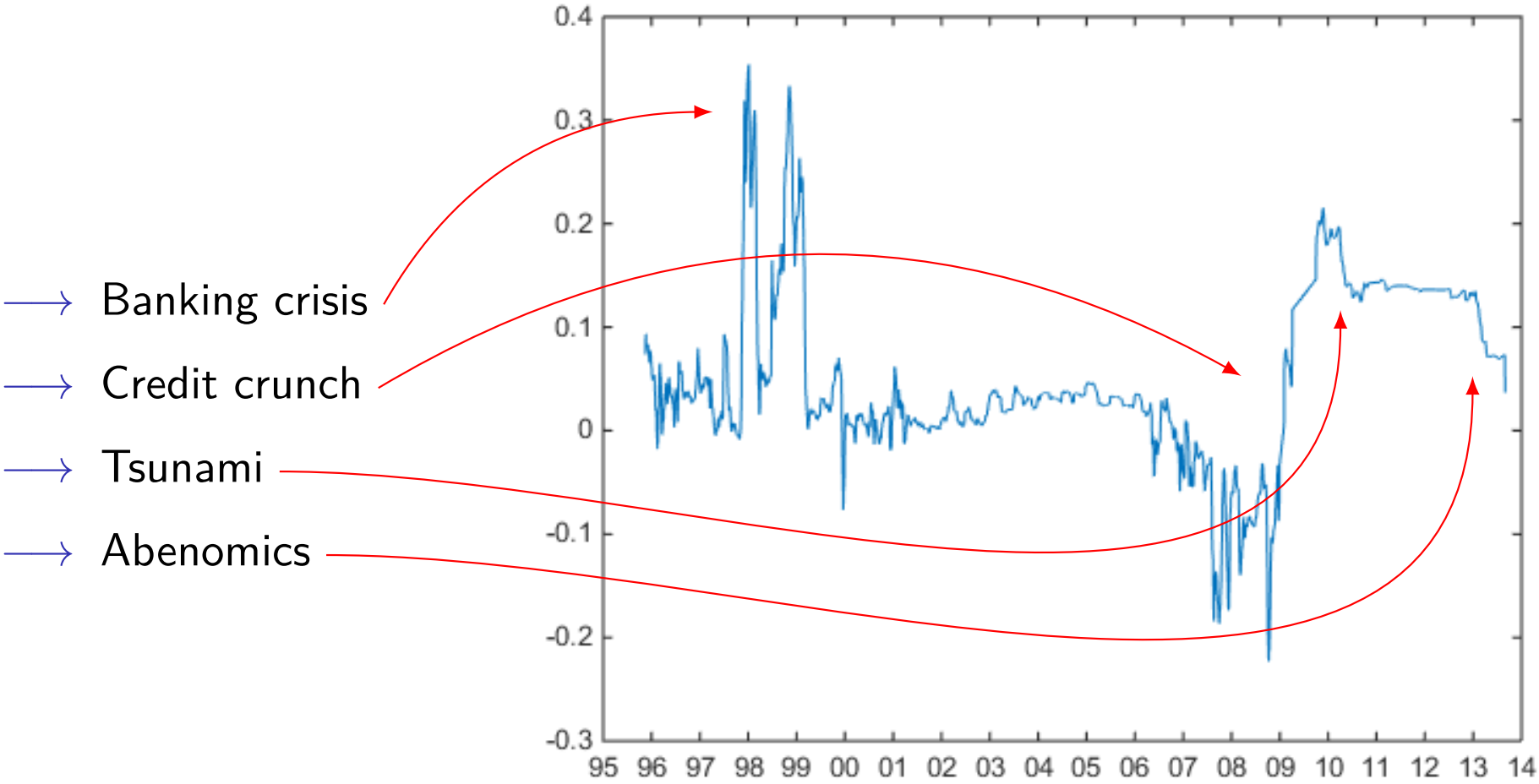


The Japanese Banking Crisis – I

- As disussed in Covrig et al. (2004) and in Shabani et al. (2016), in the aftermath of the 1989 stock market crash, the Japanese economy slowed down entering a prolonged slump.
- The soundness of the Japanese banking system weakened culminating with several highly important financial institutions defaulting in 1997.
- Insolvency in the banking sector highlighted the increasing inability of Japanese banks to access unsecured funds in foreign currencies, and to a lesser degree also in Yen.
- The failure of the CIP led to the emergence of the so-called *Japan Premium*, namely

  $\longrightarrow$ a premium on borrowing costs of Japanese banks in the international financial markets.

