TP1 ~ Mathématique actuarielles IARD I

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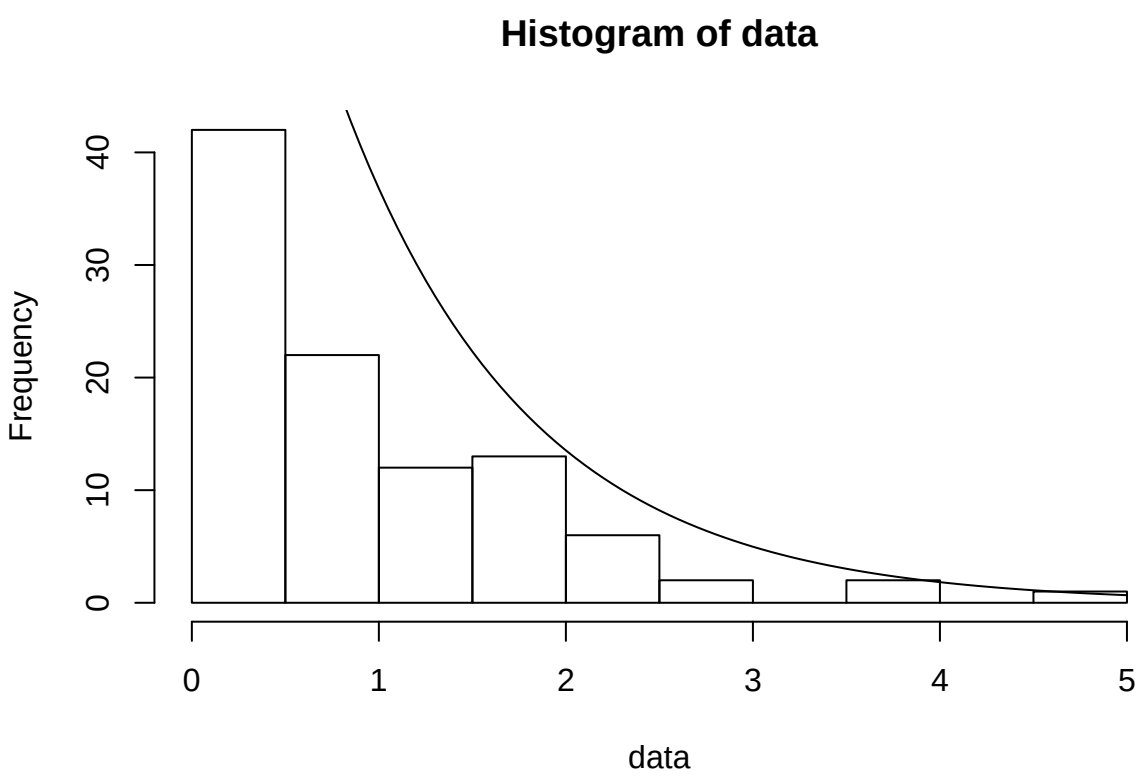
a) Le coéficient d'asymétrie estimé:

```
n <- 100
data <-  rexp(n, 1)


mu <- mean(data)
sd <- sd(data)
mu_2 <- mean(data^2)
mu_3 <- mean(data^3)
cof_asymetrie <- (mu_3 - 3 * mu_2 * mu + 2 * mu^3)/sd^3



hist(data);curve(dexp(x,1) * n, add = TRUE)
```



c)Coeficient d'asymétrie théorique

Les moments de la loi expodentiel sont donnés par:

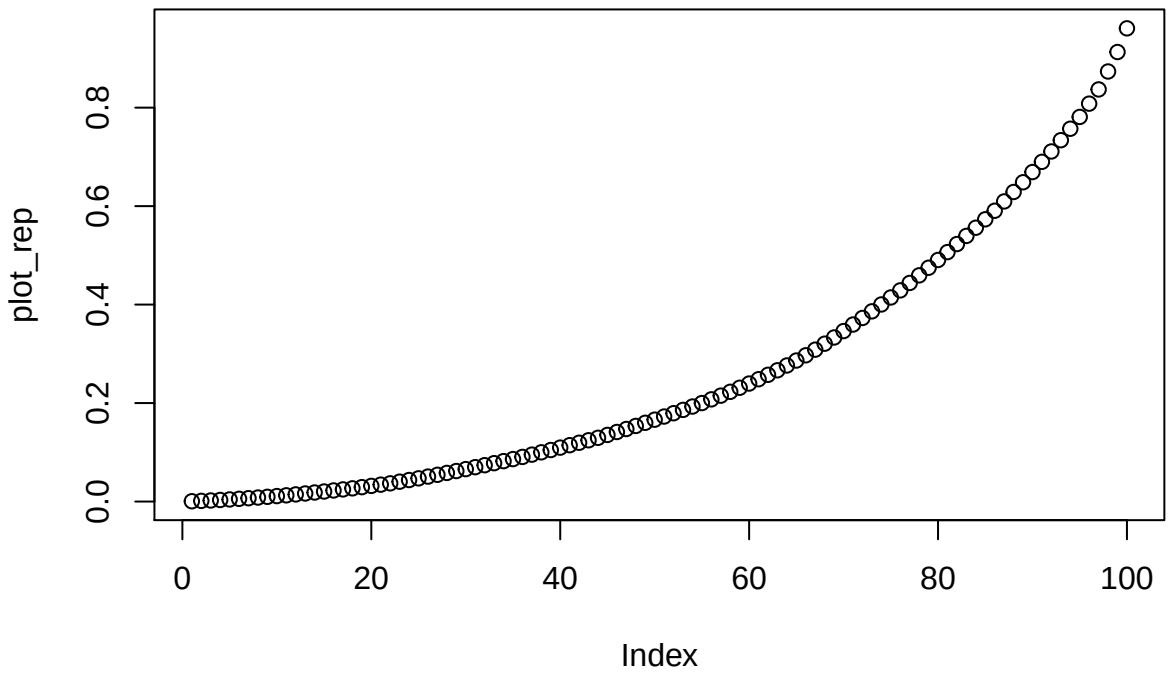
$$
\begin{aligned}
E[x] &= M_x^{'}(t)\Big|_{t=0} \\
&= \frac{d}{dt}\left(\frac{\theta}{\theta - t}\right)\Big|_{t=0} \\
&= \left(\frac{\theta}{(\theta - t)^B}\right)\Big|_{t=0} \\
&= \frac{1}{\theta}
\end{aligned}
\qquad
\begin{aligned}
E[x^2] &= M_x^{''}(t)\Big|_{t=0} \\
&= \frac{d}{dt}\left(\frac{\theta}{(\theta - t)^2}\right)\Big|_{t=0} \\
&= \left(\frac{-2\theta}{(\theta - t)^3}\right)\Big|_{t=0} \\
&= \frac{2}{\theta^2}
\end{aligned}
\qquad
\begin{aligned}
E[x^3] &= M_x^{'''}(t)\Big|_{t=0} \\
&= \frac{d}{dt}\left(\frac{-2\theta}{(\theta - t)^3}\right)\Big|_{t=0} \\
&= \left(\frac{6\theta}{(\theta - t)^4}\right)\Big|_{t=0} \\
&= \frac{6}{\theta^3}
\end{aligned}
$$

