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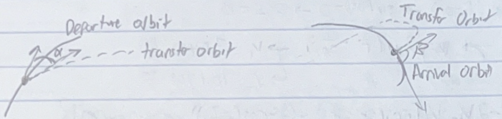
Problem 2)

given: Plane change maneuver:

Departure	$a_1 = 7000$ Km	$e_1 = 0$	$I_1 = 60°$
Arrival	$a_2 = 70000$ Km	$e_2 = 0$	$I_2 = 20°$

Assume: $\Omega = \omega =$ constant $0°$; Hohmann-like transfer (departure @ perigee, arrival @ apogee of T.O.)
Also means a_T & e_T can be calculated as if it were co-planar

Observations from given figure: ΔV_1 & ΔV_1 apply plane change and shape change simultaneously



A1) $a_T = ?$, $e_T = ?$

- Because both orbits are circular ($e_1 = e_2 = 0$), $a_1 = r_1 = 7000$ Km and $a_2 = r_2 = 70000$ Km
- Assuming departure @ perigee & arrival @ apogee of the transfer orbit: $\Delta f = 180°$ because perigee and apogee are on opposite ends of the transfer orbit.
  ↑ assume elliptic T.O.

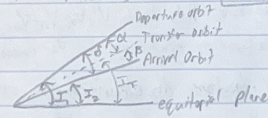
$a_T = \frac{1}{4}\left(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\Delta f)}\right) \rightarrow \boxed{a_T = 38{,}500 \text{ km}}$

same process as problem 1.6

$r = \frac{P}{1 + e\cos f} \rightarrow e_T = \frac{P_T}{r_1} - 1$, $P_T = \frac{km - 2akl}{4am - 2akl^2}$, $\begin{cases} k = r_1 r_2(1-\cos\Delta f) \\ m = r_1 r_2(1+\cos\Delta f) \\ l = r_1 + r_2 \end{cases} \Rightarrow \begin{cases} k = 9.8\times10^8 \text{ Km}^2 \\ m = 0 \text{ Km}^2 \\ l = 7.7\times10^5 \text{ Km} \end{cases}$

$\rightarrow P_T = 1.2727\times10^4$ Km $\rightarrow e_T = \frac{1.2727\times10^4 \text{ km}}{7\cdot10^3 \text{ km}} - 1 \rightarrow \boxed{e_T = 0.812}$

A2) $I_T = ?$ given $\alpha = 20° = \Delta I_1$



(Side View of orbit planes)

Observations:
- $\rightarrow \delta = \alpha + \beta = I_1 - I_2$
- $\rightarrow I_T = I_2 + \beta$
- $\rightarrow I_T = I_1 - \alpha \rightarrow I_T = (60°) - (20°) \rightarrow \boxed{I_T = 40°}$

- Calculating other angles for future use:

$\delta = I_1 - I_2 \rightarrow \delta = 60° - 20° \rightarrow \delta = 40°$
$\delta = \alpha + \beta \rightarrow \beta = \delta - \alpha \rightarrow \beta = 40° - 20° \rightarrow \beta = 20° = \Delta I_2$

A3) $\Delta V_1 = ?$, $\Delta V_2 = ?$

General equation: $\Delta V^2 = V_f^2 + V_i^2 - 2V_f V_i\left[\cos(\Delta\gamma) - (1 - \cos(\Delta I))\cos(\gamma_f)\cos(\gamma_i)\right]$,

↓ next page

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Problem 2 - A3 (continued)

- From Departure → Transfer Orbit: • $\Delta I = \Delta I_1 = \alpha = 20^\circ$
  - $V_i = V_1$
  - $V_f = V_{1T}$

  } Can determine using energy equation

- $\gamma_1 = ?$ $\tan \gamma_1 = \frac{e_1 \sin f_1}{1 + e_1 \cos f_1}$. We can state that $\gamma_1 = 0$ because it is a function of the true anomaly. We can arbitrarily set the true anomaly to anything such as 0° because on a circular orbit the direction of $\hat{e}$ doesn't matter. ∴ $\sin(0^\circ) = 0 \rightarrow \gamma_1 = \tan^{-1}(0) = 0^\circ$
- → $\gamma_{1T} = ?$ Departure @ perigee ($f = 0^\circ$) → $\sin(0^\circ) = 0 \rightarrow \gamma_{1T} = 0$ Similarly γ_{1T}
- → $V_1 = ?$ $\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \rightarrow V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)}$ → $V_1 = 7.5460 \ \frac{km}{s}$
  → $V_{1T} = 10.1751 \ \frac{km}{s}$

  ($a = r$ for a circular orbit)

$$\sqrt{\Delta V_1^2} = \text{sqrt}\{V_{1T}^2 + V_1^2 - 2V_{1T} \cdot V_1 [\cos(\cancel{\gamma_{1T}} - \cancel{\gamma_1}) - (\cancel{1} - \underbrace{\cos(\Delta I_1)}_{\alpha})\underbrace{\cos(\cancel{\gamma_{1T}})\cos(\cancel{\gamma_1})}_{1}]\} \longrightarrow$$

→ $\Delta V_1 = \sqrt{V_{1T}^2 + V_1^2 - 2V_{1T}V_1\cos(\alpha)} \rightarrow \boxed{\Delta V_1 = 4.0216 \ \frac{km}{s}}$

V_f V_i

- From Transfer Orbit → Arrival Orbit: $\Delta I_2 = \beta = -20^\circ$ (from part A2)
- → using the same thought process as above: $\gamma_2 = 0^\circ$, $\gamma_{2T} = 0^\circ$, $V_2 = 2.3863 \ \frac{km}{s}$, $V_{2T} = 1.0175 \ \frac{km}{s}$
- → Plugging in these values into $\Delta V_2 = \sqrt{V_2^2 + V_{2T}^2 - 2V_2 V_{2T}\cos(\beta)} \rightarrow \boxed{\Delta V_2 = 1.4719 \ \frac{km}{s}}$

Part B

1) Steps:
   1. Iterate through a list of α values from $-50 \rightarrow +50$ in increments of 3°
   2. At each iteration, calculate ΔV_i. Note that $V_1, V_2, V_{1T}, V_{2T}, \beta, \gamma_1, \gamma_{1T}, \gamma_2, \gamma_{2T}$ are all constant (I do this step using a function that does all the calculations.)
   3. At the end of all the iterations do element-wise vector addition and store the result for plotting.

2)
- Plot α on x-axis & the total ΔV on the y-axis
- Have Matlab find and print the minimum ΔV and the corresponding α based on it's index

The result was a minimum ΔV of $4.105 \ \frac{km}{s}$ @ $\alpha = 1^\circ$. (should be 0° but it's skipped because we iterate in steps of 3°)

3) The process for plotting the orbits is the same as in problem 1.6 except we need to calculate the z-coordinates: $r_z = r \cdot \sin I$