



## near-circular LEO

Problem 1)

$$\text{given: } \mu = 39800 \frac{\text{km}^3}{\text{s}^2}$$

Departure	$a_1 = 8000 \text{ km}$	$e_1 = 0.01$	$f_1 = 30^\circ = \frac{\pi}{6} \text{ rad}$
Arrival	$a_2 = 27000 \text{ km}$	$e_2 = 0.6$	$f_2 = 210^\circ = \frac{7\pi}{6} \text{ rad}$

no plane change

$$(\Delta I = 0, \Delta \Omega = 0)$$

HEO

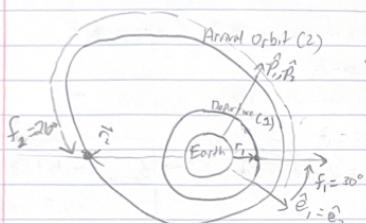
1)  $\alpha_+ = ?$  (transfer orbit semi-major axis)

$$a_{mn,T} = \frac{1}{q}(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\alpha_f)}) , \quad r_1 = ?, \quad r_2 = ?, \quad \alpha_f = ?$$

$$r_1 = \frac{a(1-e^2)}{1+e\cos f} \rightarrow r_1 = \frac{a_1(1-e_1^2)}{1+e_1\cos f_1} \rightarrow r_1 = \frac{(8000)(1-0.01^2)}{1+(0.01)\cos(30^\circ)} \text{ km} \rightarrow r_1 = 7930.519605 \text{ km}$$

$$r_2 = \frac{a_2(1-e_2^2)}{1+e_2\cos f_2} \rightarrow r_2 = \frac{(27000)(1-0.6^2)}{1+(0.6)\cos(210^\circ)} \text{ km} \rightarrow r_2 = 35971.16628 \text{ km}$$

$\star f_2 - f_1$ , because it is in different frames



The departure orbit is near-circular.  
 $\therefore$  We can approximate that it really is circular. For circular orbits, we can choose the orientation of the  $\hat{e}, \hat{p}$  frame.

I chose to align it with that of the high-eccentricity orbit.

Because  $\hat{e}_1 = \hat{e}_2$ , we are able to simply subtract  $f_2 - f_1$  to find  $\Delta f$

$$\Delta f = f_2 - f_1 \rightarrow \Delta f = 210^\circ - 30^\circ \rightarrow \Delta f = 180^\circ$$

$$a_{mn,T} = \frac{1}{q}(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\Delta f)})$$

$$a_{mn,T} = \frac{1}{q}((7930) + (35971)) + \sqrt{(7930)^2 + (35971)^2 - 2(7930)(35971) \cdot \cos(180^\circ)}$$

$$a_{mn,T} = 21950.5 \text{ km}$$

\* note that the decimal places were omitted in working, but used in the calculations

2)  $P = ?$  given  $P = \frac{\text{Km} - 2akE}{4\pi m - 2aE^2}$        $K = r_1 r_2 (1 - \cos \Delta f) \rightarrow K = (7930)(35971)(1 - \cos(180^\circ)) \rightarrow K = 5.7054 \times 10^{18} \text{ km}^2$   
 $m = \pi r_1 r_2 (1 + \cos \Delta f) \rightarrow m = (7930)(35971)(1 + \cos(180^\circ)) \rightarrow m = 0 \text{ km}^2$   
 $E = r_1 + r_2 \rightarrow E = 7930 + 35971 \rightarrow E = 43902 \text{ km}$

Plugging in  $K, m, E, a_{mn,T} \rightarrow P = 12995.8582 \text{ km}$

3)  $\Delta V$  (Departure orbit  $\rightarrow$  transfer orbit)

• Using the energy equation, we can calculate the velocity at each point

• because  $\Delta f = 180^\circ$ , the velocity impulses will be parallel to the direction its moving in already.

$$\therefore \Delta V = |V_{\text{orbit}} - V_{\text{transfer, orbit}}| \quad (\text{the vectors are already aligned})$$

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## Problem 1.3 (continued)

$$\frac{1}{2}V^2 - \frac{M}{r} = -\frac{M}{2a} \rightarrow \frac{1}{2}V^2 = \frac{M}{r} - \frac{M}{2a} \rightarrow V = \sqrt{2M\left(\frac{1}{r} - \frac{1}{2a}\right)} \rightarrow V = 2M\left(\frac{1}{r} - \frac{1}{2a}\right)^{1/2} = 7.1203 \text{ km/s}$$

$$V_1 = 2M\left(\frac{1}{r_1} - \frac{1}{2a}\right)^{1/2} \rightarrow V_1 = 9.0755 \text{ km/s}$$

Note that when moving from the departure orbit to transfer orbit, radius = constant for impulse

$$\Delta V_1 = V_{1T} - V_1 \rightarrow \Delta V_1 = 1.9552 \text{ km/s}$$

- 4) The same energy equation can be used to find the  $\Delta V$  to go from the transfer orbit to the arrival orbit.

$$\text{This process yields: } V_{2T} = 2.0009 \text{ km/s} \rightarrow \Delta V_2 = 0.7193 \text{ km/s}$$

$$V_2 = 2.7202 \text{ km/s}$$

for a minimum energy transfer with  $\Delta f = 180^\circ$

- 5) Because  $\Delta f = 180^\circ$ , the transfer orbit will have its perigee at  $\vec{r}_1$  & apogee at  $\vec{r}_2$ .

Going from perigee  $\rightarrow$  apogee is exactly half an orbit.

Therefore the time it takes is equal to one half the orbital period,  $T_p$ .

$$T_p = 2\pi\sqrt{\frac{a^3}{M}} \rightarrow t_{\text{transfer}} = \pi\sqrt{\frac{a^2}{M}} = 2\pi\sqrt{\frac{21951^{3/2}}{39860}} \text{ s} \rightarrow t_{\text{transfer}} = 16183 \text{ seconds}$$

$\approx 4.5$  hours

- 6) Plotting:

1. get the radius at each point of the orbit by using this equation and iterating through  $f$  values:

$$r = \frac{a(1-e^2)}{1+e\cos f} \quad (\text{This can be used for all orbits given that we know the eccentricity})$$

2. To plot, we need  $x$  and  $y$  coordinates. So, convert  $r$  &  $f$  to  $x$  &  $y$ :

$$\begin{aligned} \vec{r} &= r\cos\hat{x} + r\sin\hat{y} \\ \vec{r} &= r\cos\hat{x} + r\sin\hat{y} \end{aligned}$$

Note we still need  $e$  of the transfer orbit:

$$r = \frac{p}{1+e\cos f} \quad \vec{r}_1 = \frac{p}{1+e\cos f_1} \rightarrow e_T = \frac{p}{r_1} - 1 \rightarrow e_T = \frac{12996}{7930} - 1 \rightarrow e_T = 0.6327$$

$$r = r_1 @ f = 0$$

This also does not account for the orientation of the transfer orbit.

However, we know that perigee is at  $\vec{r}_1$  and apogee at  $\vec{r}_2$  so we can just subtract  $30^\circ$  ( $\Delta f$ ) in the argument for the angle.

- 7)  $\hat{e}$ ,  $\beta$  does not align between the departure and transfer orbit. This is evident on the figure.

The angle between  $\hat{e}_1$  and  $\hat{e}_{\text{transfer}}$  =  $30^\circ$  because it is the same as  $\Delta f$