EXERCISE 1

DEVIATION = REAL VALUE − PREDICTED VALUE

TIME	METHOD A	ERROR	METHOD B	ERROR
1999	2150	-30	2010	110
2000	2345	-45	2278	22
2001	2800	-75	2650	75

REAL VALUES → 2120 ; 2300 ; 2725

(A) METHOD B PROVIDES THE BEST FORECASTS, BECAUSE THE OBJECTIVE IS TO MITIGATE THE DEVIATIONS. IN THIS CASE, B HAS POSITIVE ERRORS.

(B) MAD → MEAN ABSOLUTE DEVIATION

$$\text{METHOD A} = \frac{\sum |RV - PV|}{n} = 50 \qquad \text{METHOD B} = 63$$

MES → MEAN SQUARED ERROR

$$\text{METHOD A} = \frac{\sum (RV - PV)^2}{n} = 2850 \qquad \text{METHOD B} = 6069.66$$

IN THE EVALUATION, METHOD A STRICTLY PROVIDES BETTER FORECASTS THAN METHOD B ⚠

B2

- MAPE $= \dfrac{\left[\dfrac{(30+45+75)}{7145}\right] \cdot 100}{3}$

$\rightarrow 0.699$ VERY GOOD ACCURACY

- TRACKING SIGNAL

$\rightarrow \dfrac{(30+45+75)}{50} = \boxed{3}$

NO SIGNS OF BIAS

EXERCISE 2

$H_0 = |Z| > Z_{\alpha/2}$ TRENDS

$H_A = |Z| < Z_{\alpha/2}$ NO TRENDS

$Z_{\alpha/2} = 1.96$

DANIEL TEST

TIME	NUMBERS	b	RANK	d	d^2
1999.II	19871	1	18	18	324
1999.III	14134	2	8	6	36
1999.IV	18504	3	18	15	225
2000.I	17903	4	17	13	169
2000.II	15177	5	11	6	36
2000.III	12064	6	4	-2	4
2000.IV	16127	7	14	7	49
2000I	15893	8	12	4	16
2001.II	17578	9	16	7	49
2001.III	11961	10	3	-7	49
2001.IV	15902	11	13	2	4
2002.I	17401	12	15	3	9
2002.II	14257	13	9	-4	16
2002.III	10266	14	2	-12	144
2002.IV	13608	15	7	-8	64
2003.I	14985	16	10	-6	36
2003.II	12923	17	6	-11	121
2003.III	9814	18	1	-17	289
2003.IV	12390	19	5	-14	196
				TOT.	1836

$$\tau = 1 - \frac{6 \cdot 1836}{19 \cdot (19^2 - 1)} \rightarrow \frac{11016}{6840} \rightarrow 1 - 1.610S = \boxed{-0.610S}$$

$$Z = \sqrt{19 - 1} \cdot \tau = -2.5801$$

→ THEREFORE, THERE'S NO TREND

KRUSKAL WALLIS TEST

TIME	NUMBERS	RANK	I	II	III	IV
1999.II	19871	19	/	19		
1999.III	14134	8			8	
1999.IV	18504	18				18
2000.I	17903	17	17			
2000.II	15177	11		11		
2000.III	12064	4			4	
2000.IV	16127	14				14
2001.I	15893	12	12			
2001.II	17578	16		16		
2001.III	11961	3			3	
2001.IV	15902	13				13
2002.I	17401	15	15			
2002.II	14257	9		9		
2002.III	10266	2			2	
2002.IV	13608	7				7
2003.I	14985	10	10			
2003.II	12923	6		6		
2003.III	9914	1			1	
2003.IV	12390	5				5

