

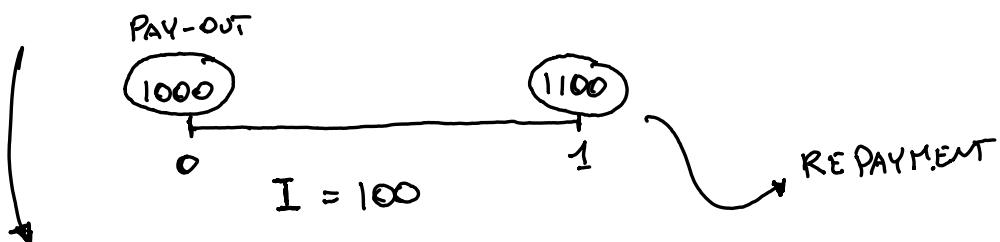
FINANCIAL TRANSACTION

AGREEMENT BETWEEN 2 PARTIES EXCHANGING MONEY IN 2 DIFFERENT TIMES.

INTEREST

CHARGE UPON THE AMOUNT

BORROWER
LENDER



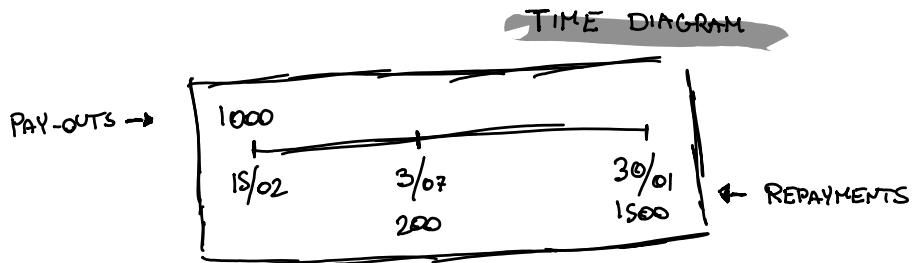
WHY CHARGING
INTEREST ?

TIME PREFERENCE THEORY

INHABITANTS PREFER TO HAVE CASH NOW, INSTEAD OF WAITING, BECAUSE THEY ARE PETULANT

TRANSACTION ELEMENTS

- PERSONAL → ACTIVE PARTY → LENDER
PASSIVE PARTY → BORROWER
- FORMAL → DEALS
 - (1) WHEN ?
 - (2) HOW ?
 - (3) CAUTION ?
- MATERIAL → THE AMOUNT OF MONEY INVOLVED.



- { (1) COMMON OPERATION
(2) COMPLEX OPERATION

BASICALLY, THE DIFFERENCE IS JUST // THE AMOUNTS OF OPERATION ON A TIME DIAGRAM.

FINANCIAL CAPITAL

→ TIME EXPRESSED
IN YEARS

(C, T)

USING AN ACCUMULATION FUNCTION

$$f(T, T') = 1 + 0,03(T' - T)$$

- LET'S SUPPOSE YOU LEND 1000 \$ IN $T = 2$
WHAT WILL BE THE AMOUNT AT $T' = 5$?

$$f(T, T') = \frac{c'}{c}$$



$$c' = f(T, T') c$$



$$= 1000 \cdot (1 + 0,03(5 - 2))$$

$$= 1000 \cdot (1,09)$$

$$= \boxed{1080}$$

- THE FUNCTION DEPENDS ONLY ON THE
 LENGTH T

INTEREST

$$C' - C = IT$$

KEEP IN MIND THAT THIS FORMULA DOESN'T GET INTO ACCOUNT THE LENGTH. !

INTEREST RATE

$$(1) = \frac{C' - C}{C}$$



UNIT PRICE

THIS TOOL IS
MORE EFFECTIVE
THAN THE PREVIOUS
ONE.

NOMINAL INTEREST RATE

$$i(T, T') = \frac{C' - C}{C \cdot t}$$

LENGTH OF
A TRANSACTION !
 $(T' - T)$

EXERCISE 1

$$C = 2500$$

$$C' = 2750$$

$$t = 2$$

$$I = 250$$

$$\text{INTEREST RATE} = 0,1\%$$

$$\text{NOMINAL IR} = 0,05\%$$

FORMULAS
ARE IN THE
PREVIOUS PAGE

FINANCIAL REGIMES

① SIMPLE INTEREST

② COMPOUND INTEREST WITH CONSTANT RATES

③ COMPOUND INTEREST WITH DIFFERENT RATES

④ DISCOUNT INTEREST

SIMPLE INTEREST



$$C' = C \cdot (1 + i \cdot t)$$

① $IT = C \cdot i \cdot t$

② INTEREST RATE = $i \cdot t$

③ NOMINAL INTEREST RATE = i

EXERCISE 3

$$C = 2200 \quad C' = 2265 \quad t = 5$$

$$2265 = 2200 \cdot (1 + 5i)$$

$$0,02954 = 5i \rightarrow i = 0,0058\%$$

EXERCISE 4

$$C = 1500 \quad C' = 1620 \quad i = 0,06$$

$$1620 = 1500 \cdot (1 + 0,06t)$$

$$1,08 = 1 + 0,06t$$

$$0,08 = 0,06t \rightarrow t = 1\frac{1}{3}$$

Exercise 6

$$C + SO = C \cdot (1 + 0.04 \cdot 6)$$

$$C + SO = 1.24 \cdot C$$

$$1.24C - C = SO \rightarrow C(1.24 - 1) = SO$$

$$0.24C = SO \rightarrow C = 208.33$$

Exercise 7

$$t = ? \quad i = 4.25\% \quad 2C = ?$$

$$100 = 50 \cdot (1 + 0.0425 \cdot t)$$

$$2 = 1 + 0.0425 \cdot t$$

$$t = \frac{1}{0.0425} \rightarrow t = 23\frac{1}{2} \text{ years}$$

EXERCISE 9

$$1) \quad C' = C(1 + i t)$$

$$2480 = 2400(1 + it)$$

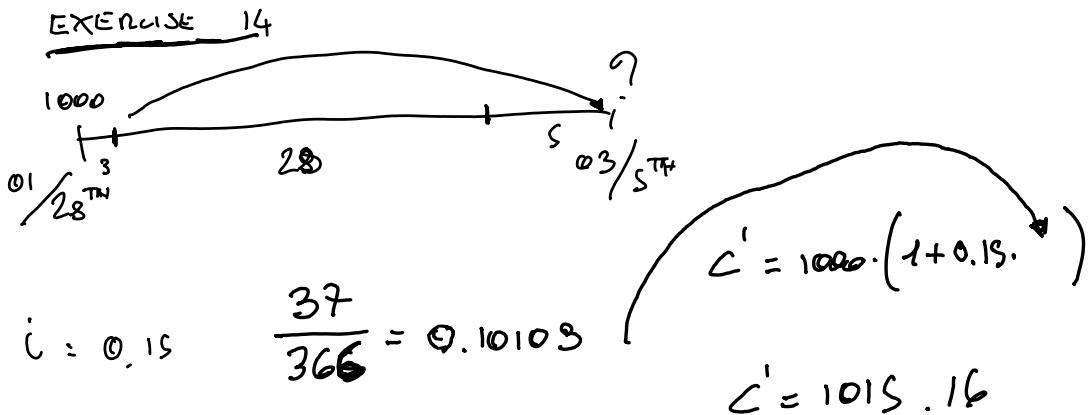
$$2) \quad C' = 1000(1 + i 2t)$$

$$it = \frac{2480}{2400} - 1$$

$$it = 0,033$$

$$C' = 1000(1 + 2 \cdot 0,033)$$

$$C' = 1000 \cdot 1,066 \rightarrow C' = 1066,66$$



EXAMPLE 16

$$C = 2000 \quad C' = ? \quad i = 0,08$$

1
07/17th 08/10th

a) Time \rightarrow $\frac{13 + 31 + 31 + 10}{365}$

↓

JUNE = 30
JULY = 31
AUGUST = 28
SEPTEMBER = 10

$$t = 0.23287671 \rightarrow C' = 2000 \cdot (1 + 0,08 \cdot t)$$

$$C' = 2037.26$$

b) $t = \frac{13 + 30 + 30 + 10}{360} \rightarrow 0.23055556$

$$C' = 2036.88$$

c) $t = \frac{13 + 31 + 31 + 10}{360} \rightarrow 0.2361111$

$$C' = 2037.77$$

EXAMPLE 17

$$C = 3500 \quad i = 0.08$$

09/17th 01/20th

SEPTEMBER	30
OCT	31
NOV	30
DEC	31
JAN	20

a) $t = \frac{105}{360} + \frac{20}{365}$



$$0.34213386$$

$$C^1 = 3595.797$$

b) $t = \frac{13 + 30 + 30 + 30 + 20}{360} \rightarrow 0.3416667$

$$C^1 = 3595.66$$

c) $t = \frac{13 + 31 + 30 + 31 + 20}{360} \rightarrow 0.34722222$

$$C^1 = 3597.22$$

COMPOUND INTEREST

- THE INTEREST EARNED AT THE END OF ONE PERIOD IS AUTOMATICALLY REINVESTED TO EARN AN ADDITIONAL INTEREST.

ACCUMULATION
FUNCTION

$$\rightarrow f(T, T') = \frac{C'}{C}$$

$$\frac{(1 + i_p)^h}{(1 + i_p)} = (1 + i_p)^h$$

- {
- ① INTEREST $\rightarrow C \cdot i \cdot p$
 - ② INTEREST RATE $\rightarrow i \cdot p$
 - ③ NOMINAL INTEREST RATE $\rightarrow i$
- }

$m \rightarrow \frac{1}{p} =$ FREQUENCY OF CAPITALIZATION IN 1 YEAR.

EXAMPLE 18

- $P = \frac{1}{2}$ $T = \emptyset$ $T' = 4$

$$m = \frac{1}{\frac{1}{2}} = 2 \quad t = 4 \quad h = m \cdot t = 8$$

- $P = \frac{1}{4}$ $T = \emptyset$ $T' = 2$

$$m = 4 \quad t = 2 \quad h = 8$$

- $P = \frac{1}{3}$ $T = \emptyset$ $T' = 5$

$$m = 3 \quad t = 5 \quad h = 15$$

$$m = \frac{1}{P} \quad i = I \cdot m \quad h = m \cdot t$$

$$I = \frac{i}{m}$$

SO, IN COMPOUND INTEREST THE
FUNCTION IS

$$C' = C(1 + I)^h$$

EXERCISE 20

$$C = 1800 \quad C' = 1923.78 \quad I = 0.03$$

$$1923.78 = 1800(1 + 0.03)^h$$

$$\frac{1923.78}{1800} = 1.03^h \rightarrow \ln 1.03^h = \ln \frac{1923.78}{1800}$$

$$h = \frac{\ln 1.03}{\ln \frac{1923.78}{1800}}$$

M = 2.25 years

EXERCISE 21

$$C' = 32.168 \quad t = 18 \quad I = 6.2\% \quad C = ?$$

$$C' = C \left(1 + 0.06I\right)^t$$

$$\hookrightarrow \frac{32.168}{1.062^{18}} = C$$

$$\hookrightarrow 14716.527$$

EXERCISE 22

$$2C = C \cdot (1+I)^{10}$$

$$2 = (1+I)^{10} \rightarrow (2) = \left((1+I)^{10} \right)^{\frac{1}{10}}$$

$$\hookrightarrow \sqrt[10]{2} = 1 + I$$

$$1.07177 - 1 = I$$

$$0.07177 = I$$

EXERCISE 23

$$C' = 1000 \cdot (1+0.05)^4$$

$$\hookrightarrow 1215.50$$

$$I = 215.5$$

EXERCISE 25)

$$C = 1750 \quad t = 6\frac{1}{2}$$

- MONTH 0.01

$$C' = 1750 \cdot (1 + 0.01)^{78} \rightarrow C' = 3802.81$$

- SEASONS 0.02

$$C' = 1750 \cdot (1 + 0.02)^{24} \rightarrow C' = 2928.48$$

- BIANNUAL 0.01

$$C' = 1750 \cdot (1 + 0.01)^{3/4} \rightarrow C' = 1807.51$$

- ANNUAL 0.01

$$C' = 1750 \cdot (1 + 0.01)^{6\frac{1}{2}} \rightarrow C' = 1866.92$$

- TRIANNUAL / FOUR MONTH

$$C' = 1750 \cdot (1 + 0.01)^{\frac{11}{6}} \rightarrow C' = 1782.21$$

EXERCISE 26

$$C = 3000 \quad t = 8 \quad i = 0.05$$

- ANNUAL

$$\hookrightarrow 3000 \cdot (1.05)^8 \rightarrow C' = 4432.26$$

- QUARTERLY $\rightarrow 0.05/4 \rightarrow 0.0125$

$$\hookrightarrow 3000 \cdot (1.0125)^{32} \rightarrow C' = 4464.39$$

- MONTHLY $\rightarrow 0.05/12 \rightarrow 0.00416$

$$\hookrightarrow 3000 \cdot (1.00416)^{96} \rightarrow C' = 4468.9$$

DISCOUNT FUNCTION

THAT'S TO SAY $\rightarrow C' = C \cdot (1+i)^t$

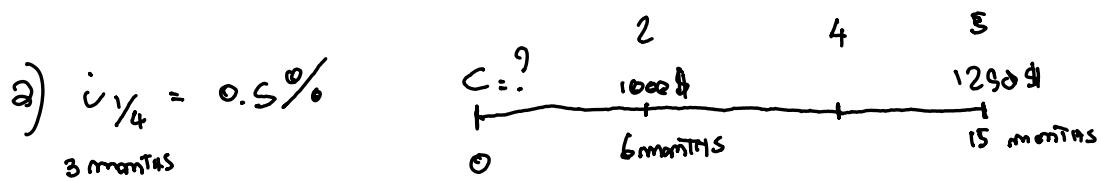
$$\text{so} \Rightarrow C = \frac{C'}{(1+i)^t}$$

