Equation sheet NAME (print):________________________________

$\boldsymbol{N}$ frame unit vectors $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ $\boldsymbol{B}$ frame unit vectors $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$

Vectors $\vec{r} = r_{N1}\hat{n}_1 + r_{N2}\hat{n}_2 + r_{N3}\hat{n}_3$ (expressed in $\boldsymbol{N}$ frame) $\vec{r} = r_{B1}\hat{b}_1 + r_{b2}\hat{b}_2 + r_{B3}\hat{b}_3$ (in $\boldsymbol{B}$ frame)

Vector components $\underline{r}^N = [r_{N1} \quad r_{N2} \quad r_{N3}]^{\mathrm{T}}$ $\underline{r}^B = [r_{B1} \quad r_{B2} \quad r_{B3}]^{\mathrm{T}}$

Rotation Matrices / Direction Cosine Matrices (DCM)

Relation for unit vectors $\begin{Bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{Bmatrix} = \begin{bmatrix} \hat{b}_1 \bullet \hat{n}_1 & \hat{b}_1 \bullet \hat{n}_2 & \hat{b}_1 \bullet \hat{n}_3 \\ \hat{b}_2 \bullet \hat{n}_1 & \hat{b}_2 \bullet \hat{n}_2 & \hat{b}_2 \bullet \hat{n}_3 \\ \hat{b}_3 \bullet \hat{n}_1 & \hat{b}_3 \bullet \hat{n}_2 & \hat{b}_3 \bullet \hat{n}_3 \end{bmatrix} \begin{Bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \end{Bmatrix}$ or $\{\hat{b}\} = [R^{NB}]\{\hat{n}\}$

Also for vector components $\begin{Bmatrix} r_{B1} \\ r_{B2} \\ r_{B3} \end{Bmatrix} = [R^{NB}] \begin{Bmatrix} r_{N1} \\ r_{N2} \\ r_{N3} \end{Bmatrix}$ or $\{\underline{r}^B\} = [R^{NB}]\{\underline{r}^N\}$

Inverse $[R^{BN}] = [R^{NB}]^{-1} = [R^{NB}]^T$ $\{\underline{r}^N\} = [R^{BN}]\{\underline{r}^B\}$

Rotation matrices for rotation about single axes ($\boldsymbol{N}$ -> $\boldsymbol{B}$)

$\underline{R}_1^{NB}(\phi_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & \sin\phi_1 \\ 0 & -\sin\phi_1 & \cos\phi_1 \end{bmatrix}$ $\underline{R}_2^{NB}(\phi_2) = \begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 \\ 0 & 1 & 0 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{bmatrix}$ $\underline{R}_3^{NB}(\phi_3) = \begin{bmatrix} \cos\phi_3 & \sin\phi_3 & 0 \\ -\sin\phi_3 & \cos\phi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Composing multiple rotations (two shown) $\{\underline{r}^B\} = [R^{KB}]\{\underline{r}^K\} = [R^{KB}][R^{NK}]\{\underline{r}^N\} = [R^{NB}]\{\underline{r}^N\}$

Angular velocities add $\vec{\omega}^{B/N} = \dot{\phi}_1 \hat{u}_{\phi_1\,axis} + \dot{\phi}_2 \hat{u}_{\phi_2\,axis} + \dot{\phi}_3 \hat{u}_{\phi_3\,axis}$ (each about appropriate unit vector)

Express in $\boldsymbol{B}$ frame $\vec{\omega}^{B/N} = \omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3$

Transport theorem ${}^N\frac{d}{dt}(\vec{r}) = {}^B\frac{d}{dt}(\vec{r}) + \vec{\omega}^{B/N} \times \vec{r}$ (velocity)

$${}^N\frac{d^2}{dt^2}(\vec{r}) = {}^B\frac{d^2}{dt^2}(\vec{r}) + {}^B\frac{d}{dt}\left(\vec{\omega}^{B/N}\right) \times \vec{r} + 2\vec{\omega}^{B/N} \times {}^B\frac{d}{dt}(\vec{r}) + \vec{\omega}^{B/N} \times \left(\vec{\omega}^{B/N} \times \vec{r}\right) \quad \text{(acceleration)}$$

${}^N\dot{\vec{r}} = \dot{r}_{N1}\hat{n}_1 + \dot{r}_{N2}\hat{n}_2 + \dot{r}_{N3}\hat{n}_3$ ${}^B\dot{\vec{r}} = \dot{r}_{B1}\hat{b}_1 + \dot{r}_{b2}\hat{b}_2 + \dot{r}_{B3}\hat{b}_3$ $\vec{\omega}^{B/N} \times \vec{r} = \begin{Bmatrix} \hat{b}_1(-\omega_3 r_{B2} + \omega_2 r_{B3}) \\ +\hat{b}_2(\omega_3 r_{B1} - \omega_1 r_{B3}) \\ +\hat{b}_3(-\omega_2 r_{B1} + \omega_1 r_{B2}) \end{Bmatrix}$

