DSP Practice Test #1.C

Name: ______________________ Start Time: _______________

Problem 1:

For the system with the input-output relation: $y[n] = T\{x[n]\} = x[n] \underbrace{\sum_{k=0}^{\infty} \left(\delta[n-k]\right)}_{u[n]}$

determine the following properties of the system

$= x[n] \cdot (\delta[n] + \delta[n-1] + \delta[n-2] \cdots)$

A) The system is Linear/ Nonlinear / Not Enough Info

$= x[n] \cdot u[n]$

Linear

$y_3[n] = (\alpha x_1 + \beta x_2) u[n] = \alpha(x_1[n] u[n]) + \beta(x_2[n] u[n])$

$= \alpha y_1[n] + \beta y_2[n]$

B) The system is Causal / Noncausal / Not Enough Info

CAUSAL

$y[n] = f(x[n])$ only current input

C) The system is Time-variant / Time-Invariant / Not Enough Info

VARIES

$y_i[n] = x[n-m] \neq y_o[n-m] = x[n-m]\, u[n-m]$

D) The system is Stable / Unstable / Not Enough Info

$y[n] = x[n] \cdot$ constant

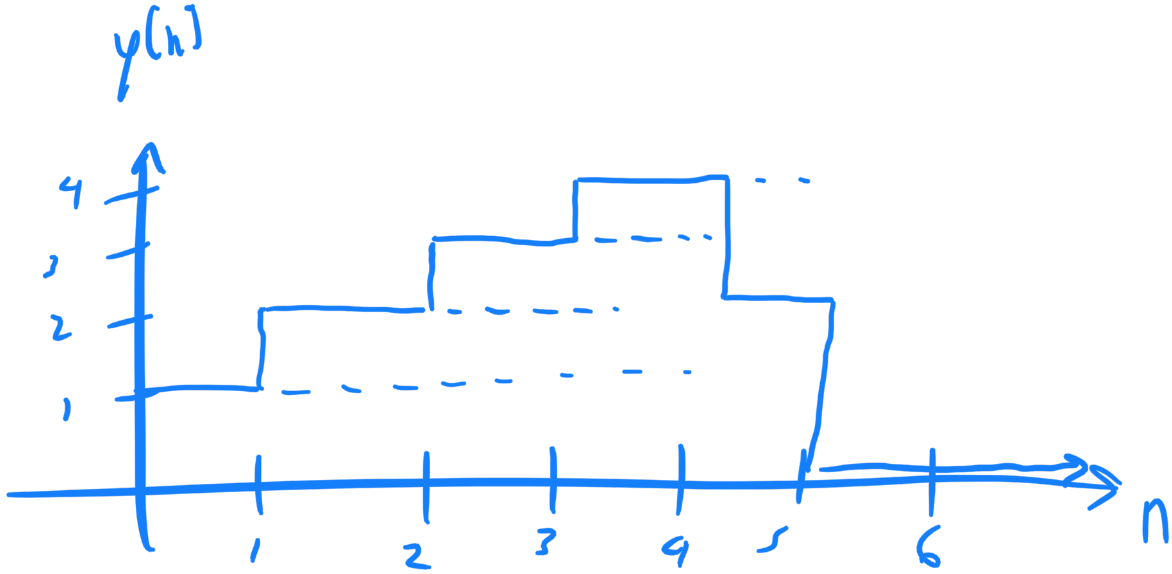
Problem 2:

An LTI system has an impulse response defined by:

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - 2\delta[n-5].$$

Determine and sketch the output $y[n]$ when the input is $x[n] = u[n]$

$y[n] = u[n] + u[n-1] + u[n-2] + u[n-3] - 2u[n-4] - 2u[n-5]$



Problem 3:

Consider an LTI system defined by the difference equation

$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$

A) Determine the impulse response of this system, $h[n]$

$h[n] = -2\,\delta[n] + 4\,\delta[n-1] - 2\,\delta[n-2]$

B) Determine the frequency response of this system. Express your answer in the form $H\left(e^{j\omega}\right) = A\left(e^{j\omega}\right)e^{j\omega n_d}$,

where $A\left(e^{j\omega}\right)$ is a real function of ω, and n_d is the delay. Explicitly specify $A\left(e^{j\omega}\right)$ and the delay n_d of this system.

$H(z) = -2 + 4z^{-1} - 2z^{-2} = -2z^{-1}(z^{+1} + 2 + z^{-1})$

$H(w) = -2e^{-jw}(2 + e^{jw} + e^{-jw}) = -2e^{-jw}(2 + 2\cos(w))$

$H(w) = \underbrace{(-4 - 4\cos(w))}_{A}\, e^{jw\overset{n_d}{(-1)}}$

C) Suppose that the input to the system is $x[n] = 1 + e^{j0.5\pi n}$, $n \in \mathbb{Z}$.

Use the frequency response function to determine the corresponding output $y[n]$. Simplify your answer as much as possible.

$y[n] = H(e^{j0}) \cdot 1 + H(e^{j.5\pi})\, e^{j.5\pi n}$

Problem 4:

Consider a random signal $x[n] = s[n] + e[n]$, where $s[n]$ and $e[n]$are independent, zero-mean stationary random signals with autocorrelation functions $\phi_{ss}[m]$and $\phi_{ee}[m]$, respectively.

A) Determine the mean of $x[n]$, μ_x

$$E\{x[n]\} = E\{s[n] + e[n]\} = \mu_s + \mu_e = 0 + 0 = 0$$

$$\boxed{\mu_x = 0}$$

B) Determine the autocorrelation function $\phi_{xx}[m]$, in terms of $\phi_{ss}[m]$ and $\phi_{ee}[m]$. Simplify your expression as much as possible.

$$E\{x[n]x[n+m]\} = E\{(s[n] + e[n])(s[n+m] + e[n+m])\}$$

$$= E\{s[n]s[n+m]\} + E\{s[n]\} \cdot E\{e[n+m]\} + E\{e[n]\} \cdot E\{s[n+m]\} + E\{e[n]e[n+m]\}$$

$$= \phi_{ss}[m] + \underbrace{\mu_s\mu_e + \mu_e\mu_s}_{} + \phi_{ee}[m]$$

$$\boxed{\phi_{xx}[m] = \phi_{ss}[m] + \phi_{ee}[m]}$$