EXERCISE 1

QN

	P2: A'	B'	C'	D'
P1: A	5, 4	4, 4	4, 5	2, 4
B	3, 7	8, 7	5, 8	10, 6
C	2, 10	7, 6	4, 6	8, 5
D	4, 4	5, 8	4, 10	10, 8

1) STRATEGY $\boxed{C'}$ STRICTLY DOMINATES $\boxed{B'}$ AND $\boxed{D'}$
THEREFORE, BOTH ELIMINATED.

2) STRATEGY $\boxed{A}$ STRICTLY DOMINATES $\boxed{C}$ AND $\boxed{D}$
SO, OUT.

3) BACK TO PLAYER 2 STRATEGIES
STRATEGY $\boxed{C'}$ DOMINATES $\boxed{A'}$ SO,
$\boxed{A'}$ OUT ⚠

4) PLAYER 1 HAS STRATEGY $\boxed{B}$
WHICH DOMINATES $\boxed{A}$ ONE, THEREFORE OUT.

5) FINALLY, THE SOLUTION OF THIS GAME
IS $S_B \in BR(S_{C'})$

EXERCISE 2

P_2

ON

P_1

	R'_1	R'_2	R'_3	R'_4	R'_5
R_1	5,5	2,8	5,5	4,6	6,4
R_2	8,2	5,5	8,2	6,4	7,3
R_3	5,5	2,8	5,5	4,6	6,4
R_4	6,4	4,6	6,4	5,5	8,2
R_5	4,6	3,7	4,6	2,8	5,5

1) PLAYER 2's STRATEGY $\boxed{R'_2}$ DOMINATES $\boxed{R'_3}$, $\boxed{R'_5}$ AND $\boxed{R'_1}$;

2) PLAYER 1's STRATEGY $\boxed{R_2}$ STRICTLY DOMINATES ALL THE OTHERS $[R_1; R_3; R_4; R_5]$;

3) PLAYER 2's STRATEGY $\boxed{R'_4}$ IS WEAKER THAN $\boxed{R'_2}$;

4) GAME's SOLUTION

$$\boxed{R_2 \in BR(R'_2)}$$

EXERCISE 3

SOLVED THANKS TO PROFESSOR

P2

P1	1'	2'	3'	4'	5'
1	2.5, 2.5	1, 4	1, 4	1, 4	1, 4
2	4, 1	2.5, 2.5	2, 3	2, 3	2, 3
3	4, 1	3, 2	2.5, 2.5	3, 2	3, 2
4	4, 1	3, 2	2, 3	2.5, 2.5	4, 1
5	4, 1	3, 2	2, 3	1, 4	2.5, 2.5

- BY USING THE PROCESS OF ELIMINATION, YOU REACH THE NASH EQUILIBRIUM WHICH IS

$$\boxed{S_3 \in BR(S_3')}$$

EXERCISE 4

DEMAND FUNCTIONS

$$\begin{cases} Q_1 = 10 - P_1 + P_2 \\ Q_2 = 10 - P_2 + P_1 \end{cases}$$

- TOTAL MARKET CAPACITY = 20

P_1 \ P_2	2	4	8	10
2	20 ; 20	24 ; 32	32 ; 32	36 ; 20
4	32 ; 24	40 ; 40	56 ; 48	64 ; 40
8	32 ; 32	48 ; 56	80 ; 80	96 ; 80
10	20 ; 36	40 ; 64	80 ; 96	100 ; 100

* BEFORE YOU FILL THE CHART, DON'T FORGET AN ESSENTIAL STEP.

THE PROFIT → $\pi = P \cdot q$

P_1 \ P_2	8	10
8	80 ; 80	96 ; 80
10	80 ; 96	100 ; 100

- IN THIS CASE THERE ARE 2 EQUILIBRIUM

$$\begin{cases} ① \; s^1_8 \in Br(s^2_8) \quad \text{INEFFICIENT} \\ ② \; s^*_{10} \in Br(s^2_{10}) \quad \text{EFFICIENT} \end{cases}$$

EXERCISE 5

	P_2: 1	2	3
P_1: 1	0;1	9;0	2;3
2	5;9	7;3	1;7
3	7;5	10;10	3;5

- PURE STRATEGY:

- THE 3RD STRATEGY OF PLAYER 1 STRICTLY DOMINATES ALL THE OTHERS;
- THE NASA EQUILIBRIUM IS: $\boxed{S_3 \in BR(S_2)}$

- MIXED STRATEGY:

- SINCE STRATEGY $\boxed{3}$ FOR PLAYER 1, HENCE THERE IS NO EQUILIBRIUM IN MIXED STRATEGY

EXERCISE 6

R

L \ R	0	0.25	0.5	0.75	1
0	0.5, 0.5	0, 0.75	0, 0.5	0, 0.25	0, 0
0.25	0.75, 0	0.25, 0.25	$0.08\overline{3}$, $0.1\overline{6}$	0, 0	-0.05, -0.2
0.5	0.5, 0	$0.1\overline{6}$, $0.08\overline{3}$	0, 0	-0.10, -0.15	$-0.1\overline{6}$, $-0.\overline{3}$
0.75	0.25, 0	0, 0	-0.15, -0.10	-0.25, -0.25	-0.32, -0.43
1	0, 0	-0.2, -0.05	$-0.3\overline{3}$, $-0.1\overline{6}$	-0.43, -0.32	-0.5, -0.5

- 0.25 STRATEGY FOR PLAYER R KILLS (0.5 ; 0.75 ; 1) ;
- SAME FOR PLAYER L ;
- THIS IS A PRISONER DILEMMA GAME BECAUSE PLAYERS END UP WITH THE INEFFICIENT EQUILIBRIUM (0.25 ; 0.25)

EXERCISE 7

* 2 PLAYERS CASE.

B

A	15 \$	30 \$	N
15 \$	0 ; 0	-15 ; 0	15 ; 0
30 \$	0 ; -15	-15 ; -15	0 ; 0
N	0 ; 15	0 ; 0	0 ; 0

- SOLVE THE PUZZLE USING THE CONCEPT OF WEAKLY DOMINATED STRATEGY.

* 3 PLAYERS CASE.

C = 15

A \ B	15	30	N
15	-15, -15, -15	-15, 0, -15	0, 0, 0
30	0, -15, -15	-15, -15, -15	0, 0, -15
N	0, 0, 0	0, 0, -15	0, 0, 15

C = 30

A \ B	15	30	N
15	-15, -15, 0	-15, -15, -15	-15, 0, 0
30	-15, -15, -15	-20, -20, -20	-15, 0, -15
N	0, -15, 0	0, -15, -15	0, 0, 0

C = N

A \ B	15	30	N
15	0, 0, 0	-15, 0, 0	15, 0, 0
30	0, -15, 0	-15, -15, 0	0, 0, 0
N	0, 15, 0	0, 0, 0	0, 0, 0

* AGAIN, USE WEAKLY DOMINATED STRATEGIES

EXERCISE 8

A \ B	1	2	3
1	5, -5	20, -20	0, $\boxed{0}$
2	0, $\boxed{0}$	10, -10	20, -20
3	10, $\boxed{-10}$	30, -30	20, -20

• PLAY THE SIMPLE GAME → $S_3^A \in B_R(S_1^B)$

A \ B	1	2	3	
1	5	20	0	0
2	0	10	20	0
3	10	30	20	<u>10</u>
	<u>10</u>	30	20	

THERE'S A RELATION BETWEEN THE NASH EQUILIBRIUM AND THE SAFETY STRATEGY.

B

A

	1	2	3
1	20, -20	0, $\boxed{0}$	20, -20
2	0, $\boxed{0}$	10, -10	20, -20
3	20, $\boxed{-20}$	30, -30	20, $\boxed{-20}$

- NASH EQUILIBRIUM $\rightarrow R_3^A \in Br(R_1^B)$
  $\rightarrow R_3^A \in Br(R_3^B)$

- WEAKLY DOMINATED STRATEGY EQUI. $\rightarrow R_3^A \in Br(R_3^B)$

B

A

	1	2	3	
1	20	0	20	0
2	0	10	20	0
3	20	30	20	20
	20	30	20	

TWO SAFETY STRATEGIES

B

A

	1	2	3
1	20, -20	0, 0	20, -20
2	0, 0	10, -10	20, -20
3	20, -20	30, -30	0, 0

- NO NASA EQUILIBRIUM

B

A

	1	2	3	
1	20	0	20	0
2	0	10	20	0
3	20	30	0	0
	20	30	20	

NO SAFETY STRATEGIES

NASH EQUILIBRIUM REMARKS

1. IT'S NOT DINAMICAL BUT STATICAL
2. IT IS SELF-ENFORCING
   ↳ ASSUMPTION OF THE OTHER PLAYER
3. IT MIGHT HAPPEN THAT YOU HAVE NO SOLUTION IN A GAME
4. A GAME MIGHT HAVE MORE EQUILIBRIUMS.
   ↳ BROWSE ABOUT (FOCAL POINTS)
5. WEAKLY OR STRICTLY DOMINATED STRATEGIES CAN OCCUR.