$R_1 = 45 \rightarrow 2025/3 \rightarrow 675$

$R_2 = 46 \rightarrow 2116/4 \rightarrow 529$

$R_3 = 44 \rightarrow 1936/4 \rightarrow 484$

$R_4 = 33 \rightarrow 1089/4 \rightarrow 272.25$

$R_5 = 22 \rightarrow 484/4 \rightarrow 121$

$\boxed{2081.25}$

$$H = \frac{12}{19(19-1)} \cdot 2081.25 - 3(19+1)$$

$$\vdots$$

$$= 73.02631 - 60$$

$$\vdots$$

$$= \boxed{13.02631}$$

$$\chi^2 = 7.81$$

$$H > \chi^2$$

↳ THE SERIES HAS SEASONALITÉS ⚠

- NAIVE METHOD

TIME	NUMBERS	FORECAST
1999.II	19871	
1999.III	14134	18871
1999.IV	18504	14134
2000.I	17903	18504
2000.II	15177	17903
2000.III	12064	15177
2000.IV	16127	12064
2000I	15883	16127
2001.II	17578	15883
2001.III	11961	17578
2001.IV	15902	11961
2002.I	17401	15902
2002.II	14257	17401
2002.III	10266	14257
2002.IV	13608	10266
2003.I	14985	13608
2003.II	12923	14985
2003.III	9314	12923
2003.IV	12390	9314

- SIMPLE MEAN

$$\bar{X} = \frac{\sum x_i}{n} \rightarrow 280858 / 19 = \boxed{14782}$$

EXERCISE IN CLASS

PRICES	Y	t	RANK	d	d^2
1987.I	141	1	1	0	0
1987.II	162	2	2	0	0
1988.I	180	3	5	2	4
1988.II	164	4	3	-1	1
1989.I	171	5	4	-1	1
1989.II	206	6	7	1	1
1990.I	183	7	6	-1	1
1990.II	207	8	8	0	0
1991.I	218	9	9	0	0
1991.II	229	10	10	0	0

$TOT = 8$

$$\tau = 1 - \frac{6 \cdot 8}{10(10^2 - 1)} \rightarrow 1 - \frac{48}{990} \rightarrow 1 - 0.0545454$$

$$\rightarrow \boxed{0.9454545}$$

$$Z = \sqrt{10 - 1} \cdot \tau = \boxed{2.8363635}$$

THERE'S TRENDS ⚠

$Z_{\alpha/2} = 1.96$

CLASS EXERCISE

$K = 4$ // $\omega = 0.4$

YEAR	Y_T	NAÏVE	MEAN	M.A	SES
1989	150	/	147.5		/
1990	132	150	⋮		150
1991	168	132	⋮		142
1992	125	168	⋮	143.75	152.4
1993	115	125	⋮	135	141.4
1994	110	115	⋮	129.5	130.84
1995	189	110	⋮	134.75	122.5
1996	193	189	⋮	151.75	149.1
1997	205	193	⋮	⋮	166.6
1998	187	193	⋮	⋮	181.9

- WITH YEARLY DATA, YOU CANNOT APPLY

WALLIS TEST

→ KEEP IN MIND THAT THIS TYPE OF TEST IS APPLICABLE JUST WHEN THE TIME SERIES SHOWS SEASONS.

CLASS EXERCISE 2

PRICES	Y_t	t	t^2	$Y \times t$	FORECAST
1987.I	141	1	1	141	147.81
1987.II	162	2	4	324	156.54
1988.I	180	3	9	540	165.27
1988.II	164	4	16	656	174
1989.I	171	5	25	855	182.73
1989.II	206	6	36	1236	191.46
1990.I	193	7	49	1351	200.19
1990.II	207	8	64	1656	208.92
1991.I	213	9	81	1962	217.65
1991.II	229	10	100	2290	226.38
1992.I	225	11			235.11
1992.II	237	12			243.84
$\sum$	1871	55	385	11011	

MSE = 74.92 MAD = 8.5

$$\beta_1 = \frac{11011 - 187.1 \cdot 55}{385 - 5.5 \cdot 55} = 720.5/82.5 = 8.73$$

