Melik Demirel – AERSP 450 – HW 2

Problem 1

GIVEN:

Consider minimum energy transfers between a Low Earth Orbit (LEO) and a High Elliptical orbit (HEO) with Earth as the central body ($\mu = 3.986 \cdot 10^5 \frac{\text{km}^3}{\text{s}^2}$), where both orbits lying in the same plane. All calculations may be done in the $\hat{e} - \hat{p} - \hat{h}$ frame.

Before:

1. $a_1 = 8000 \text{ km}$
2. $e_1 = 0.01$
3. $f_1 = 30°$ (In the departure orbit, not transfer)

After:

1. $a_2 = 27000 \text{ km}$
2. $e_2 = 0.6$
3. $f_2 = 210°$ (In the arrival orbit, not transfer)

1. Analytic:

(a) GIVEN: $a_1, e_1, f_1, a_2, e_2, f_2, \mu$

(b) TO-FIND:

Transfer orbit minimum energy semimajor axis $(\boldsymbol{a_T = a_{min}})$

(c) SOLUTION PROCESS:

The minimum energy semimajor axis for a transfer orbit is given by the equation:

$$a_{min} = \frac{1}{4}\left(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2\, r_1\, r_2 \cos \Delta f}\right)$$

Of course, this means we need to find r_1, r_2 and Δf:

$$r_1 = \frac{a_1(1 - e_1^2)}{1 + e_1 \cos f_1} = \frac{8000(1 - (0.01)^2)}{1 + (0.01)\cos(30°)} = 7930.520 \text{ km}$$

$$r_2 = \frac{a_2(1 - e_2^2)}{1 + e_2 \cos f_2} = \frac{27000(1 - (0.6)^2)}{1 + (0.6)\cos(210°)} = 35971.166 \text{ km}$$

Finding Δf requires a comprehensive understanding of the geometry. Δf is the change in anomaly on the transfer orbit, meaning we cannot immediately use the true anomalies f_1 and f_2 given on the departure and arrival orbits. However, since eccentricity (and therefore radii) are given to us as scalars and not vectors, we can assume alignment of the departure and arrival orbits in the following way to help us determine f_1 and f_2:

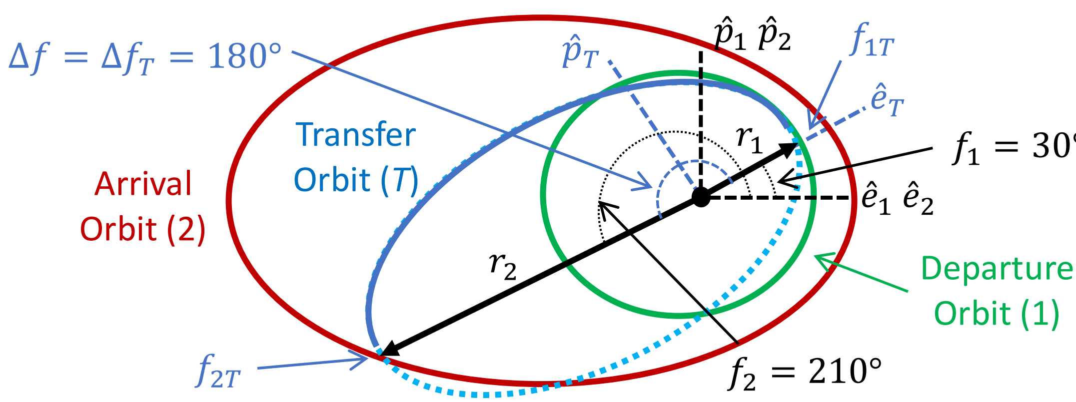

Figure 1.1 – Sketch of orbit geometry

As seen in the diagram above (*Figure 1.1*), if the $\hat{e}_1 - \hat{p}_1$ frame of the departure orbit and the $\hat{e}_2 - \hat{p}_2$ frame of the arrival orbit are aligned perfectly on top of each other, then f_1 and f_2 will be relative to the same frame ($\hat{e}_1 - \hat{p}_1$ or $\hat{e}_2 - \hat{p}_2$). However, the $\hat{e}_T - \hat{p}_T$ frame of the transfer orbit is not necessarily aligned with $\hat{e}_1 - \hat{p}_1$ or $\hat{e}_2 - \hat{p}_2$; given the geometry above, $\hat{e}_T - \hat{p}_T$ is actually rotated counter-clockwise along the $\hat{h}$ axis from $\hat{e}_1 - \hat{p}_1$ by a value of $f_1 = 30°$. With the assumption of this geometry in place, we have that the difference of f_1 and f_2, the true anomaly of the departure orbit and arrival orbit respectively, be equal to the difference in the true anomalies of the transfer orbit. Thus, for Δf, we have:

$$\Delta f = f_2 - f_1 = f_{2T} - f_{1T} = 180°$$

This is true even though $f_2 \neq f_{2T}$ and $f_1 \neq f_{1T}$. Plugging in values, we have:

$$a_T = a_{min} = \frac{1}{4}\Big((7930.520) + (35971.166) + \sqrt{(7930.520)^2 + (35971.166)^2 - 2(7930.520)(35971.166)\cos(180°)}\Big) = \mathbf{21950.843\ km}$$

2. Analytic:

(a) GIVEN: $a_1, e_1, f_1, a_2, e_2, f_2, \mu, a_T, \Delta f$, *Figure 1.1*

(b) TO-FIND:

***Transfer orbit semi-latus rectum** $(\boldsymbol{p} = \boldsymbol{p_T})$, given $a = a_T = a_{min}$ from part 1 and the formulae:*

$$k = r_1\, r_2(1 - \cos \Delta f)$$

$$m = r_1\, r_2(1 + \cos \Delta f)$$

$$l = r_1 + r_2$$

$$p = \frac{k\, m - 2\, a_{min}\, k\, l}{4\, a_{min}\, m - 2\, a_{min}\, l^2}$$

(c) SOLUTION PROCESS:

All the values we need were found in part 1, so we can immediately proceed by plugging values in:

$$k = (7930.520)(35971.166)(1 - \cos(180°)) = 570540084.587\ \text{km}^2$$

$$m = (7930.520)(35971.166)(1 + \cos(180°)) = 0\ \text{km}^2$$

$$l = (7930.520) + (35971.166) = 43901.686\ \text{km}$$

$$p = p_T = \frac{(570540084.587)(0) - 2(21950.843)(570540084.587)(43901.686)}{4(21950.843)(0) - 2(21950.843)(43901.686)^2} = \mathbf{12995.858\ km}$$

3. Analytic:

(a) GIVEN: $a_1, e_1, f_1, a_2, e_2, f_2, \mu, a_T, \Delta f, p_T$, *Figure 1.1*

(b) TO-FIND:

$\boldsymbol{\Delta v_1} = \boldsymbol{\Delta v}$ ***of departure point to transfer orbit (and explain the process).***

(c) SOLUTION PROCESS:

For all the following, let the subscript [1] be used for the departure orbit, and [*1T*] be used for the transfer orbit. To find Δv_1, we must calculate the magnitude of the change in velocity vectors at the point of single impulse between the departure orbit and the transfer orbit. Continuing to consider the assumed geometry seen in *Figure 1.1*, we use the $\hat{r} - \hat{\theta}$ frame velocity vector description to find the two velocities before and after the impulse:

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} = \frac{h \cdot e \cdot \sin f}{r(1 + e \cdot \cos f)}\hat{r} + \frac{h}{r}\hat{\theta}$$

$$p = \frac{h^2}{\mu} \to h = \sqrt{\mu\, p}$$

So, **before the impulse** we have:

$$\overrightarrow{v_1} = \frac{h_1\, e_1 \sin f_1}{r_1(1 + e_1 \cos f_1)}\hat{r} + \frac{h_1}{r_1}\hat{\theta}, \qquad h_1 = \sqrt{\mu\, a_1(1 - e_1^2)}$$

