AMM Notes

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1 Automatic Market Makers (AMM) Liquidity

An AMM pool will always satisfy the equation:

$$A_n B_n = k \tag{1}$$

where A_n is the number of token A in the pool, and B_n is the number of token B. K is held constant.

For example, if we start a pool with 100 of token A, and 1 of token B then $k = 100$. If someone goes to buy 20 of token A from the pool then this would reduce A_n to 80, requiring that B_n increases to $B_n = \frac{100}{80} = 1.25$. This means that those 20 of token A would cost 0.25 of token B. A price A/B of $0.25/20 = 0.0125$.

We can generalise this further, with an initial pool state:

$$A_1 B_1 = k \tag{2}$$

and initial price $P_1 = B_1/A_1$.

The number of tokens A in the pool changes by a factor x so that $A_2 = xA_1$, e.g. someone buys 10% of tokens then $A_2 = 0.9A_1$. As k is constant, the updated pool will be

$$A_1 B_1 = A_2 B_2 = x A_1 B_2 \tag{3}$$

so

$$x A_1 B_2 = A_1 B_1 \tag{4}$$

then

$$B_2 = \frac{B_1}{x} \tag{5}$$

Makes sense as:

$$A_2 B_2 = x A_1 \frac{B_1}{x} = A_1 B_1 \tag{6}$$

The updated price P_2 will be

$$P_2 = B_2/A_2 = \frac{\frac{B_1}{x}}{xA_1} = \frac{1}{x^2}\frac{B_1}{A_1} = \frac{P_1}{x^2} \tag{7}$$

For example, if you buy 20% of the remaining tokens in the pool, the updated price will be an increase of $\frac{1}{(1-0.2)^2} = 1.5625$, a +56.25% increase.
Or if you sell heaps of token A to the pool, increasing its token A balance by 10x, then this is a price change of $\frac{1}{10^2} = 0.01$, a 99% decrease in price.

1.1 Money received from selling X tokens

How many of token B will we receive if we sell some amount y of token A? Let $xA_1 = A_1 + y$ so $x = 1 + \frac{y}{A_1}$.

$$k = xA_1\frac{B_1}{x} = (1 + \frac{y}{A_1})A_1\frac{B_1}{1 + \frac{y}{A_1}} = (A_1 + y)B_1\frac{1}{1 + \frac{y}{A_1}} = A_2 * B_2 = k \quad (8)$$

Hence $A_2 = A_1 + y$ (as defined) and $B_2 = B_1(\frac{1}{1+\frac{y}{A_1}})$. In otherwords, selling $A_2 - A_1 = y$ of token A, gives z of token B:

$$z = B_2 - B_1 = B_1(\frac{1}{1 + \frac{y}{A_1}} - 1) \quad (9)$$

For example, if there is a pool with 20 of token A and 5 of token B, and we sell 2 of token A then we get

$$z = B_1(\frac{1}{1 + \frac{y}{A_1}} - 1) = 5(\frac{1}{1 + \frac{2}{20}} - 1) = -0.45454... \quad (10)$$

Checking:

$$20 * 5 = 100 = k \quad (11)$$

$$(20 + 2) * (5 - 0.4545..) = 100 = k \quad (12)$$