EXERCISE 8

① \ ②	$\frac{2}{5}$ X	$\frac{3}{5}$ $1-X$
$\frac{2}{5}$ P	$\underline{2}$, -2	-1, $\underline{1}$
$\frac{3}{5}$ $1-P$	-1, $\underline{1}$	$\underline{1}$, -1

- PURE NASH EQUILIBRIUM : <u>THERE'S NONE</u>

- MIXED STRATEGY :

$$EU_1 = (2)\cdot X + (-1)\cdot(1-X) = -X + 1 - X$$
$$2X + X - 1 = -2X + 1$$
$$3X - 1 = -2X + 1$$
$$5X = 2 \qquad X = 2/5 \quad 1-X = 3/5$$

$$EU_2 = -2P + 1 - P = P - 1 + P$$
$$-3P + 1 = 2P - 1$$
$$5P = 2 \qquad P = 2/5 \quad 1-P = 3/5$$

$$EU_1 = \frac{2}{5}\cdot\frac{2}{5}(2) + \frac{2}{5}\cdot\frac{3}{5}\cdot(-1) + \frac{3}{5}\cdot\frac{2}{5}(-1) + \frac{3}{5}\cdot\frac{3}{5}$$
$$\frac{8}{25} - \frac{6}{25} - \frac{6}{25} + \frac{9}{25} = \boxed{\frac{1}{5}}$$

$$EU_2 = \frac{2}{5}\cdot\frac{2}{5}(-2) + \frac{2}{5}\cdot\frac{3}{5} + \frac{3}{5}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{3}{5}(-1)$$
$$-\frac{8}{25} + \frac{6}{25} + \frac{6}{25} - \frac{9}{25} = \boxed{-\frac{1}{5}}$$

EXERCISE 10

		② $\frac{2}{3}$ (x) RP	$(1-x)$ $\frac{1}{3}$ SP
① $\frac{2}{3}$	(y) RP	7.500 ; 7.500	7500 ; 8500
$\frac{1}{3}$	$(1-y)$ SP	8500 ; 7500	5500 ; 5500

a) PURE STRATEGY $\longrightarrow$ 2 STRATEGIES

$\rightarrow E_1(SP; RP)$

$\rightarrow E_2(RP; SP)$

b) MIXED STRATEGY

$$EU_1 = \frac{2}{3}\cdot\frac{2}{3}\cdot 7500 + \frac{1}{3}\cdot\frac{2}{3}\, 7500 + \frac{1}{3}\cdot\frac{2}{3}\cdot 8500 + \frac{1}{3}\cdot\frac{1}{3}\cdot 5500$$

$$EU_2 = \frac{2}{3}\cdot\frac{2}{3}\cdot 7500 + \frac{2}{3}\cdot\frac{1}{3}\, 7500 + \frac{1}{3}\cdot\frac{2}{3}\cdot 8500 + \frac{1}{3}\cdot\frac{1}{3}\cdot 5500$$

EXERCISE 11

(1) \ (2)	$\frac{5}{6}$ A (x)	P $(1-x)$ $\frac{1}{6}$
$\frac{5}{6}$ A (y)	10 ; 10	20 ; 12
P $\frac{1}{6}$ $(1-y)$	12 ; 20	6 ; 6

a) PURE STRATEGY EQUILIBRIUM $\rightarrow \begin{cases} E_1 = (P ; A) \\ E_2 = (A ; P) \end{cases}$

b)

$$EU_1 = \left(\frac{5}{6} \cdot \frac{5}{6} \cdot 10 + \frac{5}{6} \cdot \frac{1}{6} \cdot 20\right) + \left(\frac{1}{6} \cdot \frac{5}{6} \cdot 12 + \frac{1}{6} \cdot \frac{1}{6} \cdot 10\right)$$

$$\frac{250}{36} + \frac{100}{36} + \frac{60}{36} + \frac{10}{36} = 11.\overline{66}$$

$$EU_2 = \left(\frac{5}{6} \cdot \frac{5}{6} \cdot 10 + \frac{5}{6} \cdot \frac{1}{6} \cdot 20\right) + \left(\frac{1}{6} \cdot \frac{5}{6} \cdot 12 + \frac{1}{6} \cdot \frac{1}{6} \cdot 10\right)$$

$$\frac{250}{36} + \frac{100}{36} + \frac{60}{36} + \frac{10}{36} = 11.\overline{66}$$

EXERCISE 12

		② 1/3 (x) O	1/3 (y) □	1/3 (1-x-y) X
① 1/3 (t)	O	0, 0	-1, 1	1, -1
1/3 (z)	□	1, -1	0, 0	-1, 1
1/3 (1-t-z)	X	-1, 1	1, -1	0, 0

a) PURE STRATEGY → NO EQUILIBRIUM

b) MIXED STRATEGY

EXPECTED PAYOFF = $\emptyset$

EXERCISE 13

① \ ②	DOVE	HAWK
DOVE	$\frac{1}{2}V$, $\frac{1}{2}V$	0, V
HAWK	V, 0	$\frac{1}{2}V-c$, $\frac{1}{2}V-c$

a) $\frac{1}{2}V - c > 0 \Rightarrow [V > 2c]$

b) $\frac{1}{2}V - c = 0 \Rightarrow [V = 2c]$

c) $\frac{1}{2}V - c < 0 \Rightarrow [V < 2c]$

a) $\frac{1}{2}V - C > 0$

$\frac{1}{2}V$, $\frac{1}{2}V$	0, $\underline{V}$
$\underline{V}$, 0	$\underline{\frac{1}{2}V - C}$, $\underline{\frac{1}{2}V - C}$

FIGHTING (HAWK) IS A STRICTLY DOMINANT STRATEGY; SINCE $\left[\frac{1}{2}V - C < \frac{1}{2}V\right]$, THIS IS NOT THE BEST RESPONSE.

b) $\frac{1}{2}V - C = 0$

$\frac{1}{2}V$, $\frac{1}{2}V$	$\underline{0}$, $\underline{V}$
$\underline{V}$, $\underline{0}$	$\underline{\frac{1}{2}V - C}$, $\underline{\frac{1}{2}V - C}$

3 EQUILIBRIUMS

c) $\frac{1}{2}V - C < \cancel{0}$

	X	$1-X$
Y	$\frac{1}{2}V$, $\frac{1}{2}V$	$\underline{0}$, $\underline{V}$
$1-Y$	$\underline{V}$, $\underline{0}$	$\frac{1}{2}V-C$, $\frac{1}{2}V-C$

- 2 EQUILIBRIUMS IN PURE STRATEGIES ;

- MIXED STRATEGIES :

$$\hookrightarrow EU_1 = \frac{1}{2}V\cdot(X) = V\cdot(X) + \frac{1}{2}V - C(1-X)$$

$$\cancel{\frac{1}{2}VX} = \cancel{VX} + \frac{1}{2}V - C - \cancel{\frac{1}{2}V} - CX$$

$$CX = \frac{1}{2}V - C \rightarrow \boxed{X = \frac{\frac{1}{2}V - C}{C}}$$

$$1 - X = 1 - \frac{\frac{1}{2}V - C}{C} = \boxed{\frac{\frac{1}{2}V}{C}}$$

HOW TO PLAY IF THE GAME PRESENTS MULTI-PLAYERS?

$n = 10$

$t = $ (STATE VARIABLE)

n OF PLAYERS SELECTING A STRATEGY.

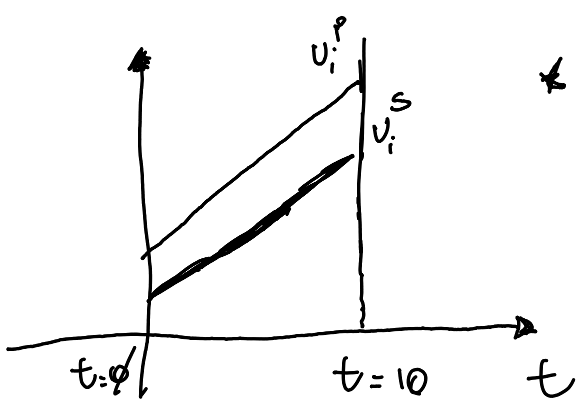
$a = 40 \qquad (P_i;\ P_i;\ S_i;\ S_i;\ P_i;\ S_i;\ S_i;\ P_i;\ P_i;\ P)$

$G.P = \quad t = 6$

$G.S = \quad t = 4$

$U_i^P = 60 + 20 \cdot (t - 1)$

$U_i^S = 40 + 20 \cdot t$



* IN THIS CASE EQUILIBRIUMS ARE AT THE EXTREMES ⚠

EXERCISE 14

a)

③ P

① \ ②	P	S
P	100, 100, 100	80, $40+\alpha$, 80
S	$40+\alpha$, 80, 80	$20+\alpha$, $20+\alpha$, 60