Where:

- h_1 is positive because we are considering the counterclockwise trajectory in *Figure 1.1*

Then, **after the impulse**, we have:

$$\overrightarrow{v_{1T}} = \frac{h_{1T}\, e_{1T} \sin f_{1T}}{r_{1T}\,(1 + e_{1T}\, \cos f_{1T})}\hat{r} + \frac{h_{1T}}{r_{1T}}\hat{\theta}$$

$$p_{1T} = a_{1T}(1 - e_{1T}^2) \to e_{1T} = \sqrt{1 - \frac{p_{1T}}{a_{1T}}}, \qquad h_{1T} = \sqrt{\mu\, a_{1T}(1 - e_{1T}^2)}$$

Where:

- $f_{1T} = \arccos\left(\frac{p_T - r_1}{e_T\, r_1}\right)$, with the geometry of *Figure 1.1*
- h_{1T} is positive because we are considering the counterclockwise trajectory in *Figure 1.1*
- $r_{1T} = r_1$ because the central body is the Earth for both the transfer orbit and the departure orbit
- $a_{1T} = a_T = a_{min}$ as was found in part 1
- $p_{1T} = p_T = p$ as was found in part 2

As such, the equations after the impulse simplify to:

$$\overrightarrow{v_{1T}} = \frac{h_{1T}\, e_{1T} \sin f_{1T}}{r_1\,(1 + e_{1T}\, \cos f_{1T})}\hat{r} + \frac{h_{1T}}{r_1}\hat{\theta}$$

$$e_{1T} = \sqrt{1 - \frac{p_T}{a_T}}, \qquad h_{1T} = \sqrt{\mu\, a_T(1 - e_{1T}^2)}$$

Plugging in values we have:

$$h_1 = \sqrt{(3.986 \cdot 10^5)(8000)(1 - (0.01)^2)} = 56466.637\,\frac{\text{km}^2}{\text{s}}$$

$$\overrightarrow{v_1} = \frac{(56466.637)\,(0.01)\sin(30°)}{(7930.520)(1 + (0.01)\cos(30°))}\hat{r} + \frac{(56466.637)}{(7930.520)}\hat{\theta} = \left((0.035)\hat{r} + (7.120)\hat{\theta}\right)\frac{\text{km}}{\text{s}}$$

$$e_{1T} = \sqrt{1 - \frac{(12995.858)}{(21950.843)}} = 0.6387$$

$$f_{1T} = \arccos\left(\frac{(12995.858) - (7930.520)}{(0.6387)\,(7930.520)}\right) = 0°$$

$$h_{1T} = \sqrt{(3.986 \cdot 10^5)(21950.843)(1-(0.6387)^2)} = 71973.252 \frac{\text{km}^2}{\text{s}}$$

$$\overrightarrow{v_{1T}} = \frac{(71973.252)\,(0.6387)\sin(0°)}{(7930.520)(1+(0.6387)\cos(0°))}\hat{r} + \frac{(71973.252)}{(7930.520)}\hat{\theta} = \Big((0)\hat{r} + (9.075)\hat{\theta}\Big)\frac{\text{km}}{\text{s}}$$

Finally, we find the magnitude of the difference:

$$\Delta v_1 = \Delta v = |\overrightarrow{v_{1T}} - \overrightarrow{v_1}| = \sqrt{\big((0)-(0.035)\big)^2 + \big((9.075)-(7.120)\big)^2} = \mathbf{1.956}\,\frac{\mathbf{km}}{\mathbf{s}}$$

4. Analytic:

(a) GIVEN: a_1, e_1, f_1, a_2, e_2, f_2, μ, a_T, Δf, p_T, *Figure 1.1*

(b) TO-FIND:

$\boldsymbol{\Delta v_2 = \Delta v}$ ***of transfer orbit to arrival orbit (and explain the process).***

(c) SOLUTION PROCESS:

For all the following, let the subscript [2] be used for the arrival orbit, and [*2T*] be used for the transfer orbit. To find Δv_2, we must calculate the magnitude of the change in velocity vectors at the point of single impulse between the arrival orbit and the transfer orbit. Continuing to consider the assumed geometry seen in *Figure 1.1*, we again use the $\hat{r} - \hat{\theta}$ frame velocity vector description to find the two velocities before and after the impulse:

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} = \frac{h \cdot e \cdot \sin f}{r(1+e\cdot\cos f)}\hat{r} + \frac{h}{r}\hat{\theta}$$

$$p = \frac{h^2}{\mu} \rightarrow h = \sqrt{\mu\, p}$$

So, **after the impulse** we have:

$$\overrightarrow{v_2} = \frac{h_2\, e_2 \sin f_2}{r_2(1+e_2\cos f_2)}\hat{r} + \frac{h_2}{r_2}\hat{\theta}, \qquad h_2 = \sqrt{\mu\, a_2(1-e_2^2)}$$

Where:

- h_2 is positive because we are considering the counterclockwise trajectory in *Figure 1.1*

And, **before the impulse**, we have:

$$\overrightarrow{v_{2T}} = \frac{h_{2T}\, e_{2T} \sin f_{2T}}{r_{2T}\,(1+e_{2T}\,\cos f_{2T})}\hat{r} + \frac{h_{2T}}{r_{2T}}\hat{\theta}$$

$$p_{2T} = a_{2T}(1-e_{2T}^2) \rightarrow e_{2T} = \sqrt{1-\frac{p_{2T}}{a_{2T}}}, \qquad h_{2T} = \sqrt{\mu\, a_{2T}(1-e_{2T}^2)}$$

Where:

- $f_{2T} = \arccos\left(\frac{p_T - r_2}{e_T\, r_2}\right)$ based on the geometry of *Figure 1.1*
- h_{2T} is positive because we are considering the counterclockwise trajectory in *Figure 1.1*
- $r_{2T} = r_2$ because the central body is the Earth for both the transfer orbit and the departure orbit
- $a_{2T} = a_T = a_{min}$ as was found in part 1
- $p_{2T} = p_T = p$ as was found in part 2

As such, the equations before the impulse simplify to:

$$\overrightarrow{v_{2T}} = \frac{h_{2T}\, e_{2T} \sin f_{2T}}{r_2\,(1 + e_{2T}\ \cos f_{2T})}\hat{r} + \frac{h_{2T}}{r_2}\hat{\theta}$$

$$e_{2T} = \sqrt{1 - \frac{p_T}{a_T}}, \qquad h_{2T} = \sqrt{\mu\, a_T(1 - e_{2T}^2)}$$

Plugging in values we have:

$$h_2 = \sqrt{(3.986 \cdot 10^5)(27000)(1 - (0.6)^2)} = 82992.819 \frac{\text{km}^2}{\text{s}}$$

$$\overrightarrow{v_2} = \frac{(82992.819)\,(0.6)\sin(210°)}{(35971.166)(1 + (0.6)\cos(210°))}\hat{r} + \frac{(82992.819)}{(35971.166)}\hat{\theta} = \left((-1.441)\hat{r} + (2.307)\hat{\theta}\right)\frac{\text{km}}{\text{s}}$$

$$e_{2T} = \sqrt{1 - \frac{(12995.858)}{(21950.843)}} = 0.6387$$

$$f_{2T} = \arccos\left(\frac{(12995.858) - (35971.166)}{(0.6387)\,(35971.166)}\right) = 180°$$

$$h_{2T} = \sqrt{(3.986 \cdot 10^5)(21950.843)(1 - (0.639)^2)} = 71973.252 \frac{\text{km}^2}{\text{s}}$$

$$\overrightarrow{v_{2T}} = \frac{(71973.252)\,(0.6387)\sin(180°)}{(35971.166)(1 + (0.6387)\cos(180°))}\hat{r} + \frac{(71973.252)}{(35971.166)}\hat{\theta} = \left((0)\hat{r} + (2.001)\hat{\theta}\right)\frac{\text{km}}{\text{s}}$$