The Japanese Banking Crisis – II



Japan premium. Difference in % between the 3m JBA Tibor rate minus the 3m ICE (ex BBA) JPY Libor rate.

Market Dislocations after the Credit Crunch – I

- Since the financial crisis of 2007 banks and financial institutions, which were so far considered as non-defaultable corporations, started being suspicious about the liquidity availability and credit worthiness of their counterparties.
- Borrowing money, even for short maturities (under one year), became more expensive, as banks charged their counterparties higher rates for unsecured lending.
- The shortage of funding sources forced central banks to adopt a number of non-standard measures to support financing conditions and credit flows both in domestic and foreign currencies.
- Despite these efforts market frictions and dislocations in single-currency and FX markets strengthened.

Market Dislocations after the Credit Crunch – II

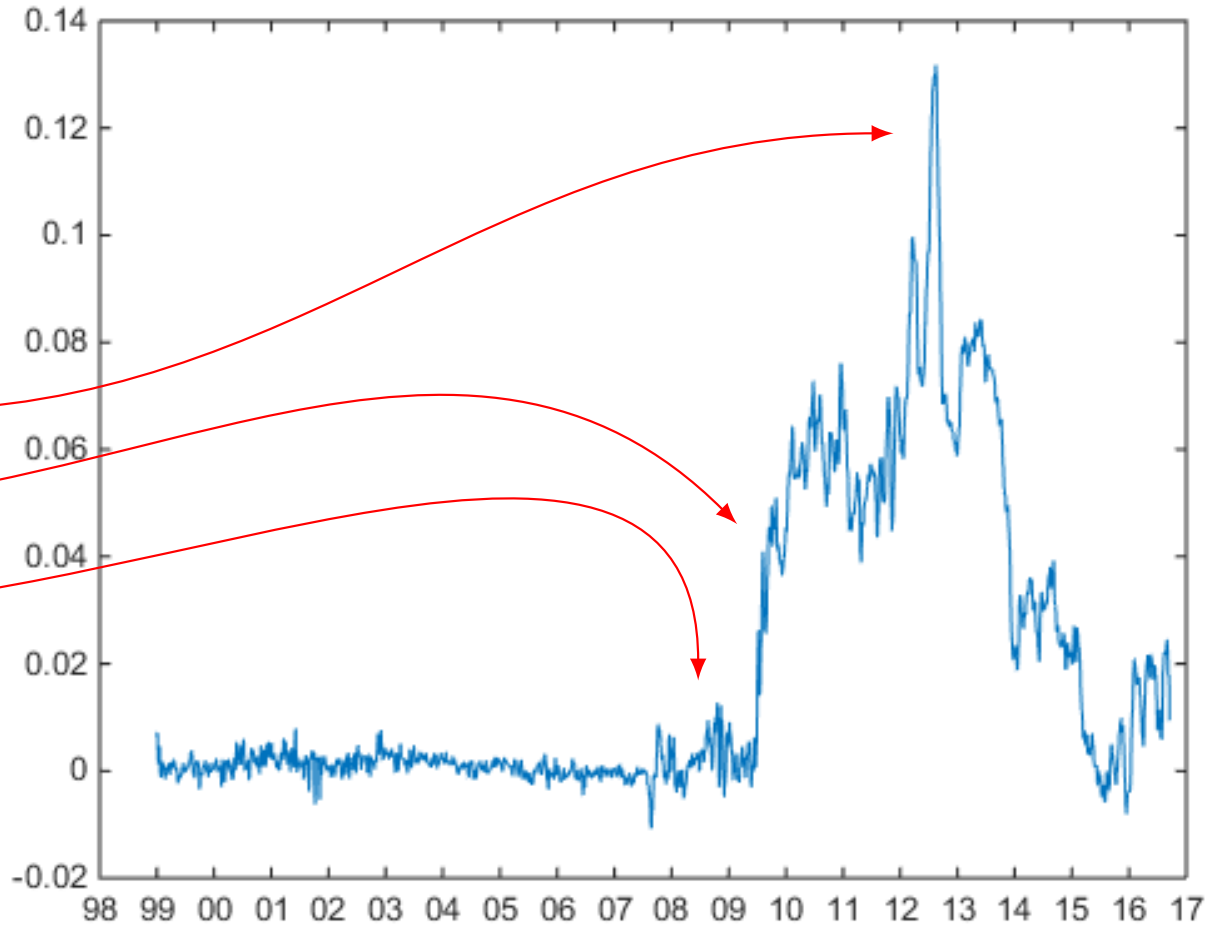
- The failure of CIP between USD and EUR, GBP and JPY as discussed in Baba et al. (2008) can be ascribed to several facts.
  - $\longrightarrow$ The market perceives EU financial institutions riskier than US ones.
  - $\longrightarrow$ The shortage in USD of EU financial institutions leads to one-sided order flows concentrated on USD borrowing.
  - $\longrightarrow$ It is difficult to size the borrowing costs in the money market by means of Libor rates.
  - $\longrightarrow$ Liquidity peaks in the USD market do not correspond to the time frame during which EU financial institutions are obliged to fulfill USD payments (mismatching market opening times).
- We can name *US Premium* the additional costs faced by non-US institutions to fund in USD.