③ S

① \ ②	P	S
P	80, 80, $40+\alpha$	60, $20+\alpha$, $20+\alpha$
S	$20+\alpha$, 60, $20+\alpha$	α, α, α

b) IF $\boxed{\alpha = 40}$

P

	P	S
P	<u>100</u>, <u>100</u>, <u>100</u>	80, 80, 80
S	80, 80, 80	60, 60, 60

S

	P	S
P	80, 80, 80	60, 60, 60
S	60, 60, 60	40, 40, 40

BY APPLYING RESPECTIVELY THE DOMINATIONS BETWEEN STRATEGIES → THE EQUILIBRIUM IN PURE STRATEGY IS

$$E^* = (P; P; P)$$

c) IF $\boxed{\omega = 70}$

③ P

① \ ②	P	S
P	100, 100, 100	80, 110, 80
S	110, 80, 80	90, 90, 60

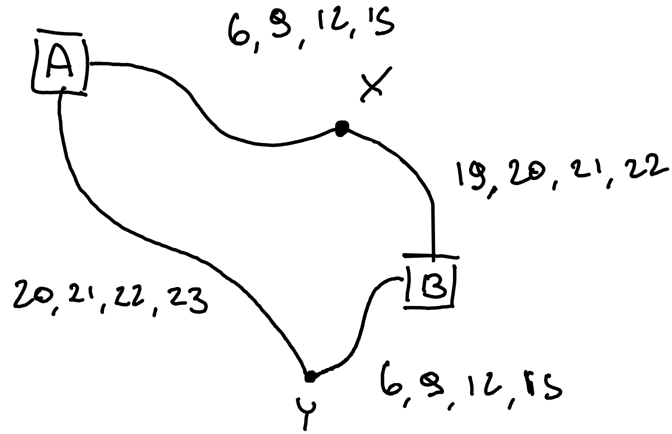
③ S

① \ ②	P	S
P	80, 80, 110	60, 90, 90
S	90, 60, 90	70, 70, 70

- APPLYING THE PROCESS OF DOMINATION, PLAYER 3 HAS NO INTEREST IN BEING PRODUCTIVE;
- THERE'S NO COOPERATION. THAT'S TO SAY, MUTUAL DEFECTION IS THE ONLY STRONG EQUILIBRIUM IN THE GAME.

THE DILEMMA, EVEN THOUGH COOPERATION PROVIDES A BETTER OUTCOME THAN DEFECTION, IS NOT THE RATIONAL OUTCOME BECAUSE COOPERATION, FROM A SELF-INTEREST PERSPECTIVE, IS IRRATIONAL.

EXERCISE 15



a) IF FOUR PEOPLE WOULD TAKE ALL THE SAME ROUTE, IT TAKES EXACTLY 37 - 38 MINUTES NO MATTER WHICH ONE EACH TO GET THERE. THUS, ONE OF THEM HAS INCENTIVE IN DEFECT BECAUSE, HE WOULD TAKE EITHER 25 OR 26 MINUTES TO REACH THE FINAL DESTINATION.

THAT BEING SAID, FOR ANY OTHER ALLOCATION OF PEOPLE TO ROUTES, AT LEAST ONE PERSON CAN REDUCE TRAVEL TIME BY SWITCHING ROUTES.

b) AFTER ANALYSED THE PREVIOUS POINT, IT SEEMS THAT THE EQUILIBRIUM IS 2 PEOPLE EACH ROUTE. IN THAT CASE EVERYONE WILL TAKE 29 - 30 MINUTES TO REACH ~~THE~~ DESTINATION. IT IS AN EQUILIBRIUM BECAUSE THERE'S NO BETTER ALLOCATION WITHOUT INCREASING THE TRAVEL TIME.

③ = X

① \ ②	X	Y
X	-37, -37, -37	-29, -26, -29
Y	-26, -29, -29	-30, -30, -25

③ = Y

① \ ②	X	Y
X	-28, -29, -26	-25, -30, -30
Y	-30, -25, -30	-34, -34, -34

THERE ARE THREE NASH EQUILIBRIUMS IN PURE STRATEGIES. $[E_i = (P_1 ; P_2 ; P_3)]$

$E_1 = (X ; Y ; X)$

$E_2 = (Y ; X ; X)$ $\quad$ $E_3 = (X ; X ; Y)$

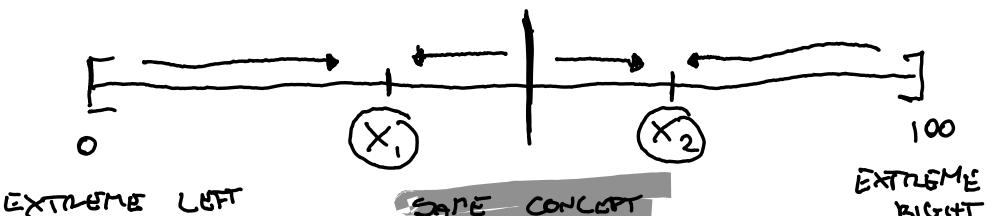
MARKET COMPETITION

- BERTRAND CONCEPT
- COURNOT CONCEPT

ALREADY MASTERED

POLITICAL COMPETITION

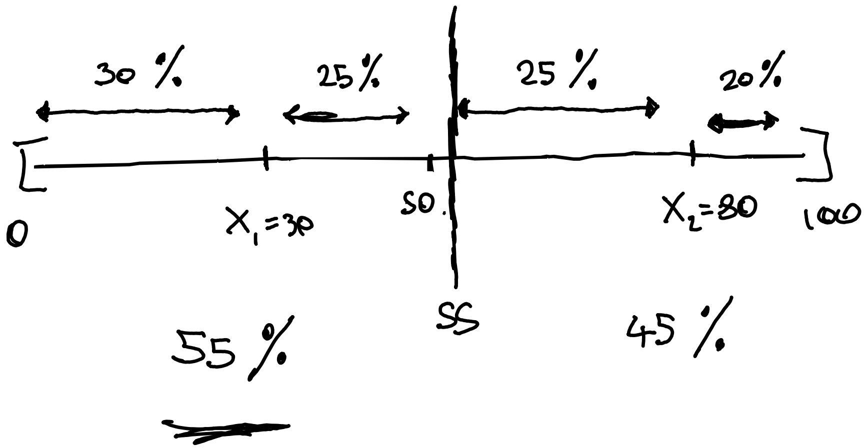
- HOTELLING - LINE CONCEPT



SAME CONCEPT OF MARGINAL CONSUMER

NEXT PAGE FOR A REAL APPLICATION

IS $(x_1 ; x_2) = (30 ; 80)$ A NE ?

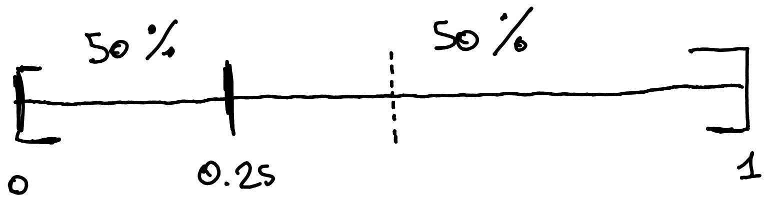


CANDIDATE 1
IS THE WINNER

★ BUT THIS
IS NOT A NE

- SINCE THE VOTERS ARE DISTRIBUTED UNIFORMELLY THE EQUILIBRIUM IS IN THE MIDDLE WHEN $(x_1 ; x_2) = (50 ; 50)$

NON - UNIFORMELY DISTRIBUTION VARIANT



- IF $X_1 = X_2 = 0.5$

THIS IS NOT A NE, BECAUSE CANDIDATES HAVE INTEREST IN GETTING CLOSER TO THE CROWD.