THEREFORE , $\textcircled{25} = (1+i)^{-t}$
DISCOUNT
FUNCTION

QUESTION 6

$$PV = ? \quad 1000\$ = 6 \text{ months}$$

$$1250 \$ = 15 \text{ months}$$



$$\frac{1250}{1.005^5} + \frac{1000}{1.005^2} = \underline{2209.28}$$

b) $i_1 = 1\%$

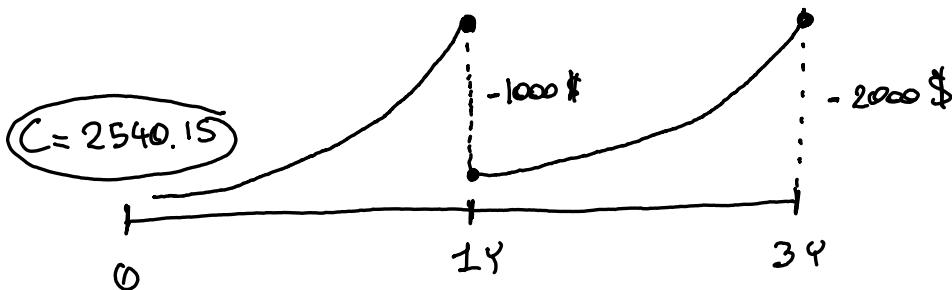
$$\frac{1250}{1.01^{5/4}} + \frac{1000}{1.01^{0.5}} = \underline{2229.58}$$

EXERCISES 31

$$C = ? \quad 1^{\text{st}} \text{ year} = 1000 \$ \quad 3^{\text{rd}} \text{ year} = 2000 \$$$

$$i_r = 7.5\%$$

$$C = \frac{1000}{1.075} + \frac{2000}{1.075^3} = 2540.15 \$$$



$$\left[2540.15 \cdot (1.075)^1 - 1000 \right] \cdot (1 + 0.075)^3 - 2000 = 0$$

EXERCISE 32

$$C = 500 \text{ } \text{B} \rightarrow C = 4000 \text{ } \text{B} \text{ in 30 years}$$

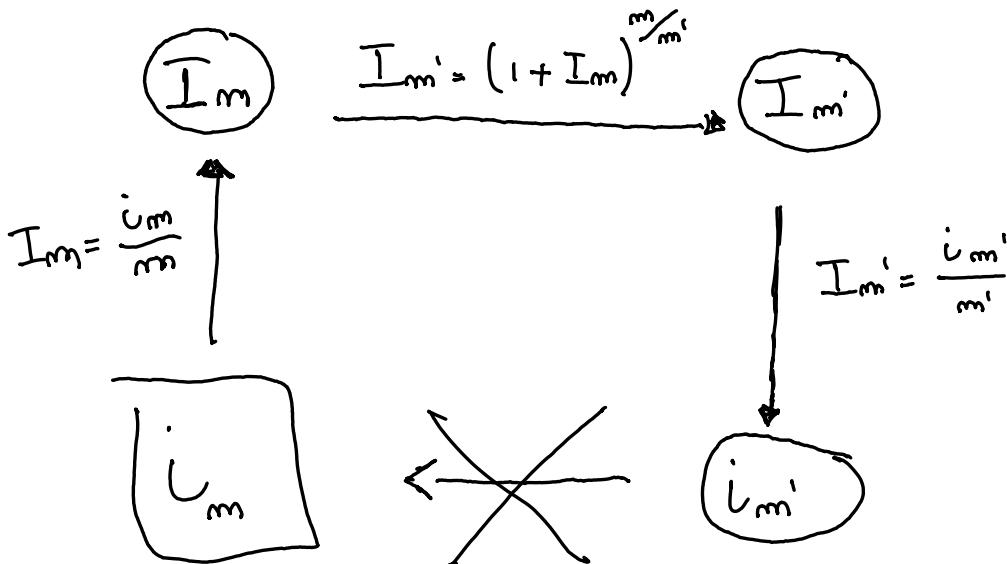
? FIND PV OF 3 10.000 \$ IN 20, 40, 60 years

$$4000 = 500 \cdot (1 + i)^{30}$$

$$8^{100} = (1 + i)^{30} \rightarrow i = 0.0717$$

so,

$$C = \frac{10.000}{1.0717^{60}} + \frac{10000}{1.0717^{40}} + \frac{10000}{1.0717^{20}} = \underline{3287.04}$$



EXERCISE 37

a)

$$i_4 = 8\% \rightarrow I_4 = \frac{i_4}{4} \rightarrow I_4 = \frac{0.08}{4}$$

$I_1 = ?$

\downarrow

$I_1 = (1 + 0.02)^4 - 1 = \underline{\underline{0.08243}}$

b) $i_2 = ? \quad I_1 = 0.08$

$$I_2 = (1 + I_1)^{\frac{1}{2}} - 1 = (1 + 0.08)^{\frac{1}{2}} - 1 =$$

$i_2 = 2 \cdot 0.03323$

\downarrow

$\underline{\underline{0.06646}}$

0.03323

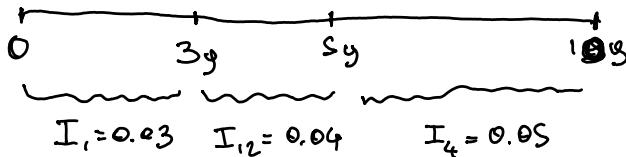
c) $i_2 = 0.08 \quad i_4 = ? \rightarrow I_2 = \frac{i_2}{2} = 0.04$

$$I_4 = (1 + 0.04)^{\frac{1}{4}} - 1 \rightarrow 0.019803$$

$\underline{\underline{i_4 = 0.0792156}}$

EXERCISE 45

$$C = \$26 \text{ #}$$

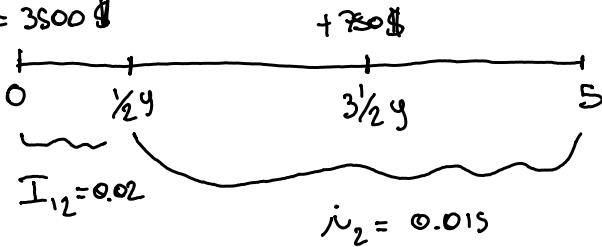


$$C = 26 \cdot (1 + 0.03)^3 \cdot (1 + 0.04)^{24} \cdot (1 + 0.05)^{20}$$

→ $\boxed{6138.72 \text{ $}}$

EXAMPLE 46

$$C = \$500 \text{ #}$$



$$\downarrow$$

$$I_2 = \frac{0.015}{2} = 0.0075$$

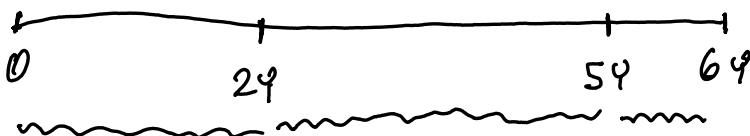
$$AV = 500 \cdot (1 + 0.02)^6 \cdot (1 + 0.0075)^9 + 750 \cdot (1 + 0.0075)^3$$

→ $\boxed{4982.74 \text{ $}}$

EXERCISE 47

$$C = 46000 \text{ \$}$$

$$D = 3000 \text{ \$}$$



$$I_1 = 4\%$$

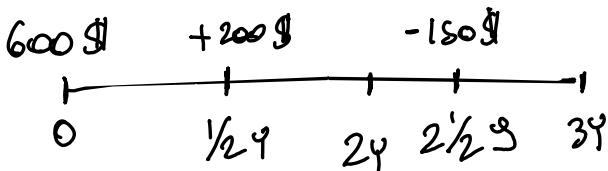
$$I_4 = 2\%$$

$$I_{12} = 1\%$$

$$C' = \left[46000 \cdot (1.04)^2 + 3000 \right] \cdot (1.02)^{12} \cdot (1.01)^{12}$$

$$\hookrightarrow 11387.48 \text{ \$}$$

QUESTION 9



$$I_2 = 1\% \quad i_{1,2} = 0.5\%$$

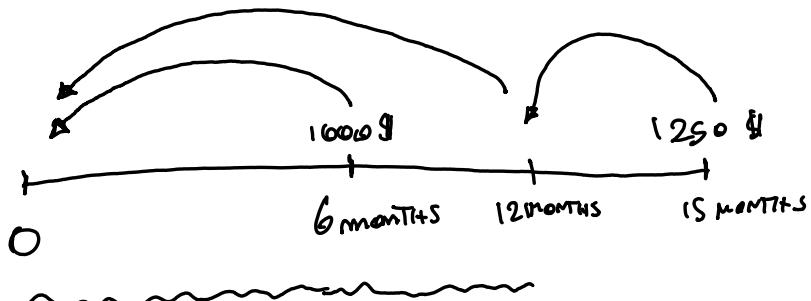
$$I_{1,2} = \frac{0.005}{12} = 0.00041\bar{6}$$

$$C = 600 \cdot (1 + 0.01)^4 \cdot (1 + 0.00041\bar{6})^{12} + 200 \cdot (1 + 0.01)^3 \cdot (1 + 0.00041\bar{6})^2 - 150 (1 + 0.00041\bar{6})^6$$

↳ 684.2 \$

QUESTION 10 |

b)



$$i_4 = 0.01$$



$$I_4 = 0.0025$$

$$I_1 = 0.03$$

$$PV = 1000 \cdot (1.01)^{-2} + 1250 \cdot (1.03) \cdot (1.0025)^{-4}$$

↳ 223,48 \$

EXERCISE 50

$$(1+i)^3 = (1,08) \cdot (1,07) \cdot (1,06)$$

$$(1+i)^3 = 1,08 \cdot 1,07 \cdot 1,06$$

$$\hookrightarrow i = \underline{0,06996}$$

EXERCISE 53

$$C = 2500 \cdot \left(1 - 0,03 \cdot \frac{7}{12}\right)$$

$$C = \underline{2518,2}$$

EXERCISE 54

$$C' = \frac{1000}{1 - 0,05 \cdot \frac{5}{12}} = \underline{1008,4}$$

EXERCISE 55

$$725 = 775 \cdot \left(1 - d \cdot \frac{1}{4}\right)$$

$$725 = 775 - 193,75 d \quad \boxed{d = 0,26}$$

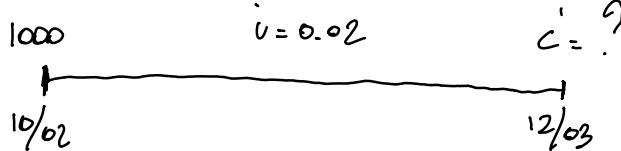
EXERCISES 59

$$C = 1040 \cdot \left(1.025 \cdot \frac{1}{2}\right) + 1560 \cdot \left(1.015 \cdot \frac{3}{4}\right) - 0.002 \cdot 2600$$

↳ 2569,45 \$

PRACTICE TEST

①



$$a) \cdot \text{IF } \frac{30}{360} \rightarrow t = \frac{61}{360}$$

$$\text{OCTOB.} = 31$$

$$\text{NOVEM} = 30$$

$$\text{DECEM} = 3$$

$$\hookrightarrow c' = 1000 \left(1 + 0.02 \cdot \frac{61}{360} \right)$$

$$\hookrightarrow \underline{\underline{c' = 1003.388}}$$

$$\cdot \text{IF } \frac{\text{ACT}}{\text{ACT}} \rightarrow t = \frac{62}{365}$$

$$\hookrightarrow c' = 1000 \cdot \left(1 + 0.02 \cdot \frac{62}{365} \right) \rightarrow \underline{\underline{c' = 1003.387}}$$

$$\cdot \text{IF } \frac{\text{ACT}}{360} \rightarrow t = \frac{62}{360}$$

$$\hookrightarrow c' = 1000 \cdot \left(1 + 0.02 \cdot \frac{62}{360} \right) \rightarrow \underline{\underline{c' = 1003.44}}$$

