DSP Practice Test #1.B

Name: _____ Start Time: _____

Problem 1:

A) Let $y[n] = \frac{1}{4}y[n-1] + x[n] + 2x[n-2]$,

Find the frequency response, $H(\omega)$, of the system

$$H(z) = \frac{1 + 2z^{-2}}{1 - \frac{1}{4}z^{-1}}$$

$$H(\omega) = \frac{1 + 2e^{-j2\omega}}{1 - \frac{1}{4}e^{-j\omega}}$$

B) Let $H(z) = \dfrac{1 + 3z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 + \frac{1}{5}z^{-1}\right)} = \dfrac{A}{1 - \frac{1}{3}z^{-1}} + \dfrac{B}{1 + \frac{1}{5}z^{-1}}$

Find the causal impulse response, $h[n]$, of the system with this frequency response.

$$A\left(1 + \tfrac{1}{5}z^{-1}\right) + B\left(1 - \tfrac{1}{3}z^{-1}\right) = 1 + 3z^{-1}$$

$$\begin{bmatrix} 1 & 1 \\ \frac{1}{5} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

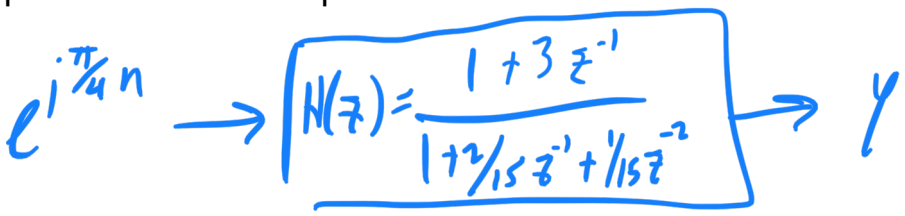
$$h[n] = A\left(\tfrac{1}{3}\right)^n u[n] + B\left(-\tfrac{1}{5}\right)^n u[n]$$

Problem 2:

Given the code below answer the following questions.

```
x = @(n)exp(1j*pi/4*n);
n=0:99;
y = filter([1,3],[1,2/15,1/15],x(n));
```

A) Explain what this sequence of MATLAB commands does



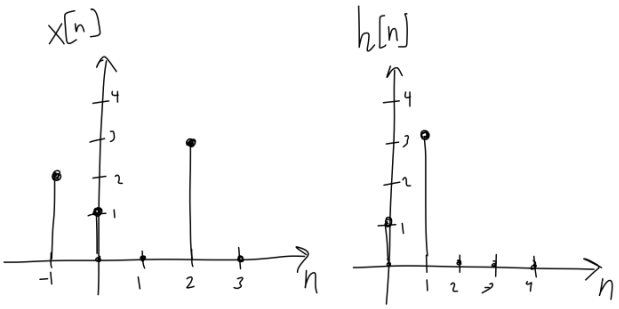
B) Write code to plot the output of the filter command.

```
plot(n,y)
```

C) Write down a closed-form expression for the output $y[n]$, given the input $x[n]$

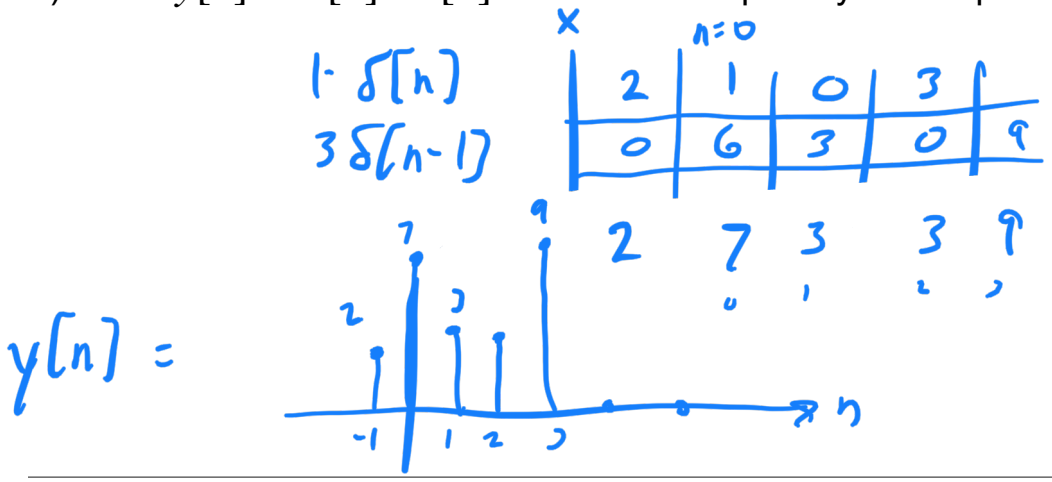
$$y[n] = H(e^{j\pi/4}) \cdot e^{j\pi/4 n}$$

Problem 3:



Let: $x[n] = 2\delta[n+1] + \delta[n] + 3\delta[n-2]$ $\quad h[n] = \delta[n] + 3\delta[n-1]$

A) Find $y[n] = x[n] * h[n]$. Show and explain your steps



B) Let $x[n] = 2\delta[n-1] - 3^n u[-n-1]$. Find the z-transform of $x[n]$, and specify the ROC. (Do not simplify the z-transform to a single term.)

$$X(z) = 2z^{-1} - \frac{-1}{1-3z^{-1}}$$

$(z \text{ any})$ $\quad |z| < 3$

$\Rightarrow$ ROC $|z| < 3$

Problem 4:

Let $y[n] = (n+1)x[n-1]$For each property, indicate by circling the appropriate term what can be shown to be true about the system. Also explain your reasoning.

A) The system is Linear/ Nonlinear / Not Enough Info

$y_3[n] = (n+1)(\alpha x_1[n] + \beta x_2[n])$

$= \alpha(n+1)x_1[n] + \beta(n+1)x_2[n]$

$= \alpha y_1[n] + \beta y_2[n]$ Yes Linear

B) The system is Causal / Noncausal / Not Enough Info

CAUSAL

$y = f(n, x[n-1])$ only past inputs

C) The system is Time-variant / Time-Invariant / Not Enough Info

VARIES

$y_1[n] = (n+1)x(n-m-1) \neq y[n-m] = (n-m+1)x[n-m-1]$

D) The system is Stable / Unstable / Not Enough Info

$x(n) = 1$ bounded → unbounded

$y = (n+1)$