101064168 - N.Ellul

1.

A) Binary System

The greatest number occurs when all the bits have a value of 1, and the lowest is when all the bits are 0. (Assuming unsigned)

∴ The max can be calculated with the sigma

$$\sum_{i=0}^{8}(2^i \cdot 1) = 1 \cdot \frac{2^{(8+1)}-1}{2-1}$$

Sigma was derived by adding the values of each bit which is 2^n where n is the bit's index

$$= \frac{2^9-1}{1}$$

$$= 2^9$$

$$= 511$$

∴ The range is 0 - 511 meaning 512 numbers can be represented

B) Hexadecimal

Using the same method as above ill be counting the range from 000000000 to FFFFFFFFF by translating F to its decimal value of 15 and using sigma.

Base 16 in Hex

$$\sum_{i=0}^{8}(15 \cdot 16^i) = 15 \cdot \frac{16^9-1}{16-1}$$

$$= \cancel{15} \cdot \frac{16^9-1}{\cancel{15}}$$

$$= 16^9 - 1$$

$$= 68\ 719\ 476\ 735$$

∴ Including 0, 9 digits can represent 68719476736 numbers in hexadecimal

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C) Octal

Octal is base 8 with a range of 000000000 to 777777777 over 9 digits.

- Using sigma I will convert this maximum number to a decimal in order to determine how many numbers it can hold

$$\sum_{i=0}^{8}(7\cdot 8^i) = 7\cdot\frac{8^9-1}{8-1}$$

$$= \cancel{7}\cdot\frac{8^9-1}{\cancel{7}}$$

$$= 8^9-1$$

$$=134217727$$

$\therefore$ By including 0, I can conclude that with 9 digits, an Octal number can represent 134217728 different base 10 numbers

2.

a) $(542)_8$

$$2\cdot 8^0 = 2$$
$$+$$
$$4\cdot 8^1 = 32$$
$$+$$
$$5\cdot 8^2 = 320$$
$$354$$

b) $(EAB1)_{16}$

E → 14, A → 10, B → 11

$$1\cdot 16^0 = 1$$
$$+$$
$$11\cdot 16^1 = 176$$
$$+$$
$$10\cdot 16^2 = 2560$$
$$+$$
$$14\cdot 16^3 = 57344$$
$$60081$$

c) $(282)_8$

↳ This is invalid because the highest base 8 digit is 7

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d) $(10110110)_2$

Two's Complement

① 01001001

② 01001010

$\hookrightarrow 2^1+2^3+2^6$

$= 74$

③ $\underline{-74}$

Unsigned Number

$$\begin{array}{cccccccc} 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

$2^7+2^5+2^4+2^2+2^1$

$= \underline{182}$

e) $(00010111)_2$

$$\begin{array}{c|cccccccc} & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ \hline 2^e & & & & 4 & 3 & 2 & 1 & 0 \end{array}$$

$2^0+2^1+2^2+2^4$

$= \underline{23}$

3.

a) $\underline{231}$

$231 - \underline{128} = 103 - \underline{64} = 39 - \underline{32} = 7 - \underline{4} = 3 - \underline{2} = 1 - 1$

$\hookrightarrow (11100111)_2$ ☆

$231 \div 8 = 28$ R7 ———— 7

$28 \div 8 = 3$ R4 ———— 47

$3 \div 8 = 0$ R3 ———— 347

$\hookrightarrow (347)_8$ ☆

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$231 \div 16 = 14$ R7 —— 7

$14 \div 16 = 0$ R14 → E —— E7

→ $(E7)_{16}$ ☆

b) 1183

Binary:

$1183 \div 2 = 591$ R1 —— 1

$591 \div 2 = 295$ R1 —— 11

$295 \div 2 = 147$ R1 —— 111

$147 \div 2 = 73$ R1 —— 1111

$73 \div 2 = 36$ R1 —— 11111

$36 \div 2 = 18$ R0 —— 011111

$18 \div 2 = 9$ R0 —— 0011111

$9 \div 2 = 4$ R1 —— 10011111

$4 \div 2 = 2$ R0 —— 010011111

$2 \div 2 = 1$ R0 —— 0010011111

$1 \div 2 = 0$ R1 —— 10010011111

→ $(10010011111)_2$ ☆

Octal:

$1183 \div 8 = 147$ R7 —— 7

$147 \div 8 = 18$ R3 —— 37

$18 \div 8 = 2$ R2 —— 237

$2 \div 8 = 0$ R2 —— 2237

→ $(2237)_8$ ☆

Hex:

$1183 \div 16 = 73$ R15 → F —— F

$73 \div 16 = 4$ R9 —— 9F

$4 \div 16 = 0$ R4 —— 49F

→ $(49F)_{16}$ ☆

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c) 1928

Binary:

1928 ÷ 2 = 964 R0 —— 0
964 ÷ 2 = 482 R0 —— 00
482 ÷ 2 = 241 R0 —— 000
241 ÷ 2 = 120 R1 —— 1000
60 ÷ 2 = 30 R0 —— 01000
30 ÷ 2 = 15 R0 —— 001000
15 ÷ 2 = 7 R1 —— 1001000
7 ÷ 2 = 3 R1 —— 11001000
3 ÷ 2 = 1 R1 —— 111001000
1 ÷ 2 = 0 R1 —— 1111001000

