"I pledge my honor that I have abided by the Stevens Honor System." N. Colonna

HW #2

Nicholas Colonna 2/12/18

1.

A	B
10 R 40 B	18 R 12 B

a) $P(\text{First Red}) = \frac{1}{2}\left(\frac{10}{50}\right) + \frac{1}{2}\left(\frac{18}{30}\right) = 0.4$

b) $P(\text{Red } 2^{nd} \,\&\, \text{Red } 1^{st}) = \frac{1}{2}\left(\frac{10}{50}\right)\left(\frac{9}{49}\right) + \frac{1}{2}\left(\frac{18}{30}\right)\left(\frac{17}{29}\right) = 0.1942$

c) $P(2^{nd} R \text{ on } 3^{rd}) = \left[\binom{2}{1}\left(\frac{10}{50}\right)\left(\frac{40}{50}\right)\right] \times \left(\frac{10}{50}\right) = 0.064$

2. Check R Markdown report in submission.

3.a) $X \sim U(a,b)$ Calculate variance of X

$$f(x) = \frac{1}{b-a}\mathbb{1}_{\{x \in a,b\}}$$

$E[X] = \frac{b+a}{2} \leftarrow$ proven in class

$$V(X) = E[X^2] - E[X]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\int_{-\infty}^{\infty} x f(x)\,dx\right)^2 = \int_a^b x^2 \frac{1}{b-a}\mathbb{1}_{[a,b]}(x)\,dx - \left(\int_a^b x \frac{1}{b-a}\mathbb{1}_{[a,b]}(x)\,dx\right)^2$$

$$= \frac{1}{b-a}\left(\frac{1}{3}x^3\right)\Big]_a^b - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{1}{3}\frac{b^3-a^3}{b-a} - \frac{b^2+a^2+2ba}{4}$$

$$= \frac{b^2+a^2+ba}{3} - \frac{b^2+a^2+2ba}{4}$$

$$= \frac{4b^2+4a^2+4ba}{12} - \frac{3b^2+3a^2+6ba}{12} = \frac{b^2+a^2-2ba}{12} = \boxed{\frac{(b-a)^2}{12} = V(X)}$$

$X \sim Exp(\lambda)$

b) $f(x) = \lambda e^{-\lambda x} \mathbb{1}_{\{x \geq 0\}}$ Calculate $E[X]$ & $V(X)$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$
$$= \cancel{\int_{-\infty}^{0} x \lambda e^{-\lambda x} \mathbb{1}_{\{x\geq 0\}} dx}^{\,0} + \int_0^{\infty} x \lambda e^{-\lambda x} \mathbb{1}_{\{x\geq 0\}} dx$$
$$= \int_0^{\infty} x \lambda e^{-\lambda x} dx$$
$u = x \quad dv = \lambda e^{-\lambda x}$
$du = 1 \quad v = -e^{-\lambda x}$
$$= -xe^{-\lambda x}\Big]_0^{\infty} - \int_0^{\infty} -e^{-\lambda x} dx$$
$$= 0 + \int_0^{\infty} e^{-\lambda x} dx = -\frac{1}{\lambda} e^{-\lambda x}\Big]_0^{\infty}$$
$$= \frac{1}{\lambda}$$
$$\boxed{\therefore E[X] = \frac{1}{\lambda}}$$

$$V(X) = E[X^2] - E[X]^2$$
$$V(X) = \int_{-\infty}^{\infty} x^2 f(x)\,dx + \Big(\underbrace{\int_{-\infty}^{\infty} x f(x)\,dx}_{E[X]}\Big)^2$$
$$= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx - \left(\frac{1}{\lambda}\right)^2$$
$u = x^2 \quad dv = \lambda e^{-\lambda x}$
$du = 2x \quad v = -e^{-\lambda x}$
$$= \left(-x^2 e^{-\lambda x}\Big]_0^{\infty} - \int_0^{\infty} -2x e^{-\lambda x} dx\right) - \left(\frac{1}{\lambda}\right)^2$$
$u = -2x \quad dv = e^{-\lambda x}$
$du = -2 \quad v = \frac{1}{\lambda} e^{-\lambda x}$
$$= \left(-x^2 e^{-\lambda x}\Big]_0^{\infty} - \left[\frac{-2x}{\lambda} e^{-\lambda x}\Big]_0^{\infty} - \int_0^{\infty} \frac{2}{\lambda} e^{-\lambda x} dx\right]\right) - \left(\frac{1}{\lambda}\right)^2$$
$$= 0 + 0 + \frac{-2}{\lambda^2} e^{-\lambda x}\Big]_0^{\infty} - \frac{1}{\lambda^2}$$
$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$
$$\boxed{\therefore V(X) = \frac{1}{\lambda^2}}$$