- IF $X_1 = 0{,}4 \quad X_2 = 0.5$

$U_1 > 50\%$ BUT THIS IS NOT A NE.
THE 2ND HAS INCENTIVE TO REACT ⚠

- IN THIS CASE, THE NE IS THE

[MEDIAN VOTER] POINT. SO, $X_1 = X_2 = 0{,}25$

PRACTICE BEFORE MID-TERM EXAM

①

P_2

P_1

	A	B	C
A	45, 45	15, 50	10, 40
B	50, 15	40, 40	15, 45
C	40, 10	45, 15	35, 35

a) STRATEGY $\boxed{B}$ STRICTLY DOMINATES STRATEGY $\boxed{A}$ FOR BOTH PLAYERS. BESIDES, STRATEGY $\boxed{C}$ DOMINATES STRATEGY $\boxed{B}$ FOR BOTH WHEN THERE'S A 2×2 MATRIX LEFT.

b) THE NE IN PURE STRATEGY IS

$$S^1_c \in Br(S^2_c)$$

(2)

a)

$\boxed{P_3} = B$

		$\boxed{P_2}$ B	$\boxed{P_2}$ NB
$\boxed{P_1}$	B	15, 15, 15	17.5, 0, 17.5
	NB	0, 17.5, 17.5	0, 0, 45

$\boxed{P_3} = NB$

		$\boxed{P_2}$ B	$\boxed{P_2}$ NB
$\boxed{P_1}$	B	17.5, 17.5, 0	45, 0, 0
	NB	0, 45, 0	0, 0, 0

b) NASH EQUILIBRIUM IN PURE STRATEGIES OCCURS WHEN

ALL THREE PLAYERS BUY A TICKET

③

$\boxed{C}$

		$\frac{4}{9}$ x TOD	$(1-x)$ $\frac{5}{9}$ TOM
$3/9$ a $\boxed{S}$	TOD	5, $\underline{10}$	$\underline{8}$, 4
$(1-a)$ $6/9$	TOM	$\underline{10}$, 5	4, $\underline{8}$

a) NO NE IN PURE STRATEGIES ⚠

b) • $U_1 = 5x + 8(1-x) = 10x + 4(1-x)$

$\hookrightarrow 5x + 8 - 8x = 10x + 4 - 4x$ $\qquad x = 4/9$

$8 - 3x = 6x + 4 \rightarrow 9x = 4 \rightarrow (1-x) = 5/9$

• $U_2 = 10a + 5(1-a) = 4a + 8(1-a)$

$\hookrightarrow 10a + 5 - 5a = 4a + 8 - 8a$ $\qquad a = 3/9$

$5a + 5 = -4a + 8 \rightarrow 9a = 3 \rightarrow (1-a) = 6/9$

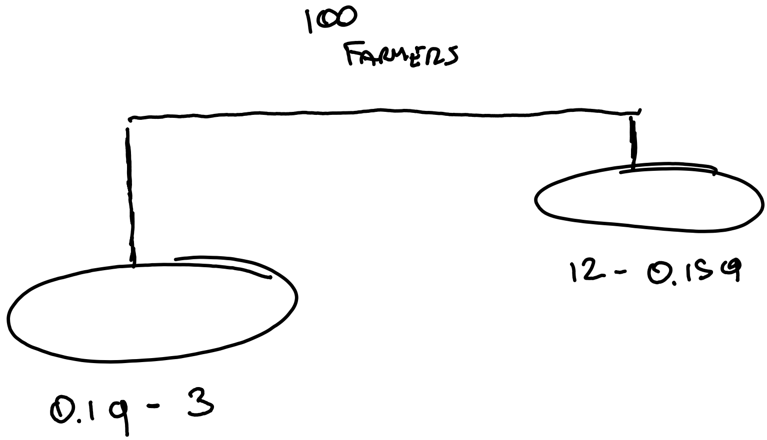
• $U_1 = \left(\frac{3}{9} \cdot \frac{4}{9} \cdot 5\right) + \left(\frac{3}{9} \cdot \frac{5}{9} \cdot 8\right) + \left(\frac{6}{9} \cdot \frac{4}{9} \cdot 10\right) + \left(\frac{6}{9} \cdot \frac{5}{9} \cdot 4\right)$

$\hookrightarrow \frac{60}{81} + \frac{120}{81} + \frac{240}{81} + \frac{120}{81} \rightarrow \frac{540}{81} \rightarrow 6.\overline{66}$

• $U_2 = \left(\frac{4}{9} \cdot \frac{3}{9} \cdot 10\right) + \left(\frac{5}{9} \cdot \frac{3}{9} \cdot 4\right) + \left(\frac{4}{9} \cdot \frac{6}{9} \cdot 5\right) + \left(\frac{5}{9} \cdot \frac{6}{9} \cdot 8\right)$

$\hookrightarrow \frac{120}{81} + \frac{60}{81} + \frac{120}{81} + \frac{240}{81} = \frac{540}{81} \rightarrow 6.\overline{66}$

④



a) • $12 - 0.15 \cdot 100 \rightarrow$ UTILITY OF $\boxed{-3}$

IT'S NOT BECAUSE FARMERS HAVE INCENTIVE TO DEFECT AND GET A BETTER RESULT.

b)

q	DEEP	SHALLOW
49	1.9	4.35
50	2	4.5
51	2.1	4.65

IT IS NOT BECAUSE FARMERS SITUATED IN DEEP AQUIFERS WANT TO GO IN SHALLOW ONES.

c) EQ $\rightarrow$ $12 - 0.15q = 0.1q - 3$

$$0.25q = 15$$

$$\boxed{q^* = 60}$$

PROBLEM SET 5

1

$$U_1 = a_1 \cdot (10 + a_2 - a_1)$$

$$U_2 = a_2 \cdot (10 + a_1 - a_2)$$

$$U_1 = 10a_1 + a_1 a_2 - a_1^2$$

$$\hookrightarrow 10 + a_2 - 2a_1 = \emptyset$$

$$2a_1 = 10 + a_2$$

$$\boxed{a_1 = 5 + 0.5\, a_2} \rightarrow$$ REACTION CURVE

$$\shortparallel$$

$$\boxed{a_2 = 5 + 0.5\, a_1}$$

$$\begin{cases} a_1 = 5 + 0.5\, a_2 \\ a_2 = 5 + 0.5\, a_1 \end{cases} \quad \begin{cases} a_1 = 5 + 2.5 + 0.25\, a_1 \\ // \end{cases} \quad \begin{cases} a_1 = 10 \\ a_2 = 10 \end{cases}$$

PLAYERS'S PAYOFF AT EQUILIBRIUM

⚠

2 $P = 140 - (q_1 + q_2)$ $CT_i = 20 q_i$

a) $\Pi_1 = (140 - q_1 - q_2) \cdot q_1 - 20 q_1$

$\to 140 - 2q_1 - q_2 - 20 = \emptyset$

$\to 120 - q_2 = 2q_1$

$\to \boxed{q_1^* = 60 - 0.5 q_2}$

SAME FOR FIRM 2

$\to \boxed{q_2^* = 60 - 0.5 q_1}$

b) $\begin{cases} q_1 = 60 - 0.5 q_2 \\ q_2 = 60 - 0.5 q_1 \end{cases}$ $\begin{cases} q_1 = 60 - 30 + 0.25 q_1 \\ // \end{cases}$ $\begin{cases} q_1 = 40 \\ q_2 = 40 \end{cases}$

c) $\begin{cases} q_1 = 60 - 0.5 q_2 - 0.5 q_3 \\ q_2 = 60 - 0.5 q_1 - 0.5 q_3 \\ q_3 = 60 - 0.5 q_1 - 0.5 q_2 \end{cases}$ $q_1 = q_2 = q_3 = 30$

(3) $P = 140 - Q_T$

$Q_T = q_1 + q_2$

$CT_i = 20 q_i$

d.) $\Pi = P \cdot Q - CT$

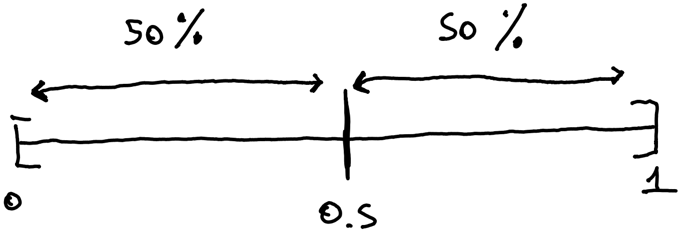
e) $P = c'$ $\rightarrow$ $P = 20$

$Q_T = 120$ $q_1 = q_2 = 60$

THIS IS THE NE, BECAUSE IN BERTRAND, THE FIRMS ENGAGE IN A PRICE UNDER-CUTTING BATTLE. (BERTRAND PARADOX) ⚠

4

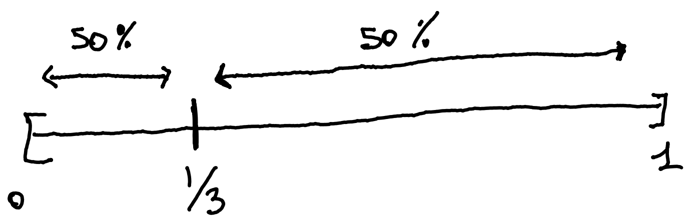
a)



- IN THIS SITUATION, BOTH CANDIDATES CHOOSE TO PLACE IN THE MIDDLE SO TO SPLIT EQUALLY THE VOTER [MEDIAN VOTER POINT]

THAT'S A NE EQUILIBRIUM

b)

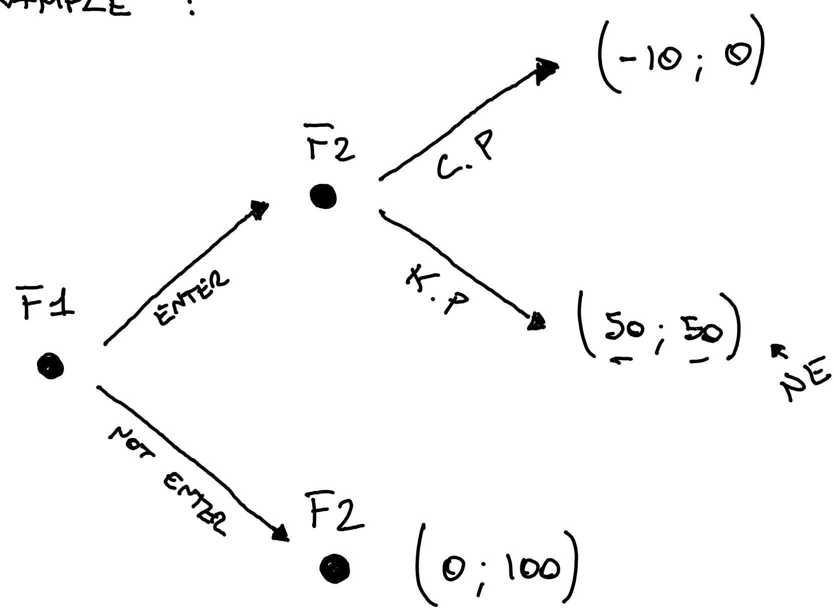


- THE EQUILIBRIUM IS JUST RIGHT AT THE POINT $\boxed{1/3}$

DYNAMIC GAMES WITH C. INFO.

