Homework 4

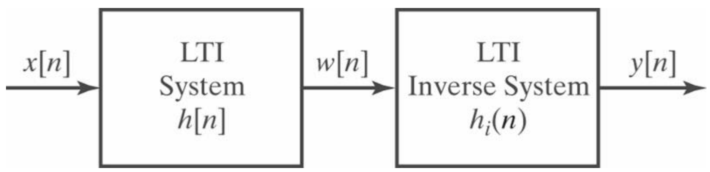
(Total 240 pts)

Due 5:00 pm on July 28, 2020 (Tuesday)

Note: Submit two files ('hw4.pdf' and 'q5.m') on Canvas.

1. (20 pts) The impulse response of an LTI system is $h[n] = \delta[n] + \delta[n-4]$.
   (a) Determine analytically the group delay associated with the system. Show your derivations.
   (b) Use the *grpdelay* function in Matlab to verify your answer in (a). Include the Matlab scripts and the plot for group delay.

2. (20 pts) Consider the cascade of an LTI system with its inverse system shown below:



   The impulse response of the first system is $h[n] = \delta[n] + 2\delta[n-1]$.
   (a) Determine the impulse response $h_i[n]$ of a stable inverse system for $h[n]$.
   (b) Is the inverse system causal?

3. (60 pts) A causal LTI system has the system function

$$H(z) = \frac{(1-e^{\frac{j\pi}{3}}z^{-1})(1-e^{-\frac{j\pi}{3}}z^{-1})(1+1.1765z^{-1})}{(1-0.9e^{\frac{j\pi}{3}}z^{-1})(1-0.9e^{-\frac{j\pi}{3}}z^{-1})(1+0.85z^{-1})}.$$

   (a) Sketch the pole-zero diagram. You can use the *zplane* function in Matlab. But make sure you mark the values of the poles and zeros on the plot.
   (b) What is the ROC for the system function?
   (c) Plot the magnitude and phase response of the system using the *freqz* function in Matlab.
   (d) Check whether the following statements are true or false about the system. Justify your answers.
      (i) The system is stable. **True ( ) False ( ). Why?**
      (ii) Because the system function has a pole at angle $\frac{\pi}{3}$, the magnitude of the frequency response has a peak at approximately $\omega = \pi/3$.
      True ( ) False ( ). Why?
      (iii) The system is a minimum-phase system. **True ( ) False ( ). Why?**

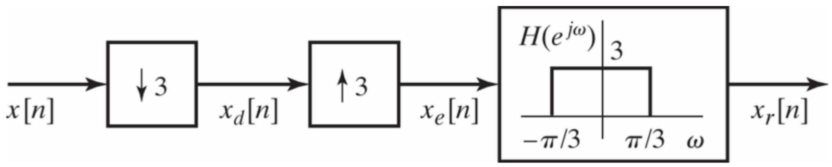
4. (40 pts) Consider the stable LTI system with system function

$$H(z) = \frac{1-4z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}}.$$

   The system function $H(z)$ can be factored such that $H(z) = H_{min}(z)H_{ap}(z)$, where $H_{min}(z)$ is a minimum-phase system, and $H_{ap}(z)$ is an all-pass system, i.e., $|H_{ap}(e^{j\omega}) = 1|$.

(a) Sketch the pole-zero diagram of $H(z)$.
(b) Determine $H_{min}(z)$ and its ROC.
(c) Determine $H_{ap}(z)$ and its ROC.
(d) Sketch the pole-zero diagrams of $H_{min}(z)$ and $H_{ap}(z)$.

5. (100 pts) Programming Assignment: Write MATLAB scripts to implement the following system considered in HW3:



(a) Generate a discrete-time input sequence $x[n] = \frac{1}{4} sinc\left(\frac{n}{4}\right)$, for $n = -1000$ to 1000, with a step size of 1. Visualize this sequence $x[n]$ by using the *plot* command. Show the plot.
(b) Generate the sequence $x_e[n]$ by down-sampling and then up-sampling the input sequence, as shown in the system above. Show the plot of $x_e[n]$.
(c) Design a low-pass filter with unity gain: *filt = fir1(nfilt, 1/3, 'low')*, where the filter order *nfilt* $= 10$. Show the plot of the filter coefficients *filt*.
(d) Show the frequency response of the above filter using the *freqz* command.
(e) Obtain the reconstructed sequence $x_r[n]$ by filtering $x_e[n]$: *xr = scale*filter(filt, 1, xe)* where *scale* = 3 corresponding to the gain of $H(e^{j\omega})$.
(f) Show in the same plot the input and reconstructed sequences using:
figure; plot(x); hold on; plot(xr, 'r')
(g) Compensate the delay introduced by the FIR filter using:
delay = mean(grpdelay(filt));
x_trunc = x(1: end - delay);
xr_shift = xr((delay + 1): end);
Show in the same plot the truncated input sequence (*x_trunc*) and the reconstructed sequence that is delay-compensated (*xr_shift*).
(h) Calculate the Mean Square Error (MSE) between the truncated input sequence (*x_trunc*) and the delay-compensated reconstructed sequence (*xr_shift*).
(i) Now, experiment with various filter orders such that *nfilt* $= 20, 50, 100, 200.$ Go through steps (c) through (h) to obtain the corresponding MSE values between the truncated input sequence (*x_trunc*) and the delay-compensated reconstructed sequence (*xr_shift*). Fill in the table below with your answers.

Filter Order	**10**	**20**	**50**	**100**	**200**
MSE					

(j) What conclusion can you draw regarding the relations between the filter order and the input reconstruction error, as well as the delays introduced by filtering?
(k) Show your MATLAB scripts used and also submit them in a single file ('q5.m') to Canvas.