Project 2: Chromatic Dispersion Compensation Using Complex-Valued All-Pass Filter

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1 Idea and Motivation

The Chromatic Dispersion (CD) in optical channels has an all-pass behavior, i.e., only the phase of a signal is changed due to CD. Using IIR or FIR filters, CD can be compensated. In this project, you will get familiar with the idea of CD compensation, especially with complex-valued IIR filters, and design a compensation filter.

2 Preparation Exercises

Read through the provided paper and answer the following questions

Discretization Using the provided continuous CD channel model

$$H_{\mathrm{CD}}(\Omega) = \exp\left(-j\frac{\lambda_0^2}{4\pi c}DL\Omega^2\right),$$

derive the channel model $H_{\mathrm{CD}}(\omega)$ for the sampled signal (sampling rate B).

Ideal Equalizer Derive the frequency domain response of the ideal equalizer for such a channel.

Frequency Sampling Consider the following values

- $B = 56$ GHz
- $\lambda_0 = 1550$ nm
- $D = 16$ ps/nm/km
- $L = 23$ km

and calculate the channel response $h_{\mathrm{CD}}[n]$ for the first 256 sampling points.

Number of Taps Consider the channel response $h_{\mathrm{CD}}[n]$. How many taps are needed for equalization with an FIR filter? Justify your answer.

Group Delay Derive the expression for the total group delay

$$\tau_{\mathrm{IIR}}(\omega) = \sum_{i=1}^{N_{\mathrm{IIR}}} \frac{1-\rho_i^2}{1+\rho_i^2-2\rho_i\cos(\omega-\theta_i)}$$

by applying

$$\tau_{\mathrm{IIR}}(\omega) = -\frac{\partial}{\partial\omega}\phi_{\mathrm{IIR}}(\omega).$$

Group Delay Extrema Consider the case $N_{\text{IIR}} = 1$. Derive the maximum and minimum value of τ_{IIR} as a function of ρ. What does this imply for ρ?

Integer Factor Derive $\beta = \lceil 2\alpha\pi \rceil$.

Desired Phase Response Derive the expression for the desired phase response.

3 Practical Examples

Optimization Framework Implement the optimization framework, which computes the optimal values for the parameters ρ_i, θ_i and ϕ_0 of the CD compensation filter in the MATLAB function
`[rho,theta,phi] = optimization_framework(alpha).`

Hints:

- The input argument `alpha` is the parameter $\alpha = \lambda_0^2 B^2 DL/\left(4\pi c\right)$ characterizing the CD channel.
- The output arguments `rho`, `theta` and `phi` are the column vector containing the N_{IIR} radii ρ_i of the poles, the column vector containing the N_{IIR} angles θ_i of the poles and the phase correction term ϕ_0, respectively.
- Implement the Abel-Smith algorithm in the function
`[rho,theta] = abel_smith(alpha,N_IIR).`
For the division of the area under the group delay curve, use the sub-function
`omega_seg = abel_smith_divide(alpha,N_IIR).`
- For solving each of the three optimization problems, use the MATLAB solver `fmincon()` with the algorithm `active-set` and the constraints $0 \leq \rho_i \leq 0.98$, $-\pi \leq \theta_i \leq \pi$ as well as $-\pi \leq \phi_0 \leq \pi$.
  - Write the function
  `y = MSE_GD(x,alpha,beta,N_IIR),`
  which computes the MSE in the group delay MSE_{GD} for `x = [rho;theta]`, as well as the function
  `y = MSE_TP(x,alpha,beta,N_IIR),`
  which computes the MSE in the transfer function $\text{MSE}_{\text{trans. phase}}$ for `x = [rho;theta;phi]`. Approximate the integrals by discretizing $\omega \in [-\pi, \pi)$ into 2^{14} equally spaced points and using the trapezoidal rule.
  - Create the cost functions of the optimization problems from these functions by using anonymous functions and pass a handle for them to `fmincon()` as can be seen in the following simple example:
  The solution $(x_{1,opt}, x_{2,opt})$ to the optimization problem

  $$\min_{(x_1,x_2)\in\mathbb{R}^2} a_1x_1^2 + a_2x_2^2 \quad \text{s.t.} \quad 0 \leq x_1 \leq 1, 2 \leq x_2 \leq 3, \quad a_1, a_2 \in \mathbb{R}$$

  can be obtained by
  `x_opt = fmincon(@(x)(fun(x,a)),x0,[],[],[],[],[0;2],[1;3],[],options),`
  where `x` $= [x_1, x_2]^{\text{T}}$, `x_opt` $= [x_{1,opt}, x_{2,opt}]^{\text{T}}$ and `a` $= [a_1, a_2]^{\text{T}}$. The cost function $f(x_1, x_2) = a_1x_1^2 + a_2x_2^2$ is implemented in the following MATLAB function:

```
function y = fun(x,a)
  y = a(1)*x(1)^2+a(2)*x(2)^2;
end
```

The vector `x0` is the initial guess for the solution `x_opt` and the Algorithm `active-set` is used by passing the variable `options = optimoptions('fmincon','Algorithm','active-set')`.

CD Compensation Using the provided function `main_program()`, implement a CD channel, CD equalization and calculate the bit error rate curve.

Hints:

- The optimal values for the parameters ρ_i, θ_i and ϕ_0 of the CD compensation filter are computed by the optimization framework implemented in the previous task and written into the variables `rho`, `theta` and `phi`, respectively.
- Implement the channel in the function `h_tot = impulse_response_channel(alpha,freq_pts)`.
- Implement the equalization in the function `output = conv_anyinput_allpass_equalizer(rho,theta,phi,input)`. For filtering use the function `filter()`.
- For calculating the bit error rate curve, the function `bit_error_rate(h_tot,index,rho,theta,phi)` is provided.

4 Lab Report

Write a report in which you make use of the previous tasks. The report should contain theoretical insight into the problem which you got by the preparation questions as well as the practical aspects you learned by the practical examples. You may put derivations, etc., into the appendix, but you should answer all questions of the previous tasks in the report.