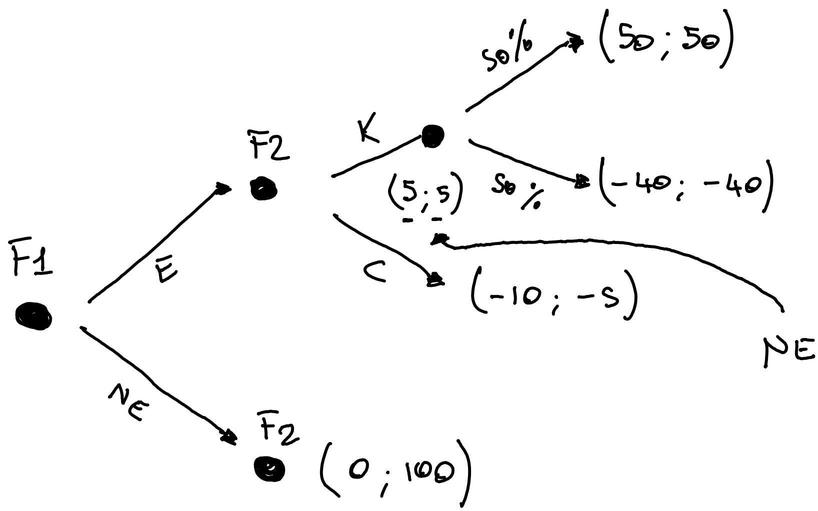
- COMPONENTS :
  - PLAYERS
  - TIME AND SEQUENCES OF THE DECISIONS
  - CHANCE
  - PLAYERS KNOW ABOUT PREVIOUS DECISIONS
  - PAYOFFS

- EXAMPLE :



- BACKWARD INDUCTION

- EXAMPLE WITH ALEATORY VARIABLE :



- IF PLAYERS ARE NEUTRAL TO RISK, THEY ANALYZE RANDOM EVENT TAKING INTO ACCOUNT THE EXPECTED MONETARITY OF THE EVENT.

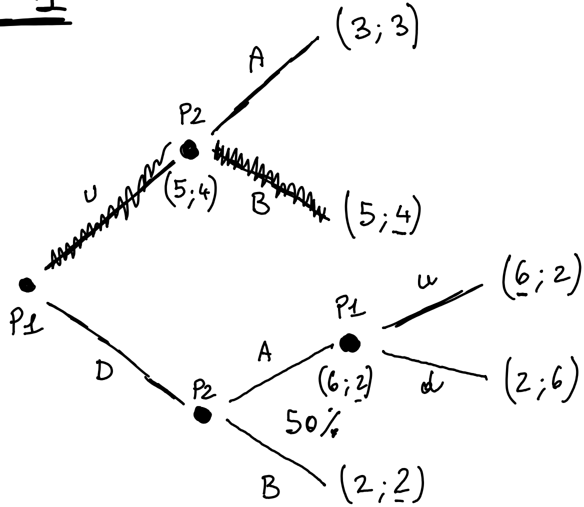
$$EU_i = \frac{1}{2} \cdot 50 + \frac{1}{2} \cdot (-40) = \boxed{5}$$

- **THERE ARE 2 TYPES OF EQUILIBRIUM**

1. SUPPORTED BY BACKWARD INDUCTION METHOD
2. SUPPORTED BY <u>THREATS</u> → IRRATIONAL BEHAVIOUR

PROBLEM SET 6

- EXERCISE 1



	A	B
Dw	6,2	2,2
Dd	2,6	2,2

a) POTENTIALLY, THERE ARE 2 POSSIBLE STRATEGIES.

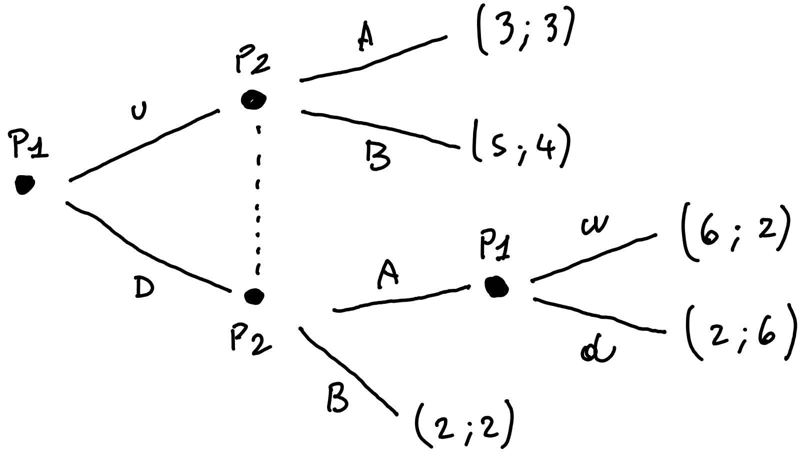
1. $E_1 = (U; B)$ E_2 IF $= (D; A; w)$

b)

P2

P1	AA	AB	BA	BB
Uw	3, 3	3, 3	5, 4	5, 4
Ud	3, 3	3, 3	5, 4	5, 4
Dw	6, 2	2, 2	6, 2	2, 2
Dd	2, 6	2, 2	2, 6	2, 2

THERE ARE FOUR NASH EQUILIBRIUMS



c) P1 MOVES FIRST;
P2 WITHOUT KNOWING P1 MOVES DECIDES BETWEEN A OR B;
IF P1 CHOOSES D, FOLLOWED BY P2 WHO CHOOSES A AND P1 CHOOSES u, THE GAME ENDS.

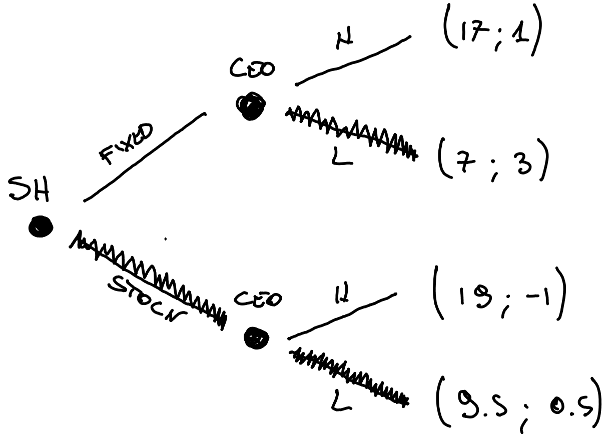
d)

	A	B
Uu	3 ; 3	5 ; 4
Ud	3 ; 3	5 ; 4
Du	6 ; 2	2 ; 2
Dd	2 ; 6	2 ; 2

THERE ARE THREE NASH EQUILIBRIUMS ⚠

EXERCISE 2

a)



b) BACKWARD INDUCTION EQUILIBRIUM

$\hookrightarrow E_i = (\text{STOCK} ; \text{LOW})$

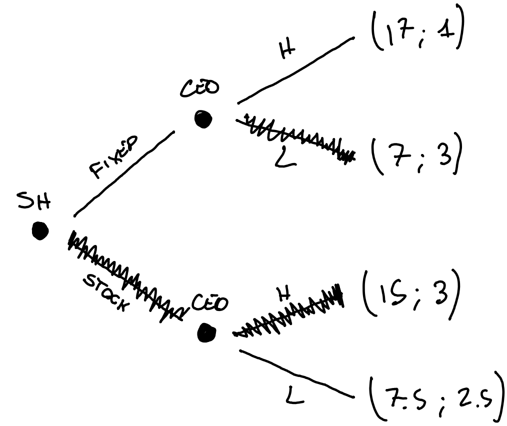
c)

