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*The assignment is to be sent using the link provided in canvas. Please generate your report using R Markdown, as presented in class (all relevant code needs to be visible).*

**Problem 1. (30 points)** Assume we have two boxes of balls. In box A, we have 50 balls in total and 10 of them are red. In box B, we have 30 balls in total and 18 of them are red. Now we randomly select a box and draw two balls out of it without replacement.

- (a) What's the probability to have first ball red?
- (b) Given that the first draw is a red ball, what's the probability to have second draw is a red ball as well?
- (c) Assuming we are drawing balls from box A five times with replacement. What's the probability to draw the second red ball at third try?

**Problem 2. (50 points)** For this problem, use the Yahoo! Finance website (<http://finance.yahoo.com/>) or a Bloomberg terminal. Choose an equity and download its daily prices for the whole year of 2017.

- (a) Read the input file into a variable and print its first lines of data using the command `head`.
- (b) Study how the `plot` command works (*Hint*: try typing `?plot` in R), and make a plot of the daily close prices (if you downloaded data from Yahoo! Finance, use the Adj. Close column). Please include a title and labels for each of the axes in the plot.
- (c) With the close prices once again, compute the corresponding series of log-returns. Make another plot and report the summary statistics using the command `summary`.  
*Hint 1:*  $r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln S_t - \ln S_{t-1}$   
*Hint 2:* The use of the function `diff` may make your life much easier.
- (d) Do a box plot of the log-returns and verify if there are any outliers.
- (e) Do a normal qqplot with the log-returns series and comment your results.

**Problem 3: (20 points)** In this problem, you need to calculate the expression of expectation and variance for  $X$  under different probability distribution:

- (a) Assume  $X \sim \mathbf{U}(a, b)$ , i.e.,

$$f(x) = \frac{1}{b-a} \mathbf{1}_{\{x \in (a,b)\}}$$

calculate the variance of  $\mathbf{X}$ .

- (b) Assume  $X \sim \mathbf{Exp}(\lambda)$ ,

$$f(x) = \lambda e^{-\lambda x} \mathbf{1}_{\{x \geq 0\}}$$

Calculate the expectation and variance of  $\mathbf{X}$ .

(c) Assume  $X \sim \mathbf{Poisson}(\lambda)$ ,

$$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \text{ when } x \in \{0, 1, 2, \dots\}.$$

Calculate the expectation of  $\mathbf{X}$ .

Bonus (5 points) Assume  $X \sim \mathbf{Poisson}(\lambda)$ , calculate the variance of  $\mathbf{X}$ .