DSP Practice Test #1-A

Name: ______________________ Start Time: _______________

Problem 1:

Let $y[n] = \begin{cases} 0, & n < 0 \\ \sum_{i=0}^{n} x[i], & n \geq 0 \end{cases}$

A) Is the system linear? Explain

Yes linear $x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$$y_3[n] = \sum_{i=0}^{n} (\alpha x_1[i] + \beta x_2[i]) = \alpha \sum_{i=0}^{n} x_1[n] + \beta \sum_{i=0}^{n} x_2[n]$$

$$y_3[n] = \alpha y_1[n] + \beta y_2[n]$$

B) Is the system causal? Explain

Yes

$$y[n] = f(x[n], x[n-1], \ldots, x[0])$$

only past inputs $\therefore$ CAUSAL

C) Is the system stable? Explain.

No $x[n] = 1$

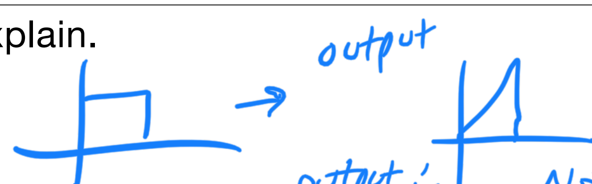
$$y[n] = \sum_{i=0}^{n} 1 = n$$

for bounded input $x[n] = 1$ output $y[n] = n$ is unbounded

NOT STABLE

D) Is the system shift-invariant? Explain.

No for example if input is



but shifted

NOT SAME $\therefore$ NOT shift inv.

Problem 2:

A system is completely characterized by its impulse response $h[n] = e^{-3n}u[n+1]$

A) Is the system linear? Explain

$y[n] = x[n] * h[n]$ yes convolution is linear

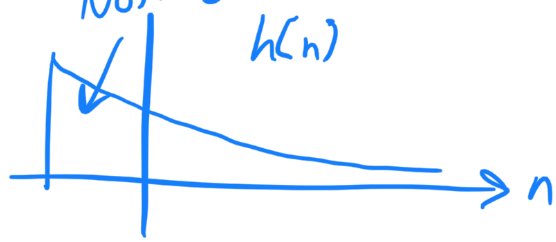
$y_3[n] = (\alpha_1 x_1[n] + \beta x_2[n]) * h[n]$

$y_3 = \alpha(x_1[n] * h[n]) + \beta(x_2[n] * h[n]) = \alpha y_1[n] + \beta y_2[n]$

B) Is the system causal? Explain

NO

NON CAUSAL



C) Is the system stable? Explain.

yes

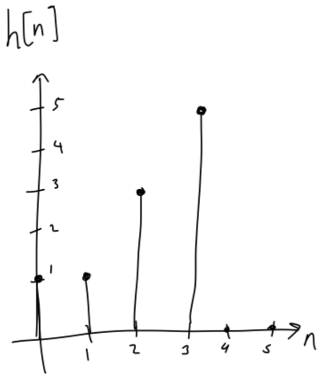
$$\sum_{-\infty}^{\infty} h[n] = \sum_{-1}^{\infty} e^{-3n} = \frac{(e^{-3})^{-1} - (e^{-3})^{\infty+1}}{1 - e^{-3}}$$

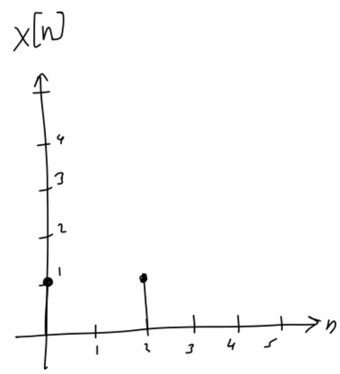
$$= \frac{e^{3}}{1 - e^{-3}} < \infty \therefore \text{STABLE}$$

D) Is the system shift-invariant? Explain.

yes convolution is time/shift invariant

Problem 3:





Let: $h[n] = \delta[n] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$ $\;x[n] = \delta[n] + \delta[n-2]$

A) Find $x[n] * h[n]$

x[0]

	n=0	1	2	3	4	5
$\delta[n]$	1	0	1			
$\delta[n-1]$	0	1	0	1		
$2\delta[n-2]$	0	0	2	0	2	
$3\delta[n-3]$ +	0	0	0	3	0	3
$y[n]$	1	1	3	4	2	3

$y[n] = \delta[n] + \delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 2\delta[n-4] + 3\delta[n-5]$

B) Find the DTFT of $h[n]$

$H(e^{jw}) = 1e^{j0w} + 1e^{-jw} + 2e^{-j2w} + 3e^{-j3w}$

$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn}$

Problem 4:

Given the causal system represented by the following difference equation:

$$y[n] = \frac{7}{10}y[n-1] - \frac{1}{10}y[n-2] + x[n] + x[n-1]$$

A) Determine the frequency response, $H(\omega)$

$$H(z) = \frac{1 + z^{-1}}{1 - \frac{7}{10}z^{-1} + \frac{1}{10}z^{-2}} = \frac{1 + z^{-1}}{(1 - \frac{1}{5}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

$$\boxed{H(\omega) = \frac{1 + e^{-j\omega}}{1 - \frac{7}{10}e^{-j\omega} + \frac{1}{10}e^{-2j\omega}}} =$$

B) Determine the impulse response, $h[n]$, of the system in found in the previous part.

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{5}z^{-1})(1 - \frac{1}{2}z^{-1})} = \frac{A}{1 - \frac{1}{5}z^{-1}} + \frac{B}{1 - \frac{1}{2}z^{-1}}$$

$$A(1 - \tfrac{1}{2}z^{-1}) + B(1 - \tfrac{1}{5}z^{-1}) = 1 + z^{-1}$$

$$\begin{pmatrix} 1 & 1 \\ -\frac{1}{2} & -\frac{1}{5} \end{pmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}$$

$A =$

$B =$

$$h[n] = A\left(+\tfrac{1}{5}\right)^n u[n] + B\left(+\tfrac{1}{2}\right)^n u[n]$$