CEO

SH	H H	H L	L H	L L
FIXED	17 ; 1	17 ; 1	7 ; 3	7 ; 3
STOCK	19 ; -1	9.5 ; 0.5	19 ; -1	9.5 ; 0.5

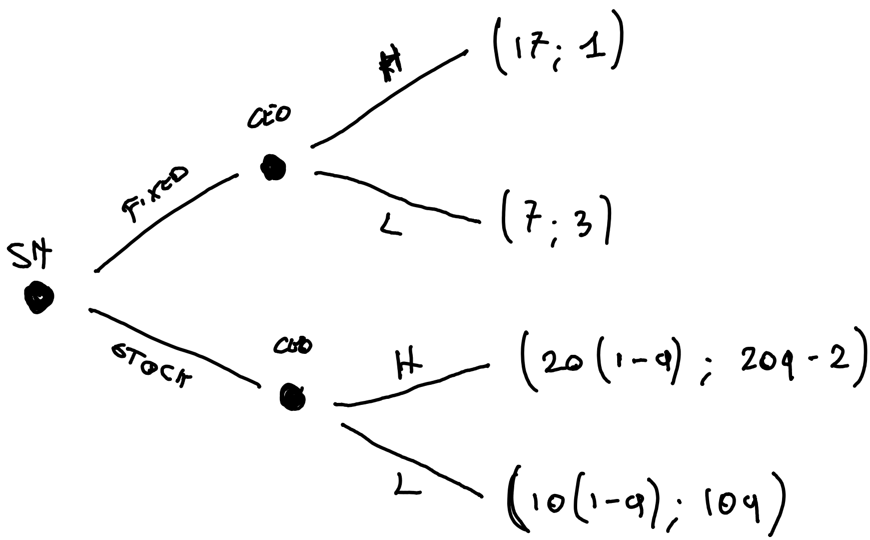
THERE'S ONE NASH EQUILIBRIUM ⚠

d)



THE EQUILIBRIUM CHANGES

↳ $E_i = (\text{STOCK}; \text{HIGH})$



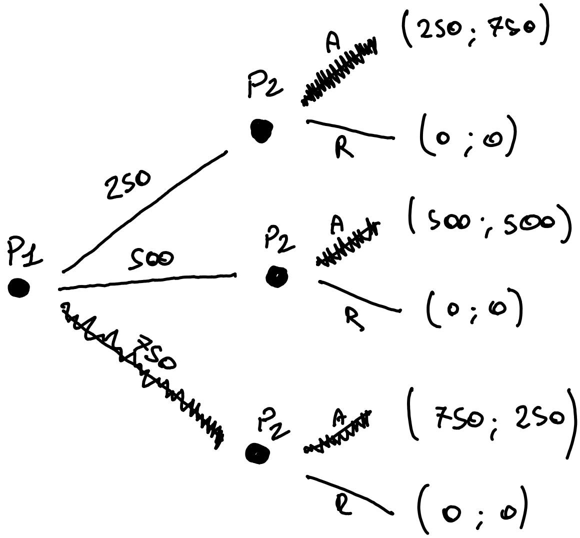
e)

$$20q - 2 \geq 10q$$

$$10q \geq 2 \rightarrow \boxed{q \geq \frac{1}{5}}$$

EXERCISE 3

a)



b) BACKWARD INDUCTION SOLUTION

↳ $E_i : (750; 250)$

$\boxed{P_2}$

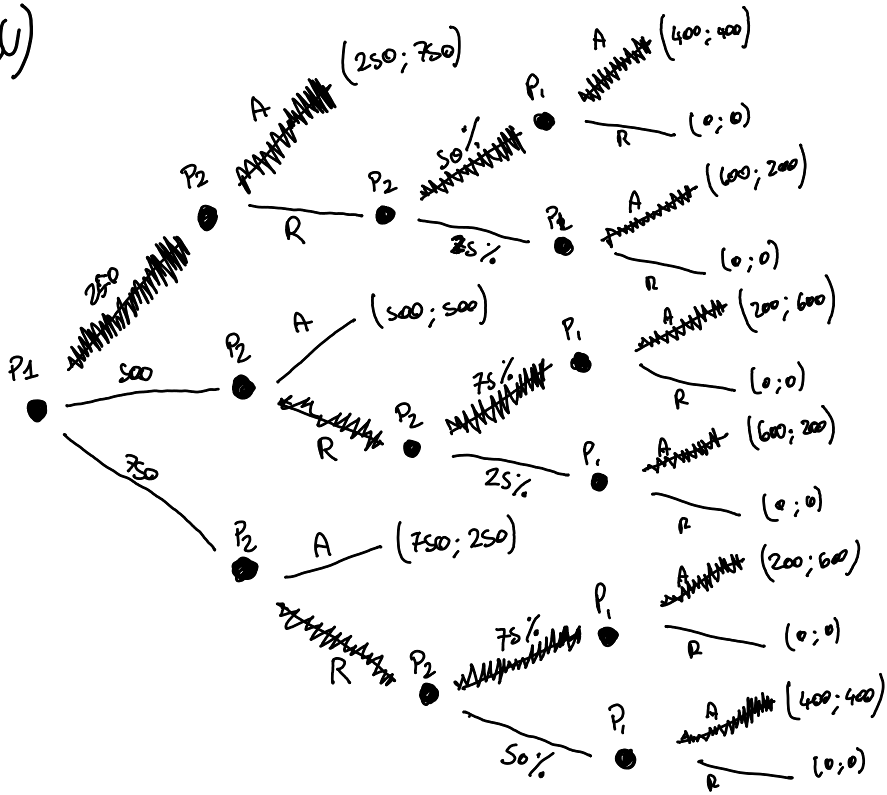
c)

$\boxed{P_1}$

	AAA	AAR	ARA	ARR	RRR	RAA	RAR	RRA
250	250; 750	250; 750	250; 750	250; 750	0; 0	0; 0	0; 0	0; 0
500	500; 500	500; 500	0; 0	0; 0	0; 0	500; 500	500; 500	0; 0
750	750; 250	0; 0	750; 250	0; 0	0; 0	750; 250	0; 0	750; 250

SUBGAME EQUILIBRIUM ⚠

d)



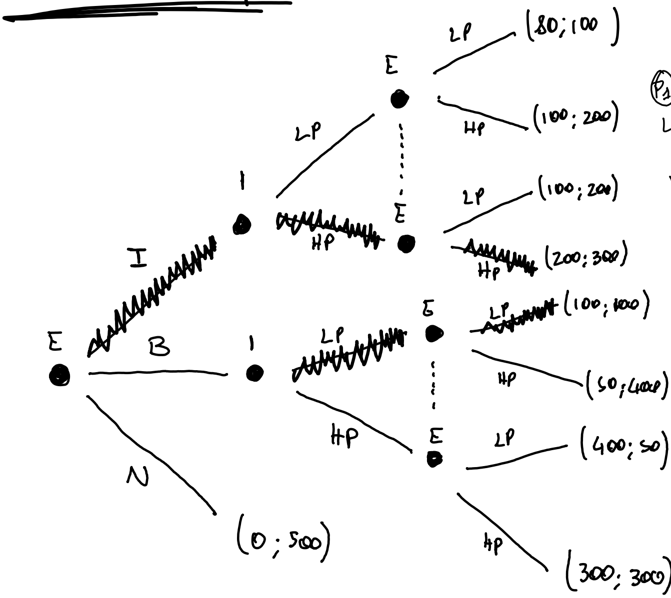
- BACKWARD INDUCTION $\rightarrow$ $E_i = (250; 750)$

- $P_1 = 3 \cdot 2^6$
  $P_2 = 3^3$

- SUBGAME EQUILIBRIUM $(250; 750)$

- INTRODUCING A $\boxed{\text{THREAT}}$

EXERCISE 4



P1 rows, P2 columns:

	LP	HP
LP	80 ; 100	100 ; 200
HP	100 ; 200	200 ; 300

P1 rows, P2 columns:

	LP	HP
LP	100 ; 100	400 ; 50
HP	50 ; 400	300 ; 300

- E MOVES FIRST ;
  FOLLOWED BY I ;
  THEN AGAIN E

- BACKWARD INDUCTION SOLUTION

  $\hookrightarrow E_i = (HP ; HP ; I)$

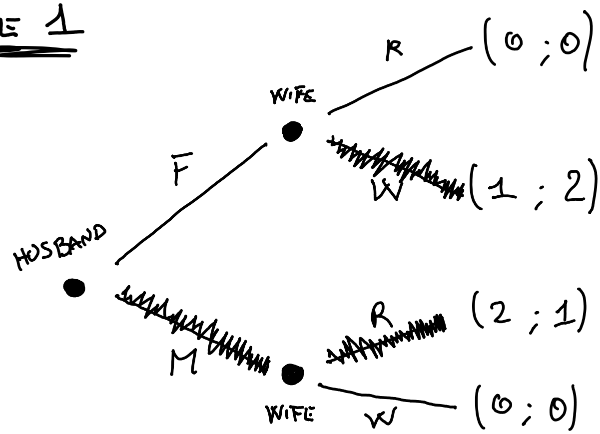
0, 500	0, 500	0, 500	0, 500
0, 500	0, 500	0, 500	0, 500
0, 500	0, 500	0, 500	0, 500
0, 500	0, 500	0, 500	0, 500
300, 300	300, 300	50, 400	50, 400
300, 300	300, 300	50, 400	50, 400
400, 50	400, 50	100, 100	100, 100
400, 50	400, 50	100, 100	100, 100
200, 300	100, 200	200, 300	100, 200
100, 200	80, 100	100, 200	80, 100
200, 300	100, 200	200, 300	100, 200
100, 200	80, 100	100, 200	80, 100