Market Dislocations after the Credit Crunch – III

→ Rating downgrade

→ Greek crisis

→ Credit crunch



US premium for EUR. Difference in % between the 3m EMMI (ex EBF) Euribor rate minus the 3m ICE (ex BBA) EUR Libor rate.

Market Dislocations after the Credit Crunch – IV

- The failure of the CIP has direct consequences also in derivative option prices.
- Dislocations may produce additional costs in funding and hedging, possibly leading to severe liquidity shortages.
  $\longrightarrow$ See the IMF working paper by Barkbu and Ong (2010).
- Funding costs depend on the funding strategies adopted by investors.
  $\longrightarrow$ Funding policies are a collection of different strategies, driven not only by financial factors.
- The dislocations we are dealing with are not counterparty-specific but systemic.
  $\longrightarrow$ We focus on a domestic investor who can fund in foreign currencies only by means of FX spot, forward, and cross-currency swap contracts.
  $\longrightarrow$ In Fujii and Takahashi (2015) the authors suggest to treat funding costs coming from market dislocations as additional FVA terms.

Talk Outline

Collateral Funding in the FX Market – I

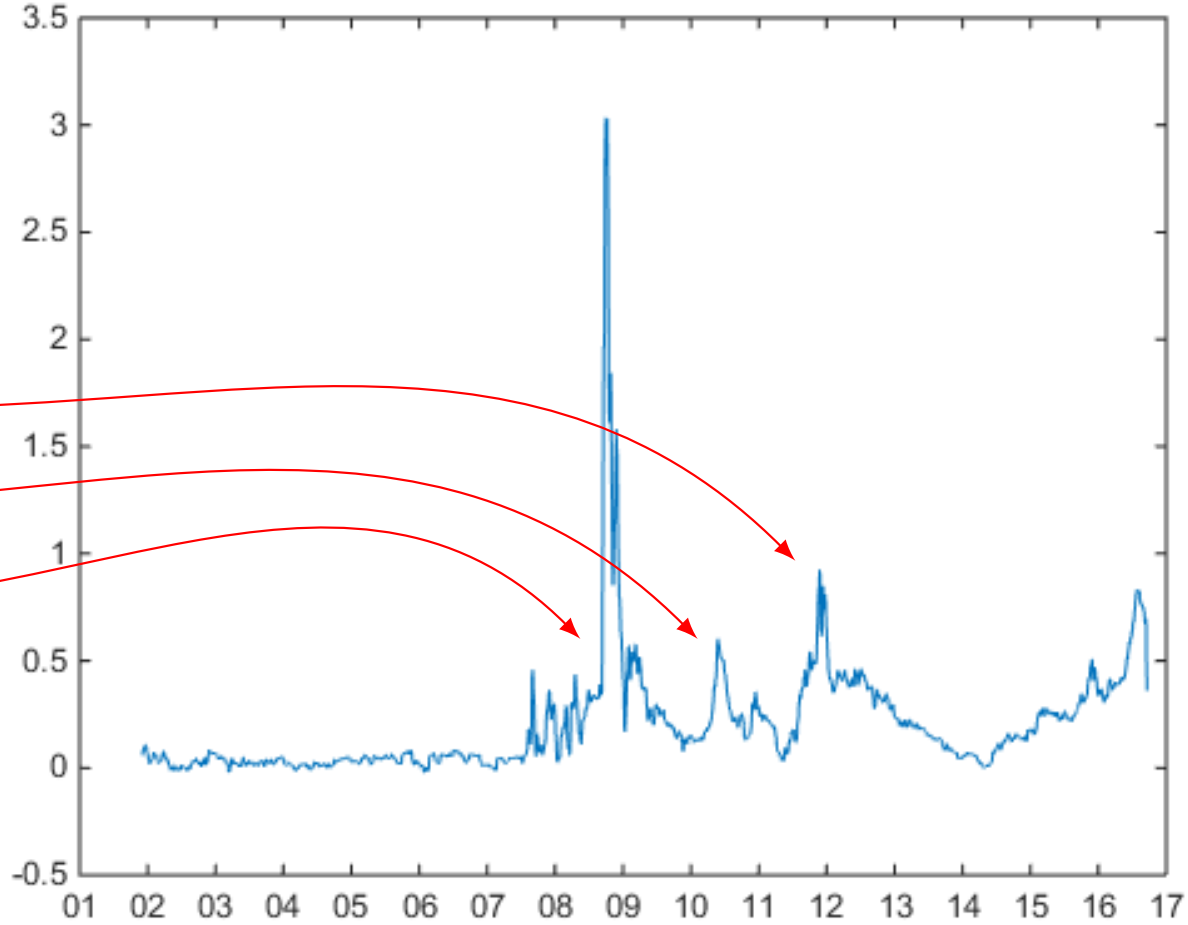
- Liquid market derivatives are usually collateralized with collateral assets remunerated at overnight rate (OIS curve).
- If we assume that our foreign funding policy is implemented by trading in the FX market, we can calculate a cross-currency spread by comparing:
  - $\longrightarrow$ foreign collateral rates implied by the CIP relationship (off-shore rates), and
  - $\longrightarrow$ foreign collateral rates quoted in the foreign money markets (on-shore rates).
- We expect to spot again the market dislocations described by the previous analyses, although the impact of macro-economic events may be quantitatively different.