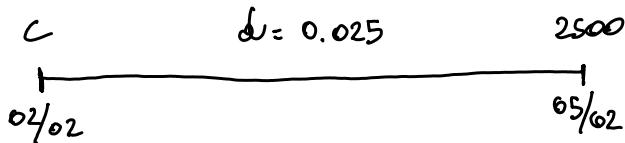
$$\cdot \text{IF } \frac{\text{ACT}}{365} = \text{IF } \frac{\text{ACT}}{\text{ACT}} = \underline{\underline{c' = 1003.387}}$$

b)

$$1.005 = 1000 \left(1 + i \cdot \frac{62}{365} \right)$$

$$\underline{\underline{i = 0.02943}}$$

(2)



$$\frac{ACT}{360} \rightarrow \frac{27+31+30+2}{360} = \frac{90}{360} = \frac{1}{4}$$

a) $C = 2500 \left(1 - 0.025 \cdot \frac{1}{4}\right)$

$$\hookrightarrow \underline{C = 2484.375}$$

b) $D\bar{T} = 2500 - 2484.375 = \underline{15,625}$

$$D(T, T') = 15,625 / 2500 = \underline{0.00625}$$

$$d(T, T') = 0.00625 / t = \underline{0.025}$$

c) $i = \frac{d}{1-d \cdot t} = \frac{0.025}{1-0.00625} = \underline{i = 0.025157}$

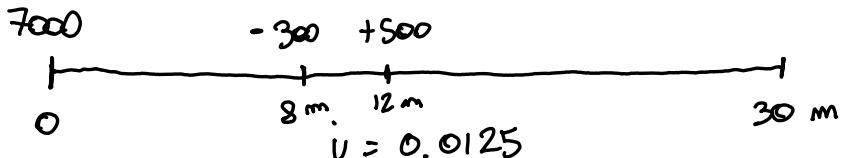
d) $C = 2500 \cdot \left(1 - 0.025 \cdot \frac{1}{4}\right) - 2500 \cdot 0.005$

$$\hookrightarrow \underline{C = 2471.875}$$

$$2500 = 2471.875 \left(1 + i \cdot \frac{1}{4}\right)$$

$$\hookrightarrow \underline{i = 0.04551}$$

(3)



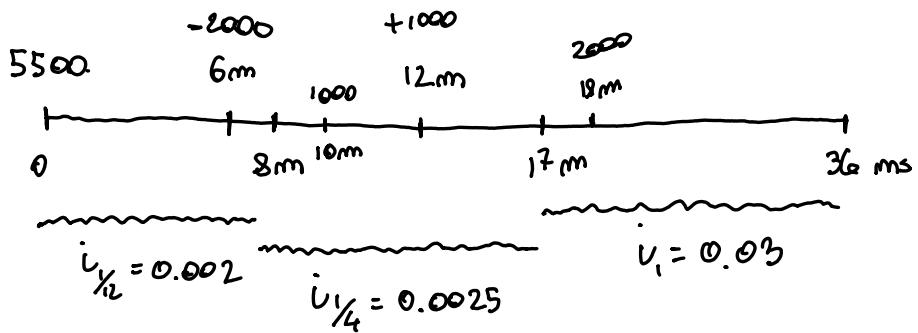
11

$$I = 0.001$$

$$C' = 7000 \cdot (1.001)^{30} - 300 \cdot (1.001)^{22} + 500 \cdot (1.001)^{18}$$

$$\hookrightarrow \underline{c' = 7415.48}$$

④



a) ④ $c' = 5500 \left(1.002\right)^8 \cdot \left(1.0025\right)^3 \cdot \left(1.03\right)^{18/12}$

$\hookrightarrow c' = 5900.42$

② $5900.42 = 5500 \left(1+i\right)^3$

$\hookrightarrow i = 0.0257$

b) ① $c' = 5500 \left(1.002\right)^8 \cdot \left(1.0025\right)^3 \cdot \left(1.03\right)^{18/12} -$

 $2000 \left(1.002\right)^2 \cdot \left(1.0025\right)^3 \cdot \left(1.03\right)^{18/12} +$
 $1000 \left(1.0025\right)^{5/3} \cdot \left(1.03\right)^{18/12}$

② $c' = c \left(1+i\right)^3$

c) $\frac{2000}{\left(1.03\right)^{12} \cdot \left(1.0025\right)^3} \cdot \left(1.002\right)^8 + \frac{1000}{\left(1.0025\right)^{18/3} \cdot \left(1.002\right)^8} -$

(5)



a) $i\% ?$

$$17980 = 9800(1+i)^3 + 3300(1+i)^2 + 4500(1+i)^{1/3}$$

ON



SPARE TIME PRACTICE



$$2750 = 2500 (1 + i \cdot 2)$$

$$\hookrightarrow i = 0.05$$

$$\text{INTEREST} = 2750 - 2500 = 250$$

$$I = 250 / 2500 = 0.1$$

$$i = 0.05$$

(2)

$$C' = 3500 (1 + 0.02 \cdot 3)$$

$$\hookrightarrow C' = 3710$$

(3)

$$2265 = 2200 (1 + i \cdot 5)$$

$$\hookrightarrow i = 0.0055$$

$$\textcircled{4} \quad 1620 = 1500 (1 + 0.06 \cdot t)$$

$$\hookrightarrow t = 1. \bar{33} \rightarrow \frac{1}{\text{YEAR}} \text{ AND } 4 \text{ MONTHS}$$

$$\textcircled{5} \quad 220 = 200 \left(1 + i \cdot 3\frac{1}{2}\right)$$

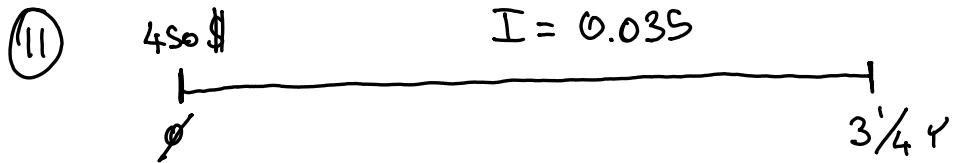
$$\hookrightarrow i = 0.028$$

$$\textcircled{6} \quad C + 50 = C (1 + 0.04 \cdot 6)$$

$$50 = 0.24 C \rightarrow C = 208. \bar{33}$$

$$\textcircled{7} \quad 2C = C (1 + 0.0425 \cdot t)$$

$$2 = 1 + 0.0425t \rightarrow t = \underline{\underline{23.52}}$$



$$C' = 450 (1 + 0.035 \cdot 13/4)$$

$$\hookrightarrow C' = 501, 18$$

$$\text{INTEREST} \rightarrow 501, 18 - 450 = \underline{\underline{51, 18}}$$

$$I \rightarrow 51, 18 / 450 = \underline{\underline{0.11371}}$$

$$i \rightarrow 0,035$$

ANNUITY

→ SAME CONCEPT
OF RENTS 

- TEMPORAL ANNUITY :

↳ THERE'S A DEADLINE.

- PERPETUITY :

↳ ENDLESS

- CONSTANT ANNUITY :

↳ TERMS ARE EQUAL

- NON-LEVEL ANNUITY :

↳ TERMS ARE NOT EQUAL

- IMMEDIATE :

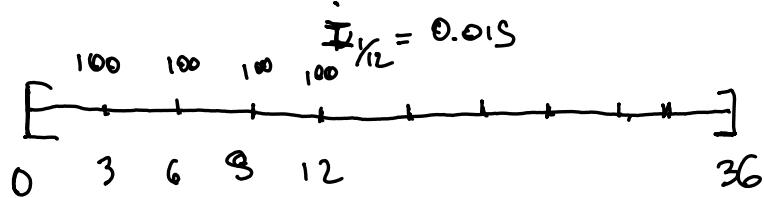
↳ AT THE BEGINNING

- DUCE :

↳ AT THE END

$$V_0 = C \cdot \frac{1 - (1 + I)^{-n}}{I}$$

EXERCISE 7)

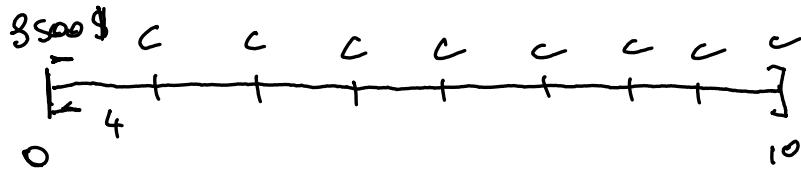


$$I_{1/4} = 0.045$$

$$V_0 = 100 \cdot \frac{1 - (1 + 0.045)^{-12}}{0.045}$$

$\hookrightarrow \$11.86$

EXERCISE 8)



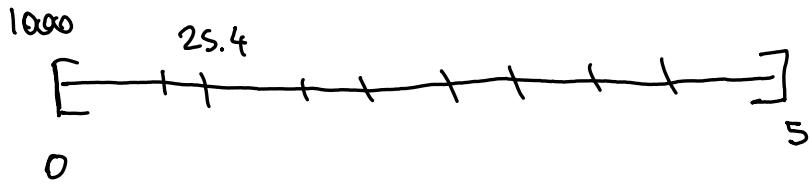
$$I_{1/4} = 0.025$$

$$\hookrightarrow I_1 = (1 + 0.025)^4 - 1$$

$\hookrightarrow 0.1038$

$$8500 = C \frac{1 - (1 + I)^{-10}}{I}$$

EXERCISE 9



$$I_{12} = ?$$

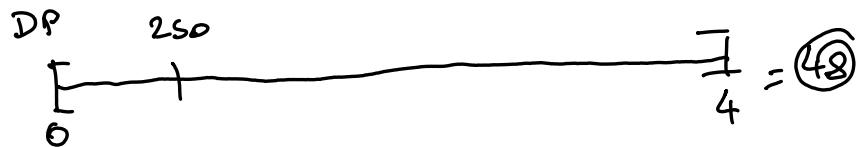
$$1000 = 25.4 \cdot \frac{1 - (1 + I_{12})^{-60}}{I_{12}}$$

$$\rightarrow I_{12} = 0.015$$

EXERCISE 10

NEED IN MIND THAT
THE PRICE OF THE CAR
IS $P_{car} = DP + V_0$

EXERCISE 12



$$U_{12} = 0.18$$

$$\downarrow \\ 0.015$$

$$P_{can} = DP + V_0$$

$$10000 = DP + (V_0)$$

$$\hookrightarrow 250 \cdot \frac{1 - (1.015)^{-48}}{0.015}$$

$$V_0 = 8510.64 \$$$

$$DP = 1489.36 \$$$

EXERCISE 15



a) $I_1 = 0.03$

$$\hookrightarrow I_{1,2} = (1 + 0.03)^{I_{1,2}} - 1$$

$$\hookrightarrow I_{1,2} = 0.002466$$

$$V_0 = 167,127^{'44}$$

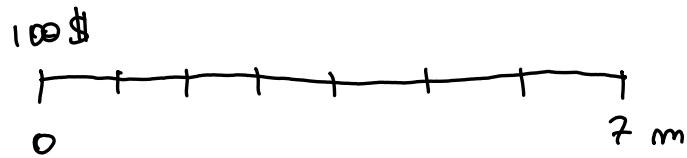
b) $i_{1,2} = 0.03 \rightarrow I_{1,2} = 0.0025$

$$V_0 = 166\ 957^{'07}$$

a.) $V_f = 167,127^{'44} \left(1 + 0.002466\right)^{60}$

b.) $V_f = 166\ 957^{'07} \left(1 + 0.0025\right)^{60}$

PROBLEM



$$i_{12} = 0.12$$

$$\hookrightarrow I_{12} = 0.01$$

→ PV

$$\hookrightarrow 100 \cdot \frac{1 - (1 + 0.01)^{-7}}{0.01} \cdot (1 + 0.01)$$

$$\hookrightarrow 679.55 \text{ \$}$$

→ AV

$$\hookrightarrow 679.55 \cdot (1.01)^7$$

$$\hookrightarrow 728.57 \text{ \$}$$

EXERCISE 24

$$I_r = 0.08 \quad C = 1000$$

$$\therefore V_g = \left(1000 \cdot \frac{1 - (1 + 0.08)^{-25}}{0.08} \right) \cdot (1.08) \cdot (1.08)^{25}$$

↳ 78.954'41 \$

EXERCISE 25

$$78.954'4 = C \cdot \frac{1 - (1 + 0.08)^{-15}}{0.08}$$

↳ C = 9224'2 \$

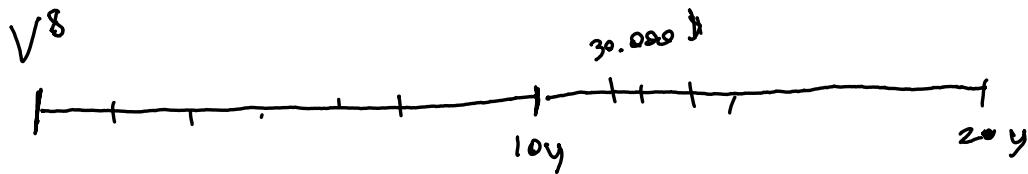
REFERRED ANNUITY

$$V_0 = V_0 \cdot (1 + I)^{-n}$$

↓

$$C \cdot \frac{1 - (1 + I)^{-n}}{I}$$

QUESTION 7



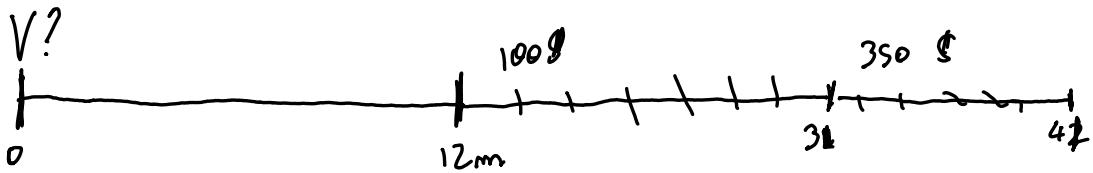
$$i = 0.03$$

$$\hookrightarrow 0.03/12 = 0.0025 \rightarrow (1 + 0.0025)^{12} = \underline{\underline{0.03041}}$$

$$V^8 = 25000 \cdot \frac{1 - (1 + 0.03041)^{-10}}{0.03041} +$$

$$30000 \cdot \frac{1 - (1 + 0.03041)^{-10}}{0.03041} \cdot (1 + 0.03041) \cdot (1 + 0.03041)^{-9}$$

