Problem 3: Lambert's Problem

a) Givens

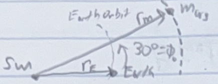
Need to get to mars on a T.O. who's T.O.F $\leq$ 183 days (1.58112×10^6 s)

$1 A.U. = 149.6 \times 10^6$ km

Assumptions: • circular, co-planar orbits ✓

• $r_E = 1 A.U.$, $r_m = 1.524 A.U.$, $\mu_{sun} = 1.327 \times 10^{11} \frac{km^3}{s^2}$

• $\phi_0 = 40°$

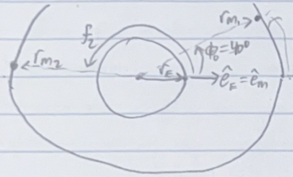


1) b) To find: Δf

c) Solution Process:

For circular orbits, $a = r$ and $e = 0$ $\rightarrow$ $a_E = r_E = 1 A.U.$, $e_E = 0$; $a_m = r_m = 1.524 A.U.$, $e_M = 0$

Because it's a circular obit, we can define $f = 0°$ in any direction. Will choose to align it w/ $\vec{r}_E$



$\theta_{M_0} = 40°$

$\theta_{E_0} = 0° = f_1$

$\longrightarrow$ $f_1 = 40°$

$f_2 = ?$

$f_2 = \omega_{orbit} \times \Delta t$, $\omega_{orbit} = \frac{2\pi}{T_P}$ and $\Delta t = T.O.F$ $\Big\}\longrightarrow$

for circular orbit: $T_P = 2\pi\sqrt{\frac{r^3}{\mu}}$

$\rightarrow$ $\Delta f = \frac{2\pi}{2\pi\sqrt{\frac{r_m^3}{\mu_s}}}(T.O.F) \rightarrow \Delta f = \sqrt{\frac{1.327 \times 10^{11}\ km^3/s^2}{(1.524 \times 149.6 \times 10^6\ km)^3}} \times (1.58112 \times 10^6\ sec) \rightarrow \boxed{\Delta f = 2.3712\ rad = 135.8624°}$

2) a) given: everything above

b) To find: a_T & e_T

c) Solution Process:

$a_{min,T} = a_T = \frac{1}{4}\left(r_E + r_m + \sqrt{r_E^2 + r_m^2 - 2r_E r_m \cos\Delta f}\right) \rightarrow \boxed{a_T = 1.8219 \times 10^8\ km}$

e_T can be found using the same process as problem 1.6) & also problem 2A1)

$r = \frac{P}{1+e\cos f} \rightarrow e_T = \frac{P_T}{r_1} - 1$, $P_T = \frac{km - 2akl}{4am - 2akl^2}$, $\begin{cases} k = r_1 r_2(1-\cos\Delta f) \\ m = r_1 r_2(1+\cos\Delta f) \\ l = r_1 + r_2 \end{cases} \Rightarrow \begin{cases} k = 5.8585 \times 10^{16}\ km^2 \\ m = 9.6296 \times 10^{15}\ km^2 \\ l = 3.7759 \times 10^8\ km \end{cases}$ $\longrightarrow$

$\rightarrow$ Plugging in those values $\rightarrow P_T = 1.6683 \times 10^8$ Km

$\therefore e_T = \frac{1.6683 \times 10^8\ km}{1.496 \times 10^8\ km} - 1 \rightarrow \boxed{e_T = 0.1152}$

3) a) given: everything above

b) To find: ΔV_{Total}

c) Solution Process: $\Delta V_{Total} = |\Delta\vec{V}_1 + \Delta\vec{V}_2|$, $\Delta V_1 = ?$, $\Delta V_2 = ?$

• Finding ΔV_1 ... $\frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \rightarrow V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} \rightarrow V_E = \sqrt{2\mu_s\left(\frac{1}{r_1} - \frac{1}{2a}\right)} = 29.7831$ km/s

$V_{ET} = \sqrt{2\mu_s\left(\frac{1}{r_1} - \frac{1}{2a_T}\right)} = 32.3371$ km/s

$\Delta V_1 = \sqrt{V_{ET}^2 + V_E^2 - 2V_{ET}V_E\cos(\gamma_{ET} - \gamma_E)}$, $\gamma_{ET} = ?$, $\gamma_E = ?$

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Recall that for a circular orbit we can choose anywhere for $\hat{e}$ to point because there is no perigee to point to. Therefore, we can make $f=0$. Thus, $\gamma_E = 0°$ and $\gamma_M = 0°$.

~~$\gamma_{ET} = \tan^{-1}\left(\frac{e_T \sin f_{ET}}{1+e_T\cos f_{ET}}\right)$, $r = \frac{p}{1+e\cos f} \Rightarrow f = \cos^{-1}\left(\frac{1}{e}\left(\frac{p}{r}-1\right)\right) \Rightarrow f_{ET} =$~~

$$\therefore \Delta V_E = \sqrt{V_{ET}^2 + V_E^2 - 2V_{ET}V_E} \rightarrow \Delta V_E = 2.5542 \frac{km}{s}$$

- The same process can be repeated for getting ΔV_2 (ΔV_m)

$$V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} \rightarrow V_{mT} = 20.8738 \text{ km/s}, \quad V_m = 24.1255 \text{ km/s}$$

$$\Delta V = \sqrt{V_f^2 + V_i^2 - 2V_fV_i} \rightarrow \Delta V_M = 3.2517 \frac{km}{s}$$

$$\Delta V_{Total} = \Delta V_E + \Delta V_m \rightarrow \boxed{\Delta V_{Total} = 5.8059 \frac{km}{s}}$$

4) Plotting Orbit

a) given: everything above

b) To find: Plot

c) Solution Process:

- Convert radii and true anomaly to x & y coordinates for plotting. Same process as problem 1.6 or problem 2.B3:

$X_E = r_E \cos F$, $Y_E = r_E \sin F$ ← r_E, F are both known (and constant)

$X_m = r_m \cos F$, $Y_m = r_m \sin F$ ← r_m, F are both known (and constant)

$X_T = r_T \cos F$, $Y_T = r_T \sin F$ ← use energy to get r_T @ each f value

- These calculations are easily vectorized and plotted