Collateral Funding in the FX Market – II

→ Rating downgrade

→ Greek crisis

→ Credit crunch



Cross-currency spread. Difference in % between the rate implied via CPI from the 3m EUR OIS rate minus the 3m USD OIS rate.

Derivative Pricing – I

- Market dislocations and collateral funding costs have an impact on hedging and funding strategies and, in turn, on derivative pricing when foreign currencies are involved.
- How can we price a derivative contract whose cash flows or collateral assets are expressed in a foreign currency ?
- We restrict our analysis with the following assumptions, see Moreni and Pallavicini (2015).
  - $\longrightarrow$ We consider that foreign cash can be funded only by trading in the FX market.
  - $\longrightarrow$ Derivative contracts are perfectly collateralized, namely the collateralization procedure is able to prevent any loss in case of default of one of the two counterparties.
  - $\longrightarrow$ Collateral assets may be re-hypothecated, see Brigo et al. (2011).

Derivative Pricing – II

- Market dislocations prevent us to follow the classical path where the domestic risk-neutral measure is equivalent to the foreign risk-neutral measure.
- According to Piterbarg (2010) we can price perfectly collateralized contract, such as FX swaps, by discounting at the collateral rate.

$$V_t^{\text{FXswap}} := \mathbb{E}_t \left[\left(\frac{\chi_T}{X_t(T;e)} - 1 \right) D(t,T;e) \right]$$

where $D(t,T;e)$ is the domestic EUR OIS discount factor, χ_T the FX spot rate, and $X_t(T;e)$ the FX forward rate.

- The above expectation is taken under the domestic risk-neutral measure.

Basis Curves – I

- The FX forward rate is quoted by the market so that the FX swap is at par, namely

$$X_t(T;e) = \frac{\mathbb{E}_t[\,\chi_T D(t,T;e)\,]}{\mathbb{E}_t[\,D(t,T;e)\,]} = \mathbb{E}_t^{T;e}[\,\chi_T\,]$$

where the last expectation is taken under the collateralized forward measure $\mathbb{Q}^{T;e}$ defined by the Radon-Nikodym derivative

$$\left.\frac{d\mathbb{Q}^{T;e}}{d\mathbb{Q}}\right|_t := \frac{D(0,t;e)P_t(T;e)}{P_0(T;e)}\,, \quad P_t(T;e) := \mathbb{E}_t[\,D(t,T;e)\,]$$

Basis Curves – II

- By following Moreni and Pallavicini (2015) we can use FX forward rates to define a new pricing measure: the collateralized foreign measure $\mathbb{Q}^b$.