Finally, we find the magnitude of the difference:

$$\Delta v_2 = \Delta v = |\overrightarrow{v_2} - \overrightarrow{v_{2T}}| = \sqrt{\left((-1.441) - (0)\right)^2 + \left((2.307) - (2.001)\right)^2} = \mathbf{1.473}\frac{\mathbf{km}}{\mathbf{s}}$$

5. Analytic:

(a) GIVEN: a_1, e_1, f_1, a_2, e_2, f_2, μ, a_T, Δf, p_T, *Figure 1.1*

(b) TO-FIND:

Time of transfer: the time taken by the spacecraft to travel between the departure and arrival locations on the transfer orbit $(\mathbf{\Delta t})$

(c) SOLUTION PROCESS:

In lecture 11, the time of transfer is given as half the time-period of the transfer orbit. Thus:

$$\Delta t = T_p = \frac{1}{2}\left(2\pi\sqrt{\frac{a_{min}^3}{\mu}}\right)$$

Plugging values in:

$$\Delta t = \frac{1}{2}\left(2\pi\sqrt{\frac{(21950.843)^3}{3.986 \cdot 10^5}}\right) = \mathbf{16182.963\ s}$$

6. (Numeric) PLOT:

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(a) GIVEN: $a_1, e_1, f_1, a_2, e_2, f_2, \mu, a_T, \Delta f, p_T,$ *Figure 1.1*
(b) TO-FIND:

The departure orbit, the arrival orbit, and the transfer trajectory.

(c) SOLUTION PROCESS:

(See the MATLAB code at the end of this part…)

We construct the plots for the departure (subscript 1) and arrival orbits (subscript 2) as a polar-coordinate plot, where R_1 and R_2 are **radii as a function** of the true anomaly, F_1 and F_2 (where $F_1 = [0, 360]°$ and $F_2 = [0, 360]°$):

$$R_1 = \frac{a_1(1 - e_1^2)}{1 + e_1 \cos F_1}$$

$$R_2 = \frac{a_2(1 - e_2^2)}{1 + e_2 \cos F_2}$$

So, for the x and y coordinates of the plot, we have:

$$x_1 = R_1 \cos F_1\,, \qquad y_1 = R_1 \sin F_1$$
$$x_2 = R_2 \cos F_2\,, \qquad y_2 = R_2 \sin F_2$$

For the transfer orbit (subscript T), we start with a similar scheme, where we know $p_T = p$ from part 2:

$$p_T = a_T(1 - e_T^2) \rightarrow e_T = \sqrt{1 - \frac{p_T}{a_T}}$$

$$R_T = \frac{a_T(1 - e_T^2)}{1 + e_T \cos F_T}$$

$$(x_T)_{1,2} = R_T \cos F_T\,, \qquad (y_T)_{1,2} = R_T \sin F_T$$

In our formulation so far, there is no distinction between the values for x_1, y_1, x_2, y_2 and $(x_T)_{1,2}, (y_T)_{1,2}$ yet, given we haven't exactly specified how the values swept out by F_1 and F_2 (as they vary from 0° to 360° on the departure and arrival orbits) differ from the values swept out by F_T (as it varies from 0° to 180° on the transfer orbit). If we were to plot $(x_T)_{1,2}$ and $(y_T)_{1,2}$ while sweeping F_T, we would incorrectly obtain values for a transfer orbit that has not yet been tilted into position by the difference between the $\hat{e}_2 - \hat{p}_2 = \hat{e}_1 - \hat{p}_1$ frame and the $\hat{e}_T - \hat{p}_T$ frame. So, we must introduce a rotation matrix (DCM), using the angle of rotation between the arrival and departure frame and the transfer frame, which can be found to be equivalent to the value of $f_1 = 30°$ from the geometry in *Figure 1.1*. Thus, we have:

$$e_T = \sqrt{1 - \frac{p_T}{a_T}}, \qquad R_T = \frac{a_T(1 - e_T^2)}{1 + e_T \cos F_T}$$

$$(x_T)_{1,2} = R_T \cos F_T\,, \qquad (y_T)_{1,2} = R_T \sin F_T$$

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} \cos(f_1) & -\sin(f_1) \\ \sin(f_1) & \cos(f_1) \end{bmatrix} \begin{bmatrix} (x_T)_{1,2} \\ (y_T)_{1,2} \end{bmatrix}$$

Where:

- $F_T = [f_{1T}, f_{2T}]° = [0°, \Delta f] = [0°, 180°]$, given the geometry in *Figure 1.1.*
- x_T and y_T are the true x and y coordinates of the transfer orbit.

- $(x_T)_{1,2}$ and $(y_T)_{1,2}$ are placeholder coordinate values "in the $\hat{e}_2 - \hat{p}_2 = \hat{e}_1 - \hat{p}_1$ frame," before rotation.
- $a_T = a_{min}$ as was found in part 1.
- $f_1 = 30°$, the amount of rotation on the $\hat{h}$ axis from the $\hat{e}_2 - \hat{p}_2 = \hat{e}_1 - \hat{p}_1$ frame to the $\hat{e}_T - \hat{p}_T$ frame, as described in the geometry in *Figure 1.1.* See part 7 for further discussion and reasoning.

Furthermore, the following plot shows the departure orbit, the arrival orbit, and the transfer trajectory:

