Stevens Institute of Technology | **Homework 4**
QF 202 – Introduction to Financial Time Series | **Due Date: April 24th, 2018**

The assignment is to be sent using the link provided in canvas. Please generate your report using R Markdown, as presented in class (all relevant code needs to be visible).

Problem 1. (20 points) Given the following AR(2) model:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + a_t \, , \; a_t \sim N(0, \sigma^2)$$

Compute:

(a) $\mathbf{E}[r_t \mid \mathcal{F}_{t-1}]$.

(b) $\mathbf{Var}[r_t \mid \mathcal{F}_{t-1}]$.

Problem 2. (80 points) Using the same AR(2) model from Problem 1, answer:

(a) Compute $\mathbf{E}[r_{t+2} \mid \mathcal{F}_t]$.

(b) Evaluate your result from **(a)** for the following values:

- $\phi_0 = 0.001$, $\phi_1 = 0.5$, and $\phi_2 = 0.3$;
- $r_t = 0.015$, and $r_{t-1} = 0.005$;
- $\sigma^2 = 0.0001$.

(c) Using the provided AR(2) model and the values provided in **(b)**, generate 1000 realizations of r_{t+2}, compute the mean and compare your results with the previous answer.

(d) Now, instead of generating 1000 realizations of r_{t+2}, use the same setup to create a time series with 2000 values (r_{t+1}, r_{t+2}, ..., r_{t+1999}, r_{t+2000}). Report the summary statistics using the command `summary`.

(e) Compute the PACF of the time series created in **(d)**. What is the recommended AR order given the results of the PACF?

(f) Use R's `ar` function to estimate the parameters of the time series created in **(d)**. Do these parameters approximate the real (known) parameter values?

Bonus 1. (10 points) Using the same AR(2) model from Problem 1, compute:

(a) $\mathbf{E}[r_t \mid \mathcal{F}_{t-2}]$.

(b) $\mathbf{Var}[r_t \mid \mathcal{F}_{t-2}]$.

Bonus 2. (10 points) Given the following AR(1) model:

$$r_t = \phi_0 + \phi_1 r_{t-1} + a_t \,,\; a_t \sim N(0, \sigma^2)$$

And assuming r_t *stationary*, compute:

(a) $\mathbf{E}[r_t]$ (the unconditional expected value).

(b) $\mathbf{Var}[r_t]$ (the unconditional variance).

Bonus 3: (10 points) Prove that

$$\gamma_1 = Cov(r_t, r_{t-1}) = E[(r_t - \mu)(r_{t-1} - \mu)]$$

, and for a general l,

$$\gamma_l = E[(r_t - \mu)(r_{t-l} - \mu)]$$