$\bar{Y} = 1871/10 = 187.1$

$\bar{t} = 55/10 = 5.5$

$\beta_0 = 187.1 - 8.73 \cdot 5.5 = 139.08$

$Y = \beta_1 x + \beta_0$

K=2

PRICES	Y_t	MA	MA'	T	B_1	$\hat{Y}$
1987.I	141	-	-	-	-	-
1987.II	162	151.5	-	-	-	-
1988.I	180	171	161.25	180.8	19.5	-
1988.II	164	172	171.5	172.5	1	200.2
1989.I	171	167.5	169.75	165.25	-5	173.5
1989.II	206	188.5	178	199	21	160.25
1990.I	193	199.5	194	205	11	220
1990.II	207	200	199.75	200.25	6.5	216
1991.I	218	212.5	206.25	218.75	12.5	200.75
1991.II	229	223.5	218	229	11	231.25
1992.I	225					240
1992.II	237					251

MAD = 14.5

MSE = 210.5

HOLT EXPONENTIAL SMOOTHING FORMULAS

$$\hat{T}_T = \omega \cdot (Y_T) + (1-\omega) \cdot Y_{T-1}$$

$$B_1 = \gamma \cdot (\hat{T}_T - \hat{T}_{T-1}) + (1-\gamma) \cdot B_{1(T-1)}$$

$0.7 \cdot (50) + (1-0.7) \cdot 48.7$

$35 + 14.61 = \boxed{48.61}$ $\hat{T} = 2007$

$0.2\,(48.61 - 47.75) + (0.8) \cdot 0.36$

$0.372 + 0.768 = 1.14$

2008 = $48.61 + 1.14 = 50.75$

2009 = $48.61 + 1.14 \times 2 = 51.89$

2010 = $48.61 + 1.14 \times 3 = 53.03$

FORECASTING LEAGUE EX 1

YEAR	SALES	t	RANK	d	d^2
2012	120	1	4	3	9
2013	115	2	1	-1	1
2014	118	3	2	-1	1
2015	121	4	5	1	1
2016	123	5	6	1	1
2017	119	6	3	-3	9
					TOTAL. 22

10 DANIEL TEST

$$\tau = 1 - \frac{6 \cdot 22}{6 \cdot (6^2 - 1)} \rightarrow 1 - \frac{132}{6 \cdot 35}$$

$$1 - 0.62857$$

$$0.37143$$

$$|Z| = \sqrt{6-1} \cdot \tau$$

$$0.83$$

$Z_{\alpha/2} = 1.96$

SINCE $|Z| < Z_{\alpha/2}$

I CANNOT REJECT THE NULL HYPO.

THEREFORE, THERE'S NO TREND.

11 WALLIS TEST

- I CANNOT COMPUTE SUCH TEST BECAUSE, THE TIME SERIES HAS NO PERIODS IN A YEAR - SPAN. THEREFORE, THERE'S NO SEASONALITY

- FOR WHAT I HAVE SEEN THE FORECAST FITS IN THE 1ST TIME SERIES MODEL. WITH NEITHER TREND NOR SEASONALITY.

12 3 YEARS MOVING AVERAGES

YEAR	SALES	M.A	e	e^2	|e|
2012	120	/	/	/	/
2013	115	/	/	/	/
2014	118	/	/	/	/
2015	121	117.66	3.34	11.15	3.34
2016	123	118	5	25	5
2017	113	118	1	1	1

MSE = 11.15 MAD = 3.34

MSE = 13 MAD = 3
2016-2017 2016-2017

13 5 YEARS MOVING AVERAGE

YEAR	SALES	M.A	e	e^2	|e|
2012	120	/	/	/	/
2013	115	/	/	/	/
2014	118	/	/	/	/
2015	121				
2016	123				
2017	119				

- HOW CAN I COMPUTE SUCH AVERAGE IF ON MY SAMPLE PERIOD I HAVE JUST 4 YEARS.

14 $2016 = 122$ $\omega = 0.3$

YEAR	SALES	FORECAST	e	e^2	$\|e\|$
2012	120	/	/	/	/
2013	115	120	-5	25	5
2014	118	118.5	-0.5	0.25	0.5
2015	121	118.35	2.65	7.02	2.65
2016	122	119.14	2.86	8.18	2.86
2017	119	119.14	-0.14	0.0196	0.14
2018	?	119.14			

MSE = 10.75 MAD = 2.71

MSE_{2018} = MAD_{2018} =