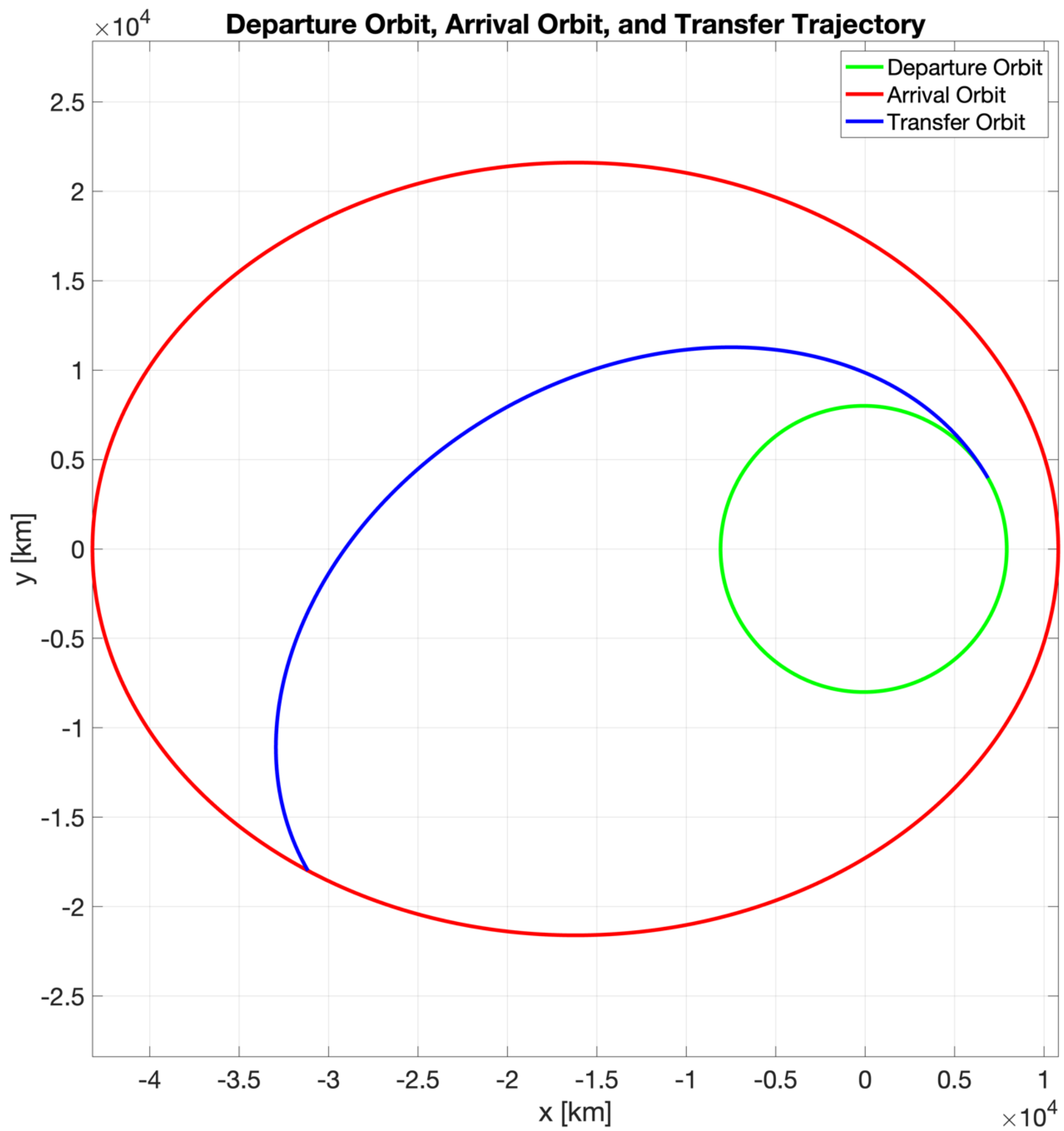


Figure 1.2

AERSP 450 HW 2 - Melik Demirel - The following is the code for Problem 1 Part 6:

```
% Author: Melik Demirel
% mcd5703
% PSU ID: 952718091
% This code solves HW2 Problem 1
clc; close all; clear;

%% Givens
mu = 3.986*10^5; % km^3*s^-2
% Before
a1 = 8000; % km
e1 = 0.01;
f1 = 30; % deg
% After
a2 = 27000; % km
e2 = 0.6;
f2 = 210; % deg

%% Calculations for PART 1:
% r1 and r2 =
r1 = (a1 * (1-(e1^2))) / (1+e1 * cosd(f1));
fprintf('r1 = %.3f km.\n', r1);
r2 = (a2 * (1-(e2^2))) / (1+e2 * cosd(f2));
fprintf('r2 = %.3f km.\n', r2);
% delta f
Df = f2 - f1;
fprintf('Δf = %.3f °.\n', Df);
% amin
aT = (1/4)*(r1 + r2 + sqrt(r1^2 + r2^2 - 2 * r1 * r2 * cosd(Df)));
fprintf('a_min = %.3f km.\n', aT);


%% Calculations for Part 2:

% k, m and l
k = r1 * r2 * (1-cosd(Df));
fprintf('k = %.3f km^2.\n', k);
m = r1 * r2 * (1+cosd(Df));
fprintf('m = %.3f km^2.\n', m);
l = r1 + r2;
fprintf('l = %.3f km.\n', l);

% p
pT = ((k*m)-(2*aT*k*l)) / ((4*aT*m) - (2*aT*(l^2)));
fprintf('p = %.3f km.\n', pT);


%% Calculations for part 3:
disp('---')
disp('1 to transfer:')
% Finding v1
h1 = sqrt(mu * a1 * (1-e1^2));
v1 = [(h1*e1*sind(f1))/(r1*(1+e1*cosd(f1))) ; h1/r1];
fprintf('h1 = %.3f km^2/s.\n', h1);
```

```
fprintf('v1 = [%.3f r, %.3f θ] km/s.\n', v1(1), v1(2));

% Finding v1T
e1T = sqrt(1-(pT/aT));
f1T = acosd((pT-r1)/(r1*e1T));
h1T = sqrt(mu * aT * (1-e1T^2));
v1T = [(h1T*e1T*sind(f1T))/(r1*(1+e1T*cosd(f1T))) ; h1T/r1];
fprintf('e1T = %.4f.\n', e1T);
fprintf('f1T = %.3f °.\n', f1T);
fprintf('h1T = %.3f km^2/s.\n', h1T);
fprintf('v1T = [%.3f r, %.3f θ] km/s.\n', v1T(1), v1T(2));

% Finding the difference
Dv1 = norm(v1T - v1);
fprintf('Δv1 = %.3f km/s.\n', Dv1);

%% Calculations for part 4:
disp('---')
disp('transfer to 2:')
% Finding v2
h2 = sqrt(mu * a2 * (1-e2^2));
v2 = [(h2*e2*sind(f2))/(r2*(1+e2*cosd(f2))) ; h2/r2];
fprintf('h2 = %.3f km^2/s.\n', h2);
fprintf('v2 = [%.3f r, %.3f θ] km/s.\n', v2(1), v2(2));

% Finding v2T
e2T = sqrt(1-(pT/aT));
f2T = acosd((pT-r2)/(r2*e2T));
h2T = sqrt(mu * aT * (1-e2T^2));
v2T = [(h2T*e2T*sind(f2T))/(r2*(1+e2T*cosd(f2T))) ; h2T/r2];
fprintf('e2T = %.4f.\n', e2T);
fprintf('f2T = %.3f °.\n', f2T);
fprintf('h2T = %.3f km^2/s.\n', h2T);
fprintf('v2T = [%.3f r, %.3f θ] km/s.\n', v2T(1), v2T(2));

% Finding the difference
Dv2 = norm(v2 - v2T);
fprintf('Δv2 = %.3f km/s.\n', Dv2);

%% Calculations for part 5:
disp('---')
Dt = (1/2) * (2*pi*sqrt((aT^3)/mu));
fprintf('Δt = %.3f s.\n', Dt);

%% Plot for part 6:

% Independent Variables
F1 = 0:0.5:360;
F2 = 0:0.5:360;
FT = 0:0.5:Df;

% Radii (and eccentricity)
R1 = (a1*(1-e1^2))./(1+e1.*cosd(F1));
R2 = (a2*(1-e2^2))./(1+e2.*cosd(F2));
eT = sqrt(1-(pT/aT));
```

```matlab
RT = (aT*(1-eT^2))./(1+eT.*cosd(FT));

% x and y coordinates
% Departure orbit
x1 = R1 .* cosd(F1);
y1 = R1 .* sind(F1);
% Arrival orbit
x2 = R2 .* cosd(F2);
y2 = R2 .* sind(F2);
% Transfer orbit
xT = RT .* cosd(FT);
yT = RT .* sind(FT);

% Rotate each point into position
R3 = [cosd(f1), -sind(f1); sind(f1), cosd(f1)];
for i = 1:length(FT)
    transfer = R3 * [xT(i); yT(i)];
    xT(i) = transfer(1);
    yT(i) = transfer(2);
end

% Plot
fig = figure(Position=[0,0,1000,1000]);
plot(x1,y1,'g', LineWidth=3);
hold on;
plot(x2,y2, 'r', LineWidth=3);
plot(xT,yT,'b', LineWidth=3);
axis equal
grid on;
xlabel('x [km]');
ylabel('y [km]');
title('Departure Orbit, Arrival Orbit, and Transfer Trajectory');
legend('Departure Orbit', 'Arrival Orbit', 'Transfer Orbit');
fontsize(fig, 'scale', 2)

%% Export figure

% Export graph
FileType = '.png';
SN1 = 'P1p6';
ST1 = [SN1, FileType];
FN1 = fullfile(ST1);
exportgraphics(fig, FN1, 'Resolution', 300, 'BackgroundColor', 'white');
```