$X \sim \text{Poisson}(\lambda)$

c) $f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$, when $x \in \{0, 1, 2, \ldots\}$ Calculate $E[X]$

$$E[X] = \sum_{x=0}^{\infty} x f(x) = \sum_{x=0}^{\infty} x \frac{\lambda^x}{x!} e^{-\lambda}$$

$$= 0 + \sum_{x=1}^{\infty} x \frac{\lambda^x}{x!} e^{-\lambda}$$

$$= \sum_{x=1}^{\infty} \cancel{x} \frac{\lambda^x}{\cancel{x}(x-1)!} e^{-\lambda} = \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} e^{-\lambda}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$ want to manipulate to match denominator $(x-1)$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = \lambda e^{-\lambda} \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots + \frac{\lambda^\infty}{\infty!} \right]$$

looks like $\frac{\lambda^x}{x!}$

$$= \lambda e^{-\lambda} \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^x}{x!}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$= \lambda e^{-\lambda} [e^{\lambda}]$$

$$= \lambda$$

$$\boxed{\therefore E[X] = \lambda}$$

<u>Bonus:</u> Calculate $V(X)$ of X if $X \sim \text{Poisson}(\lambda)$

Assignment 2

Nicholas Colonna

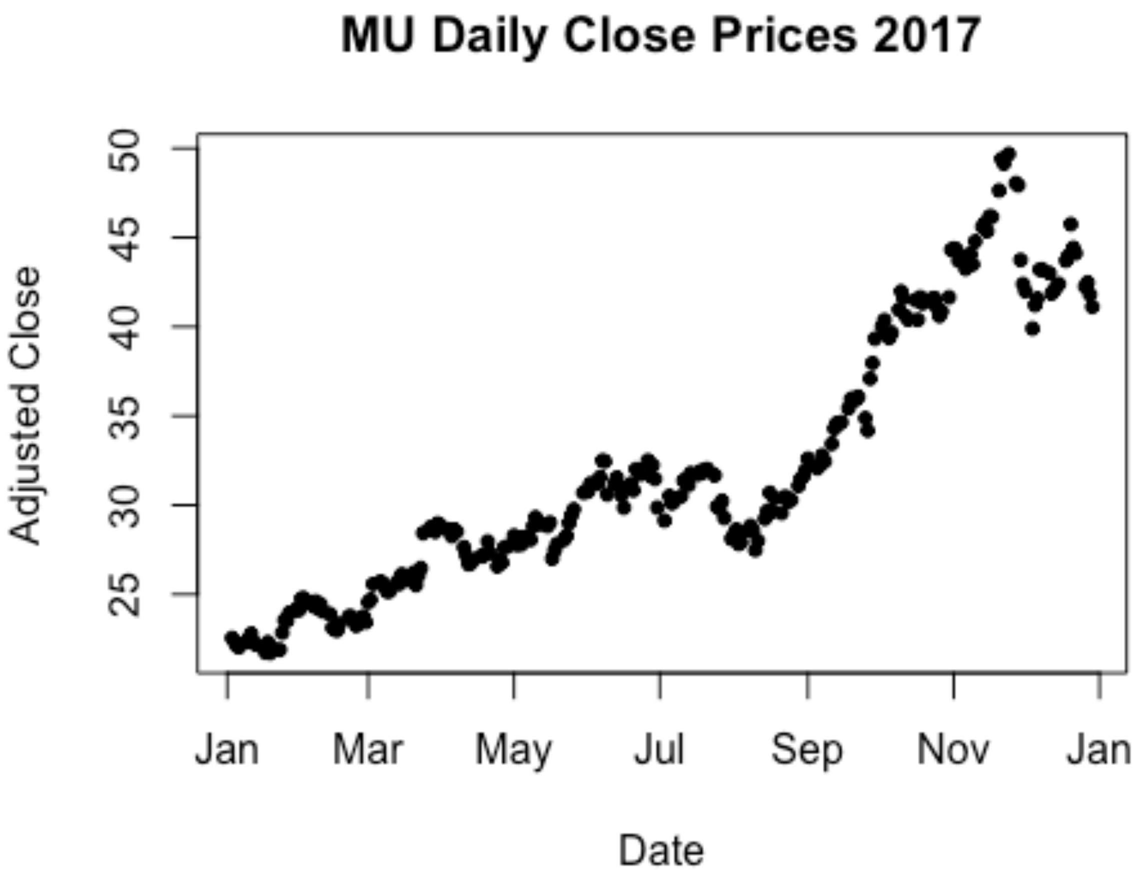
2/11/2018

Problem 2

```
#(a) Read the input file into a variable and print its first lines of data
using the command head.
micron <- read.csv('MU.csv', header=TRUE)
micron$Date <- as.Date(micron$Date, "%m/%d/%y")
head(micron)
```

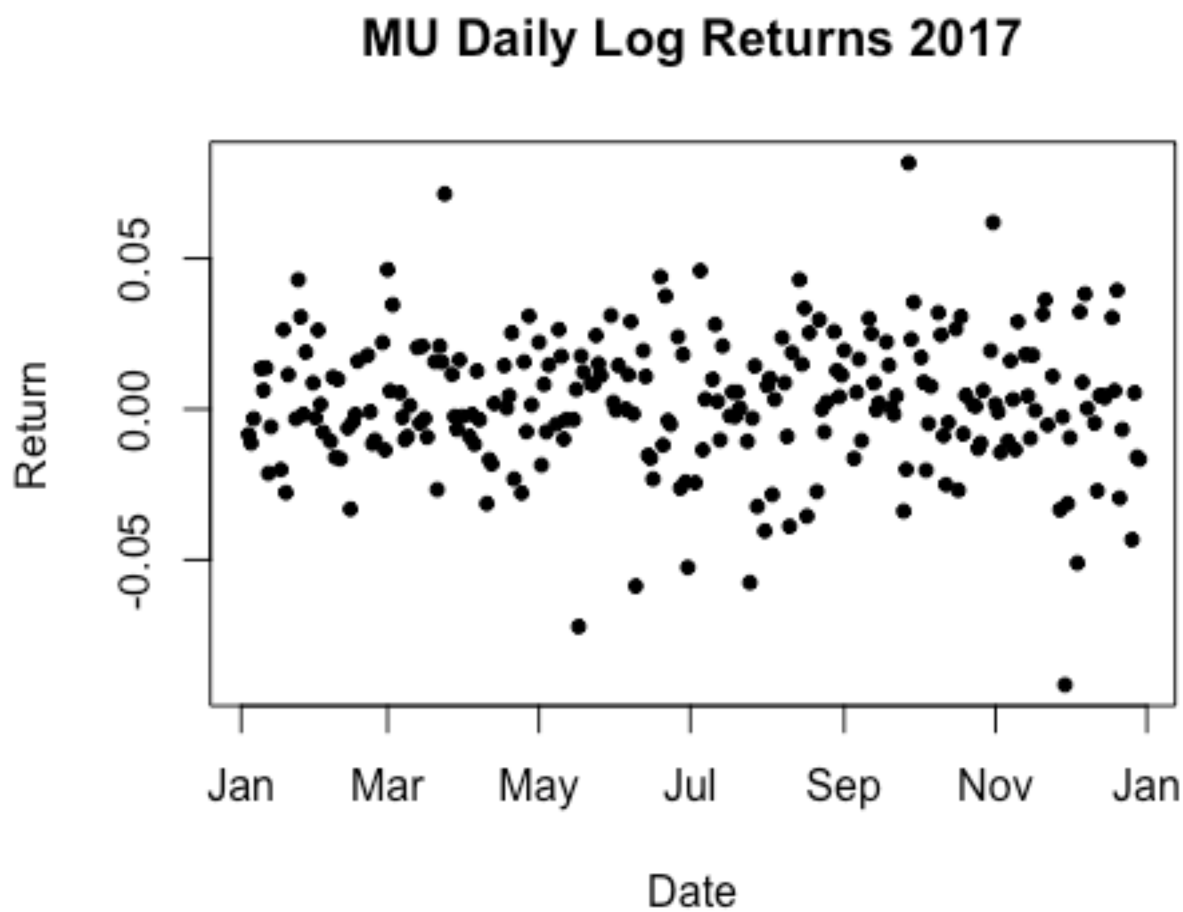
```
##         Date Adj.Close
## 1 2017-01-03     22.55
## 2 2017-01-04     22.36
## 3 2017-01-05     22.11
## 4 2017-01-06     22.04
## 5 2017-01-09     22.34
## 6 2017-01-10     22.48
```

```r
#(b) Plot the daily close prices
plot(micron$Date, micron$Adj.Close, main='MU Daily Close Prices 2017',
xlab='Date', ylab='Adjusted Close', pch=20)
```