- THERE ARE FOUR NASA EQUILIBRIUMS ⚠

PROBLEM SET 7

EXERCISE 1

a)



b) BACKWARD INDUCTION SOLUTION

↳ $E_1 = ($MEAT ; RED WINE$)$

P2

c)

P1	R R	R W	W R	W W
MEAT	2 ; 1	2 ; 1	0 ; 0	0 ; 0
FISH	0 ; 0	1 ; 2	0 ; 0	1 ; 2

$\boxed{\text{3 NE}}$

d) $S_H = ($MEAT$)$ $S_W = ($RED ; WHITE$)$

e) FISH WITH WHITE WINE IS AN EQUILIBRIUM BUT HUSBAND DOESN'T WANT TO CHANGE TO MEAT BECAUSE WIFE IS THREATENING HIM ⚠

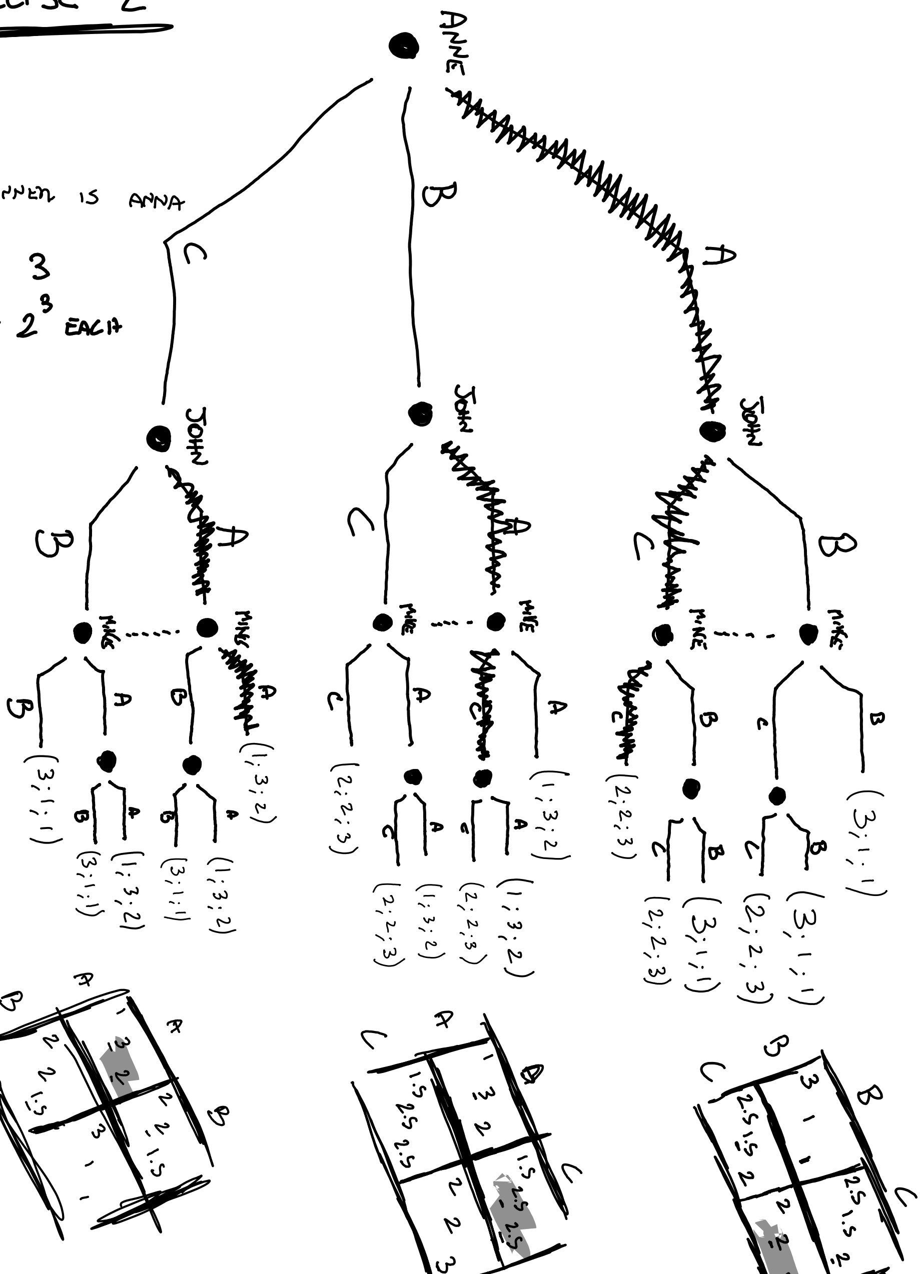
EXERCISE 2

a)

b) THE WINNER IS ANNA

c) ANNE = 3

MIKE AND JOHN = 2^3 EACH



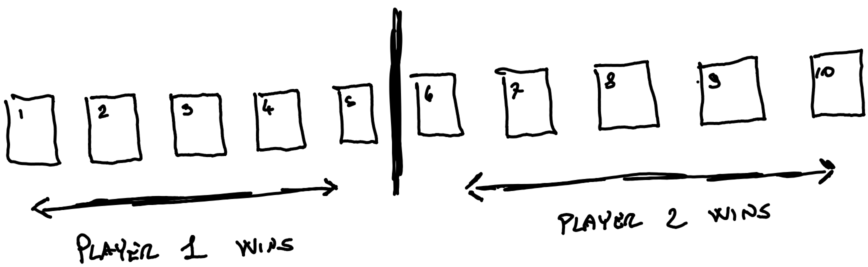
d) ANNE = (A)

JOHN = (C ; A ; A)

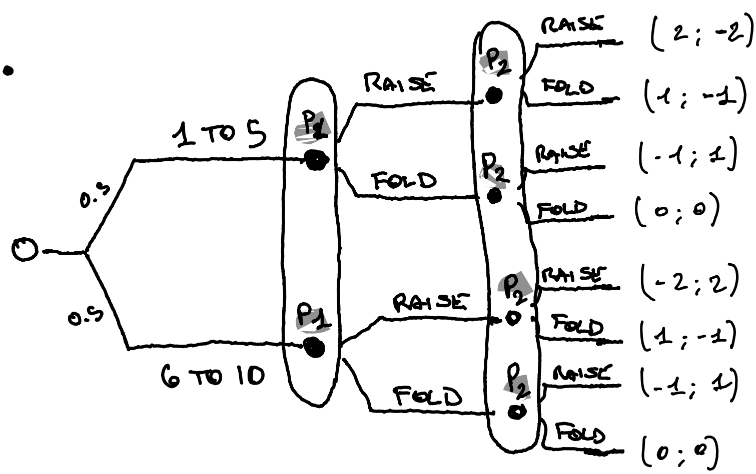
MIKE = (C ; C ; A)

GAMES WITH INCOMPLETE INFORMATION

- PICK TEN CARDS.
- ENTRY FEE OF 1$.



- ONCE DRAWN A CARD, PLAYERS CAN RAISE THE BET OR FOLD.
-



	R	F
R	0 ; 0	1 ; -1
F	-1 ; 1	0 ; 0

PROBLEM SET 8

RECALL STACKELBERG CONCEPT

EXERCISE 1

a)

$$\Pi_m = P \cdot q - 10 \cdot q$$

$$\Pi_R = 200 - \left(\frac{q}{100}\right) \cdot q - P_x \cdot q$$

$$200 - \frac{q^2}{100} - P_x \cdot q \rightarrow = 200 - \frac{q}{50} - P_x = \emptyset$$

$$= q = (200 - P_x) \cdot 50$$

$$= \boxed{q = 10000 - 50 P_x}$$

REACTION CURVE

$$\Pi_m = P_x \cdot (10000 - 50 P_x) - 10 \cdot (10000 - 50 P_x)$$

$$= 10000 P_x - 50 P_x^2 - 100000 + 500 P_x$$

$$= 10000 - 100 P_x + 500 = \emptyset \rightarrow P_x = 10500/100$$

$$= \boxed{105} \qquad \boxed{q^* = 4750}$$

$$\Pi_m = 451250 \ \$$$

$$\Pi_R = 225625 \ \$$$

b)

$$\Pi_m = 200 - \left(\frac{q}{100}\right) \cdot q - 10 \cdot q$$

$$= 200 - \frac{q}{50} - 10 = \emptyset \rightarrow q = 9500 \qquad P = 105$$

$$\Pi = 9500 \cdot 105 - 10 \cdot 9500$$

$$\rightarrow 902500 \ \$$$

EXERCISE 2

RECALL COURNOT CONCEPT

$P = a - q_1 - q_2$

$f_1 = \frac{2a^3}{81}$

[PROFIT FUNCTIONS]