7. <u>(Numeric) PLOT:</u>

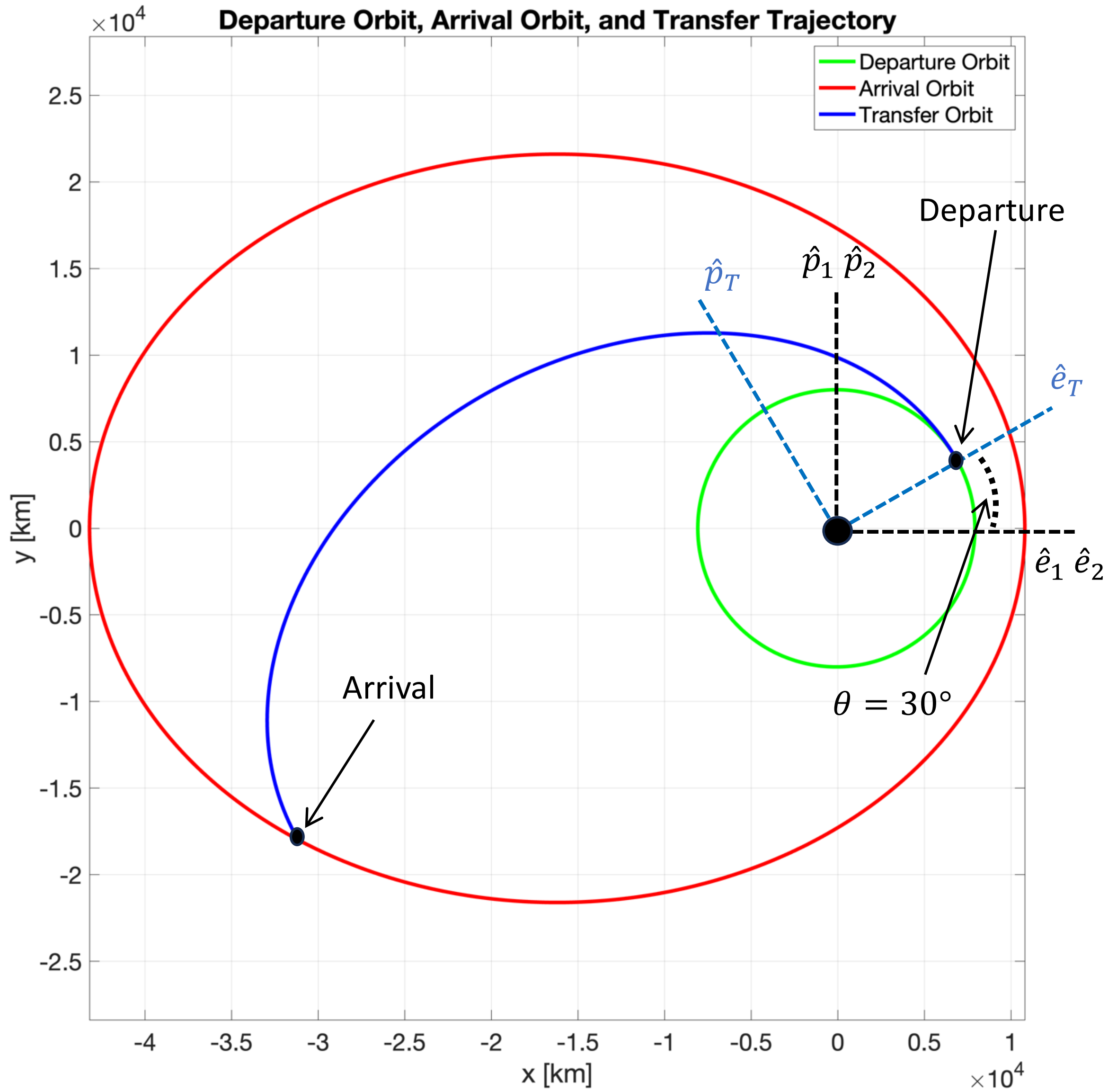
(a) GIVEN: a_1, e_1, f_1, a_2, e_2, f_2, μ, a_T, Δf, p_T, *Figure 1.1, Figure 1.2*

(b) TO-FIND:

Mark the departure location, and arrival location. Sketch the $\hat{e}$ and $\hat{p}$ directions for the transfer orbit and the departure orbit. Are they the same? If not, write the value of the angle between the two $\hat{e}$ directions. Explain how you arrived at this value.

(c) SOLUTION PROCESS:

The marked plot is shown below:



Where:

- $\hat{e}_1, \hat{e}_2$ and $\hat{p}_1, \hat{p}_2$ describe the frame of the departure and arrival orbits
- $\hat{e}_T$ and $\hat{p}_T$ describe the frame of the transfer orbit
- $\theta = f_1 = 30°$ is the angle between the transfer orbit frame and the departure/arrival orbits frame.

No, the $\hat{e}$ and $\hat{p}$ directions are not the same for the transfer and departure/arrival orbits. The value of the angle between the two $\hat{e}$ directions is $\theta = f_1 = 30°$. We derive this value from *Figure 1.1*, where the angle between the frames is a byproduct of assuming that the $\hat{e}_2 - \hat{p}_2$ frame equals the $\hat{e}_1 - \hat{p}_1$ frame, knowing (calculated in part 3 and 4) that the departure occurs at the perigee of the transfer orbit ($f_{1T} = 0°$), and that the true anomaly at the single impulse maneuver occurs at $f_1 = 30°$ on the departure orbit.

Problem 2

Aersp 450 - HW2 - Melik Demirel

$\boxed{\text{Problem II:}}$ $\mu = 398600\ km^3/s^2$

Departure: $a_1 = 7000\ km$, $e_1 = 0$, $I_1 = 60°$

Arrival: $a_2 = 70{,}000\ km$, $e_2 = 0$, $I_2 = 20°$

Givens

Assumptions:

→ $\Omega, \omega = 0°$

→ Hohmann-like transfer: depart at perigee of of transfer orbit arrive at apogee of transfer orbit. Also, a and e of the transfer orbit can be calculated as though departure and arrival orbits are coplanar.

figure:

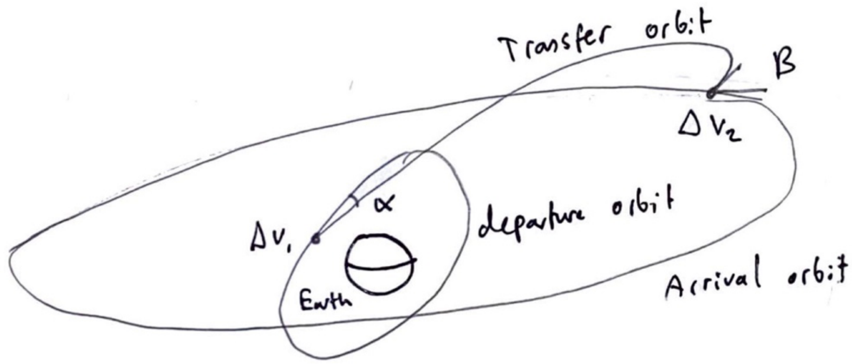


figure 2.1

Observations:

→ ∞ many Hohmann transfers based on α an β

→ Constraint: $\alpha + \beta = \delta$. $\delta = I_2 - I_1$ deg

→ Both Δv_1 and Δv_2 are applied to make shape and plane changes depending on α, β and orbit geometry.

