Simuleasy

Nicholas Farrel

March 2021

Problema 1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC. The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at R. Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC.

Problema 2. Let $n \geq 3$ be an integer. Let t_1, t_2, ..., t_n be positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \cdots + t_n)\left(\frac{1}{t_1} + \frac{1}{t_2} + \cdots + \frac{1}{t_n}\right).$$

Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$.

Problema 3. Given a triangle ABC satisfying $AC + BC = 3 \cdot AB$. The incircle of triangle ABC has center I and touches the sides BC and CA at the points D and E, respectively. Let K and L be the reflections of the points D and E with respect to I. Prove that the points A, B, K, L lie on one circle.

Problema 4. Find all pairs of integers a, b for which there exists a polynomial $P(x) \in \mathbb{Z}[X]$ such that product $(x^2 + ax + b) \cdot P(x)$ is a polynomial of a form

$$x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0$$

where each of $c_0, c_1, \ldots, c_{n-1}$ is equal to 1 or -1.