$$\left.\frac{d\mathbb{Q}^b}{d\mathbb{Q}}\right|_t := \frac{\chi_t}{\chi_0} D(0,t;e - b^f(e))$$

where we define the basis rate $b_t^f(e)$

$$b_t^f(e)\,dt := e_t\,dt - \mathbb{E}_t\left[\frac{d\chi_t}{\chi_t}\right]$$

- If we use the above measure in the definition of FX forward rate, we can define the effective foreign funding curve (or basis curve) as

$$P_t^f(T;e) = \frac{X_t(T;e)}{\chi_t} P_t(T;e)\,, \quad P_t^f(T;e) := \mathbb{E}_t^b\left[\, D(t,T;b^f(e))\,\right]$$

Master Pricing Equations – I

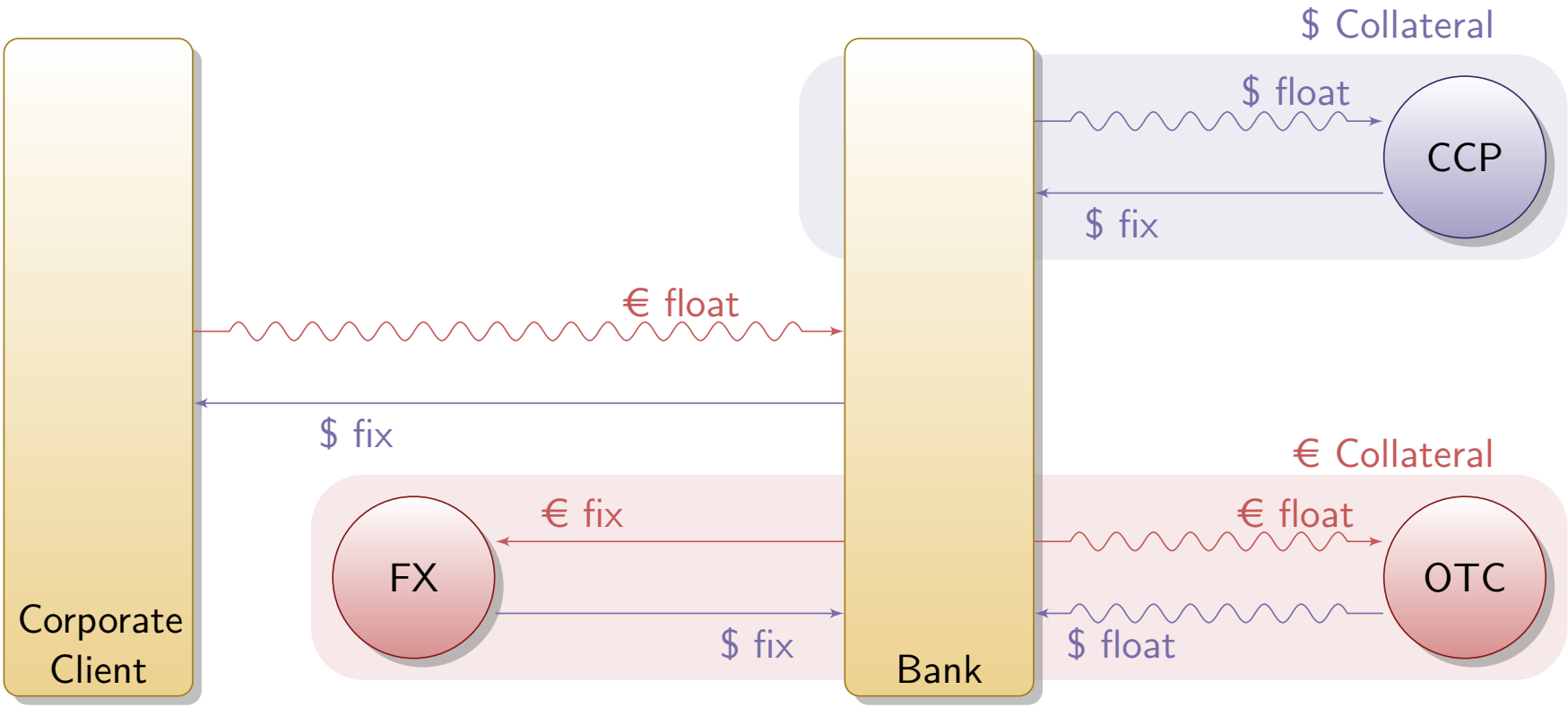
- We can reformulate the classical pricing theory to derivative contracts according to the currency of contractual cash flows and collateral accounts (always in case of perfect collateralization).

