DSP Practice Test #1.D

Name: ______________________ Start Time: _______________

Problem 1:

For the system with the input-output relation:

$x[n] * e^{-2n}u[n] = x[n] * h[n]$

$$y[n] = T\{x[n]\} = \sum_{k=-\infty}^{\infty} \left(x[k]e^{-2(n-k)}u[n-k] \right)$$ determine the properties of the system

---

A) The system is Linear/ Nonlinear / Not Enough Info

conv is linear

B) The system is Causal / Noncausal / Not Enough Info

conv is CAUSAL

C) The system is Time-variant / Time-Invariant / Not Enough Info

conv is T.I.

D) The system is Stable / Unstable / Not Enough Info

$$\sum_{n=-\infty}^{\infty} |h[n]| = \frac{e^0}{1-e^{-2}} < \infty$$

∴ STABLE

Problem 2:

An LTI system has an impulse response defined by:

$h[n] = -\delta[n+8] + \delta[n-1] + e^{-n}u[n].$

Determine and sketch the output $y[n]$ when the input is $x[n] = u[n]$

$$h[n] = \underbrace{\left(-\delta[n+8] + \delta[n-1]\right)}_{h_1[n]} + \underbrace{\left(e^{-n}u[n]\right)}_{h_2[n]}$$

$$y_1[n] = h_1[n] * x_1[n] = -u[n+8] + u[n-1]$$

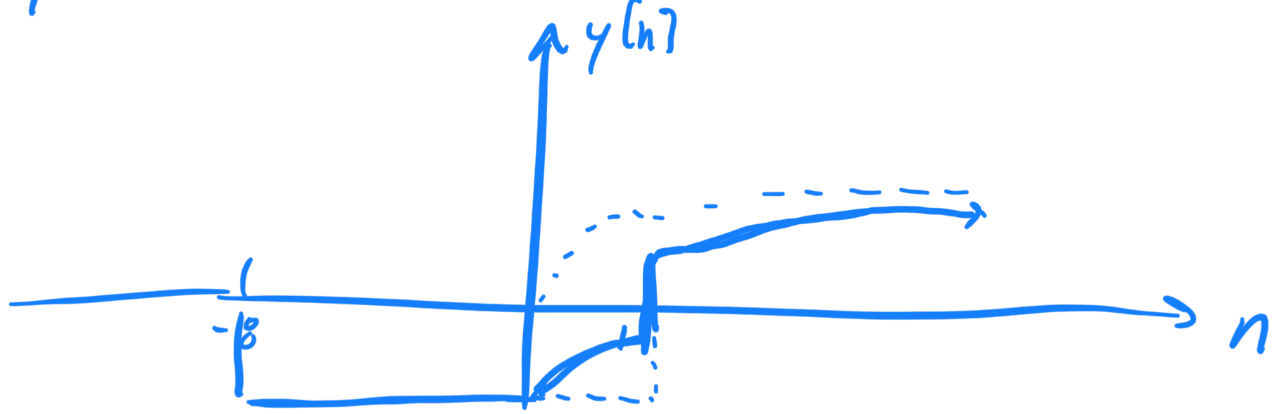
$$y_2[n] = e^{-n}u[n] * u[n] \Longrightarrow \frac{1}{1-\left(\frac{1}{e}\right)z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{A}{1-\left(\frac{1}{e}\right)z^{-1}} + \frac{B}{1-z^{-1}}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -\frac{1}{e} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \frac{1}{e}$$
$$B = 1 - \frac{1}{e}$$

$$y_2[n] = Ae^{-n}u[n] + Bu[n]$$
$$\frac{1}{e}e^{-n}u[n] + \left(1-\frac{1}{e}\right)u[n]$$

$$y[n] = -u[n+8] + u[n-1] + \frac{1}{e}e^{-n}u[n] + \left(1-\frac{1}{e}\right)u[n]$$



Problem 3:

Consider an LTI system defined by the difference equation

$$y[n] - y[n-2] = -2x[n] + 5x[n-1] - 2x[n-2]$$

A) Determine the z-transform of this system $H(z)$

$$H(z) = \frac{-2 + 5z^{-1} - 2z^{-2}}{1 - z^{-2}}$$

B) Determine the frequency transform of this system $H\left(e^{j\omega}\right)$

$$H(e^{j\omega}) = \frac{-2 + 5e^{-j\omega} - 2e^{-2j\omega}}{1 - e^{-2j\omega}}$$

C) Suppose that the input to the system is

$x[n] = -3 + e^{j0.2\pi n} + \cos(.3\pi n) + (-1)^n,\ n \in \mathbb{Z}$

$$y[n] = H(0)(-3) + H(.2\pi)e^{j.2\pi n} + H(.3\pi)\left(\tfrac{1}{2}e^{j.3\pi n}\right) + H(-.3\pi)\left(\tfrac{1}{2}e^{-j.3\pi n}\right) + H(\pi)(-1)^n$$

D) Determine the impulse response of this system, $h[n]$

$$H(z) = \frac{-2 + 5z^{-1} - 2z^{-2}}{1 - z^{-2}} = \frac{-2 + 5z^{-1} - 2z^{-1}}{(1 - z^{-1})(1 + z^{-1})} = A + \frac{B}{1 - z^{-1}} + \frac{C}{1 + z^{-1}}$$

$$h[n] = A\delta[n] + B\,u[n] + C(-1)^n u[n]$$

Problem 4:

Consider a random signal $x[n] = s[n] + e[n]$, where $s[n]$ and $e[n]$are stationary random signals

$E\{(s-\mu_s)(e-\mu_e)\} = \rho\sigma_s\sigma_e$

$E\{se\} = \rho\sigma_s\sigma_e + \mu_s\mu_e$

- with non-zero means, μ_s and μ_e respectively
- Are non-independent with covariances $K = \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_e \\ \rho\sigma_s\sigma_e & \sigma_e^2 \end{bmatrix}$ $E\{s^2\} = \sigma_s^2 + \mu_s^2$
- with autocorrelation functions $\phi_{ss}[m]$and $\phi_{ee}[m]$,
- and crosscorrelation function $\phi_{se}[m] = \phi_{es}[m]$.

A) Determine the mean of $x[n]$, μ_x

$E\{x[n]\} = E\{s[n] + e[n]\} = \boxed{\mu_s + \mu_e}$

B) Determine the standard deviation of of $x[n]$, σ_x

$E\{(x-\mu_x)^2\} = \sigma_x^2$

$E\{x^2\} = \sigma_x^2 + \mu_x^2$

$E\{(s+e)^2\} = E\{s^2 + 2se + e^2\} = (\sigma_s^2 + \mu_s^2) + 2(\rho\sigma_s\sigma_e + \mu_s\mu_e) + (\sigma_e^2 + \mu_e^2)$

C) Determine the autocorrelation function $\phi_{xx}[m]$, in terms of $\phi_{ss}[m]$, $\phi_{ee}[m]$, and $\phi_{se}[m]$. Simplify your expression as much as possible.

$\sigma_x^2 = (\sigma_s^2 + \mu_s^2) + 2(\rho\sigma_s\sigma_e + \mu_s\mu_e) + (\sigma_e^2 + \mu_e^2) - \mu_x^2$

$= \sigma_s^2 + \cancel{\mu_s^2} + 2\rho\sigma_s\sigma_e + \cancel{2\mu_s\mu_e} + \sigma_e^2 + \cancel{\mu_e^2} - \cancel{\mu_s^2} - \cancel{2\mu_s\mu_e} - \cancel{\mu_e^2}$

$\sigma_x^2 = \sigma_s^2 + 2\rho\sigma_s\sigma_e + \sigma_e^2$

$\boxed{\sigma_x = \sqrt{\sigma_s^2 + 2\rho\sigma_s\sigma_e + \sigma_e^2}}$

$\Phi_{xx}[m] = E\{x[n]x[n+m]\} = E\{s[n]s[n+m] + s[n]e[n+m] + e[n]s[n+m] + e[n]e[n+m]\}$

$= \boxed{\phi_{ss}[m] + 2\phi_{se}[m] + \phi_{ee}[m]}$