• QUESTION 8



$$t = 3\frac{1}{2} \text{ years}$$

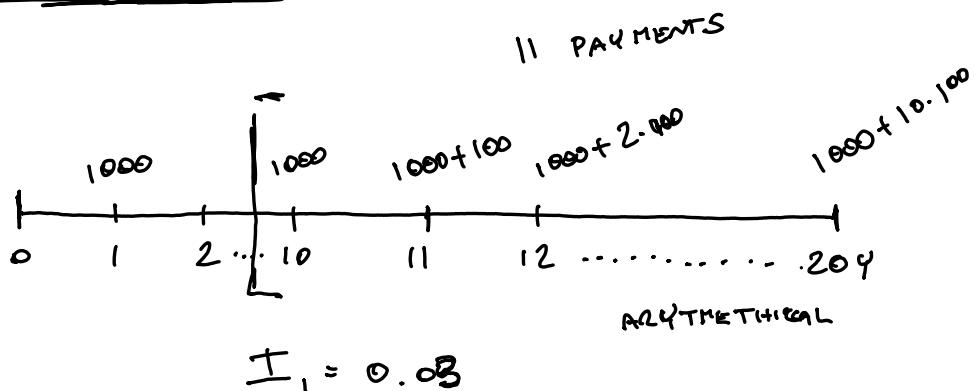
$$I_1 = 0.03$$

$$\hookrightarrow I_{12} = (1 + I)^{\frac{1}{2}} - 1$$

$$\hookrightarrow 0.002466$$

$$V = 100 \cdot \frac{1 - (1 + I_{12})^{-20}}{I_{12}} \cdot (1 + I_{12})^{-31} + \\ 350 \cdot \frac{1 - (1 + I_{12})^{-10}}{I_{12}} \cdot (1 + I_{12})^{-31}$$

EXAMPLE 531



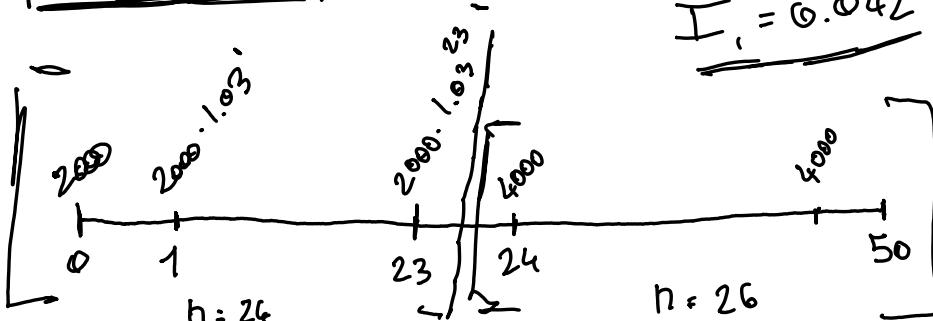
- $\boxed{A1}$ $V_0 = 1000 \cdot \frac{1 - (1 + 0.03)^{-9}}{0.03}$

- $\boxed{A2}$ $C_1 = 1000 \quad H = 100 \quad n = 11$

$$V_0 = \left[\left(1000 + 11 \cdot 100 + \frac{100}{0.03} \right) \cdot \frac{1 - (1 + 0.03)^{-11}}{0.03} - \frac{11 \cdot 100}{0.03} \right] \cdot (1 + 0.03)^{-9}$$

- ADD $\boxed{A1} + \boxed{A2}$

EXAMPLE 50



GEOMETRICAL

• A1

$$V_0 = \left[\frac{2000 \cdot 1 - (1 + 0.042)^{-24} \cdot 1.03^{24}}{1 + 0.042 - 1.03} \right] \cdot (1 + 0.042)$$

• A2

$$V_0 = 4000 \cdot \frac{1 - (1 + 0.042)^{-26}}{0.042} \cdot (1 + 0.042)^{-23}$$

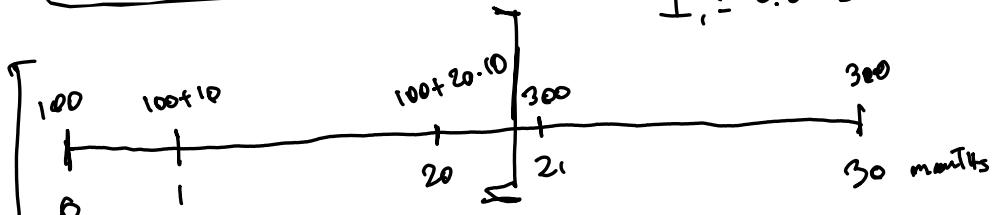
• ADD UP $(\underline{\overline{A1}} \text{ AND } \underline{\overline{A2}}) = V_0$

• CAPITALIZE $\rightarrow V_0 \cdot (1 + 0.042)^{50}$

QUESTION

12

$$I_1 = 0.025$$



$$n = 21 \quad i = 10 \quad C = 100$$

$$I_{1,2} = (1 + 0.025)^{1/2} - 1$$

$$\hookrightarrow 0.002059$$

(A)

A1

$$V_0 = \left[\left(100 + \frac{10}{I_{1,2}} + 10 \cdot 21 \right) \cdot \frac{1 - (1 + I_{1,2})^{-21}}{I_{1,2}} - \frac{10 \cdot 21}{I_{1,2}} \right] \cdot (1 + I_{1,2})$$

A2

$$V_0 = 300 \cdot \frac{1 - (1 + I_{1,2})^{-10}}{I_{1,2}} \cdot (1 + I_{1,2})^{-20}$$

(B)

• $\boxed{A_1}$

$$V_0 = \left[100 + \frac{10}{0.004} + 10 \cdot 21 \right] \cdot \frac{1 - (1 + 0.004)^{-21}}{0.004} - \frac{10 \cdot 21}{0.004} \cdot (1 + 0.004)$$

• $\boxed{A_2}$

$$V_0 = 300 \cdot \frac{1 - (1 + 0.005)^{-10}}{0.005} \cdot (1 + 0.004)^{-20}$$

• ADD $\boxed{A_1}$ AND $\boxed{A_2}$

$$\cdot V_0 \cdot (1 + 0.004)^{20} \cdot (1 + 0.005)^{10}$$



YOU WILL FIND
THE ACCUMULATED
VALUE.

PERPETUITY

ANNUITY WITH AN INFINITE TERM.

- CONSTANT ANNUITY = $\frac{C}{I_m}$

- GEOMETRIC PROGRESSION = $\frac{C}{1 + I_m - q}$

- ARITHMETIC PROGRESSION = $\left(C + \frac{H}{I_m}\right) \cdot \frac{1}{I_m}$

EXAMPLE 41

$$V = 10 \cdot \frac{1 - (1 + 0.06)^6}{0.06} = 166.66 \text{ \$}$$

EXAMPLE 42

$$I_{1,2} = 0.015$$

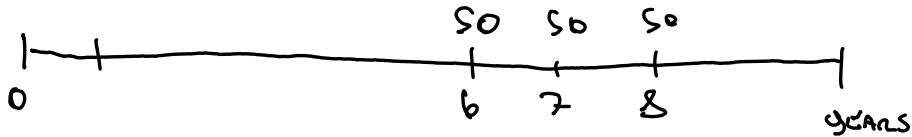
$$\hookrightarrow I_1 = (1 + 0.015)^2 - 1 = 0.195618$$

$$V_0^{\infty} = \frac{c}{I_1} \cdot (1 + I_1) = 226.13 \text{ \$}$$

EXAMPLE 43

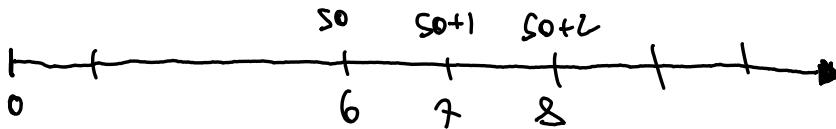
$$I_1 = 0.1$$

(a)



$$V_0^\infty = \frac{C}{I_1} \cdot (1 + I_1)^{-5}$$

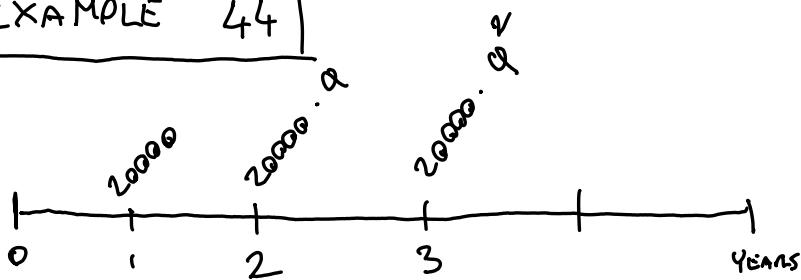
(b)



$$C = 50 \quad H = 1$$

$$V_0^\infty = \left[\left(50 + \frac{1}{0.1} \right) \cdot \frac{1}{0.1} \right] \cdot (1 + 0.1)^{-5}$$

EXAMPLE 44

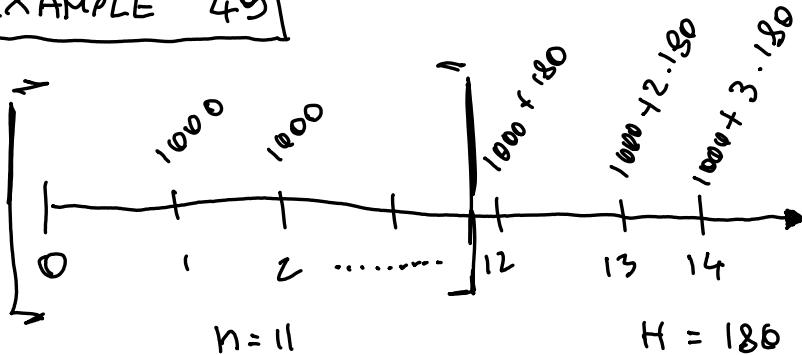


$$I_4 = 0.01 \rightarrow I_1 = (1 + 0.01)^4 - 1 = [0.0406040]$$

$$q = [1.03]$$

$$V_0 = 20000 \cdot \frac{1}{1 + I_1 - 1.03}$$

EXAMPLE 49]



[A1]

$$V_0^{\infty} = 1000 \cdot \frac{1 - (1 + I_r)^{-11}}{I_r}$$

[A2]

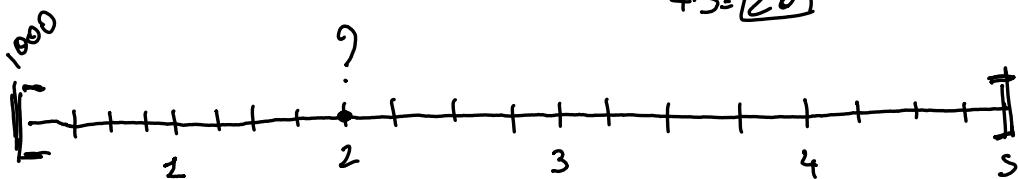
$$V_0^{\infty} = \left[\left(1180 + \frac{180}{I_r} \right) \cdot \frac{1}{H_r} \right] \cdot (1 + I_r)^{-11}$$

OUT STANDING LOAN BALANCE

CHECN ON SLIDES



EXERCISE



$$i_4 : 0.06 \rightarrow I_4 = 0.015$$

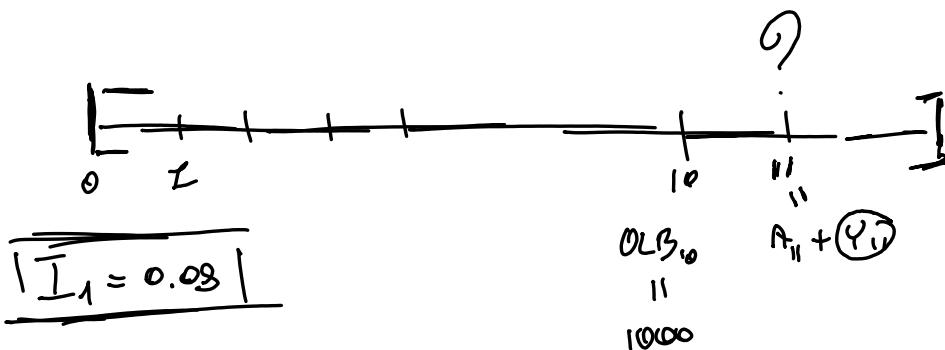
$$10000 = [\alpha] \cdot \frac{1 - (1 + I_4)^{-20}}{I_4}$$