π_t	C_t	Pricing Formula
d	d	$V_t = \int_t^T \mathbb{E}_t[\, D(t,u;c)\, d\pi_u \,]$
d	f	$V_t = \int_t^T \mathbb{E}_t\big[\, D(t,u;c^f - b^f(e) + e)\, d\pi_u \,\big]$
f	d	$V_t^f = \int_t^T \mathbb{E}_t^b\big[\, D(t,u;c + b^f(e) - e)\, d\pi_u^f \,\big]$
f	f'	$V_t^f = \int_t^T \mathbb{E}_t^b\Big[\, D(t,u;c^{f'} - b^{f'}(e) + b^f(e))\, d\pi_u^f \,\Big]$

Master Pricing Equations – II

- Foreign market risks are usually quoted in foreign markets.
  $\longrightarrow$ In particular, accrual rates for foreign collateral accounts are quoted in foreign money markets.
- Market quotes of foreign contracts are mainly traded by players which can access foreign money markets without restrictions.
- Thus, an investor, which can fund in a foreign currency only via FX swaps, cannot calibrate foreign risk dynamics to such market quotes.
- We do not have a unique price because of market incompleteness.
  $\longrightarrow$ Pricing techniques developed for incomplete markets should be used to price foreign contracts.

An Example of a Cross-Currency Trade – I



A EUR-based bank borrows USD to a corporate client by hedging market risks both with bilateral and cleared products. Additional hedging in the FX market is required to match the collateral flows.

An Example of a Cross-Currency Trade – II

- We simplify the example by considering only interest-rate swaps of one period. We focus on the USD floating leg.
- We can write the price of the leg within the bilateral contract as

$$V_t^{f,\mathrm{OTC}} := \alpha\, \mathbb{E}_t^b \left[D(t, T_1; b^f(e)) L_{T_0}^f(T_1) \right]$$

while the same leg in the cleared contract is given by

$$\begin{aligned} V_t^{f,\mathrm{CCP}} &:= \alpha\, \mathbb{E}_t^b \left[D(t, T_1; e^f) L_{T_0}^f(T_1) \right] \\ &= V_t^{f,\mathrm{OTC}} + \frac{\alpha}{\chi_t} \int_t^{T_1} du\, \mathbb{E}_t \left[D(t, u; e)\, \chi_u\, V_u^{f,\mathrm{CCP}} \left(e_u^f - b_u^f(e) \right) \right] \end{aligned}$$

where α is the year-fraction, and L^f is the USD Libor rate.

- The second term on the right-hand side represents the additional exposure to FX market risks due to foreign collateralization.

An Example of a Cross-Currency Trade – III

- The additional contribution depends on the spread $s_t^f(e) := e_t^f - b_t^f(e)$.
  $\longrightarrow$ Such spread represents the funding costs due to market dislocations of a EUR-based institution to fund in USD via FX swaps.

$$V_t^{f,\mathrm{CCP}} - V_t^{f,\mathrm{OTC}} = \frac{\alpha}{\chi_t} \int_t^{T_1} du\, \mathbb{E}_t \left[D(t,u;e)\, \chi_u\, s_u^f(e)\, V_u^{f,\mathrm{CCP}} \right]$$

- We do not have enough market quotes to calibrate the dynamics of s_t^f under $\mathbb{Q}^b$, and so under $\mathbb{Q}$.
  $\longrightarrow$ The practical choice of using quotes from the USD money market may lead to mis-price funding costs.
  $\longrightarrow$ We notice that funding initial margins required by the CCP introduces additional funding costs.
- Moreover, cross-currency trades in practice usually span over long maturities and include renotioning. Both these features lead to price uncertainties due to market incompleteness.

