AERSP 309 Sec 001 – MELIK DEMIREL – FINAL EXAM FORMULA SHEET – 2:30-4:20 – Chambers 101.

Orbits

$R_3^{NB}(\phi_3) = \begin{bmatrix} c\phi_3 & s\phi_3 & 0 \\ -s\phi_3 & c\phi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos(x) = -\sin(x)$

I have completed this work with integrity

$\underline{R}_1^{NB}(\phi_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & \sin\phi_1 \\ 0 & -\sin\phi_1 & \cos\phi_1 \end{bmatrix}$ $\underline{R}_2^{NB}(\phi_2) = \begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 \\ 0 & 1 & 0 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{bmatrix}$ $\{\underline{r}^B\} = [R^{KB}]\{\underline{r}^K\} = [R^{KB}][R^{NK}]\{\underline{r}^N\} = [R^{NB}]\{\underline{r}^N\}$

$\boxed{\text{atan2}} = \begin{cases} x=y=0 & \text{undefined} \\ x=0,\ y<0 & \pi \\ x=0,\ y>0 & -\pi/2 \\ & +\pi/2 \end{cases}$ $\begin{cases} x<0,\ y<0 & \arctan(y/x)-\pi \\ x<0,\ y\geq 0 & \arctan(y/x)+\pi \\ x>0 & \arctan(y/x) \end{cases}$

Gravitation $\vec{F}_G = -\frac{Gm_1m_2}{r^2}\frac{\vec{r}}{r}$ $V_G = -\frac{Gm_1m_2}{r}$ $V_{esc} = \sqrt{\frac{2\mu}{r}}$ $T+V=0$

position vector $\vec{r}$ and velocity vector $\vec{v} = \dot{\vec{r}}$ $V = |\vec{v}|$

$\varepsilon = \frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$ $V = \sqrt{2\left(\varepsilon + \frac{\mu}{r}\right)}$ $r = \frac{2\mu}{V^2 - 2\varepsilon}$ $\mu = GM$ $\boxed{\mu_\oplus = 3.986 \times 10^5\ \text{km}^3/\text{s}^2}$

$\vec{h} = \vec{r} \times \hat{v} = \vec{r} \times \dot{\vec{r}}$ $h = |\vec{h}|$ $h = r\,V_T = r^2\dot{\upsilon}$ (nu) $T_{\text{next time}} = T_0 + T_{\text{now}}$

$\vec{e} = \frac{1}{\mu}(\dot{\vec{r}} \times \vec{h}) - \frac{\vec{r}}{r}$ $e = |\vec{e}|$ $e = \sqrt{1 + \frac{2h^2\varepsilon}{\mu^2}}$

$a = -\frac{\mu}{2\varepsilon}$ $p = h^2/\mu = a(1-e^2)$ $\upsilon = \cos^{-1}\left(\frac{p-r}{er}\right)$

$\upsilon = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right)$ $(0 \leq \text{angle} \leq \pi)$ if $\vec{r} \cdot \hat{v} < 0$, $\upsilon = -\upsilon + 2\pi$ (heading towards perigee)

$\tan\left(\frac{\upsilon}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$ $\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\upsilon}{2}\right)$

$M = E - e\sin E = \sqrt{\frac{\mu}{a^3}}(t - T_0)$ $t_0 = t - \sqrt{\frac{a^3}{\mu}}M$ $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$i = \cos^{-1}\left(\frac{\vec{h}\cdot\hat{k}}{h}\right)$ (principal value: $0 \leq \text{angle} \leq \pi$)

Line of nodes, $\hat{n} = \frac{\hat{k}\times\vec{h}}{|\hat{k}\times\vec{h}|}$ (if $i = 0$, $\hat{n} = \hat{\imath}$) $\hat{n} = \hat{\imath}\cos\Omega + \hat{\jmath}\sin\Omega = \hat{\imath}\,n_1 + \hat{\jmath}\,n_2$

$\Omega = \tan^{-1}(n_2, n_1)$ (2-argument arctangent: $-\pi \leq \text{angle} \leq \pi$)

$\omega = \cos^{-1}\left(\frac{\hat{n}\cdot\vec{e}}{e}\right)$ (returns $0 \leq \text{angle} \leq \pi$) if $\vec{e} \cdot \hat{k} < 0$, $\omega = -\omega + 2\pi$ (perigee south of equator)