$T = \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}mv^2$ (inertial velocity) $\vec{H} = \vec{r} \times m\vec{v}$ (about inertial origin, mass center)

Two-body problem center of mass $\vec{R}_c = \frac{m_1\vec{R}_1 + m_2\vec{R}_2}{m_1 + m_2}$ Equation of motion $\ddot{\vec{R}}_c = \vec{0}$

relative motion $\vec{r} \equiv \vec{R}_2 - \vec{R}_1$ Equation of relative motion $\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}$

Gravitation $\vec{F}_G = -\frac{Gm_1m_2}{r^2}\frac{\vec{r}}{r}$ $V_G = -\frac{Gm_1m_2}{r}$

position vector $\vec{r}$ and velocity vector $\vec{v} = \dot{\vec{r}}$ $V = |\vec{v}|$

$\varepsilon = \frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$ $V = \sqrt{2\left(\varepsilon + \frac{\mu}{r}\right)}$ $r = \frac{2\mu}{V^2 - 2\varepsilon}$ μ = GM $Gm_{\oplus} = 3.986 \times 10^5$ km³/s²

$\vec{h} = \vec{r} \times \vec{v} = \vec{r} \times \dot{\vec{r}}$ $h = |\vec{h}|$ $h = r\,V_T = r^2\dot{\upsilon}$ (nu)

$\vec{e} = \frac{1}{\mu}\left(\dot{\vec{r}} \times \vec{h}\right) - \frac{\vec{r}}{r}$ $e = |\vec{e}|$ $e = \sqrt{1 + \frac{2h^2\varepsilon}{\mu^2}}$

$a = -\frac{\mu}{2\varepsilon}$ $p = h^2/\mu = a(1 - e^2)$ $\upsilon = \cos^{-1}\left(\frac{p - r}{e\,r}\right)$

$\upsilon = \cos^{-1}\left(\frac{\vec{e}\bullet\vec{r}}{e\,r}\right)$ $(0 \le \text{angle} \le \pi)$ if $\vec{r}\bullet\vec{v} < 0$, $\upsilon = -\upsilon + 2\pi$ (heading towards perigee)

$\tan\left(\frac{\upsilon}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$ $\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\upsilon}{2}\right)$

$M = E - e\sin E = \sqrt{\frac{\mu}{a^3}}\,(t - T_0)$ $t_0 = t - \sqrt{\frac{a^3}{\mu}}\,M$ $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$i = \cos^{-1}\left(\frac{\vec{h}\bullet\hat{k}}{h}\right)$ (principal value: $0 \le \text{angle} \le \pi$)

Line of nodes, $\hat{n} = \frac{\hat{k}\times\vec{h}}{|\hat{k}\times\vec{h}|}$ (if $i = 0, \hat{n} = \hat{\imath}$) $\hat{n} = \hat{\imath}\cos\Omega + \hat{\jmath}\sin\Omega = \hat{\imath}\,n_1 + \hat{\jmath}\,n_2$

$\Omega = \tan^{-1}(n_2, n_1)$ (2-argument arctangent: $-\pi \le \text{angle} \le \pi$)

$\omega = \cos^{-1}\left(\frac{\hat{n}\bullet\vec{e}}{e}\right)$ (returns $0 \le \text{angle} \le \pi$) if $\vec{e}\bullet\hat{k} < 0$, $\omega = -\omega + 2\pi$ (perigee south of equator)

$\dot{\Omega} = -\frac{3nJ_2R_\oplus^2}{2a^2(1-e^2)^2}\cos i$ $\dot{\omega} = \frac{3nJ_2R_\oplus^2}{2a^2(1-e^2)^2}\left(\frac{5}{2}\sin^2 i - 2\right)$

$r = \frac{p}{1+e\cos\upsilon}$ $\vec{r} = r\,\hat{r}$ $\vec{r} = \hat{p}\,(r\cos\upsilon) + \hat{q}\,(r\sin\upsilon)$ $\vec{v} = \left(-\sqrt{\frac{\mu}{p}}\sin\upsilon\right)\hat{p} + \sqrt{\frac{\mu}{p}}(e + \cos\upsilon)\,\hat{q}$

$\vec{v} = \dot{\vec{r}} = V_R\,\hat{r} + V_T\,\hat{t} = \dot{r}\,\hat{r} + r\dot{\upsilon}\,\hat{t}$ $V_R = \dot{r} = \sqrt{\frac{\mu}{p}}\,e\sin\upsilon$ $V_T = r\dot{\upsilon} = \sqrt{\frac{\mu}{p}}\,(1 + e\cos\upsilon)$