Talk Outline

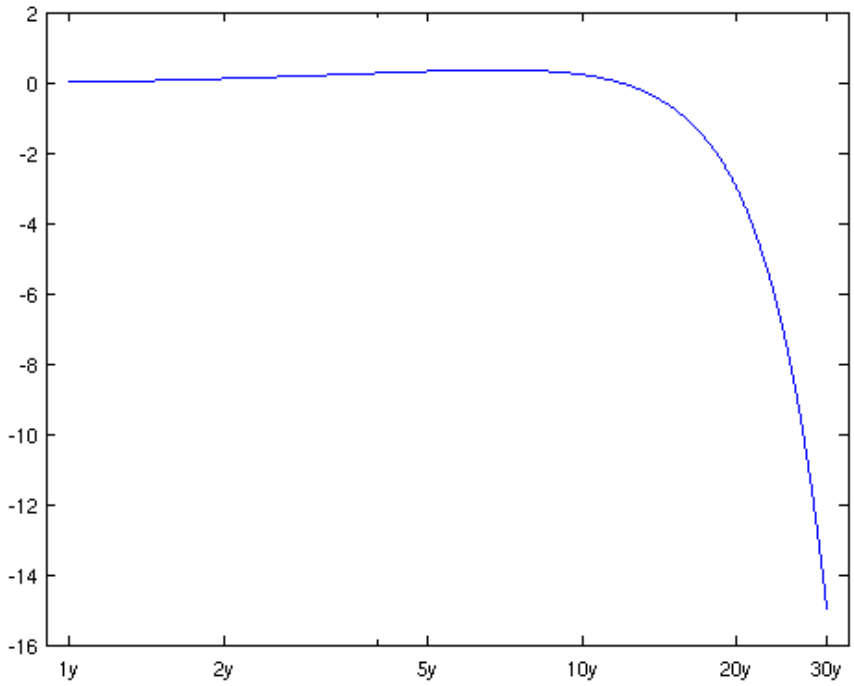
Changing the Funding Currency – I

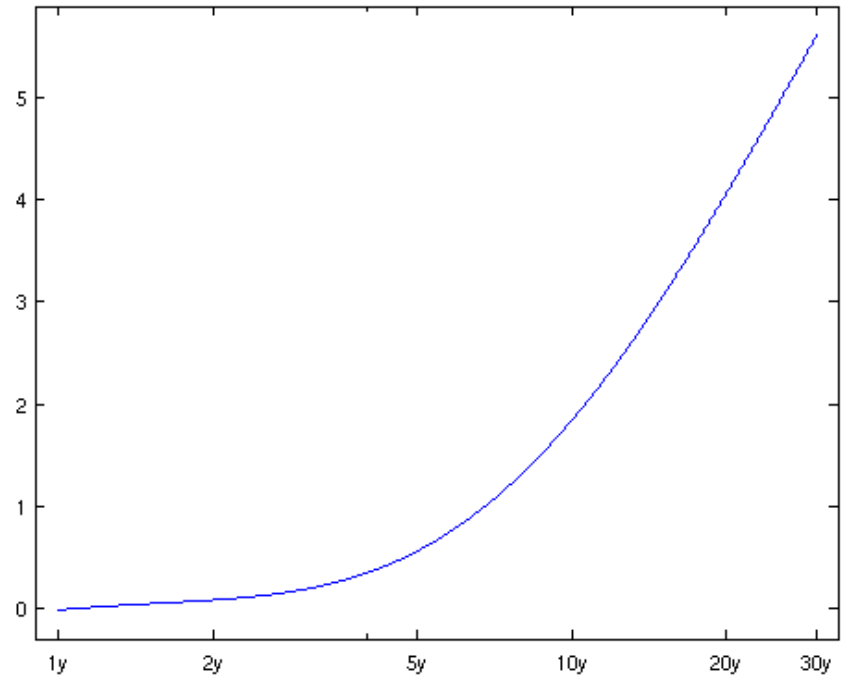
- FX and CCS market quotes are scarce and incomplete (no clear hint of collateralization standard).
  - $\longrightarrow$ Market participants use some approximated procedures.
- A solution usually adopted consists of four curves:
  - $\longrightarrow$ a domestic discounting curve given by the domestic collateral rate;
  - $\longrightarrow$ a domestic forwarding curve obtained from single-currency standard floaters quoted in the domestic money market;
  - $\longrightarrow$ a foreign-currency forwarding curve obtained from single-currency standard floaters quoted in the foreign-currency money market;
  - $\longrightarrow$ an implied foreign currency discounting curve (basis curve) obtained from FX swaps and marked-to-market cross-currency swaps (MtM-CCS).
- All convexity adjustments, correlations and funding costs are implicitly incorporated in the basis curve.
  - $\longrightarrow$ Numerical investigations lead to price uncertainties well below the bid-ask spread.

Changing the Funding Currency – II

- We can size price uncertainties due to market dislocations by comparing quotes of cross-currency products from the point of view of investors with different funding policies.
- As a first example we consider the currency pair EUR/USD.
  - $\longrightarrow$ We bootstrap implied foreign discount curves from MtM-CCS with renotioning on the USD leg.
  - $\longrightarrow$ We price constant-notional cross-currency swaps (CN-CCS), which are not usually quoted by the market.
- We repeat the calculations starting from the same set of market quotes, by adopting first the point of view of a EUR investor, then of a USD investor.