$\dot{\Omega} = -\frac{3n J_2 R_\oplus^2}{2a^2(1-e^2)^2}\cos i$ $\dot{\omega} = \frac{3n J_2 R_\oplus^2}{2a^2(1-e^2)^2}\left(\frac{5}{2}\sin^2 i - 2\right)$

$r = \frac{p}{1+e\cos\upsilon}$ $\vec{r} = r\hat{r}$ $\vec{r} = \hat{p}\,(r\cos\upsilon) + \hat{q}\,(r\sin\upsilon)$ $\vec{v} = \left(-\sqrt{\frac{\mu}{p}}\sin\upsilon\right)\hat{p} + \sqrt{\frac{\mu}{p}}(e + \cos\upsilon)\,\hat{q}$

$\vec{v} = \dot{\vec{r}} = V_R\,\hat{r} + V_T\,\hat{t} = \dot{r}\,\hat{r} + r\dot{\upsilon}\,\hat{t}$ $\boxed{V_R = \dot{r} = \sqrt{\frac{\mu}{p}}\,e\sin\upsilon}$ $\boxed{V_T = r\dot{\upsilon} = \sqrt{\frac{\mu}{p}}\,(1 + e\cos\upsilon)}$ orbits

Perifocal to ECI $\underline{R}^{IP} = R_3(\omega)\,R_1(i)\,R_3(\Omega)$ (3-1-3 Euler rotations) $\begin{Bmatrix}\hat{p}\\ \hat{q}\\ \hat{\omega}\end{Bmatrix} = [\underline{R}^{IP}]\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix} = \begin{bmatrix}[\vec{e}/e]\\ [\hat{\omega}\times\hat{p}]\\ [\vec{h}/h]\end{bmatrix}\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix}$

Transfr

Hohmann $a_T = \frac{r_1+r_2}{2}$ $\Delta V_1 = V_{TP} - V_{1C} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T}} - \sqrt{\frac{\mu}{r_1}}$ $\Delta V_2 = V_{2C} - V_{TA} = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_T}}$

$\boxed{\text{Low thrust } \Delta V = \sqrt{\frac{\mu}{r_{1C}}} - \sqrt{\frac{\mu}{r_{2C}}}}$ Inclination $\Delta V = 2V_T\sin\left(\frac{\Delta i}{2}\right)$ (V_R: no change)

Single-impulse $\Delta V_R = V_{2R} - V_{1R}$ $\Delta V_T = V_{2T} - V_{1T}$ $\boxed{\Delta V = \sqrt{\Delta V_R^2 + \Delta V_T^2}}$

$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$ $\Delta V = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos(\phi_2 - \phi_1)}$ $\phi = \cos^{-1}\left(\frac{h}{rV}\right)$ (+quad ✓) $= \tan^{-1}\left(\frac{V_R}{V_T}\right)$

Propulsion / staging $m_p = m_0\left(1 - e^{-\Delta V/I_{sp}g}\right)$ $T = \dot{m}V_E = \dot{m}\,I_{sp}\,g$ $t_b = \frac{m_p}{\dot{m}} = \frac{W_0}{T}\,I_{sp}\,\frac{m_p}{m_0}$

$\Delta V = V_E\ln\left(\frac{m_0}{m_f}\right) = I_{sp}\,g\ln\left(\frac{m_0}{m_f}\right)$ $\pi = \frac{m_*}{m_0} = 1 - \frac{m_P/m_0}{1-\epsilon}$ $\epsilon = \frac{m_S}{m_P+m_S}$ $\frac{m_f}{m_0} = 1 - \frac{m_P}{m_0} = \epsilon + (1-\epsilon)\pi$ Propulsion

$\Delta V_{*N} = -\sum_{k=1}^{N} I_{sp\,k}\,g\,\ln(\epsilon_k + (1-\epsilon_k)\pi_k)$ $\pi_* = \prod_{k=1}^{N}\pi_k$ $\pi_k = \frac{m_{0(k+1)}}{m_{0(k)}}$

