Eclipsing Binary Stars CE - Winter 2018

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1 Introduction

The purpose of this simulation is to calculate the change in magnitude over time of an eclipsing binary star. Each star in the binary pair will be defined by its radius, mass, and temperature while their mutual orbit will be defined by their semi-major axis, eccentricity, inclination, and longitude of the node. Based on their initial velocities and positions, and the acceleration caused by their mutual gravity, their motion will be calculated using explicit Euler integration. Their positions and luminosities will be used to calculate the dip in magnitude during an eclipse based on what percentage of a star's area is being covered by the other at a given time.

2 Initial Information

2.1 Stellar Characteristics

Each star is defined by a mass in kg, a radius in meters, and a temperature in kelvin. If the star is not on the main sequence (e.g., a white dwarf), its luminosity must be input manually. Otherwise it can be calculated by the Stefan-Boltzmann Equation:

$$L = \sigma A T^4$$

where:

L is the luminosity in watts
A is the area in m^2
T is the temperature in kelvin
σ is the Stefan-Boltzmann constant ($5.670 \times 10^{-8} W \cdot m^{-2} \cdot K^{-4}$)

2.2 Semi-major axis and eccentricity

The major axis of an ellipse is a line which runs through its center and both of its foci. It is also the largest diameter. The semi-major axis a is half of this value. Similarly the minor axis of an ellipse is its smallest diameter, and the semi-minor axis b is half this value. The eccentricity e is a measure of how non-circular a conic section such as an ellipse is. An eccentricity of 0 denotes a circle, where the semi-major and semi-minor axes are equal. An eccentricity between 0 and 1 (non-inclusive) is an ellipse.

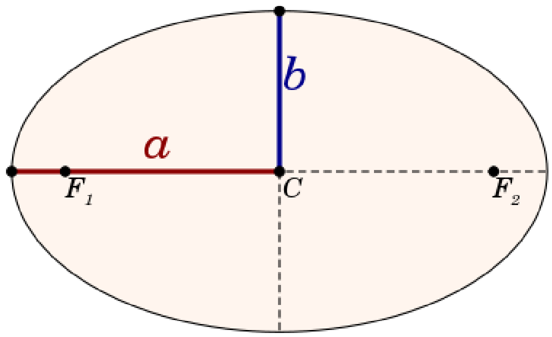

Figure 1: The axes of an ellipse (image from Wikipedia)

The eccentricity, semi-major axis, and semi-minor axis of an ellipse are related through the equation:

$$b = a\sqrt{1 - e^2}$$

2.3 Inclination and longitude of the node

The inclination i of a binary star system is the angle between its orbital plane and its plane of reference. The longitude of the node Ω of an orbit is the point between 0 and 180 degrees where the orbiting object passes through the plane of reference. For a binary star, the reference plane is always tangential to the celestial sphere, or perpendicular to a line drawn from the Earth to the binary.

3 Calculations

3.1 Initial Velocity

Based on the initial distance between the stars and the characteristics of their orbit, the initial velocity is calculated using:

$$v = \sqrt{GM\left(\frac{1}{r} - \frac{1}{a}\right)}$$

where:

v is the initial velocity in m/s
G is the gravitational constant 6.674×10^{-11} $\mathrm{m}^3 \cdot \mathrm{k}^{-1} \cdot \mathrm{s}^{-2}$
M is the sum of the stars' masses in kg
r is the initial distance between them in meters
a is the semi-major axis of their orbit in meters

3.2 Gravitational Acceleration

The magnitude of the gravitational force between the two stars can be calculated by using Newton's Law of Universal Gravitation.

$$F = \frac{Gm_1m_2}{r^2}$$

where:

F is the force in newtons
G is the gravitational constant
m_1 and m_2 are the masses of the stars in kg
r is the distance between them in meters.

From this force, the acceleration due to gravity for each star can be calculated using Newton's Third Law.

$$a = \frac{m}{F}$$

where

a is the acceleration of the star in m/s^2
m is the mass of the star in kg
F is the gravitational force acting on the star in newtons.