L \rightarrow 58.24 \\$

$$OLB_8 = 58.24 \cdot \frac{1 - (1 + I_4)^{-(20-8)}}{I_4}$$

L \rightarrow 635.25 \\$

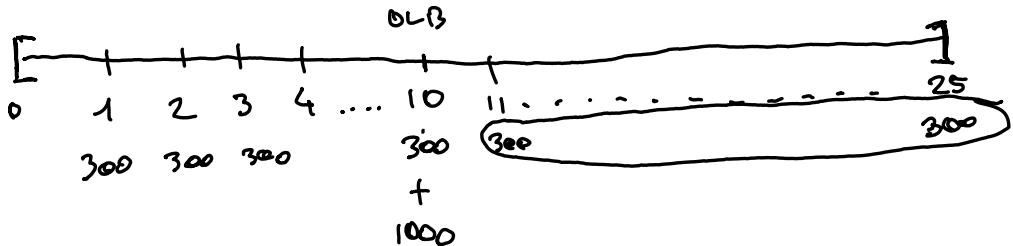
Exercise 2



$$Y_p = OLB \cdot I_m$$

$$Y_{11} = OLB \cdot I_1 = 1000 \cdot 0.03 = 30 \text{ } \frac{1}{\Omega}$$

EXTRA REPAYMENTS



$$I_1 = 0.08$$

$$\textcircled{1} \quad OLB = 300 \cdot \frac{1 - (1 + 0.08)^{-25}}{0.08}$$

$\hookrightarrow 2567.84 \$$

$$\textcircled{2} \quad (OLB - x = c')$$

$$2567.84 - 1000 = \boxed{1567.84 \$}$$

$$\textcircled{3} \quad 1567.84 = \boxed{2} \cdot \frac{1 - (1 + 0.08)^{-15}}{0.08}$$

$\hookrightarrow 2 = \boxed{233.66 \$}$

EXERCISE



$$I_1 = 0.07$$

$$\textcircled{1} \quad \text{OLB} = 400 \cdot \frac{1 - (1 + 0.07)^{-(20 - 4)}}{0.07}$$

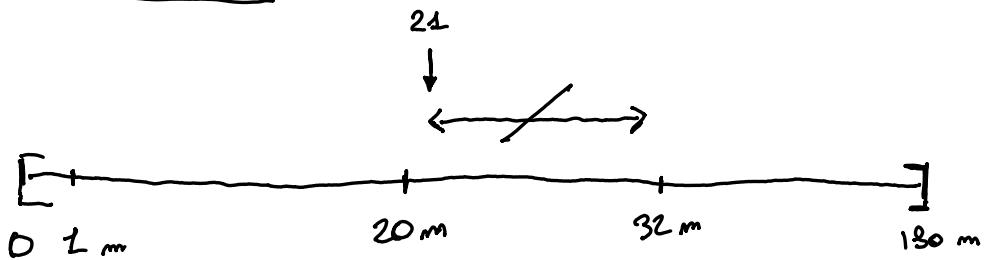
$\hookrightarrow \boxed{3778.66 \text{ \$}}$

$$\textcircled{2} \quad 3778.66 - 200 = \boxed{\underline{3578.66 \text{ \$}}}$$

$$\textcircled{3} \quad 3578.66 = \textcircled{X} \cdot \frac{1 - (1 + 0.07)^{-10}}{0.07}$$

$\hookrightarrow \boxed{495 \text{ \$}}$

EXERCISE 46



$$C = 2000 \text{ \$}$$

$$i_1 = 0.06 \rightarrow I_1 = \frac{0.06}{12} \xrightarrow{[0.005]}$$

$$I_{1,2} = (1 + 0.005)^{12} - 1 \xrightarrow{[0.000416]}$$

[1]

$$2000 \cdot \frac{1 - (1 + I_{1,2})^{-160}}{I_{1,2}} = 221951,53 \text{ \$}$$

[2]

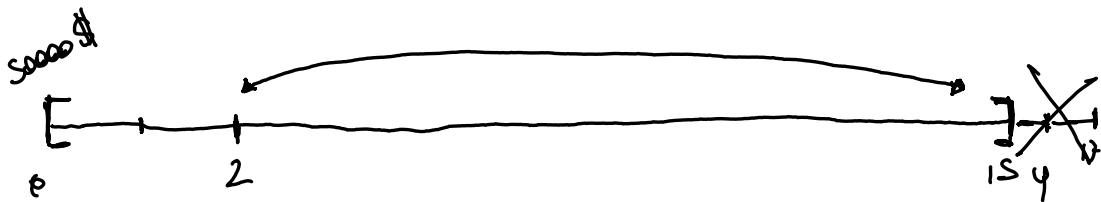
$$221951,53 \cdot (1 + I_{1,2}) = 235268,30 \text{ \$}$$

[3]

$$235268,30 = \textcircled{X} \cdot \frac{1 - (1 + I_{1,2})^{-148}}{I_{1,2}}$$

$\xrightarrow{[2234,10 \text{ \$}]}$

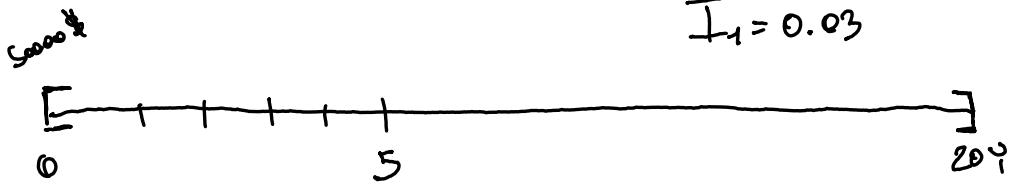
EXERCISE 49



$$I_1 = 0.03$$

$$50000 = X \cdot \frac{1 - (1 + I_2)^{-14}}{I_2} \cdot (1 + I_1)^{-1}$$

EXERCISE 50



$$I_1 = 0.03$$

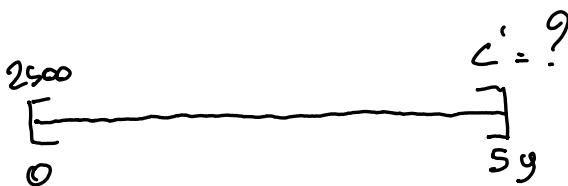
$$\cdot Y = 50000 \cdot 0.03 = 1500 \text{ $}$$

$$\cdot 50000 = \textcircled{X} \cdot \frac{1 - (1 + I_2)^{-15}}{I_2}$$

L $\rightarrow 4188.32 \text{ $}$

PROBLEM SET UNIT 4

Q.2



$$I_4 = 0.01$$

$$\begin{aligned} &= 2500 \cdot (1 + 0.01)^{20} \\ &\therefore C = 3050.47 \text{ $} \end{aligned}$$

$$\cdot AF = 0.5\% \times 2500 = 12.5 \text{ $}$$

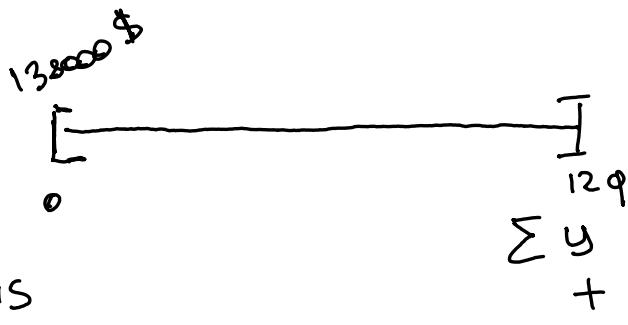
$$\cdot ME = 400 \text{ $}$$

$$2500 = 12.5 + 400 + 3050.47 (1 + I_4)^{-5}$$

$$TAE = I_1 = 0.07881$$

Q5

(a)



$$i_4 = 0.015$$

$$\hookrightarrow I_4 = 0.00375$$

$$(\sum y) = 138000 \cdot 0.00375 = 517.5 \text{ \$}$$

(b)

$$138000 = 517.5 \cdot \frac{1 - (1 + I_4)^{-12}}{I_4} + 138000 \cdot (1 + I_4)^{-12}$$

(c)

$$AF = 0.25\% \cdot 138000 = 345 \text{ \$}$$

$$ME = 400 \text{ \$}$$

TOTAL ARRANG. EXP.

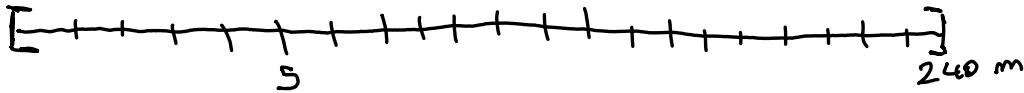
ALWAYS AN ANNUAL
EFFECTIVE INTEREST
RATE

$$138000 = 345 + 400 + 517.5 \cdot \frac{1 - (1 + I_4)^{-12}}{I_4} + 138000 \cdot (1 + I_4)^{-12}$$

Qv3

150000

$$\textcircled{a} = y + A$$



$$I_1 = 0.048$$

$$\hookrightarrow I_{12} = (1 + 0.048)^{\frac{1}{12}} = \boxed{0.003814}$$

$$\textcircled{a} \quad 150000 = \textcircled{a} \cdot \frac{1 - (1 + I_{12})^{-240}}{I_{12}}$$

$$\hookrightarrow 965,04 \text{ $}$$

b

$$OLB^{\text{mb}} = 965,04 \cdot \frac{1 - (1 + I_{12})^{-235}}{I_{12}}$$

$$\hookrightarrow 143035,9 \text{ $}$$

$$OLB^{\text{RSTW}} = 150000 \cdot (1 + I_{12})^5 - 965,04 \cdot \frac{1 - (1 + I_{12})^{-5}}{I_{12}} \cdot (1 + I_{12})^5$$

$$OLB_{\frac{5}{-}} = 143035,9 + 965,04 = 143060,94 \text{ $}$$

(c)

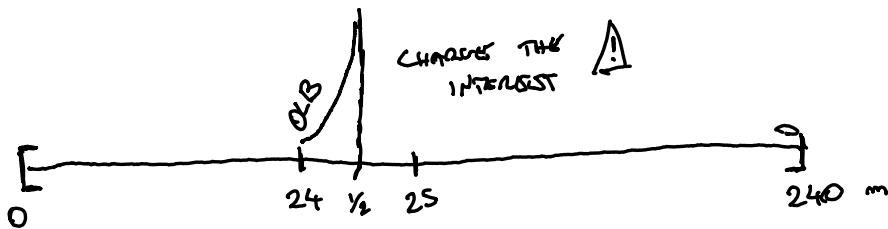
$$A_6 + y_6 = 965,04 \text{ $}$$

$$y_r = OLB_{n-1} \cdot I_n$$

$$y_6 = 148095,9 \cdot I_{12} = \underline{\underline{579,64 \text{ $}}}$$

$$\underline{\underline{A_6 = 385,39 \text{ $}}}$$

(d)



$$OLB_{24} = 965,04 \cdot \frac{1 - (1 + I_{12})^{-216}}{I_{12}} = 140511,27 \text{ $}$$

$$140511,27 \cdot (1 + I_{12})^{0,5} = \underline{\underline{140786,08 \text{ $}}}$$

8



$$AF = 0.01 \cdot 150000 = 1500 \$$$

$$ME = 150 \$$$

$$150000 = 1500 + 150 + 965,04 \cdot \frac{1 - (1 + I_{1,2})^{-240}}{I_{1,2}}$$

$$\text{TAE} = (1 + I_{1,2})^2 - 1$$

8

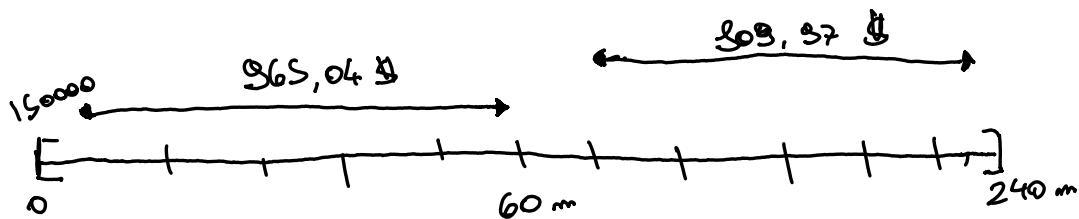
$$OLB = 965,04 \cdot \frac{1 - (1 + I_{1,2})^{-180}}{I_{1,2}} = 124501,16 \$$$

$$124501,16 - 20000 = 104501,16 \$$$

$$104501,16 = \text{@} \cdot \frac{1 - (1 + I_{1,2})^{-180}}{I_{1,2}}$$

$$\rightarrow 809,87 \$$$

BEIR



$$AF = 1500$$

$$\begin{matrix} a \\ + \\ 20000\$ \end{matrix}$$

$$MF = 150$$

$$150000 = 1500 + 150 + 965,04 \cdot \frac{1 - (1 + I_{12})^{-60}}{I_{12}} + 20000 (1 + I_{12})^{-60} + 809,97 \cdot \frac{1 - (1 + I_{12})^{-180}}{I_{12}} \cdot (1 + I_{12})^{-60}$$

BEIR

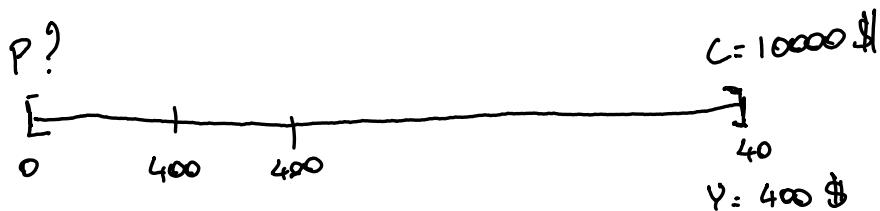
TOTAL COST BUT
IT'S NOT AN ANNUAL
INTEREST RATE.