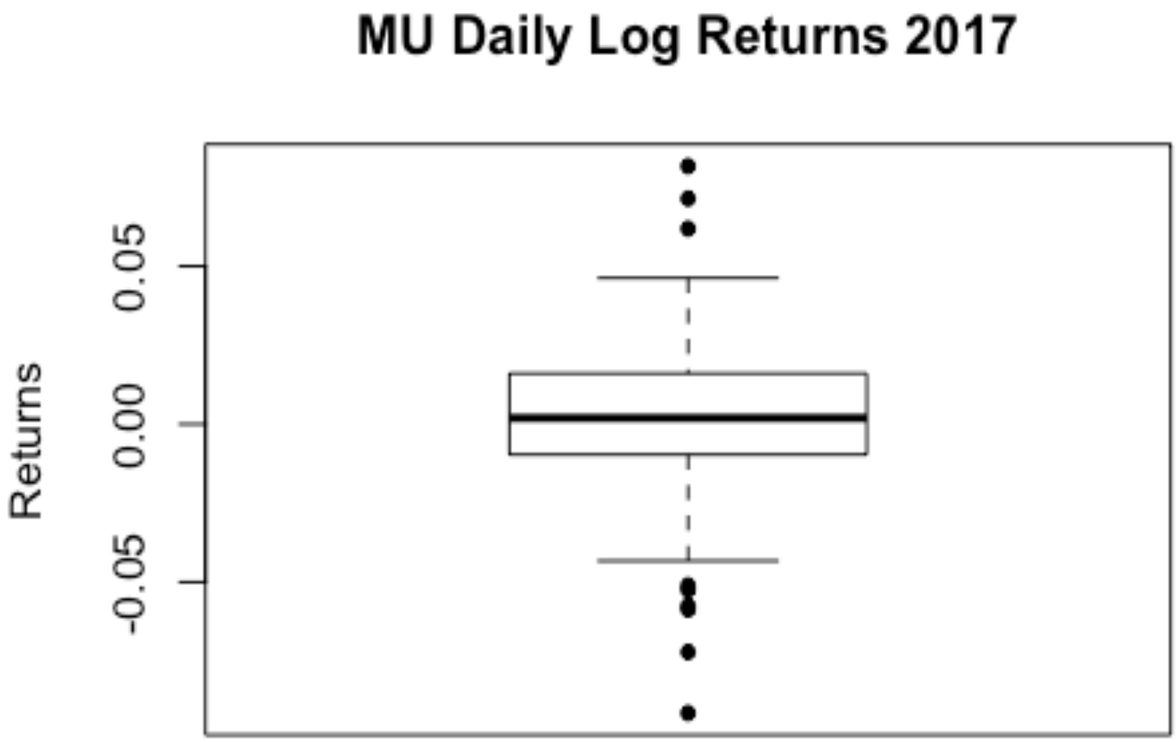

```r
#(c) Compute the corresponding series of log-returns. Make another plot and
report the summary statistics using the command summary.
micron["Returns"] <- NA
i <- 1
while(i <= length(micron$Adj.Close)){
  if(i==1){
    micron$Returns[i] <- NA
  }else{
    micron$Returns[i] <- log(micron$Adj.Close[i])-log(micron$Adj.Close[i-1])
  }
  i <- i+1
}
```

```r
plot(micron$Date, micron$Returns, main='MU Daily Log Returns 2017',
xlab='Date', ylab='Return', pch=20)
```