Orbits

- $\boxed{r_{1E}} = |\vec{r}_{1E}|$
- $\boxed{V_{1E}} = |\vec{V}_{1E}|$
- $\boxed{\varepsilon} = \frac{1}{2}(V_{1E})^2 - \frac{\mu}{r_{1E}}$
- $\boxed{a} = -\frac{\mu}{2\varepsilon}$
- $\boxed{\vec{h}} = \vec{r}_{1E} \times \vec{V}_{1E}$
- $\boxed{h} = |\vec{h}|$
- $\boxed{\vec{e}} = \frac{1}{\mu}(\vec{V}_{1E} \times \vec{h}) - \frac{\vec{r}_{1E}}{r_{1E}}$
- $\boxed{e} = |\vec{e}|$
- $\boxed{u_1} = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}_{1E}}{e\cdot r_{1E}}\right)$ $\left\{\begin{array}{l}\text{If } \vec{r}_{1E}\cdot\vec{V}_{1E} < 0\\ \text{Then } u_1 = -u_1 + 2\pi\end{array}\right.$
- $\boxed{E_1} = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{u_1}{2}\right)\right)$
- $\boxed{M_1} = E_1 - e\sin E_1$
- $\boxed{T_0} = t_1 - \sqrt{\frac{a^3}{\mu}}\,M_1$
- $\boxed{\hat{k}} = 0\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$
- $\boxed{i} = \cos^{-1}\left(\frac{\vec{h}\cdot\hat{k}}{h}\right)$ $\left\{\begin{array}{l}\text{If } i<0\\ \text{then } i = i+\pi\end{array}\right.$
- $\boxed{n_1}\hat{\imath} + \boxed{n_2}\hat{\jmath} = \frac{\hat{k}\times\vec{h}}{|\hat{k}\times\vec{h}|} = \hat{n}$
- $\boxed{\Omega} = \text{atan2}(n_2, n_1)$ $\{-\pi \leq \Omega \leq \pi$
- $\boxed{\omega} = \cos^{-1}\left(\frac{\hat{n}\cdot\vec{e}}{e}\right)$ $\left\{\begin{array}{l}\vec{e}\cdot\hat{k}<0\\ \boxed{\omega} = -\omega+2\pi\end{array}\right.$
- $\boxed{\hat{w}} = \frac{\vec{h}}{h}$
- $\boxed{M_2} = \sqrt{\frac{\mu}{a^3}}(t_2 - T_0)$
- $\boxed{E_2} - e\sin E_2 = M_2$

$\hookrightarrow E_{2n+1} \approx E_{2n} - \frac{E_{2n} - e\sin E_{2n} - M_2}{1 - e\cos E_{2n}}$

$E_{20} = M_2$ $|E_{2n+1} - E_{2n}| \leq \text{tolerance}$

- $\boxed{u_2} = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E_2}{2}\right)\right)$
- $\boxed{r_2} = \frac{a(1-e^2)}{1+e\cos(u_2)}$
- $\boxed{\vec{r}_P} = r_2\cos u_2\,\hat{p} + r_2\sin u_2\,\hat{q}$
- $\boxed{p} = a(1-e^2)$
- $\boxed{\vec{V}_P} = -\sqrt{\frac{\mu}{p}}\sin u_2\,\hat{p} + \sqrt{\mu/p}\,(e + \cos u_2)\,\hat{q}$
- $\boxed{\underline{R}^{IP}} = R_3(\omega)\,R_1(i)\,R_3(\Omega)$
- $\boxed{\underline{R}^{PI}} = (\underline{R}^{EP})^T$
- $\boxed{\vec{r}_{2E}} = \underline{R}^{PI}\,\vec{r}_P$
- $\boxed{\vec{V}_{2E}} = \underline{R}^{PI}\,\vec{V}_P$

Propulsion

- $\boxed{\Delta V} = -I_{sp}\ln\left(\frac{M_f}{M_0}\right)$
- $\boxed{M_p} = M_0 - M_f$
- $\boxed{F_{thrust}} = -\dot{m}\,V_E$ OR $\boxed{F_{thrust}} = \dot{m}\,I_{sp}g$
- $\boxed{\dot{m}} = \frac{F_{thrust}}{I_{sp}g}$
- $\boxed{t_{burn}} = \frac{M_P}{\dot{m}}$

Impulsive Burn: conditions $\frac{t_{burn}}{2\pi\sqrt{\frac{a^3}{\mu}}} << 1$

- $\boxed{\Delta V_2} = V_{*2} = \Delta V - \Delta V_1$
- $\boxed{\Delta V_1} = V_{*1}$
- $\boxed{\Delta V_n} = \Delta V - \Delta V_{n-1}\cdots$
- $\boxed{V_{*1}} = -I_{sp}\cdot g\cdot\ln(\varepsilon + (1-\varepsilon)\pi)$
- $\boxed{\pi_1} = \frac{\exp\left(\frac{-V_*}{I_{sp}g}\right) - \varepsilon}{1-\varepsilon}$