Melik Demirel

$\boxed{2.A}$ Analytic: (a) Given a_1, e_1, a_2, e_2, Hohmann-like transfer

(1) Calculate a_T and e_T of the transfer orbit. ← (b) to find

(c) Solution process:

$$r_1 = \frac{a_1(1-e_1^2)}{1+e_1\cos f_1}, \quad r_2 = \frac{a_2(1-e_2^2)}{1+e_2\cos f_2}, \quad e_1=0, \ e_2=0$$

$$\therefore \ r_1 = a_1, \quad r_2 = a_2.$$

Hohmann like transfer → $\Delta f = 180°$

$$a_T = a_{min} = \frac{1}{4}\left(r_1 + r_2 + \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\Delta f)}\right)$$

$$= \frac{1}{4}\left(2(r_1+r_2)\right) = (r_1+r_2)\frac{1}{2}$$

$$a_T = (7000 + 70000)\frac{1}{2} = \boxed{38500 \text{ km}}$$

To find e_T, we must find P_T:

$$m = r_1 r_2(1+\cos\Delta f)$$

$$k = r_1 r_2(1-\cos\Delta f)$$

$$l = r_1 + r_2$$

$$P_T = \frac{2akl - km + k\sqrt{m(8a^2 - 4al + m)}}{2al^2 - 4am}$$

$\Delta f = 180 \rightarrow \cos\Delta f = -1 \rightarrow m = 0$

$$P_T = \frac{\cancel{2a}kl}{\cancel{2a}l^2} = k/l, \quad l = r_1 + r_2, \quad k = 2r_1r_2$$

$$k = 2(7000)(70000) = \underline{980000000 \text{ km}^2}$$

$$l = 7000 + 70000 = \underline{77000 \text{ km}}$$

$$P_T = \frac{980000000}{77000} = \underline{12727.273 \text{ km}}$$

Then → $P_T = a_T(1-e_T^2) \rightarrow e_T = \sqrt{1 - \frac{P_T}{a_T}}$

$$e_T = \sqrt{1 - \frac{12727.273}{38500}} = \boxed{0.818182}$$

Melik Demirel

(2) (a) Given $\alpha = -20°, I_1, I_2$; (b) To-find inclination of transfer orbit and Explain why. find I_T. (c) solution process:

Consider the following geometry: (side view of orbits)

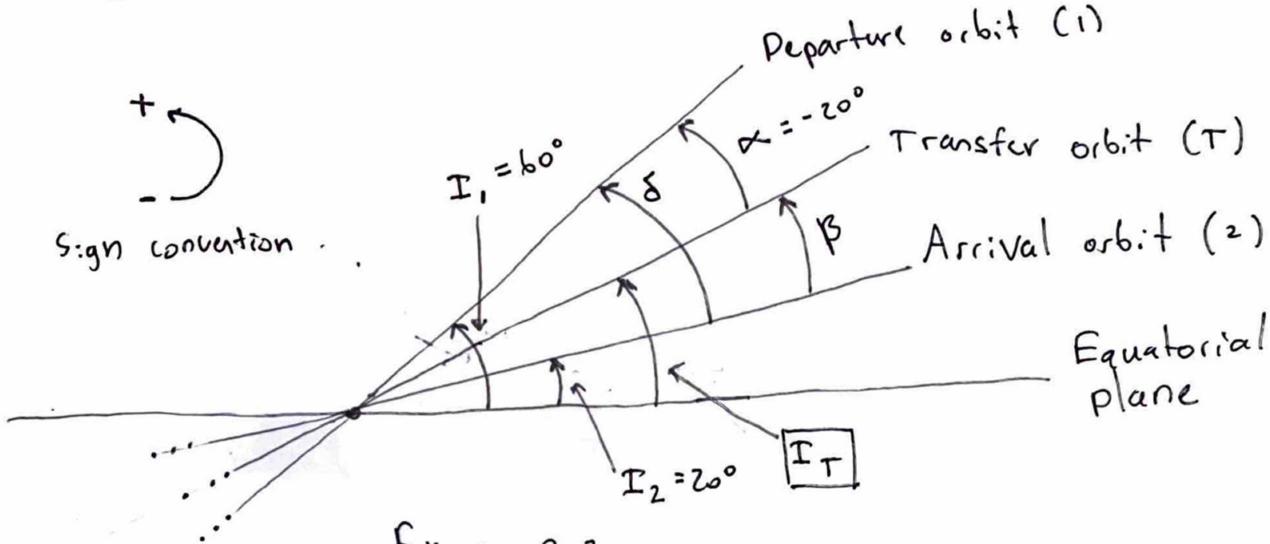


Figure 2.2

Given: $\alpha + \beta = \delta$, $\delta = I_2 - I_1$

plugging in values: $\delta = I_2 - I_1 = 20° - 60° = \underline{-40°}$

$\alpha + \beta = \delta \rightarrow -20 + \beta = -40 \rightarrow \beta = \underline{-20°}$

from the geometry, we see:

$$I_1 - |\alpha| = I_2 + |\beta| = I_T \rightarrow \left.\begin{array}{l} I_1 - |\alpha| = 60° - 20° = 40° \\ I_2 + |\beta| = 20° + 20° = 40° \end{array}\right\} = I_T$$

$\therefore \boxed{I_T = 40°}$

for the next part, it also becomes useful to define the following:

$$\Delta I_1 = \alpha = I_T - I_1 = \underline{-20°}$$

$$\Delta I_2 = \beta = I_2 - I_T = \underline{-20°}$$

3. Analytic:

(a) GIVEN: $a_1, e_1, I_1, a_2, e_2, I_2, \alpha = \Delta I_1, \beta = \Delta I_2, a_T, e_T, I_T, p_T, r_1, r_2$, *Figure 2.1, Figure 2.2, "Hohmann-like transfer"*, $\omega, \Omega, \Delta f, \mu$

(b) TO-FIND:

The value of Δv_1 and Δv_2. Explain your process in detail.

(c) SOLUTION PROCESS:

We first use the energy (Vis-viva) equation to obtain the magnitudes of velocity for the departure orbit v_1, the departing transfer orbit velocity v_{1T}, the arriving transfer orbit velocity v_{2T}, and the arrival orbit velocity v_2:

$$\varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \rightarrow v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

The radii at the point of departure are the same for the departure orbit and the transfer. This is also true at the point of arrival for the arrival and departure orbits:

$$v_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_1}\right)}, \quad v_{1T} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)}, \quad v_{2T} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_T}\right)}, \quad v_2 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_2}\right)}$$

Plugging in the given values:

$$v_1 = \sqrt{398600\left(\frac{2}{7000} - \frac{1}{7000}\right)} = 7.546\frac{\text{km}}{\text{s}}, \quad v_{1T} = \sqrt{398600\left(\frac{2}{7000} - \frac{1}{38500}\right)} = 10.175\frac{\text{km}}{\text{s}}$$

$$v_{2T} = \sqrt{398600\left(\frac{2}{70000} - \frac{1}{38500}\right)} = 1.018\frac{\text{km}}{\text{s}}, \quad v_2 = \sqrt{398600\left(\frac{2}{70000} - \frac{1}{70000}\right)} = 2.386\frac{\text{km}}{\text{s}}$$

We then use the analytical plane-change maneuver formula, as derived in lecture 12, or page 28 of the H2-Orbital Maneuvers handout, where the:

$$\Delta v = |\vec{v_f} - \vec{v_i}| = \sqrt{v_f^2 + v_i^2 - 2v_f v_i[\cos\Delta\gamma - (1 - \cos\Delta I)\cos\gamma_f\cos\gamma_i]}$$

Where:

- Subscripts i and f pertain to the after "before" and "after" orbits.
- $\Delta\gamma = \gamma_f - \gamma_i$
- $\Delta I = I_f - I_i$
- I is the corresponding orbit inclination angle.
- γ is the corresponding orbit inclination angle, given by $\tan(\gamma) = \frac{e\sin(f)}{1+e\cos(f)}$, such that $\gamma > 0°$ when the true anomaly $f < 0°$ and $\gamma < 0°$ when the true anomaly $f > 0°$.