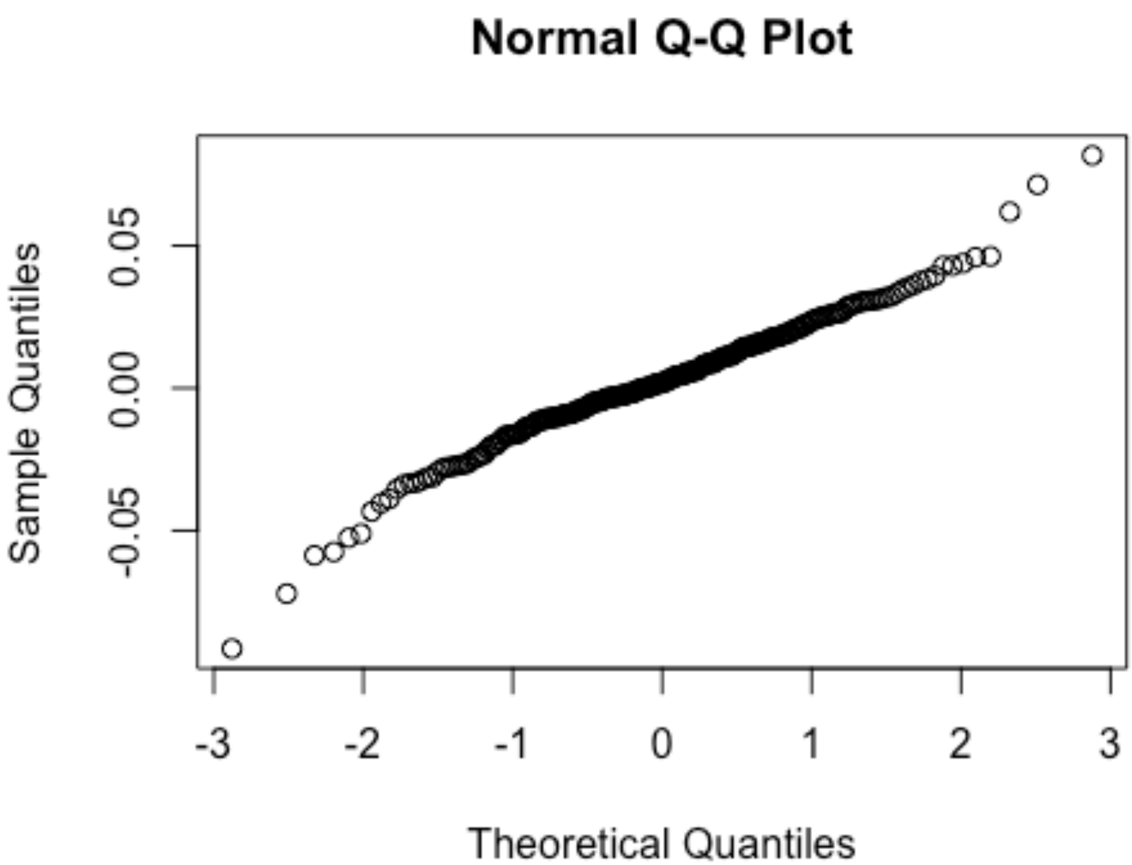

```r
summary(micron$Returns)

##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.      NA's
## -0.091480 -0.009610  0.001947  0.002403  0.016030  0.081710         1
```

```r
#(d) Do a box plot of the log-returns and verify if there are any outliers.
boxplot(micron$Returns, main='MU Daily Log Returns 2017', ylab='Returns',
pch=20)
```




```r
# Since there are a few points outside of the plot, it means that we have
outliers in the returns.
```

```r
#(e) Do a normal qqplot with the log-returns series and comment your resuts.
qqnorm(micron$Returns)
```




```r
# Since the data points fall along a straight, ascending line on the normal
qqplot, with a majority of the points in the interval [-1,1], it would be a
safe assumption to say the log-returns of Micron are normally distributed
```