$\vdots$

- $\boxed{\pi_2} = \frac{\exp(-V_*/I_{sp}g) - \varepsilon}{1-\varepsilon}$

$\vdots$

- $\boxed{\pi_n} = \frac{\exp(-V_{*n}/I_{sp}g) - \varepsilon_n}{1-\varepsilon_n}$
- $\boxed{\pi_n} = \frac{M_{*n}}{M_{0n}}$
- $\boxed{M_{0n}} = \frac{M_{*n}}{\pi_n}$
- $\boxed{M_{0n}} = M_{Pn} + M_{Sn} + M_{*n}$
- $\boxed{\varepsilon_n} = \frac{M_{Sn}}{M_{Sn} + M_{Pn}}$
- $\boxed{M_{Sn}} = \varepsilon_n(M_{0n} - M_{*n})$
- $\boxed{M_{Pn}} = M_{0n} - M_{Sn} - M_{*n}$
- $\boxed{M_{*n-1}} = M_{0n}$ $(M_{*1} = M_{02}\ \ldots)$

Orbit transfers

- $\boxed{r_2} = \left(\left(\frac{T_0}{2\pi}\right)^2\mu\right)^{\frac{1}{3}}$
- $\boxed{a_T} = \frac{r_1+r_2}{2}$
- $\boxed{\Delta V_{tot}} = \Delta V_1 + \Delta V_2$
- $\boxed{t_{transfer}} = \frac{2\pi\sqrt{\frac{a_T^3}{\mu}}}{2}$
- $\boxed{U} = \cos^{-1}\left(\left(\frac{p}{r_m} - 1\right)\frac{1}{e}\right)$
- $\boxed{U} = 360 - \downarrow$
- Flight path angle $\phi = \text{atan2}(V_R, V_T)$

Catching up orbit…

$n = \sqrt{\frac{\mu}{r^3}}$ $\Delta t_{rendezvous} = \frac{\Delta V}{\Delta P}$

$n = \sqrt{\frac{\mu}{a^3}}$

SSO: $\left(\frac{d\Omega}{dt}\right) = 2\pi\ \frac{\text{rad}}{\text{yr}}$ $\left(\frac{d\omega}{dt}\right) = 0$ $J_2 = 1.082\times10^{-3}$

$\upsilon = \cos^{-1}\left(\frac{\vec{e}\bullet\vec{r}}{e\,r}\right)$ $(0 \le \text{angle} \le \pi)$ if $\vec{r}\bullet\vec{v} < 0$, $\upsilon = -\upsilon + 2\pi$ (heading towards perigee)

$\tan\left(\frac{\upsilon}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$ $\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\upsilon}{2}\right)$

$M = E - e\sin E = \sqrt{\frac{\mu}{a^3}}\,(t - t_0)$ $t_0 = t - \sqrt{\frac{a^3}{\mu}}\,M$ $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$i = \cos^{-1}\left(\frac{\vec{h}\bullet\hat{k}}{h}\right)$ (principal value: $0 \le \text{angle} \le \pi$)

Line of nodes, $\hat{n} = \frac{\hat{k}\times\vec{h}}{|\hat{k}\times\vec{h}|}$ (if $i = 0$, $\hat{n} = \hat{\imath}$) $\hat{n} = \hat{\imath}\cos\Omega + \hat{\jmath}\sin\Omega = \hat{\imath}\,n_1 + \hat{\jmath}\,n_2$

$\Omega = \tan^{-1}(n_2, n_1)$ (2-argument arctangent: $-\pi \le \text{angle} \le \pi$)

$\omega = \cos^{-1}\left(\frac{\hat{n}\bullet\vec{e}}{e}\right)$ (returns $0 \le \text{angle} \le \pi$) if $\vec{e}\bullet\hat{k} < 0$, $\omega = -\omega + 2\pi$ (perigee south of equator)

$\dot{\Omega} = -\frac{3nJ_2R_\oplus^2}{2a^2(1-e^2)^2}\cos i$ $\dot{\omega} = \frac{3nJ_2R_\oplus^2}{2a^2(1-e^2)^2}\left(\frac{5}{2}\sin^2 i - 2\right)$