$\Pi_1 = (a - q_1 - q_2) \cdot q_1 - \frac{2a^3}{81}$

$\Pi_2 = (a - q_1 - q_2) \cdot q_2$

[REACTION CURVES]

$\begin{cases} q_1 = \frac{a - q_2}{2} \\ q_2 = \frac{a - q_1}{2} \end{cases}$ $\begin{cases} q_1 = 0.5a - 0.5q_2 \\ q_2 = 0.5a - 0.5q_1 \end{cases}$ $\begin{cases} q_1 = 0.5a - 0.25a + 0.25q_1 \\ // \end{cases}$

$\begin{cases} q_1 = \frac{a}{3} \\ q_2 = \frac{a}{3} \end{cases}$

$\Pi_1 = \frac{a^2}{9} - \frac{2a^3}{81}$

$\stackrel{!}{=} \frac{2a}{9} - \frac{6a^2}{81} \rightarrow \frac{18a - 6a^2}{81} = \emptyset$

$6a^2 = 18a$ $\boxed{a = 3}$

$\boxed{q_1 = q_2 = 1}$

COOPERATIVE GAMES

THERE ARE DIFFERENT WAYS TO SPLIT OUTCOMES.

1ST APPROACH PROPORTIONAL

$$(x_1 ; x_2) = \left(\frac{100}{300} \cdot 60 ; \frac{200}{300} \cdot 60\right)$$

2nd APPROACH EGALITARIUM

$$(x_1 ; x_2) = (50\% ; 50\%)$$

NO MATTER THE STAKE

~~GT~~

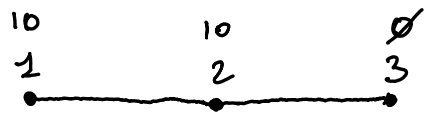
3rd APPROACH EGALITARIUM OF THE SURPLUS

SHAPLEY VALUE

GT

PAYING THE PLAYERS ACCORDING TO HER CONTRIBUTION TO THE GAME.

EXERCISE (NEIGHBORING GAMES)



$$
\left.\begin{array}{l}
V(1) = V(2) = 10 \quad V(3) = \emptyset \\
V(12) = 30 \quad V(13) = 10 \quad V(23) = 30 \\
V(123) = 50
\end{array}\right\}
$$

POSSIBLE STRATEGIES

SHAPLEY VALUE

THIS IS THE GAME CORE.

	1	2	3
123	10	20	20
132	10	40	∅
213	20	10	20
231	20	10	20
312	10	40	∅
321	20	30	∅
$SH(v) = \frac{\Sigma}{6}$	15	25	10

EXERCISE (VOTING GAMES)

VOTES REQUIRED TO WIN.

$$[W_1 = 20 \,;\; W_2 = 30 \,;\; W_3 = 50 \,;\; q = 51]$$

$$\left.\begin{array}{l} V(1\,2\,3) = 1 \\ V(1) = V(2) = V(3) = \emptyset \\ V(1\,2) = \emptyset \quad V(1\,3) = 1 \quad V(2\,3) = 1 \end{array}\right\} \text{POSSIBLE STRATEGIES}$$

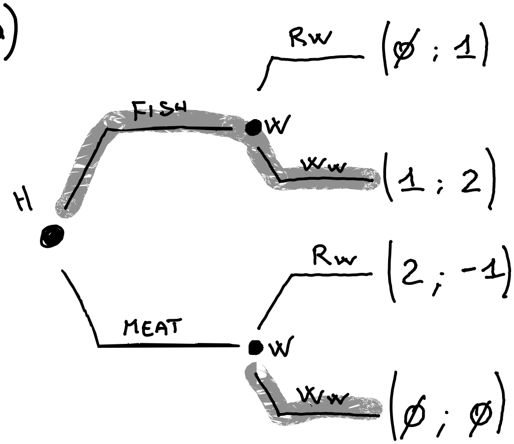
THIS IS NOT THE GAME CORE

	1	2	3
123	$\emptyset$	$\emptyset$	1
132	$\emptyset$	$\emptyset$	1
213	$\emptyset$	$\emptyset$	1
231	$\emptyset$	$\emptyset$	1
312	1	$\emptyset$	$\emptyset$
321	$\emptyset$	1	$\emptyset$
SH(V)	1/6	1/6	2/3

MOCK TEST

EXERCISE 1

a)



b) BACKWARD INDUCTION

↓

HUSBAND CHOOSES FISH AND HIS WIFE PICKS WHITE WINE.

b²) • HUSBAND HAS 2 STRAGIES

(FISH ; MEAT)

• WIFE HAS 2^2 STRATEGIES

(Rw; Rw)(Rw; Ww)(Ww; Rw)(Ww; Ww)

c)

HUSBAND \ WIFE	Rw Rw	Rw Ww	Ww Rw	Ww Ww
FISH	Ø ; 1	Ø ; 1	1 ; 2	1 ; 2
MEAT	2 ; -1	Ø ; Ø	2 ; -1	Ø ; Ø

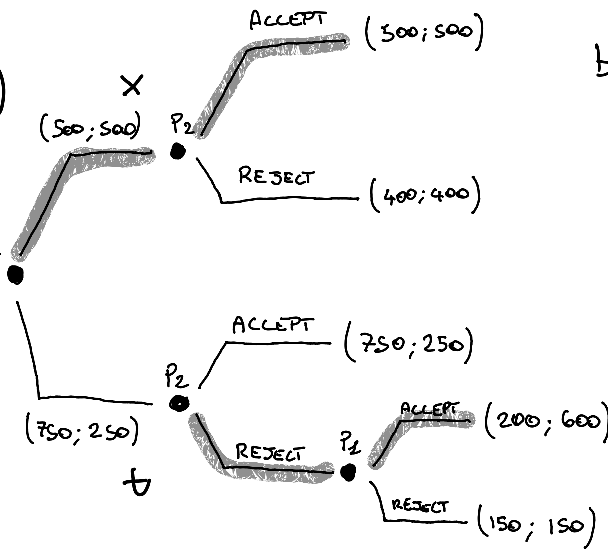
OW

d) SPNE → HUSBUND (FISH) ; WIFE (Ww ; Ww)

e) ALTHOUGH HE PREFERS MEAT, HIS WIFE DOESN'T ACCOMODATE HIS DESIRE FOR THAT GOOD.
A THREAT IN THIS CASE, IT IS NOT CREDIBLE BECAUSE HIS WIFE COULD BE ADAMANT ON HER CHOICE OF Ww.

EXERCISE 2

a)



b) CLEARLY, BY REJECTING THE SECOND PROPOPOSAL MADE BY PLAYER 1, PLAYER 2 ASSURED AN EQUAL DEAL.

ON

c) P_1 HAS 2^2 STRATEGIES

P_2 HAS 2^2 STRATEGIES

SPNE

P_1 \ P_2	ACC ACC	ACC REJ	REJ ACC	REJ REJ
X_{ACC}	500, 500	500, 500	400, 400	400, 400
X_{REJ}	500, 500	500, 500	400, 400	400, 400
t_{ACC}	750, 250	200, 600	750, 250	200, 600
t_{REJ}	750, 250	150, 150	750, 250	150, 150

EXERCISE 3

P1 \ P2	COOP	BLOCK
COOP	10, 10	5, Ø
BLOCK	Ø, 5	Ø, Ø

a) THIS IS NOT A PRISONER'S DILEMMA.
BOTH PLAYERS DECIDED TO COOPERATE IN ORDER TO REACH A COMMON GOAL. IN THIS CASE, THE EFFICIENT NASH EQUILIBRIUM.

b)

- EACH ~~PLAYER HAS 4 STRATEGIES~~.
- THIS IS AN EQUILIBRIUM, BECAUSE IT IS THE BEST OUTCOME THEY CAN GET WITH 20 EACH.
- THEY ARE NOT INTERESTED IN DEFECTION, BECAUSE IT WILL BRING A NULL PAYOFF.
- THEY ARE NOT IN EQUILIBRIUM, BECAUSE THERE IS A BETTER ALLOCATION.

EXERCISE 4

a) PLAYER 1 HAS 2 STRATEGIES
PLAYER 2 HAS 4 STRATEGIES

b)

P_1 \ P_2	AA	AB	BA	BB
A	4.5, 4.5	6.5, 6.5	7, 5	9, 7
B	7, 9	5, 7	5.5, 5.5	3.5, 3.5

c)

P_1 \ P_2	A	B
A	5, 5	10, 6
B	6, 10	3, 3