For the **departure to transfer orbit** maneuver, we have:

$$\Delta v_1 = |\overrightarrow{v_{1T}} - \overrightarrow{v_1}| = \sqrt{v_{1T}^2 + v_1^2 - 2v_{1T}v_1[\cos(\gamma_{1T} - \gamma_1) - (1 - \cos(\Delta I_1))\cos\gamma_{1T}\cos\gamma_1]}$$

Where:

- $\Delta I_1 = \alpha$
- $\gamma_1 = \arctan\left(\frac{e_1\sin(f_1)}{1+e_1\cos(f_1)}\right) = \arctan\left(\frac{0\sin(f_1)}{1+0\cos(f_1)}\right) = 0°$

However, γ_{1T} requires the following work, knowing the departure and transfer orbit have the same radius:

$$\gamma_{1T} = \arctan\left(\frac{e_T \sin(f_{1T})}{1 + e_T \cos(f_{1T})}\right)$$

$$r_1 = \frac{p_T}{1 + e_T \cos(f_{1T})} \rightarrow f_{1T} = \arccos\left(\frac{1}{e_T}\left(\frac{p_T}{r_1} - 1\right)\right) = \arccos\left(\frac{1}{0.818182}\left(\frac{12727.273}{7000} - 1\right)\right) \approx 0°$$

$$\therefore \gamma_{1T} = \arctan\left(\frac{e_T \sin(0°)}{1 + e_T \cos(0°)}\right) = 0°$$

Thus, we rewrite the equation for Δv_1:

$$\Delta v_1 = \sqrt{v_{1T}^2 + v_1^2 - 2v_{1T}v_1[\cos 0° - (1 - \cos(\alpha)) \cos 0° \cos 0°]}$$

$$\Delta v_1 = \sqrt{v_{1T}^2 + v_1^2 - 2v_{1T}v_1 \cos(\alpha)}$$

For the **<u>transfer to arrival orbit</u>** maneuver, we have:

$$\Delta v_2 = |\overrightarrow{v_2} - \overrightarrow{v_{2T}}| = \sqrt{v_2^2 + v_{2T}^2 - 2v_2 v_{2T}[\cos(\gamma_2 - \gamma_{2T}) - (1 - \cos(\Delta I_2)) \cos\gamma_2 \cos\gamma_{2T}]}$$

Where:

- $\Delta I_2 = \beta$
- $\gamma_2 = \arctan\left(\frac{e_2 \sin(f_2)}{1+e_2 \cos(f_2)}\right) = \arctan\left(\frac{0 \sin(f_2)}{1+0 \cos(f_2)}\right) = 0°$

However, γ_{2T} requires the following work, knowing the arrival and transfer orbit have the same radius:

$$\gamma_{2T} = \arctan\left(\frac{e_T \sin(f_{2T})}{1 + e_T \cos(f_{2T})}\right)$$

$$r_2 = \frac{p_T}{1 + e_T \cos(f_{2T})} \rightarrow f_{2T} = \arccos\left(\frac{1}{e_T}\left(\frac{p_T}{r_2} - 1\right)\right) = \arccos\left(\frac{1}{0.818182}\left(\frac{12727.273}{70000} - 1\right)\right) \approx 180°$$

$$\therefore \gamma_{2T} = \arctan\left(\frac{0.818182 \sin(180°)}{1 + 0.818182 \cos(180°)}\right) \approx 0°$$

Thus, we rewrite the equation for Δv_1:

$$\Delta v_2 = \sqrt{v_2^2 + v_{2T}^2 - 2v_2 v_{2T}[\cos 0° - (1 - \cos(\beta)) \cos 0° \cos 0°]}$$

$$\Delta v_2 = \sqrt{v_2^2 + v_{2T}^2 - 2v_2 v_{2T} \cos(\beta)}$$

Finally, we can plug values in:

$$\Delta v_1 = \sqrt{(10.175)^2 + (7.546)^2 - 2(10.175)(7.546) \cos(-20°)} = \mathbf{4.022 \frac{km}{s}}$$

$$\Delta v_2 = \sqrt{(2.386)^2 + (1.018)^2 - 2(2.386)(1.018) \cos(-20°)} = \mathbf{1.472 \frac{km}{s}}$$

Melik Demirel – AERSP 450 – HW 2

2.B – Numeric Tasks

1. Numeric:

(a) GIVEN: $a_1, e_1, I_1, a_2, e_2, I_2, \alpha = \Delta I_1, \beta = \Delta I_2, a_T, e_T, I_T, p_T, r_1, r_2$, *Figure 2.1, Figure 2.2, "Hohmann-like transfer"*, ω, Ω, Δf, μ, $\Delta v_1, \Delta v_2, v_1, v_{1T}, v_{2T}, v_2$, equations for Δv_1 and Δv_2

(b) TO-FIND:

Vary the value of α from -50 deg to 50 deg in increments of 3°. Calculate and store the value of total $\Delta v = \Delta v_1 + \Delta v_2$.

(c) SOLUTION PROCESS:

We have the following equations from before:

$$v_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_1}\right)}, \qquad v_{1T} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)}, \qquad v_{2T} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_T}\right)}, \qquad v_2 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_2}\right)}$$

$$\Delta v_1 = \sqrt{v_{1T}^2 + v_1^2 - 2v_{1T}v_1\cos(\alpha)}, \qquad \Delta v_2 = \sqrt{v_2^2 + v_{2T}^2 - 2v_2v_{2T}\cos(\beta)}$$

$$\beta = I_2 - I_1 - \alpha$$

We vary α from -50° to 50° with 3° steps. This only affects α and β, which in turn only affects the values of Δv_1 and Δv_2. Everything else remains constant (and has been found in previous parts). This results in the following table:

α [°]	Δv [km/s]
-50	9.2546
-47	8.8482
-44	8.4449
-41	8.0458
-38	7.652
-35	7.2648
-32	6.8856
-29	6.5162
-26	6.159
-23	5.8168
-20	5.4934
-17	5.194
-14	4.9251
-11	4.6954
-8	4.5153
-5	4.3968
-2	4.3506
1	4.3841
4	4.4983
7	4.6871
10	4.94
13	5.2448
16	5.5897
19	5.9651
22	6.3631
25	6.7779
28	7.2046
31	7.6399
34	8.0808
37	8.5252
40	8.9713
43	9.4176
46	9.8629
49	10.306

The code for this section is provided at the end of problem 2 numerical part 3, since all 3 parts utilize the same code.

Melik Demirel – AERSP 450 – HW 2

2. Numeric:

(a) GIVEN: a_1, e_1, I_1, a_2, e_2, I_2, $\alpha = \Delta I_1$, $\beta = \Delta I_2$, a_T, e_T, I_T, p_T, r_1, r_2, *Figure 2.1*, *Figure 2.2*, *"Hohmann-like transfer"*, ω, Ω, Δf, μ, Δv_1, Δv_2, v_1, v_{1T}, v_{2T}, v_2, equations for Δv_1 and Δv_2, Δv vs. α, (Analytical part 3) f_{1T}, f_{2T}

(b) TO-FIND:

Plot α v/s Δv. Can you identify the fuel-optimal transfer from all the transfers you have computed? What is the Δv for this fuel optimal transfer.

