DSP Practice Test #1.E

Name: ______________________ Start Time: _______________

Problem 1:

For each fo the following systems below specify whether or not the system is (1) Linear, (20 time-invariant, (3) causal, (4) stable, or there is not enough information. The system input is $x[n]$ and the output is $y[n]$

A) $y[n] = T\{x[n]\} = x[2n]$

Linear? Y/N	Time-Invariant? Y/N	Causal? Y/N	Stable? Y/N
Linear	VARIES	NON CAUSAL	STABLE

B) $y[n] = T\{x[n] + x[n-1]\}$

Time-Invariant? Y/N	Time-Invariant? Y/N	Causal? Y/N	Stable? Y/N
LINEAR	INVARIANT	CAUSAL	STABLE

C) $y[n] = T\{x[n]\} = \left(x\left[-|n|\right]\right)^2$

Linear? Y/N	Time-Invariant? Y/N	Causal? Y/N	Stable? Y/N
NONLINEAR	VARIES	CAUSAL	STABLE

Problem 2:

Let a causal LTI system be described by the following z-transform: $H(z) = \dfrac{1 + \frac{1}{2}z^{-1}}{1 - 2z^{-1}}$

A) Determine the frequency response of the system $H\left(e^{j\omega}\right)$

$$H(e^{jw}) = \frac{1 + \frac{1}{2}e^{-jw}}{1 - 2e^{-jw}}$$

B) Determine the difference equation relating the input and the output of the system

$y[n] - 2y[n-1] = x[n] + \frac{1}{2}x[n-1]$

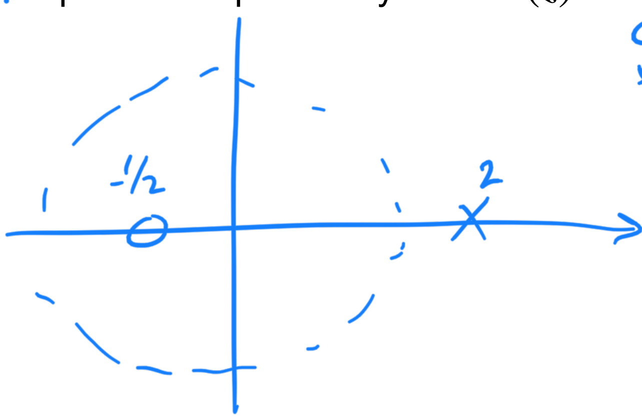
C) Plot the pole-zero plot of system $H(z)$

$1 - az^{-1} = 0$

$1 = az^{-1}$

$1/a = z^{-1}$

$z = a$



o zero

x pole

D) What is the ROC for this causal system?

$|z| > 2$

E) Is the system stable?

NO ROC does NOT include unit circle

F) Is the system causal?

yes

Problem 3:

Given an input random signal, $x[n]$, that is white with zero mean and unit variance, that is put into a system that is described by the following difference equation:
$y[n] = x[n+1] + x[n-1]$

A) Determine the impulse response $h[n]$ of the system

$h[n] = \delta[n+1] + \delta[n-1]$

B) Determine the transfer function $H\left(e^{j\omega}\right)$ of the system

$\boxed{H(e^{jw}) = e^{+jw} + e^{-jw}} = 2\cos(w)$

C) What is the autocorrelation of the input signal, $x[n]$, $\phi_{xx}[m]$?

$\phi_{xx}[m] = \sigma_x^2 \, \delta[m] = \delta[m]$

D) What is the power spectral density of the input signal, $S_{xx}(\omega)$?

$S_{xx}(w) = 1$

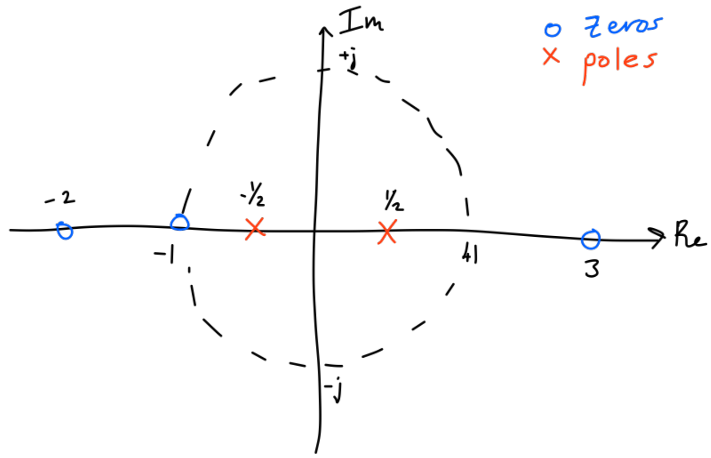
E) What is the power spectral density of the output signal, $S_{yy}(\omega)$?

$S_{yy}(w) = |H(w)|^2 \, S_{xx}(w)$

$(2\cos(w))^2 (1)$

Problem 4:

Given the following pole plot for the causal system $H(z)$



A) Determine an equation for $H(z)$ that corresponds to the pole-zero plot.

$$H(z) = \frac{(1+2z^{-1})(1+z^{-1})(1-3z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}$$

B) Is the system stable?

Yes

C) Given the input $x[n] = -30 + e^{j\pi/3n} + (-1)^n$, what is the output $y[n]$?

$$y[n] = H(e^{j0})(-30) + H(e^{j\pi/3})e^{j\pi/3\,n} + H(e^{j\pi})(-1)^n$$

$$(-1)^n = e^{j\pi n}$$