$(1111001000)_2$

Octal:

1928 ÷ 8 = 241 R0 —— 0
241 ÷ 8 = 30 R1 —— 10
30 ÷ 8 = 3 R6 —— 610
3 ÷ 8 = 0 R3 —— 3610

$(3610)_8$

Hex:

1928 ÷ 16 = 120 R8 —— 8
120 ÷ 16 = 7 R8 —— 88
7 ÷ 16 = 0 R7 —— 788

$(788)_{16}$

4. a) Unsigned Integer

↳ range 0 to $\sum_{i=0}^{n-1} 2^i$ where n is

↳ $= \frac{2^{(n-1)+1} - 1}{2-1}$

$= \frac{2^6 - 1}{1}$

∴ $= 64 - 1$

$= 63$

∴ It has a range of 0 to 63 when 6 bits are used

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b) 2's complement:

- The last digit is used to denote sign so the largest positive number is

$0\underbrace{11111}$

$\rightarrow 2^4+2^3+2^2+2^1+2^0$

max $=31$

- minimum number is the greatest negative number

$\rightarrow 100000$

$\rightarrow -32$ $\therefore$ The range is $-32 \rightarrow 31$

c) 1's complement

max positive number

$\rightarrow 011111$

$\rightarrow$ same as 2's: 31

minimum number $1\underbrace{00000}$

$\rightarrow$ Flipped max positive

$\rightarrow -31$

$\therefore$ The range is

-31 to 31.

Also note that -0 and $+0$ exist in this range.

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5.

a)
```
  1 1 1 1 1      1
   01011101
 + 10101001
 ----------
   00000110  → 6
```

b) 10110111 − 11001011

= 10110111 + 00110101

```
   11 1 11
   10110111
 + 00110101
 ----------
 11 101100  → −20
```

c)
```
      0000 0111
    × 0000 0101
    -----------
      0000 0111
    00000 00 000
  000000 1 11
  -------------
      00100011  → 35
```

6.

a) 72 = 64 + 8

One's complement

01001000

two's complement

01001000

Positive #'s are handelled normally

b) 0

One's Complement

0000000 or 11111111

two's complement

0000 0000

c) −128

one's complement

• Invalid. The range in 8 bits stops at −127.

two's complement

1000000

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d) $-5 \rightarrow 4+1$

One's Complement

1. 00000101
2. 11111010

Two's Complement

1. 00000101
2. 11111010
3. 11111011

7.

a) 01011000

unsigned:

$2^6+2^4+2^3 = 88$

One's Comp:

$2^6+2^4+2^3 = 88$

Two's Comp:

$2^6+2^4+2^3 = 88$

Excess-127:

$88-127 = -39$

Excess-127:

10101001 = 169

169 − 127 = 42

b) 10101001

unsigned:

$2^7+2^5+2^3+2^0 = 169$

One's comp:

10101001

↓

01010110

↓

$2^6+2^4+2^2+2^1 = 86$

Two's Comp:

10101001

↓

01010110

↓

01010111

↓

$-(2^6+2^4+2^2+2^1+2^0) = -87$

8. Converting to floating Point

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A) Fixed Point binary: 4.625

100.101

Normalized in floating Point

$\underline{1.00101 \times 2^{2}}$

b) Fixed Point binary: -1.25

1.01×2^{0}

9. a) 101111110010

Octal

101 111 110 010

(5 7 6 2)

$(5762)_8$

Hex

1011 1111 0010

11 15 2

B F 2

$(BF2)_{16}$

b) 000101111110

octal

000 101 111 110

(0 5 7 6)

$(576)_8$

Hex

0001 0111 1110

1 7 14

1 7 E

$(17E)_{16}$

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10. a) 8bit float

- 3 bit exponent 111
- 4 bit mantissa 1111
- 1 bit sign

0.5

0.1 in binary fixed Point

Normalized → 1.0×2^{-1}

mantissa = $\underline{0000}$

Convert exponent Using excess-3

$-1 + 3 = 2 \rightarrow \underline{010}$

sign

$0\,010\,0000 = 0.5$

b) $1\,110\,1010$

$-$

$6 - 3 = 3$

1.1010

$2^3 \times 1.1010 = 1101.0$

-13

∴ $11101010 = \underline{-13}$

11.

a) Whats the smallest positive number = 00000000

- Large negative exponent
  - 000 in excess 3 is $0-3 = \underline{-3}$
- small mantissa
  - 0000 (only implied 1 stays)

$1.000 \; 1.0 \times 2^{-3}$

$0.001 = 0.125$

∴ Smallest positive number is 0.125
Assuming 0 is not considered Positive

b) What is the smallest number? 101064168 N.Elkul

↳ The largest negative number

↳ Largest exponent → 111

↳ $7-3=\underline{4}$

excess-3

↳ Largest mantissa → 1111

implicity: $1.1111 \times 2^4 = 11111$

31

−31 Since negative bit