Dot Product of Vector with Itself

Contents

Theorem

Let $\mathbf u$ be a vector in the real vector space $\mathbb R^n$.

Then:

$$\mathbf u \cdot \mathbf u = \left\Vert \mathbf u \right\Vert^2$$

where $\left\Vert \mathbf u \right\Vert$ is the length of $\mathbf u$.

Proof 1

Let $\mathbf u = \left({u_1, u_2, \ldots, u_n}\right)$.

Then:

$$\begin{aligned}
\mathbf u \cdot \mathbf u &= u_1 u_1 + u_2 u_2 + \cdots + u_n u_n && \text{Definition of Dot Product} \\
&= u_1^2 + u_2^2 + \cdots + u_n^2 \\
&= \left({\sqrt {\sum_{i \mathop = 1}^n u_i^2} }\right)^2 \\
&= \left\Vert \mathbf u \right\Vert^2 && \text{Definition of Vector Length in } \mathbb R^n
\end{aligned}$$

$\blacksquare$

Proof 2

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Note

Because this theorem is used to prove the general (n-dimensional) case of Cosine Formula for Dot Product, this proof is circular the way we have defined the dot product.

However, some texts use the cosine formula as the definition of the dot product and derive the sum of products form as a consequence.

The two definitions are equivalent, so we have included this proof to show how the statement would be proved from the cosine definition.

Also presented as

This can also be seen presented as:

$$\|\mathbf{u}\| = (\mathbf{u} \cdot \mathbf{u})^{1/2}$$

or:

$$\|\mathbf{u}\| = u = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$

and so on.

Sources

- 1927: C.E. Weatherburn: *Differential Geometry of Three Dimensions: Volume I* ... (previous) ... (next): Introduction:Vector Notation and Formulae: Products of Vectors
- 1951: B. Hague: *An Introduction to Vector Analysis* (5th ed.) ... (previous) ... (next): Chapter II: The Products of Vectors: 2. The Scalar Product: (2.4)
- 1992: Frederick W. Byron, Jr. and Robert W. Fuller: *Mathematics of Classical and Quantum Physics* ... (previous) ... (next): Volume One: Chapter 1 Vectors in Classical Physics: 1.3 The Scalar Product