Lois de probabilité en actuariat

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0.1 Lois discrètes

Loi	$\Pr(X=x)$	$F_X(x)$	$E[X]$	$Var(X)$	$M_X(t)$	$P_X(t)$
Uniforme	$\begin{cases} \frac{1}{b-a+1} & x=a,a+1,\ldots,b \\ 0 & \text{ailleurs} \end{cases}$	$\begin{cases} 0 & x<a \\ \frac{\lfloor x\rfloor-a+1}{b-a+1} & a\leq x\leq b \\ 1 & x\geq b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{at}-e^{(b+1)t}}{(b-a+1)(1-e^t)}$	N/A
Bernouilli	$\begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & \text{ailleurs} \end{cases}$	$\begin{cases} 0 & x<0 \\ 1-p & 0\leq x\leq 1 \\ 1 & x\geq 1 \end{cases}$	p	$p(1-p)$	$(1-p)+pe^t$	$(1-p)+p^t$
Binomiale	$\begin{cases} \binom{n}{x}p^x(1-p)^{n-x} & x=0,1,2,\ldots \\ 0 & \text{ailleurs} \end{cases}$	$\sum_{k=1}^{\lfloor x\rfloor}\binom{n}{k}p^k(1-p)^{(n-k)}$	np	$np(1-p)$	$((1-p)+pe^t)^n$	$((1-p)+p^t)^n$
Poisson	$\begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & x=0,1,2,\ldots \\ 0 & \text{ailleurs} \end{cases}$	$\sum_{k=1}^{\lfloor x\rfloor}\frac{\lambda^k e^{-\lambda}}{k!}, x\geq 0$	λ	λ	$e^{\lambda(e^t-1)}$	$e^{\lambda(t-1)}$
Géométrique	$\begin{cases} p(1-p)^{(x-1)} & x=0,1,2,\ldots \\ 0 & \text{ailleurs} \end{cases}$	$\begin{cases} 0 & x<1 \\ 1-(1-p)^{\lfloor x\rfloor} & x\geq 1 \end{cases}$	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{pt}{1-(1-p)t}$
Binomiale négative	$\begin{cases} \binom{x-1}{r-1}p^r(1-p)^{x-r} & x=r,r+1,\ldots \\ 0 & \text{ailleurs} \end{cases}$	N/A	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$	$\left(\frac{pt}{1-(1-p)t}\right)^r$
Hypergéométrique	$\begin{cases} \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}} & x=0,1,\ldots,\min(m,n) \\ 0 & \text{ailleurs} \end{cases}$		$n\frac{m}{N}$	$\frac{nm}{N}\left(\frac{(n-1)(m-1)}{N-1}+1-\frac{nm}{N}\right)$		

0.2 Lois continues

Loi	$f_X(x)$	$F_X(x)$	$F_X^{-1}(x)$	$E[X]$	$Var(X)$	$M_X(t)$
Uniforme	$\begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{ailleurs} \end{cases}$	$\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x \geq b \end{cases}$	$a + (b-a) \cdot u$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{t(b-a)}$
Normale	$\begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} & x \in \mathbb{R} \\ 0 & \text{ailleurs} \end{cases}$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	$\mu + \sigma\Phi^{-1}(u)$	μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Lognormale	$\begin{cases} \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2} & x \in \mathbb{R}+ \\ 0 & \text{ailleurs} \end{cases}$	$\Phi\left(\frac{\ln x-\mu}{\sigma}\right)$	$e^{\mu+\sigma\Phi^{-1}(u)}$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$	N/A
Exponentielle	$\begin{cases} \lambda e^{-\lambda x} & x > 0, \lambda > 0 \\ 0 & \text{ailleurs} \end{cases}$	$1 - e^{-\lambda x}, x > 0$	$\frac{-\ln(1-u)}{\lambda}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, t < \lambda$
Gamma	$\begin{cases} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} & x > 0, \lambda > 0 \\ 0 & \text{ailleurs} \end{cases}$	si $\alpha \notin \mathbb{Z}$ $\frac{\Gamma(\alpha;\lambda x)}{\Gamma(\alpha)}$ $x > 0, \lambda > 0, \alpha < 0$	si $\alpha \in$ $1 - \sum_{k=0}^{\alpha-1} \frac{(\lambda x)^k e^{-\lambda x}}{k!}$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^\alpha, t < \lambda$
Khi-carré	$\begin{cases} \frac{x^{\frac{n}{2}-1} e^{-\frac{n}{2}}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} & x > 0 \\ 0 & \text{ailleurs} \end{cases}$	$\frac{\Gamma\left(\frac{n}{2};\frac{x}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$		n	$2n$	$\left(\frac{1}{1-2t}\right)^{\frac{n}{2}}, t < \frac{1}{2}$
Beta	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $0 < x < 1, \alpha > 0, \beta > 0$	$\frac{\beta(x;\alpha,\beta)}{\beta(\alpha,\beta)}$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Pareto	$\begin{cases} \frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}} & x > 0, \alpha > 0, \lambda > 0 \\ 0 & \text{ailleurs} \end{cases}$	$1 - \left(\frac{\lambda}{\lambda+x}\right)^\alpha$	$E[X^k] = \frac{\lambda^k k!}{\prod_{j=1}^k(\alpha-j)}, \alpha > k$	$\frac{\lambda}{\alpha-1}$	$\frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)}$	N/A