Bonds

(3)

$$N = 10000 \text{ \$} \quad t = 20 \text{ years} \quad C_P = 400 \text{ \$}$$

$$P = ? \quad Y_{IELD} = 6\% \quad C_{RATE} = ?$$



$$Y_{IELD_2} = \left(1 + 0.06\right)^{-2} - 1 = \boxed{0.029556}$$

$$P = 400 \cdot \frac{1 - (1 + I_2)^{-40}}{I_2} + 10000 \cdot (1 + I_2)^{-40} = 12429,62 \text{ \$}$$

$$C_{RATE} = \frac{400}{10000} = \boxed{0.04}$$

[5]

$$\text{PAR} = 1000 \$ \quad t = 20 \text{ years} \quad \text{RED} = 1050 \$$$

$$\text{YIELD} = 8.25\% \quad C = 75 \$ \rightarrow 3\%$$

$$P = 75 \cdot \frac{1 - 1.03^{20} \cdot (1 + I_1^*)^{-20}}{1 + I_1^* - 1.03} + 1050 \cdot (1 + I_1^*)^{-20}$$

L 1115.11 \$

[6]

$$\text{PAR} = 1000 \$ \quad i_2 = 0.09 \rightarrow I_2 = \frac{i_2}{2} = 0.045$$

$$\text{RED} = 1125 \$ \quad \text{YIELD} = 0.05$$



$$\text{COUPON} = 1000 \cdot 0.045 = 45 \$$$

$$P = 45 \cdot \frac{1 - (1 + 0.05)^{-h}}{0.05} + \boxed{1125 \cdot (1 + 0.05)^{-h}}$$

\downarrow
214, 14

BY SOLVING
 π, v GET
 $h = 34$

$\rightarrow 942,82 \$$

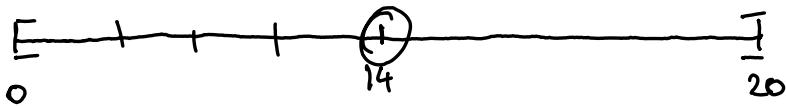
~~BOOK VALUE~~

TOTAL OUTSTANDING
FUTURE PAYMENTS.

13

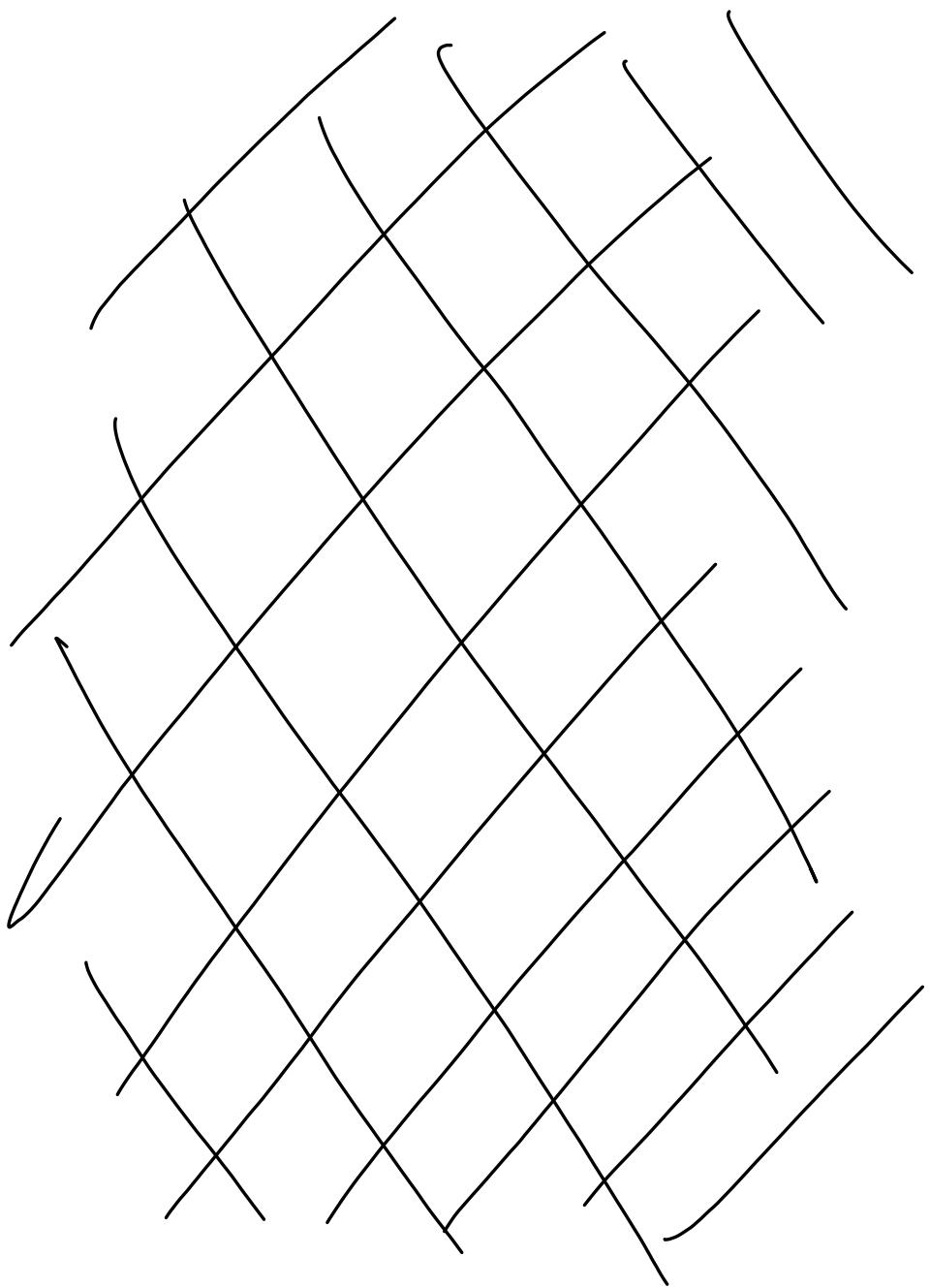
$$P_{AR} = 1000 \$ \quad t = 10 \text{ yrs} \quad C_{\text{RATES}} = 0.025$$

$$\text{YIELD} = 0.06 \quad C = 25$$



$$B_{14} = 25 \cdot \frac{1 - (1 + 0.06)^{-6}}{0.06} + 1000 \cdot (1 + 0.06)^{-6}$$

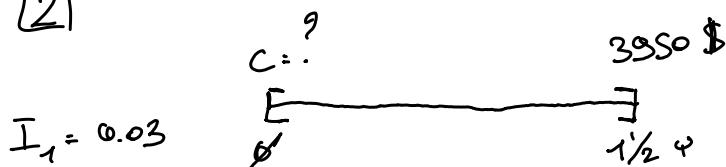
→ 827, 39 \\$



FINAL EXAM BOOT-CAMP

$$a = A + \gamma$$

[2]



$$3950 = \boxed{C} \cdot (1 + 0.03)^{\frac{3}{2}}$$

$$\hookrightarrow \boxed{3778.7 \$}$$

[7]



$$i_4 = 0.02 \rightarrow I_4 = 0.002s \rightarrow I_2 = (1 + 0.002s)^2 - 1 \rightarrow 0.005006$$

$$\therefore C = 50000 \cdot (1 + 0.005006)^{10} = \boxed{52560.27 \$}$$

$$\therefore OLB^{PRO} = 52560.27 \cdot (1 + 0.005006)^{-3} = 51778.69 \$$$

10

\$60000 \$

0

5
Y

$$i_{12} = 0.06 \rightarrow I_{12} = 0.005 \rightarrow I_4 = (1 + 0.005)^{\frac{1}{4}} - 1 \\ \hookrightarrow 0.015075$$

$$Y = 8442.07$$

LAST PAYMENT

$$\$60000 + 8442.07 = \underline{\underline{\$68442.07}}$$

20

20000 \$

0

7
5
Y

$$I_{12} = 0.048 \rightarrow I_4 = (1 + 0.048)^{\frac{1}{4}} - 1 \\ \hookrightarrow 0.011739$$

$$20000 = \textcircled{1} \cdot \frac{1 - (1 + I_4)^{-20}}{I_4}$$

$$\hookrightarrow 1128.37 \$$$

28

$$c = ?$$



$$I_r = 0.05$$

$$256 \cdot \frac{1 - (1 + 0.05)^{-10}}{0.05} = 1976.76 \text{ \$}$$

23

• 15 YEARS \rightarrow $150000 = (\textcircled{a}) \cdot \frac{1 - (1 + 0.00083)^{-180}}{0.00083}$

\downarrow

$\Rightarrow 338.03 \text{ \$}$

• 20 YEARS \rightarrow $150000 = (\textcircled{a}) \cdot \frac{1 - (1 + 0.000625)^{-240}}{0.000625}$

\downarrow

$\Rightarrow 673.24 \text{ \$}$

25



$$I_1 = 0.05 \rightarrow I_{12} = (1 + 0.05)^{1/12} - 1$$

$\hookrightarrow 0.004074$

$$200000 = \textcircled{a} \cdot \frac{1 - (1 + 0.004074)^{-120}}{0.004074}$$

$\hookrightarrow \boxed{2110.45 \text{ \$}}$

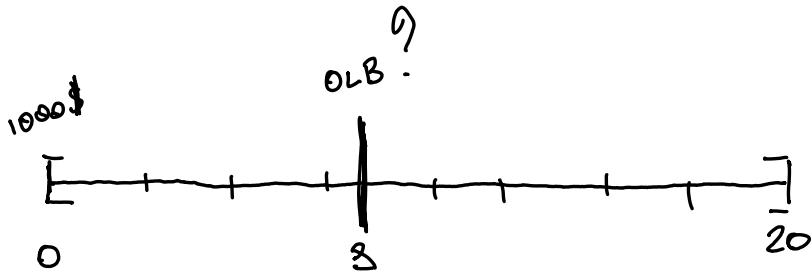
$$200000 \cdot (1 + I_{12}) - 2110.45 = \boxed{\underline{\underline{198704.35 \$}}}$$

$$2110.45 \cdot \frac{1 - (1 + 0.004074)^{-110}}{0.004074} = 186802.76 \text{ \$}$$

(brace from above)

$$\rightarrow 186802.76 + 2110.45 = \boxed{\underline{\underline{188913.21 \$}}}$$

26



$$i_4 = 6.06 \rightarrow I_4 = \frac{0.06}{4} = \boxed{0.015}$$

$$\bullet 1000 = \textcircled{a} \cdot \frac{1 - (1 + 0.015)^{-20}}{0.015}$$

$\hookrightarrow \underline{\underline{58.24 \text{ \$}}}$

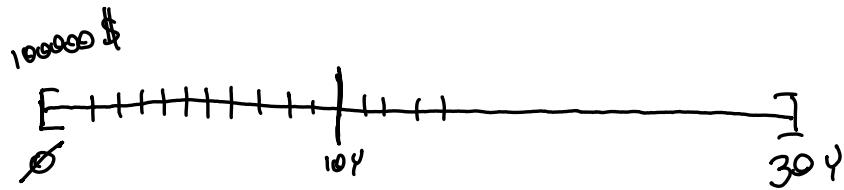
$$OLB^{P00} = 58.24 \cdot \frac{1 - (1 + 0.015)^{-12}}{0.015}$$

$\hookrightarrow \underline{\underline{635.25 \text{ \$}}}$

$$OLB^{\text{RETRO}} = 1000 \cdot (1 + 0.015)^8 - 58.24 \cdot \frac{1 - (1 + 0.015)^{-8}}{0.015} \cdot (1 + 0.015)^8$$

$\hookrightarrow \underline{\underline{635.25 \text{ \$}}}$

27



$$i_{12} = 0.08 \rightarrow I_{12} = \frac{0.08}{12} = \boxed{0.00666}$$

$$1000000 = (0) \cdot \frac{1 - (1 + 0.00666)^{-360}}{0.00666}$$

L \rightarrow **733.21 \$**

$$OLB^{P_{120}} = 733.21 \cdot \frac{1 - (1 + 0.00666)^{-240}}{0.00666}$$

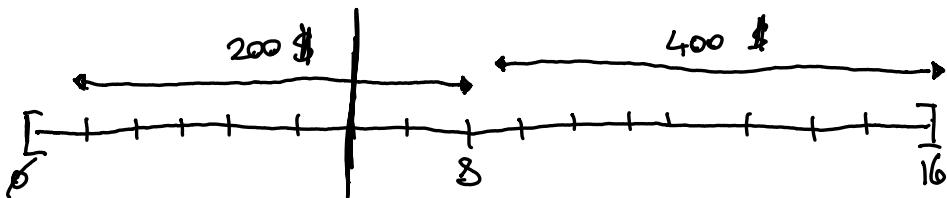
L \rightarrow **87710.60 \$**

$$OLB^{R_{120} R_{120}} = 1000000 \left(1 + 0.00666\right)^{120} - 733.21 \cdot \frac{1 - (1 + 0.00666)^{-120}}{0.00666} \cdot (1.00666)^{120}$$