$r = \frac{p}{1+e\cos\upsilon}$ $\vec{r} = r\hat{r}$ $\vec{r} = \hat{p}\,(r\cos\upsilon) + \hat{q}\,(r\sin\upsilon)$ $\vec{v} = \left(-\sqrt{\frac{\mu}{p}}\sin\upsilon\right)\hat{p} + \sqrt{\frac{\mu}{p}}(e + \cos\upsilon)\,\hat{q}$

$\vec{v} = \dot{\vec{r}} = V_R\,\hat{r} + V_T\,\hat{t} = \dot{r}\,\hat{r} + r\dot{\upsilon}\,\hat{t}$ $V_R = \dot{r} = \sqrt{\frac{\mu}{p}}\,e\sin\upsilon$ $V_T = r\dot{\upsilon} = \sqrt{\frac{\mu}{p}}\,(1 + e\cos\upsilon)$

Perifocal to ECI $\underline{R}^{IP} = R_3(\omega)\,R_1(i)\,R_3(\Omega)$ (3-1-3 Euler rotations) $\begin{Bmatrix}\hat{p}\\ \hat{q}\\ \hat{w}\end{Bmatrix} = [\underline{R}^{IP}]\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix} = \begin{bmatrix}\lfloor\vec{e}/e\rfloor\\ \lfloor\hat{w}\times\hat{p}\rfloor\\ \lfloor\vec{h}/h\rfloor\end{bmatrix}\begin{Bmatrix}\hat{\imath}\\ \hat{\jmath}\\ \hat{k}\end{Bmatrix}$

Hohmann $a_T = \frac{r_1+r_2}{2}$ $\Delta V_1 = V_{TP} - V_{1C} = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T}} - \sqrt{\frac{\mu}{r_1}}$ $\Delta V_2 = V_{2C} - V_{TA} = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_T}}$

Low thrust $\Delta V = \sqrt{\frac{\mu}{r_{1C}}} - \sqrt{\frac{\mu}{r_{2C}}}$ Inclination $\Delta V = 2V_T\sin\left(\frac{\Delta i}{2}\right)$ (V_R: no change)

Single-impulse $\Delta V_R = V_{2R} - V_{1R}$ $\Delta V_T = V_{2T} - V_{1T}$ $\Delta V = \sqrt{\Delta V_R^2 + \Delta V_T^2}$

$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$ $\Delta V = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos(\phi_2 - \phi_1)}$ $\phi = \cos^{-1}\left(\frac{h}{r\,V}\right)(+quad\checkmark) = \tan^{-1}\left(\frac{V_R}{V_T}\right)$

Propulsion / staging $m_P = m_0\left(1 - e^{-\Delta V/I_{sp}g}\right)$ $T = \dot{m}V_E = \dot{m}\,I_{sp}\,g$ $t_b = \frac{m_P}{\dot{m}} = \frac{W_0}{T}\,I_{sp}\,\frac{m_P}{m_0}$

$\Delta V = V_E\ln\left(\frac{m_0}{m_f}\right) = I_{sp}\,g\ln\left(\frac{m_0}{m_f}\right)$ $\pi = \frac{m_*}{m_0} = 1 - \frac{m_P/m_0}{1-\epsilon}$ $\epsilon = \frac{m_S}{m_P+m_S}$ $\frac{m_f}{m_0} = 1 - \frac{m_P}{m_0} = \epsilon + (1-\epsilon)\pi$

$\Delta V_{*N} = -\sum_{k=1}^{N} I_{sp\,k}\,g\,\ln(\epsilon_k + (1-\epsilon_k)\pi_k)$ $\pi_* = \prod_{k=1}^{N}\pi_k$ $\pi_k = \frac{m_{0\,(k+1)}}{m_{0\,(k)}}$

$\begin{Bmatrix}H_1\\ H_2\\ H_3\end{Bmatrix} = \begin{bmatrix}I_{11} & I_{12} & I_{13}\\ I_{21} & I_{22} & I_{23}\\ I_{31} & I_{32} & I_{33}\end{bmatrix}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}$ $T = \frac{1}{2}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}^T\begin{bmatrix}I_{11} & I_{12} & I_{13}\\ I_{21} & I_{22} & I_{23}\\ I_{31} & I_{32} & I_{33}\end{bmatrix}\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}$ $\vec{M} = \vec{r}\times\vec{F}$

