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Aersp 450 - HW2

Problem 1)

given: $\mu = 398600 \ \frac{km^3}{s^2}$

near-circular LEO

Departure	$a_1 = 8000$ km	$e_1 = 0.01$	$f_1 = 30° = \frac{\pi}{6}$ rad
Arrival	$a_2 = 27000$ km	$e_2 = 0.6$	$f_2 = 210° = \frac{7}{6}\pi$ rad

HEO

no plane change
$(\Delta I = 0, \Delta\Omega = 0)$

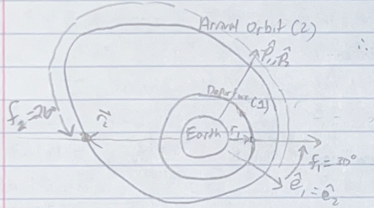
1) $a_T = ?$ (transfer orbit semi-major axis)

$\neq f_2 - f_1$ because ~~it is in~~ different frames

$$a_{min_T} = \frac{1}{4}\left(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\Delta f)}\right), \quad r_1 = ?, \ r_2 = ?, \quad \Delta f = ?$$

$$r = \frac{a(1-e^2)}{1+e\cos f} \to r_1 = \frac{a_1(1-e_1^2)}{1+e_1\cos f_1} \to r_1 = \frac{(8000)(1-0.01^2)}{1+(0.01)\cos(30°)} \text{ km} \to r_1 = 7930.519685 \text{ km}$$

$$r_2 = \frac{a_2(1-e_2^2)}{1+e_2\cos f_2} \to r_2 = \frac{(27000)(1-0.6^2)}{1+(0.6)\cos(210°)} \text{ km} \to r_2 = 35971.16628 \text{ km}$$



The departure orbit is near-circular. ∴ We can approximate that it really is circular. For circular orbits, we can choose the orientation of the $\hat{e}, \hat{p}$ frame. I chose to align it with that of the high eccentricity orbit.

Because $\hat{e}_1 = \hat{e}_2$, we are able to simply subtract $f_2 - f_1$ to find Δf

$\Delta f = f_2 - f_1 \to \Delta f = 210° - 30° \to \Delta f = 180°$

$$a_{min,T} = \frac{1}{4}\left(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\Delta f)}\right)$$

$$a_{min,T} = \frac{1}{4}\left((7930) + (35971) + \sqrt{(7930)^2 + (35971)^2 - 2(7930)(35971)\cdot\cos(180°)}\right)$$

$$\boxed{a_{min,T} = 21950.5 \text{ km}}$$

* note that the decimal places were omitted in writing, but used in the calculations

2) $p = ?$ given $p = \frac{km - 2akl}{4am - 2al^2}$,

$k = r_1r_2(1-\cos\Delta f) \to k = (7930)(35971)(1-\cos(180°)) \to k = 5.7054\times10^8 \text{ km}^2$

$m = r_1r_2(1+\cos\Delta f) \to m = (7930)(35971)(1+\cos(180°)) \to m = 0 \text{ km}^2$

$l = r_1 + r_2 \to l = 7930 + 35971 \to l = 43902$ km

Plugging in $k, m, l, a_{min,T} \to$ $\boxed{p_T = 12995.8582 \text{ km}}$

3) ΔV_1 (Departure orbit → transfer orbit)

- using the energy equation, we can calculate the velocity at each point
- because $\Delta f = 180$, the velocity impulses will be parallel to the direction it's moving in already.

∴ $\Delta V = |V_{orbit} - V_{transfer, orbit}|$ (the vectors are already aligned)

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Problem 1.3 (continued)

$$\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \rightarrow \frac{1}{2}V^2 = \frac{\mu}{r} - \frac{\mu}{2a} \rightarrow V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} \rightarrow V_1 = 2\mu\left(\frac{1}{r_1} - \frac{1}{2a_1}\right) = 7.1203 \text{ km/s}$$

$$\rightarrow V_{1T} = 2\mu\left(\frac{1}{r_1} - \frac{1}{2a_T}\right) = 9.0755 \text{ km/s}$$

note that when moving from the departure orbit to transfer orbit, radius = constant for impulse

$$\Delta V_1 = V_{1T} - V_1 \rightarrow \boxed{\Delta V_1 = 1.9552 \text{ km/s}}$$

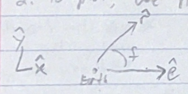
4) The same energy equation can be used to find the ΔV to go from the transfer orbit to the arrival orbit.

This process yields: $\cdot\ V_{2T} = 2.0009$ km/s
$\cdot\ V_2 = 2.7202$ km/s
$\rightarrow \boxed{\Delta V_2 = 0.7193 \text{ km/s}}$

5) Because $\Delta f = 180°$, the transfer orbit will have its perigee @ $\vec{r}_1$ & apogee @ $\vec{r}_2$ (for a minimum energy transfer with $\Delta f = 180°$).
Going from perigee $\rightarrow$ apogee is exactly half an orbit.
Therefore the time it takes is equal to one half the orbital period, T_P.

$$T_p = 2\pi\sqrt{\frac{a^3}{\mu}} \rightarrow t_{transfer} = \pi\sqrt{\frac{a_T^3}{\mu}} \rightarrow t_{transfer} = 2\pi \cdot \sqrt{\frac{21451^3}{398600}}\ s \rightarrow \boxed{t_{transfer} = 16183 \text{ seconds} \approx 4.5 \text{ hours}}$$

6) Plotting:
1. get the radius at each point of the orbit by using this equation and iterating through f values:
$$r = \frac{a(1-e^2)}{1+e\cos f}$$ ← (This can be used for all orbits given that we know the eccentricity)
2. To plot, we need x and y coordinates. So, convert r & f to x & y:



$$\vec{r} = r\cos f\,\hat{x} + r\sin y\,\hat{y}$$

Note we still need e of the transfer orbit:
$$r = \frac{p}{1+e\cos f} \rightarrow r_1 = \frac{P_T}{1+e_T} \rightarrow e_T = \frac{P_T}{r_1} - 1 \rightarrow e_T = \frac{12996}{7930} - 1 \rightarrow e_T = 0.6387$$
$r = r_1$ @ $f = 0$

This also does not account for the orientation of the transfer orbit. However, we know that perigee is @ r_1 and apogee @ r_2 so we can just subtract 30° in the argument for the angle.

7) $\hat{e}, \hat{p}$ does not align between the departure and transfer orbit. This is evident on the figure. The angle between $\hat{e}_1$ and $\hat{e}_{transfer}$ = 30° because it is the same as $f_2 - \Delta f$. This was noticed based on the geometry of the figure in part 2