Changing the Funding Currency – III





Left panel: differences in basis points between EUR/USD forward FX rates bootstrapped by a EUR investor and by a USD investor.

Righ panel: differences in basis points between EUR/USD CN-CCS par rates bootstrapped by a EUR investor and by a USD investor.

Conclusions

- In periods of market stress, covered interest parity violations occur as systemic or regional/macro-area effects.
  - $\longrightarrow$ Investors with free access only to a domestic treasury bank account will get foreign currencies through FX and cross-currency swaps.
- Deals with coupons/collateralization in foreign currency are valued taking into account the cross-currency basis
  - $\longrightarrow$ In our approach the foreign risk-neutral measure is not equivalent to domestic one thus leading to pricing equations of incomplete market modelling style – Moreni and Pallavicini (2015).
- Complex dynamical models would be needed, but there too few quotes to calibrate them.
  - $\longrightarrow$ Market practice (so far) is to neglect collateral impact on market quotes and use effective curves...
  - $\longrightarrow$ ...but some brokers are starting to discuss of different quote sets for each collateral choice.

Selected references – I

- Baba, N. and Packer, F. and Nagano, T. (2008)
  The Spillover of Money Market Turbulence to FX Swap and Cross-Currency Swap Markets. *BIS Quarterly Review* 1, 2008.
- Baba, N. and Packer, F. (2009)
  From turmoil to crisis: Dislocations in the FX swap market. *BIS Working Paper* 285, 2009.
- Barkbu, B. B. and Ong, L. L. (2010)
  FX Swaps: Implications for Financial and Economic Stability. *IMF Working Paper* 55, 2010.
- Brigo, D., Capponi, A., Pallavicini, A., Papatheodorou, V. (2011)
  Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypothecation and Netting. *IJTAF* 16 (2), 2013. Extended version available at `ssrn.com`.
- Brigo, D., Morini, M., Pallavicini A. (2013)
  "Counterparty Credit Risk, Collateral and Funding with Pricing Cases for All Asset Classes". Wiley, 2013.
- Brigo, D., Pallavicini, A. (2014)
  CCP Cleared or Bilateral CSA Trades with Initial/Variation Margins Under Credit, Funding and Wrong-Way Risks: A Unified Valuation Approach. *JFE* (2014) 1, (1). Extended version available at `ssrn.com`.
- Coffey, N. and Hrung, W. and Sarkar, A. (2009)
  Capital Constraints, Counterparty Risk, and Deviations from Covered Interest Rate Parity. *FED Working Paper* 393, 2009.
- Covrig, V., Low, B.S. and Melvin, M. (2004)
  A Yen Is Not a Yen: TIBOR/LIBOR and the Determinants of the Japan Premium. *The Journal of Financial and Quantitative Analysis* 39 (1), 2004.
- Duffie, D. (2001)
  Dynamic Asset Pricing Theory. Princeton University Press, 2001.

Selected references – II

- Fries, C.P. (2012)
  Curves and Term Structure Models: Definition, Calibration and Application of Rate Curves and Term Structure Models. Available at ssrn.com.
- Fujii, M., Shimada, Y., Takahashi, A. (2010)
  Collateral Posting and Choice of Collateral Currency. Available at ssrn.com.
- Fujii, M., Takahashi, A. (2015)
  Choice of Collateral Currency Updated: A market model for the benchmark pricing. Working paper on arXiv.org and ssrn.com.
- Giménez, E., Elices, A. and Villani, G. (2014)
  A generalized pricing and hedging framework for multi-currency fixed income desks. Working paper on arXiv.org and ssrn.com.
- Henrard, M. (2014)
  "Interest Rate Modelling in the Multi-Curve Framework". Palgrave Macmillan, 2014.
- McCloud P. (2013)
  Collateral Convexity Complexity. *Risk Magazine* 2, 2013.
- Pallavicini, A. and Brigo, D. (2013)
  Interest-Rate Modelling in Collateralized Markets: multiple curves, credit-liquidity effects, CCPs. Available at ssrn.com.
- Piterbarg, V. (2010)
  Funding beyond discounting: collateral agreements and derivatives pricing. *Risk Magazine* 2, 2010.
- Piterbarg V. (2012)
  Cooking with Collateral. *Risk Magazine* 8, 2012.
- Shabani, M., Stenfors, A. and Toporowski, J. (2016)
  Bank Risk Premia and Abenomics: The Return of the Japan Premium in the Cross-Currency Swap Market *LSE Financial Markets Group Paper Series*, 2016.