3.3 Integration

Each iteration, the current acceleration of a star due to gravity is used to calculate its current velocity, which is in turn used to calculate its current position.

$$\begin{cases} v = a \times dt + v_0 \\ p = v \times dt + p_0 \end{cases}$$

where:

dt is the time step (in seconds) used between iterations
a is the acceleration vector of the star due to gravity in m/s^2
v is the velocity vector of the star in m/s
v_0 is the velocity vector of the star from the previous iteration
p is the position vector of the star in m
p_0 is the position vector of the star from the previous iteration

3.4 Eclipse

To simulate an eclipse, the stars are treated as two circles in a 2D plane. The percentage of the eclipsed area of one of the stars is used to calculate the percent reduction of its luminosity. The resultant change in magnitude (unitless) is then calculated and compared to observed data.

$$M_* = -2.5 \log_{10} \frac{L_*}{3.0128 \times 10^{28}\,\mathrm{W}}$$

where:

M_* is the bolometric magnitude of the star.
L_* is the bolometric luminosity of the star in Watts
3.0128×10^{28} W is a constant (zero-point luminosity)

4 Results

Algol

Orbital period:	
Simulated:	2.866 days or 2d 20h 47m
Expected:	2.867 days
Percent error:	0.034%
Change in magnitude *	
Simulated:	1.38
Expected:	1.3
Percent error:	7.103%

VV Cephei

Orbital period:	
Simulated:	20.4 years or 20y 155d 17h
Expected:	20.4 years
Percent error:	0%
Change in magnitude *	
Simulated:	0.44
Expected:	0.5
Percent error:	5.682%

R Canis Majoris

Orbital period:	
Simulated:	1.178 days or 1d 4h 17m
Expected:	1.1359 days
Percent error:	3.57%
Change in magnitude *	
Simulated:	0.64
Expected:	0.64
Percent error:	0%

* magnitude is a logarithmic scale where the lower the magnitude the brighter the object. A difference in magnitude x is equal to a difference in brightness of 2.512^x fold. Percent error values are based on a comparison of the simulated vs expected changes in brightness and not a comparison of magnitudes.

Images

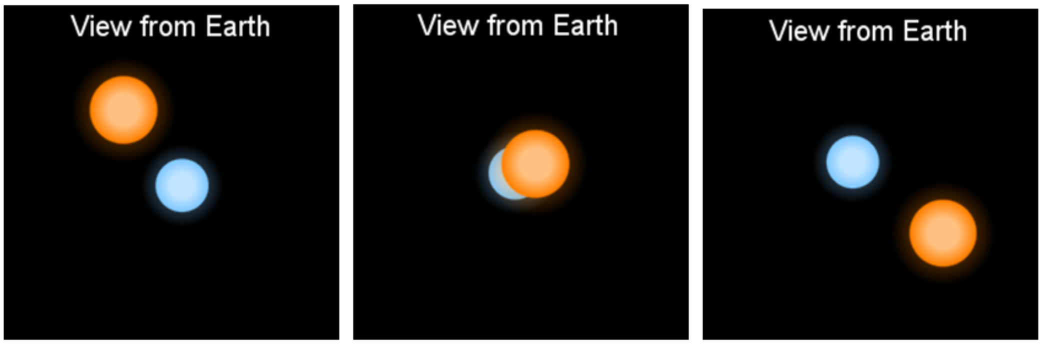

Figure 2: Simulated images of Algol

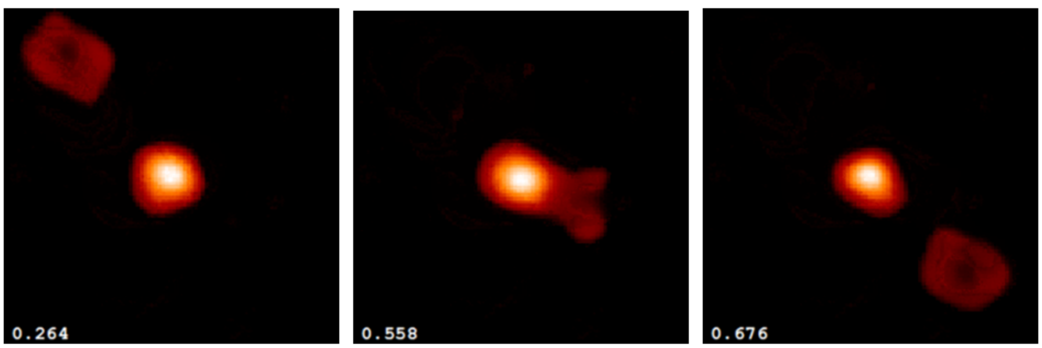

Figure 3: Images of Algol from the CHARA interferometer

5 Discussion and Conclusion

The simulation proved accurate with all of the binaries eclipsing each other, and most of the binaries falling almost within 5% error. Significant error was found with Algol which could be due to the fact that Algol A and Algol B are so close together (0.062 astronomical units) that the latter star has filled its Roche lobe. This means that the gravitational forces of Algol A on B are strong enough to cause mass to be transfered between the two and to shape Algol B into an ellipsoid shape. The simulation does not take mass transfer into account and uses spherical stars, which could explain the discrepancy. Additionally, the expected data values were analyzed from light years away, and as such they have large error margins. Data with more significant figures would improve the percent error and precision of the simulation.