Homework 3-4

First Name:
Last Name:
Red ID#:

Q1. Is the following distributive law valid? $A \oplus BC = (A \oplus B)(A \oplus C)$
Prove your answer.

Q2. Which of the following statements are always true? Justify your answers.

(a) If $A + B = C$, then $AD' + BD' = CD'$
(b) If $A'B + A'C = A'D$, then $B + C = D$
(c) If $A + B = C$, then $A + B + D = C + D$
(d) If $A + B + C = C + D$, then $A + B = D$

Q3. Given $F_1 = \sum m(0,4,5,6)$ and $F_2 = \sum m(0,3,6,7)$ find the minterm expression for $F_1 + F_2$. State a general rule for finding the expression for $F_1 + F_2$ given the minterm expansions for F_1 and F_2. Prove your answer by using the general form of the minterm expansion.

Q4.

Build the truth table where XYZ (binary) equals the number of 1's in $ABCD$:

A	B	C	D	#1's	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1
0	0	1	0	1	0	0	1
0	0	1	1	2	0	1	0
0	1	0	0	1	0	0	1
0	1	0	1	2	0	1	0
0	1	1	0	2	0	1	0
0	1	1	1	3	0	1	1
1	0	0	0	1	0	0	1
1	0	0	1	2	0	1	0
1	0	1	0	2	0	1	0
1	0	1	1	3	0	1	1
1	1	0	0	2	0	1	0
1	1	0	1	3	0	1	1
1	1	1	0	3	0	1	1
1	1	1	1	4	1	0	0

Minterm expansions:

$$X = \sum m(15)$$

$$Y = \sum m(3, 5, 6, 7, 9, 10, 11, 12, 13, 14)$$

$$Z = \sum m(1, 2, 4, 7, 8, 11, 13, 14)$$

Q5. A half adder is a circuit that adds two bits to give a sum and a carry. Complete the truth table for a half adder, and implement the half adder using only two gates. Then design a circuit which finds the negative number of a 4-bit positive input number in 2's complement format. For example, if a positive number is ABCD in 2's complement format, then the corresponding negative number would be A'B'C'D'+1. Use four half adders and any additional gates for your implementation.