$[\underline{I}^A]\{\hat{e}_i\} = \lambda_i\{\hat{e}_i\}$ $[E] = [\{\hat{e}_1\}\{\hat{e}_2\}\{\hat{e}_3\}]$ $[\underline{I}^B] = [E]^T[\underline{I}^A][E] = \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$ $[R^{AB}] = [E]^T$

$I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) = M_1$
$I_2\dot{\omega}_2 + \omega_3\omega_1(I_1 - I_3) = M_2$
$I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) = M_3$

torque − free
axisymmetric

$\vec{H} \leftrightarrow \hat{b}_3:$ $\tan\theta = \frac{I_1\omega_0}{I_3\Omega}$
$\vec{H} \leftrightarrow \vec{\omega}:$ $\tan\gamma = \frac{I_3}{I_1}\tan\theta = \frac{\omega_0}{\Omega}$

$M_{G1} = \frac{3\mu}{R^5}YZ(I_3 - I_2)$
$M_{G2} = \frac{3\mu}{R^5}ZX(I_1 - I_3)$
$M_{G3} = \frac{3\mu}{R^5}XY(I_2 - I_1)$

$\omega_{s/c} = -\frac{I_R}{I_{s/c}+I_R}\omega_R$ $\omega_{b3} = +\frac{I_R}{I_{s/c}}\omega_{R1}\sin\theta_{R2}$

$P_{sun} = \phi_S\,A_{proj}$ $P_{rad} = \sigma\epsilon\,A_{rad}\,T^4$ $\phi_S = 1371\,\frac{W}{m^2}$ $\sigma = 5.67e - 8\,\frac{W}{m^2K^4}$

N frame unit vectors $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ **B** frame unit vectors $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$

Vectors $\vec{r} = r_{N1}\hat{n}_1 + r_{N2}\hat{n}_2 + r_{N3}\hat{n}_3$ (expressed in **N** frame) $\vec{r} = r_{B1}\hat{b}_1 + r_{b2}\hat{b}_2 + r_{B3}\hat{b}_3$ (in **B** frame)

Vector components $\underline{r}^N = [r_{N1}\ \ r_{N2}\ \ r_{N3}]^T$ $\underline{r}^B = [r_{B1}\ \ r_{B2}\ \ r_{B3}]^T$

Rotation Matrices / Direction Cosine Matrices (DCM)

Relation for unit vectors $\begin{Bmatrix}\hat{b}_1\\ \hat{b}_2\\ \hat{b}_3\end{Bmatrix} = \begin{bmatrix}\hat{b}_1\bullet\hat{n}_1 & \hat{b}_1\bullet\hat{n}_2 & \hat{b}_1\bullet\hat{n}_3\\ \hat{b}_2\bullet\hat{n}_1 & \hat{b}_2\bullet\hat{n}_2 & \hat{b}_2\bullet\hat{n}_3\\ \hat{b}_3\bullet\hat{n}_1 & \hat{b}_3\bullet\hat{n}_2 & \hat{b}_3\bullet\hat{n}_3\end{bmatrix}\begin{Bmatrix}\hat{n}_1\\ \hat{n}_2\\ \hat{n}_3\end{Bmatrix}$ or $\{\hat{b}\} = [R^{NB}]\{\hat{n}\}$

Also for vector components $\begin{Bmatrix}r_{B1}\\ r_{B2}\\ r_{B3}\end{Bmatrix} = [R^{NB}]\begin{Bmatrix}r_{N1}\\ r_{N2}\\ r_{N3}\end{Bmatrix}$ or $\{\underline{r}^B\} = [R^{NB}]\{\underline{r}^N\}$

Inverse $[R^{BN}] = [R^{NB}]^{-1} = [R^{NB}]^T$ $\{\underline{r}^N\} = [R^{BN}]\{\underline{r}^B\}$

Rotation matrices for rotation about single axes (**N -> B**)