L \rightarrow **$221787.7 - 134073.01 = 87710 $$**

28

OLB?



$$i_4 = 0.1 \rightarrow I_4 = 0.025$$

$$C = 400 \cdot \frac{1 - (1 + 0.025)^{-16}}{0.025} - 200 \cdot \frac{1 - (1 + 0.025)^{-8}}{0.025}$$

$$\hookrightarrow 5222 \$ - 1434.027 \$ = \boxed{3787.97 \$}$$

$$OLB_{R20} = 200 \cdot \frac{1 - (1 + 0.025)^{-2}}{0.025} + 400 \cdot \frac{1 - (1 + 0.025)^{-8}}{0.025} \cdot (1 + 0.025)^{-2}$$

$$\hookrightarrow 385.48 + 2729.36 = \boxed{3115.33 \$}$$

$$OLB_{R20} = 1000 \cdot (1 + 0.025)^6 - 200 \cdot \frac{1 - (1 + 0.025)^{-6}}{0.025} \cdot (1 + 0.025)^6$$

→ $\boxed{3115.33 \$}$

29

$$I_1 = 0.08$$

$$A_{10} = A_1 \cdot (1 + 0.08)^{10-1}$$

:

$$100 = A_1 \cdot (1.08)^9$$

:

$$A_1 = 50.02 \text{ $}$$

$$A_5 = 50.02 (1.08)^4$$

:

$$A_5 = 63.05 \text{ $}$$

30

$$Y_{\text{INT}} = 1000 \cdot 0.08 = \boxed{80 \text{ $}}$$

38



$$i_4 = 0.12 \rightarrow [I_4 = 0.03]$$

$$C = 1000 \cdot \frac{1 - (1 + 0.03)^{-20}}{0.03}$$

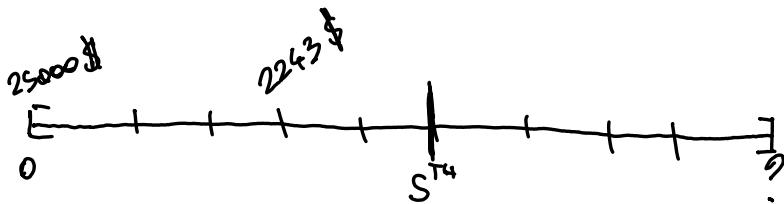
$$\hookrightarrow 14877.47 \text{ \$}$$

14877.47 \\$	1000 \\$	446.3 \\$	553.7
14323.77 \\$	1000 \\$	425.7 \\$	570.3
	1000 \\$		
	:		

6 TH

$$553.7 (1 + 0.03)^5 = 641.89 \text{ \$}$$

[32]



$$I_1 = 0.075$$

$$25000 \parallel 2243 \parallel 1875 \parallel 368$$

$$24632 \parallel 2243 \parallel 1842.4 \parallel 385.6$$

$$24236.4 \parallel 2243 \parallel 1817.73 \parallel 425.27 \quad \text{OM}$$

$$23811.13 \parallel 2243 \parallel 1787.83 \parallel 455.17$$

$$23355.86 \parallel 2243 \parallel \underline{\underline{1751.69}} \parallel \dots \dots$$

(OM)

$$\text{OLB}_4^{\text{RORO}} = 25000 \left(1+0.075\right)^4 - 2243 \cdot \frac{1-(1+0.075)^{-4}}{0.075} \cdot \left(1+0.075\right)^4$$

$\rightarrow 23355.86 \cdot (1+0.075) = \underline{\underline{1751.69 \$}}$

33)

$$1000 = \textcircled{w} \cdot \frac{1 - (1 + 0.08)^{-4}}{0.08}$$

301.92 \$

1000 || 301.92 || 80 || 221.92

778.08 || 301.92 || 62.25 || 239.67

538.41 || 301.92 || 43.67 || 258.85

279.56 || 301.92 || 22.36 || 279.56

34)

$$3500 = \textcircled{w} \cdot \frac{1 - (1 + 0.05)^{-3}}{0.05}$$

3121.27 \$

3500 || 3121.27 || 425 || 2696.27

5803.73 || 3121.27 || 290.19 || 2831.08

2972.65 || 3121.27 || 148.63 || 2972.64

[35]

$$i_{12} = 0.08 \rightarrow I_{12} = 0.0066$$

$$S_{00} = \textcircled{w} \cdot \frac{1 - (1 + 0.0066)^{-6}}{0.0066}$$

L $\rightarrow 136.43 \$$

$S_{00} \parallel 136.43 \parallel 5.28 \parallel 131.15$

$668.85 \parallel 136.43 \parallel 4.41 \parallel 132.02$

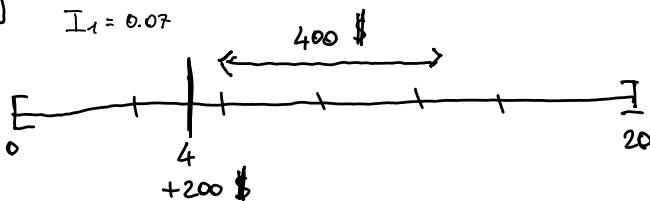
$536.83 \parallel 136.43 \parallel 3.54 \parallel 132.89$

$403.94 \parallel 136.43 \parallel 2.66 \parallel 133.77$

$270.17 \parallel 136.43 \parallel 1.78 \parallel 134.65$

$135.52 \parallel 136.43 \parallel 0.89 \parallel 135.52$

37



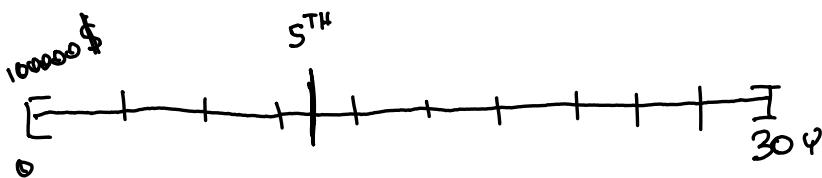
$$OLB = 400 \cdot \frac{1 - (1 + 0.07)^{-16}}{0.07} = 3778.66 \text{ \$}$$

$$3778.66 - 200 = 3578.66 \text{ \$}$$

$$3578.66 = (a) \cdot \frac{1 - (1 + 0.07)^{-10}}{0.07}$$

$$\hookrightarrow \boxed{509.52 \text{ \$}}$$

38



$$I_{12} = 0.005 \rightarrow I_1 = (1 + 0.005)^{12} - 1 = \boxed{0.06167}$$

$$1000000 = (a) \frac{1 - (1 + 0.06167)^{-30}}{0.06167}$$

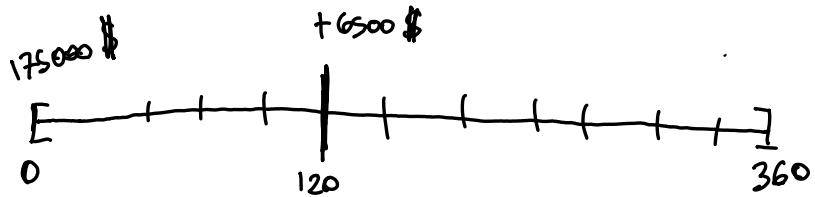
$$\hookrightarrow 7395.18 \text{ \$} \cdot \frac{1 - (1 + 0.06167)^{-25}}{0.06167}$$

$$\hookrightarrow 93053.49 - 4500 = 88553.49 \text{ \$}$$

$$88553.49 = (a) \cdot \frac{1 - (1 + 0.06167)^{-25}}{0.06167}$$

$$\hookrightarrow \boxed{7037.55 \text{ \$}}$$

39



$$I_1 = 6.05 \rightarrow I_{12} = (1 + 0.05)^{1/2} - 1 = \boxed{0.004074}$$

$$175000 = \textcircled{a} \cdot \frac{1 - (1 + 0.004074)^{-360}}{0.004074}$$

↓

$$\rightarrow 927.58 \text{ \$}$$

$$OLB^{P120} = 927.58 \cdot \frac{1 - (1 + 0.004074)^{-240}}{0.004074}$$

↓

$$\rightarrow 141869.05 - 6500 \text{ \$} = \boxed{135368.05 \text{ \$}}$$

$$135368.05 = \textcircled{a} \cdot \frac{1 - (1 + 0.004074)^{-240}}{0.004074}$$

↓

$$\rightarrow \boxed{885.08 \text{ \$}}$$

[40]

$$U_4 = 0.08 \rightarrow \boxed{I_4 = 0.02}$$



$$\theta L B_{10}^{\text{PAO}} = \omega \cdot \frac{1 - (1 + 0.02)^{-30}}{0.02}$$

$$\hookrightarrow \omega \cdot 22.39645$$

$$22.39645 \omega - X = \omega \cdot \frac{1 - (1 + 0.02)^{-30}}{0.02}$$

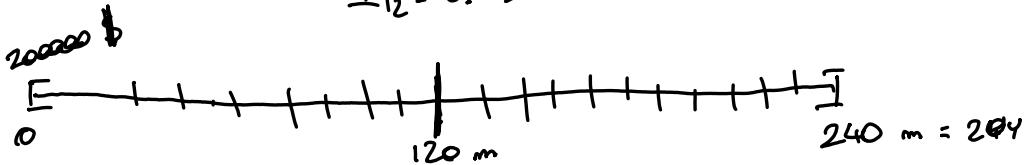
$$X = 22.39645 \omega - 22.39645 \omega$$

$$\hookrightarrow 22.39645 (\omega - 0.9\omega)$$

$$\boxed{X = 2.239645 \omega}$$

42

$$I_{12} = 0.015$$



$$200000 = \textcircled{w} \cdot \frac{1 - (1 + 0.015)^{-240}}{0.015}$$

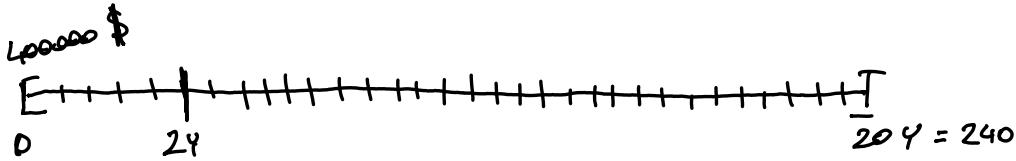
↳ 3086.62 \$

$$3086.62 \cdot \frac{1 - (1 + 0.015)^{-120}}{0.015} = \textcircled{171302.63}$$

$$3086.62 \cdot \frac{1 - (1 + 0.015)^{-60}}{0.015} = \textcircled{121551.32}$$

• 48750.72 \$

44



$$i_1 \Rightarrow I_1 = 0.00416 \rightarrow (1 + 0.00416)^{1/2} - 1 = \underline{\underline{0.00034656}}$$

$$\bullet 400000 = \textcircled{a} \cdot \frac{1 - (1 + 0.00034656)^{-240}}{0.00034656}$$

$\hookrightarrow \boxed{1737.23 \$}$

$$\bullet \frac{P_{10}}{24} = 1737.23 \cdot \frac{1 - (1 + 0.00034656)^{-216}}{0.00034656}$$

$\hookrightarrow 361480.62$

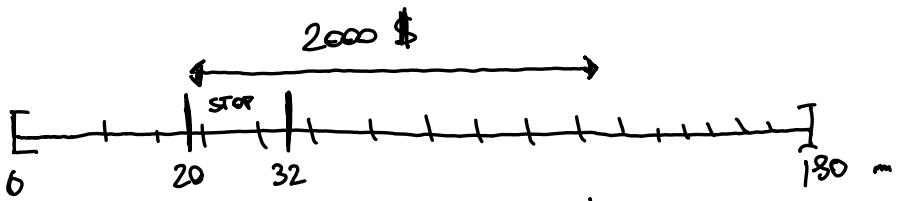
$$i_1 \Rightarrow I_1 = 0.004583 \rightarrow (1 + 0.004583)^{1/2} - 1 = \underline{\underline{0.00038114}}$$

$$\bullet 361480.62 = \textcircled{a} \cdot \frac{1 - (1 + 0.00038114)^{-216}}{0.00038114}$$

$\hookrightarrow \boxed{1743.67 \$}$

$$\bullet 400000 = 1743.67 \cdot \frac{1 - (1 + 0.00038114)^{-t}}{0.00038114}$$

46



$$i_1 \rightarrow I_{12} = 0.005 \rightarrow I_{12} = (1 + 0.005)^{1/2} - 1 = \underline{\underline{0.0004157}}$$

$$\cdot 2000 \cdot \frac{1 - (1 + 0.0004157)^{-180}}{0.0004157} = \boxed{346791.65 \text{ \$}}$$

$$\cdot \text{BLB}_{20}^{\text{P20}} = 2000 \cdot \frac{1 - (1 + 0.0004157)^{-160}}{0.0004157}$$