Perifocal to ECI $\underline{R}^{IP} = R_3(\omega)\,R_1(i)\,R_3(\Omega)$ (3-1-3 Euler rotations) $\begin{Bmatrix}\hat{p}\\ \hat{q}\\ \hat{w}\end{Bmatrix} = [\underline{R}^{IP}]\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix} = \begin{bmatrix}\lfloor\vec{e}/e\rfloor\\ \lfloor\hat{w}\times\hat{p}\rfloor\\ \lfloor\vec{h}/h\rfloor\end{bmatrix}\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix}$

Hohmann $a_T = \frac{r_1+r_2}{2}$ $\Delta V_1 = V_{TP} - V_{1C} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T}} - \sqrt{\frac{\mu}{r_1}}$ $\Delta V_2 = V_{2C} - V_{TA} = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_T}}$

Low thrust $\Delta V = \sqrt{\frac{\mu}{r_{1C}}} - \sqrt{\frac{\mu}{r_{2C}}}$ Inclination $\Delta V = 2V_T\sin\left(\frac{\Delta i}{2}\right)$ (V_R: no change)

Single-impulse $\Delta V_R = V_{2R} - V_{1R}$ $\Delta V_T = V_{2T} - V_{1T}$ $\Delta V = \sqrt{\Delta V_R^2 + \Delta V_T^2}$

$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$ $\Delta V = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos(\phi_2 - \phi_1)}$ $\phi = \cos^{-1}\left(\frac{h}{r\,V}\right)(+quad\checkmark) = \tan^{-1}\left(\frac{V_R}{V_T}\right)$

Propulsion / staging $m_P = m_0\left(1 - e^{-\Delta V/I_{sp}g}\right)$ $T = \dot{m}V_E = \dot{m}\,I_{sp}\,g$ $t_b = \frac{m_P}{\dot{m}} = \frac{W_0}{T}\,I_{sp}\,\frac{m_P}{m_0}$

$\Delta V = V_E\ln\left(\frac{m_0}{m_f}\right) = I_{sp}\,g\ln\left(\frac{m_0}{m_f}\right)$ $\pi = \frac{m_*}{m_0} = 1 - \frac{m_P/m_0}{1-\epsilon}$ $\epsilon = \frac{m_S}{m_P+m_S}$ $\frac{m_f}{m_0} = 1 - \frac{m_P}{m_0} = \epsilon + (1-\epsilon)\pi$

$\Delta V_{*N} = -\sum_{k=1}^{N} I_{sp\,k}\,g\,\ln(\epsilon_k + (1-\epsilon_k)\pi_k)$ $\pi_* = \prod_{k=1}^{N}\pi_k$ $\pi_k = \frac{m_{0\,(k+1)}}{m_{0\,(k)}}$

$\begin{Bmatrix}H_1\\ H_2\\ H_3\end{Bmatrix} = \begin{bmatrix}I_{11} & I_{12} & I_{13}\\ I_{21} & I_{22} & I_{23}\\ I_{31} & I_{32} & I_{33}\end{bmatrix}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}$ $T = \frac{1}{2}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}^T\begin{bmatrix}I_{11} & I_{12} & I_{13}\\ I_{21} & I_{22} & I_{23}\\ I_{31} & I_{32} & I_{33}\end{bmatrix}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}$ $\vec{M} = \vec{r}\times\vec{F}$

$[\underline{I}^A]\{\hat{e}_i\} = \lambda_i\{\hat{e}_i\}$ $[E] = [\{\hat{e}_1\}\{\hat{e}_2\}\{\hat{e}_3\}]$ $[\underline{I}^B] = [E]^T[\underline{I}^A][E] = \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$ $[R^{AB}] = [E]^T$

$I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) = M_1$
$I_2\dot{\omega}_2 + \omega_3\omega_1(I_1 - I_3) = M_2$
$I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) = M_3$

torque − free
axisymmetric

$\vec{H}\leftrightarrow\hat{b}_3:$ $\tan\theta = \frac{I_1\omega_0}{I_3\Omega}$

$\vec{H}\leftrightarrow\vec{\omega}:$ $\tan\gamma = \frac{I_3}{I_1}\tan\theta = \frac{\omega_0}{\Omega}$

$M_{G1} = \frac{3\mu}{R^5}YZ(I_3 - I_2)$
$M_{G2} = \frac{3\mu}{R^5}ZX(I_1 - I_3)$
$M_{G3} = \frac{3\mu}{R^5}XY(I_2 - I_1)$

$\omega_{s/c} = -\frac{I_R}{I_{s/c}+I_R}\omega_R$ $\omega_{b3} = +\frac{I_R}{I_{s/c}}\omega_{R1}\sin\theta_{R2}$

$P_{sun} = \phi_S\,A_{proj}$ $P_{rad} = \sigma\epsilon T^4$ $\phi_S = 1371\,\frac{W}{m^2}$ $\sigma = 5.67e-8\,\frac{W}{m^2K^4}$