$\underline{R}_1^{NB}(\phi_1) = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi_1 & \sin\phi_1\\ 0 & -\sin\phi_1 & \cos\phi_1\end{bmatrix}$ $\underline{R}_2^{NB}(\phi_2) = \begin{bmatrix}\cos\phi_2 & 0 & -\sin\phi_2\\ 0 & 1 & 0\\ \sin\phi_2 & 0 & \cos\phi_2\end{bmatrix}$ $\underline{R}_3^{NB}(\phi_3) = \begin{bmatrix}\cos\phi_3 & \sin\phi_3 & 0\\ -\sin\phi_3 & \cos\phi_3 & 0\\ 0 & 0 & 1\end{bmatrix}$

Composing multiple rotations (two shown) $\{\underline{r}^B\} = [R^{KB}]\{\underline{r}^K\} = [R^{KB}][R^{NK}]\{\underline{r}^N\} = [R^{NB}]\{\underline{r}^N\}$

Angular velocities add $\vec{\omega}^{B/N} = \dot{\phi}_1\hat{u}_{\phi_1\,axis} + \dot{\phi}_2\hat{u}_{\phi_2\,axis} + \dot{\phi}_3\hat{u}_{\phi_3\,axis}$ (each about appropriate unit vector)

Express in **B** frame $\vec{\omega}^{B/N} = \omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3$

Transport theorem ${}^N\frac{d}{dt}(\vec{r}) = {}^B\frac{d}{dt}(\vec{r}) + \vec{\omega}^{B/N}\times\vec{r}$ (velocity)

${}^N\frac{d^2}{dt^2}(\vec{r}) = {}^B\frac{d^2}{dt^2}(\vec{r}) + {}^B\frac{d}{dt}\left(\vec{\omega}^{B/N}\right)\times\vec{r} + 2\vec{\omega}^{B/N}\times{}^B\frac{d}{dt}(\vec{r}) + \vec{\omega}^{B/N}\times\left(\vec{\omega}^{B/N}\times\vec{r}\right)$ (acceleration)

${}^N\dot{\vec{r}} = \dot{r}_{N1}\hat{n}_1 + \dot{r}_{N2}\hat{n}_2 + \dot{r}_{N3}\hat{n}_3$ ${}^B\dot{\vec{r}} = \dot{r}_{B1}\hat{b}_1 + \dot{r}_{b2}\hat{b}_2 + \dot{r}_{B3}\hat{b}_3$ $\vec{\omega}^{B/N}\times\vec{r} = \begin{Bmatrix}\hat{b}_1(-\omega_3r_{B2} + \omega_2r_{B3})\\ +\hat{b}_2(\omega_3r_{B1} - \omega_1r_{B3})\\ +\hat{b}_3(-\omega_2r_{B1} + \omega_1r_{B2})\end{Bmatrix}$

$T = \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}mv^2$ (inertial velocity) $\vec{H} = \vec{r}\times m\vec{v}$ (about inertial origin, mass center)

Two-body problem center of mass $\vec{R}_c = \frac{m_1\vec{R}_1 + m_2\vec{R}_2}{m_1+m_2}$ Equation of motion $\ddot{\vec{R}}_c = \vec{0}$

relative motion $\vec{r} \equiv \vec{R}_2 - \vec{R}_1$ Equation of relative motion $\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}$

Gravitation $\vec{F}_G = -\frac{Gm_1m_2}{r^2}\frac{\vec{r}}{r}$ $V_G = -\frac{Gm_1m_2}{r}$

position vector $\vec{r}$ and velocity vector $\vec{v} = \dot{\vec{r}}$ $V = |\vec{v}|$

$\varepsilon = \frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$ $V = \sqrt{2\left(\varepsilon + \frac{\mu}{r}\right)}$ $r = \frac{2\mu}{V^2 - 2\varepsilon}$ $\mu = \mathrm{GM}$ $Gm_\oplus = 3.986\times10^5\ \mathrm{km^3/s^2}$

$\vec{h} = \vec{r}\times\vec{v} = \vec{r}\times\dot{\vec{r}}$ $h = |\vec{h}|$ $h = r\,V_T = r^2\dot{\upsilon}$ (nu)

$\vec{e} = \frac{1}{\mu}(\dot{\vec{r}}\times\vec{h}) - \frac{\vec{r}}{r}$ $e = |\vec{e}|$ $e = \sqrt{1 + \frac{2h^2\varepsilon}{\mu^2}}$

$a = -\frac{\mu}{2\varepsilon}$ $p = h^2/\mu = a(1 - e^2)$ $\upsilon = \cos^{-1}\left(\frac{p-r}{e\,r}\right)$