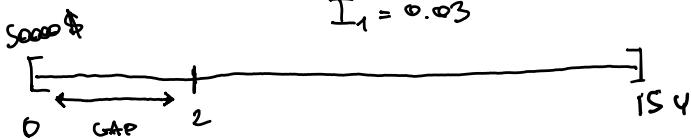
$$\hookrightarrow 308527.93 \text{ \$}$$

$$\cdot 308527.93 \cdot (1 + 0.0004157)^{12} = 311075.52 \text{ \$}$$

$$\cdot 311075.52 = \textcircled{0} \cdot \frac{1 - (1 + 0.0004157)^{-148}}{0.0004157}$$

$$\hookrightarrow \boxed{2167.62 \text{ \$}}$$

45



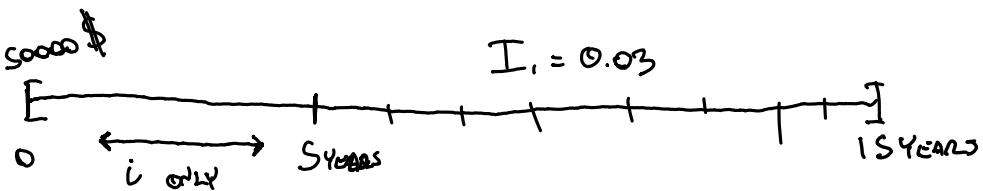
$$I_1 = 0.03$$

- $5000 \cdot (1 + 0.03) = 51500 \text{ \$}$

- $51500 = 5000 \cdot \frac{1 - (1 + 0.03)^{-14}}{0.03}$

L 4313.88 \\$

56



$$I_1 = 0.03$$

- $Y = 5000 \cdot 0.03 = 1500 \text{ \$}$

- $5000 = 1500 \cdot \frac{1 - (1 + 0.03)^{-5}}{0.03} + 5000 \cdot \frac{1 - (1 + 0.03)^{-10}}{0.03} \cdot (1 + 0.03)^{-5}$

L

BEIR

BORROWER EFFECTIVE
INTEREST RATE

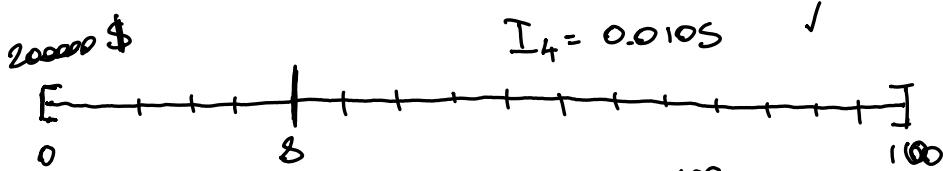
EXPRESSED AS ANNUAL INTEREST, IT REPRESENTS
THE ACTUAL COST OF A LOAN.

- ARRANGEMENT FEES ;
 - VALUATION FEE ;
 - LIFE INSURANCE ;
 - BANK CHARGES ;
- ALL INCLUDED 

EAR

EFFECTIVE
ANNUAL RATE (TAE)

QUESTION 5



$$\bullet \quad 200000 = \textcircled{0} \cdot \frac{1 - (1 + 0.0105)^{-10}}{0.0105}$$

$$\hookrightarrow 3240.03 \text{ } \$$$

$$\bullet \quad \text{DLB}_8^{10\%} = 3240.03 \cdot \frac{1 - (1 + 0.0105)^{-92}}{0.0105}$$

$$\hookrightarrow 180537.19 \text{ } \$ - 15000 \text{ } \$ = 175537.19 \text{ } \$$$

$$- \quad 175537.19 = \textcircled{0} \cdot \frac{1 - (1 + 0.0105)^{-92}}{0.0105}$$

$$\hookrightarrow 2884.96 \text{ } \$$$

$$- \quad 175537.19 = 3240.03 \cdot \frac{1 - (1 + 0.0105)^{-t}}{0.0105}$$

$$OLB_8 = 3240.03 \cdot \frac{1 - (1 + 0.0105)^{-82}}{0.0105}$$

↓

$$\rightarrow 180537.18 \$$$

$$180537.18 = \textcircled{Q} \cdot \frac{1 - (1 + 0.01175)^{-82}}{0.01175}$$

↓

$$\rightarrow 3399.35 \$$$

✓

$$180537.18 \cdot (1 + 0.0105)^4 = 198666.6769 \$$$

$$198666.6769 = \textcircled{Q} \cdot \frac{1 - (1 + 0.0105)^{-88}}{0.0105}$$

↓

$$\rightarrow 3463.99 \$$$

✓

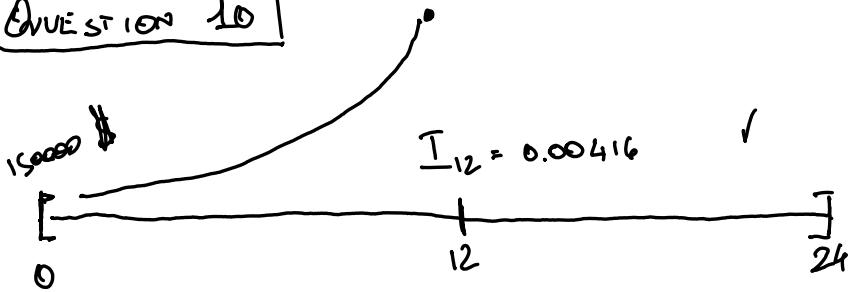
$$180537.18 \cdot 0.0105 = 2000.6 \$$$

$$180537.18 = 2000.6 \cdot \frac{1 - (1 + 0.0105)^{-4}}{0.0105} + \textcircled{Q} \cdot \frac{1 - (1 + 0.0105)^{-88}}{0.0105}$$

✓

THIS IS WHAT SHE
WAS LOOKING FOR

QUESTION 10



- $Y = 150000 \cdot 0.00416 = 625 \$$

$$150000 = \textcircled{w} \cdot \frac{1 - (1 + 0.00416)^{-12}}{0.00416}$$

$\hookrightarrow 12840.57236 \$$

IEE

$$150000 = 625 \cdot \frac{1 - (1 + I_{12})^{-12}}{I_{12}} + 12840.57 \cdot \frac{1 - (1 + I_{12})^{-12}}{I_{12}} \cdot (1 + I_{12})^{-12}$$

- $AF = 800 \$$

$$150000 = 800 + 625 \cdot \frac{1 - (1 + I_{12})^{-12}}{I_{12}} + 12840.57 \cdot \frac{1 - (1 + I_{12})^{-12}}{I_{12}} \cdot (1 + I_{12})^{-12}$$

\downarrow

$$EAR = (1 + I_{12})^{12} - 1$$

INTRODUCTION TO BONDS

- ① **COUPON AMOUNT** → FACE VALUE × COUPON RATE
- ② **REDEMPTION**
- PREMIUM → RED AM > PAR
- DISCOUNT → RED AM < PAR
- ③ **YIELD TO MATURITY** → INTEREST EARNED BY THE INVESTOR

$$P = \frac{\text{COUPON AMOUNT}}{\text{YIELD}} \cdot \frac{1 - (1 + \text{YIELD})^{-n}}{\text{YIELD}} + \frac{\text{REDEMPTION AMOUNT}}{(1 + \text{YIELD})^n}$$

EXAMPLE 2

$$t = 8 \text{ yrs} \rightarrow \textcircled{16}$$

COUPON RATE = 0.05

COUPON AMOUNT = 150 \$

$$Y_{\text{IELD}} = (1 + 0.12)^{-1/2} - 1 \Rightarrow 0.0583$$

$$P = 150 \cdot \frac{1 - (1 + 0.0583)^{-16}}{0.0583} + 2000 \cdot (1 + 0.0583)^{-16}$$

EXAMPLE 3

$$t = 20 \text{ yrs} \rightarrow \textcircled{40}$$

$$\bullet \text{ COUPON RATE} = \frac{400}{10000} \cdot 100 = 4\%$$

$$\bullet Y_{\text{IELD}} = (1 + 0.06)^{-1/2} - 1 = 0.02856$$

$$P = 400 \cdot \frac{1 - (1 + 0.02856)^{-40}}{0.02856} + 10000 \cdot (1 + 0.02856)^{-40}$$

EXAMPLE 4

$$t = 10Y \rightarrow 20$$

- COUPON AMOUNT = 10 \$
- REDEMPTION AMOUNT = 105 \$
- YIELD = 0.04

$$P = 10 \cdot \frac{1 - (1 + 0.04)^{-20}}{0.04} + 105 \cdot (1 + 0.04)^{-20}$$

EXAMPLE 5

RECALL THE GEOMETRICAL FORMULA



$$P = 75 \cdot \frac{1 - 1.03^{20} \cdot (1 + I^*)^{-20}}{1 + I^* - 1.03} + 1050 \cdot (1 + I^*)^{-20}$$

EXAMPLE 10

$$t = 10 \text{ years} \rightarrow 20$$

- COUPON RATE = 0.005
- COUPON AMOUNT = 20 \$
- REDEMPTION AMOUNT = 4120 \$

$$3800 = 20 \cdot \frac{1 - (1 + I_{12})^{-120}}{I_{12}} + 4120 \cdot (1 + I_{12})^{-120}$$
$$\text{YIELD} = (1 + I_{12})^{12} - 1$$

EXAMPLE 11

$$t = 5 \text{ years} \rightarrow 20$$

- COUPON RATE = 0.05
- COUPON AMOUNT = 10 \$
- YIELD $\rightarrow (1 + 0.05)^{1/4} - 1 \rightarrow 0.012272$

$$1075 = 10 \cdot \frac{1 - (1 + I_{14})^{-20}}{I_{14}} + R \cdot (1 + I_{14})^{-20}$$

EXAMPLE 12

$$t = 74 \rightarrow \textcircled{14}$$

- COUPON RATE = 0.05
- YIELD = 0.03
- COUPON AMOUNT = 150 \$

$$P = 150 \cdot \frac{1 - (1 + 0.03)^{-14}}{0.03} + 3000 \cdot (1 + 0.03)^{-14}$$

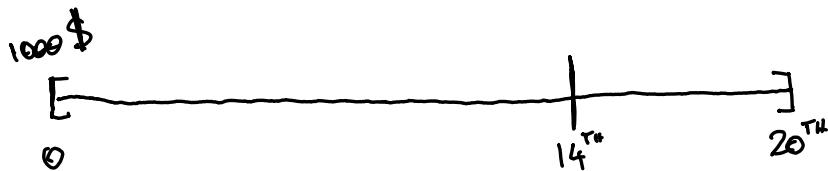
↳ $3677.76 - 3000 = \textcircled{677.76}$

PREMIUM

BOOK VALUE

IT IS THE EQUIVALENT OF
@LB BUT FOR BONDS.

EXAMPLE 13



- COUPON RATE = 0.025
- YIELD = $0.12 / 2 \rightarrow 0.06$

$$BV = 25 \cdot \frac{1 - (1 + 0.06)^{-6}}{0.06} + 1000 \cdot (1 + 0.06)^{-6}$$

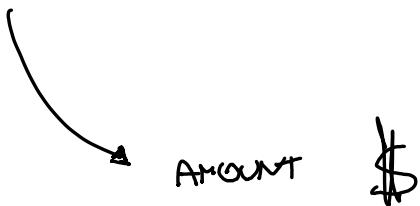
→ AMOUNT \$

EXAMPLE 14

$$t = 5 \rightarrow 10$$

- COUPON RATE = 0.035
- REDEMPTION AMOUNT = 2120 \$
- COUPON AMOUNT = 70 \$
- YIELD = $(1 + 0.035)^{\frac{1}{2}} - 1 \Rightarrow 0.02465$

$$BV = 70 \cdot \frac{1 - (1 + 0.02465)^{-6}}{0.02465} + 2120 \cdot (1 + 0.02465)^{-6}$$



AMOUNT \$

EXAMPLE IS

$$t = 10 \rightarrow 120$$

- COUPON RATE = 0.0033
- REDEMPTION AMOUNT = 1030 \$
- COUPON AMOUNT = 3.3 \$
- C = 10 \$

$$980 + 10 = 3.3 \cdot \frac{1 - (1 + I_{1,2})^{-120}}{I_{1,2}} + 1030 (1 + I_{1,2})^{-120}$$


$$YIELD = (1 + I_{1,2})^n - 1$$

$$\textcircled{2} \quad C = 2500 \quad i_1 = 0.025 \quad \hat{t} = \frac{90}{360} = \frac{1}{4}$$

$$\text{a) } 2500 \cdot \left(1 + 0.025 \cdot \frac{1}{4}\right) = 2515.625 \text{ $}$$

$$\text{b) } DT = 2515.625 - 2500 = 15.625 \text{ $}$$

$$D(T, T') =$$

$$\text{c) } d = 0.025 \rightarrow i = \frac{d}{1-dt}$$

$$\hookrightarrow i = 0.3\bar{3}$$