Alors, le coeficient d'asymétrie théorique est donnée par

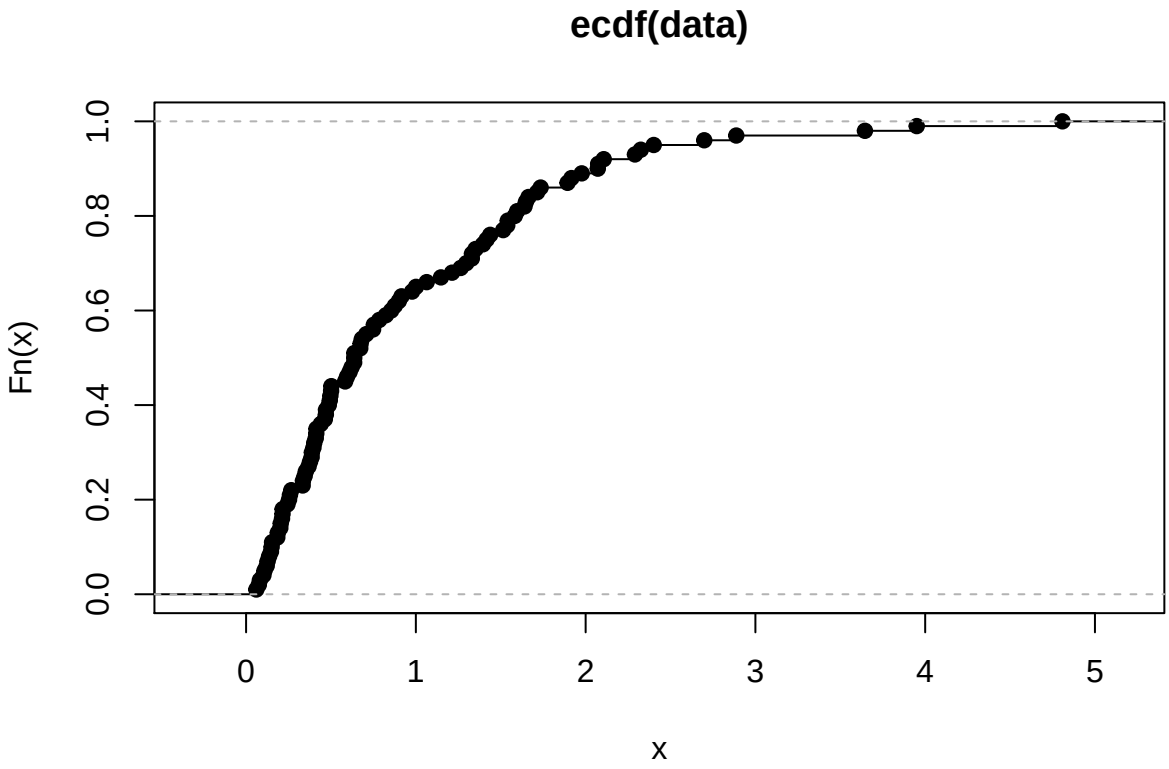
$$
\begin{aligned}
\gamma &= E\left[\frac{(x-\mu)^3}{\sigma^3}\right] \\
&= \frac{1}{\sigma^3}\left(E[x^3 - 3x^2\mu + 3x\mu^2 - \mu^3]\right) \\
&= \frac{1}{\sigma^3}\left(E[x^3] - 3\mu E[x^2] + 3\mu^2 E[x] - \mu^3\right) \\
&= \theta^3\left[\frac{6}{\theta^3} - 3\left(\frac{1}{\theta}\right)\left(\frac{2}{\theta^2}\right) + 3\left(\frac{1}{\theta}\right)^3 - \left(\frac{1}{\theta}\right)^3\right] \\
&= \theta^3\left[\frac{6}{\theta^3} - \frac{6}{\theta^3} + \frac{2}{\theta^3}\right] \\
&= 6 - 6 + 2 \\
&= 2
\end{aligned}
$$

Question 2

```
rep_emp <- function(t){
    sum(data * (data <= t)) / n
}
plot_rep <- sapply(sort(data), rep_emp)
plot(plot_rep)
```




```
plot(ecdf(data))
```


ecdf(data)
Fn(x)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
x