(c) SOLUTION PROCESS:

Based on the data in problem 2 numerical part 1, we obtain the following plots:

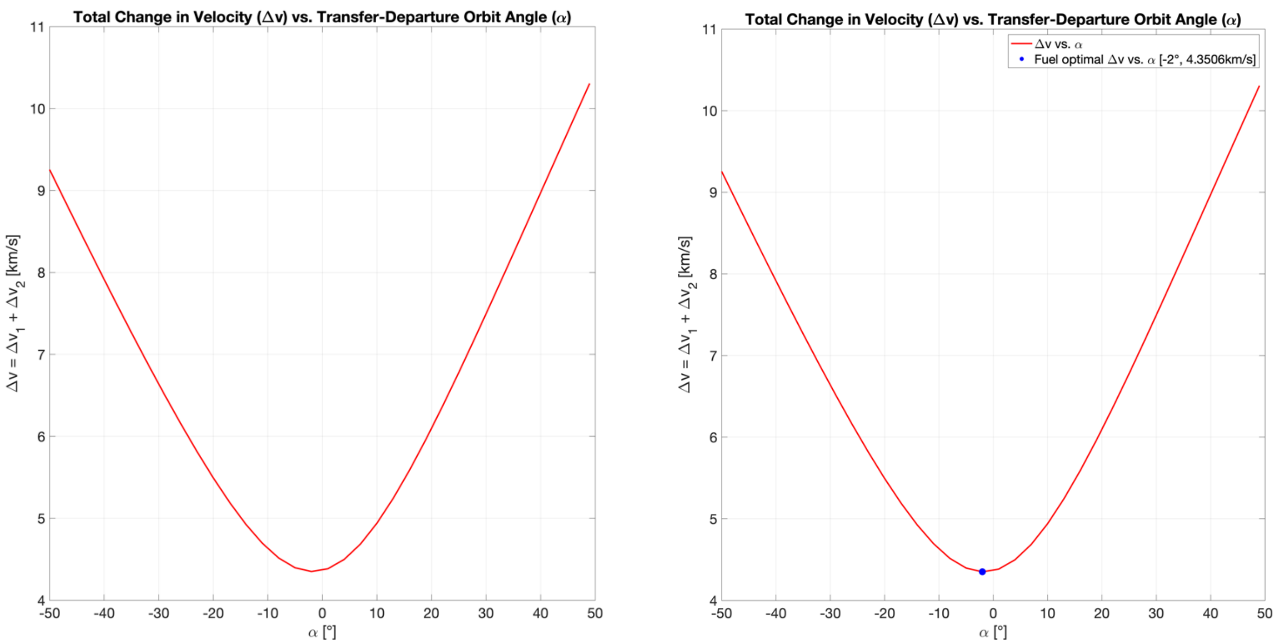


Figure 2.3

On the left, we have plotted α vs. Δv. On the right, we identify the fuel-optimal transfer as the transfer (α value) that corresponds to a minimum Δv value. This is because thrust, and therefore fuel, depends directly on Δv. Thus, the Δv result for a fuel-optimal transfer is $\boldsymbol{\Delta v_{FO} = 4.3506\ km/s}$, and the corresponding α value is $\boldsymbol{\alpha_{FO} = -2°}$.

The code for this section is provided at the end of problem 2 numerical part 3, since all 3 parts utilize the same code.

3. Numeric:

(a) GIVEN: a_1, e_1, I_1, a_2, e_2, I_2, $\alpha = \Delta I_1$, $\beta = \Delta I_2$, a_T, e_T, I_T, p_T, r_1, r_2, *Figure 2.1*, *Figure 2.2*, *"Hohmann-like transfer"*, ω, Ω, Δf, μ, Δv_1, Δv_2, v_1, v_{1T}, v_{2T}, v_2, equations for Δv_1 and Δv_2, Δv vs. α, Δv_{FO}, α_{FO}

(b) TO-FIND:

Plot the departure orbit, arrival orbit, and the fuel optimal transfer.

(c) SOLUTION PROCESS:

We begin the construction of the departure (subscript [1]) and arrival orbits (subscript [2]) as flat-plane polar-coordinate plots drawn on the equatorial plane, where R_1 and R_2 are **radii as a function** of the true anomaly, F_1 and F_2 (where $F_1 = [0, 360]°$ and $F_2 = [0, 360]°$):

$$R_1 = \frac{a_1(1 - e_1^2)}{1 + e_1 \cos F_1}, \qquad R_2 = \frac{a_2(1 - e_2^2)}{1 + e_2 \cos F_2}$$

However, since $e_1 = e_2 = 0$, we have:

$$R_1 = a_1, \qquad R_2 = a_2$$

Eventually, we will have to apply the plane-change inclinations to the plots with rotation matrices. But, before we do so, we can determine the flat plane cartesian coordinates of the orbits on the equatorial plane. Let's denote these with the subscript [P] for the having represented them on the equatorial plane:

$$(x_P)_1 = R_1 \cos F_1\,, \qquad (y_P)_1 = R_1 \sin F_1$$
$$(x_P)_2 = R_2 \cos F_2\,, \qquad (y_P)_2 = R_2 \sin F_2$$

Since the current representation is on the flat equatorial plane, we can introduce the third dimension, where z_P is simply zero for both orbits. Additionally, we can introduce further simplifications by realizing that $F_1 = F_2 = F = [0°, 360°]$, and substituting $R_1 = a_1$ and $R_2 = a_2$:

$$(x_P)_1 = a_1 \cos F\,, \qquad (y_P)_1 = a_1 \sin F\,, \qquad (z_P)_1 = 0$$
$$(x_P)_2 = a_2 \cos F\,, \qquad (y_P)_2 = a_2 \sin F\,, \qquad (z_P)_2 = 0$$

Similarly, let's build the flat-plane polar plot of the transfer orbit, denoted with subscript [T]. We begin with the transfer orbit radius R_T as a function of the varying transfer true anomaly $F_T = [f_{1T}, f_{2T}] = [0°, 180°]$, where f_{1T} and f_{2T} were found in analytical part 3, and a_T and e_T were found as constants in analytical part 1:

$$R_T = \frac{a_T(1 - e_T^2)}{1 + e_T \cos F_T}$$

We then have the flat plane cartesian coordinates for the transfer orbit in a similar manner as the arrival and departure orbits:

$$(x_P)_T = R_T \cos F_T\,, \qquad (y_P)_T = R_T \sin F_T\,, \qquad (z_P)_T = 0$$

Now, all that remains is the rotation matrix to apply the inclination plane-change to the departure (there is only an inclination change—right ascension of the ascending node and argument of perigee are given to be 0°), transfer, and arrival orbits. This will convert the x_P, y_P, z_P coordinates to the true x, y, z coordinates of the orbits. We do this with the Euler-angle rotation matrix R_1 around the first axis (for inclination):

$$\overline{\overline{R_1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Here, $\theta = I_1$ for the departure orbit and $\theta = I_2$ for the arrival orbit. For the transfer orbit, **we use the fuel-optimal value of $\boldsymbol{\alpha}$, $\boldsymbol{\alpha_{FO}}$ found in numerical part 2, to calculate $\boldsymbol{I_T}$**. Based on the geometry of *Figure 2.2*, we have that $I_T = I_1 + \alpha_{FO}$. Thus, for the transfer orbit, we have $\theta = I_T = I_1 + \alpha_{FO}$.

So, to summarize everything:

$$F = [0{,}360]°, \qquad F_T = [0{,}180]°$$

$$R_T = \frac{a_T(1 - e_T^2)}{1 + e_T \cos F_T}$$

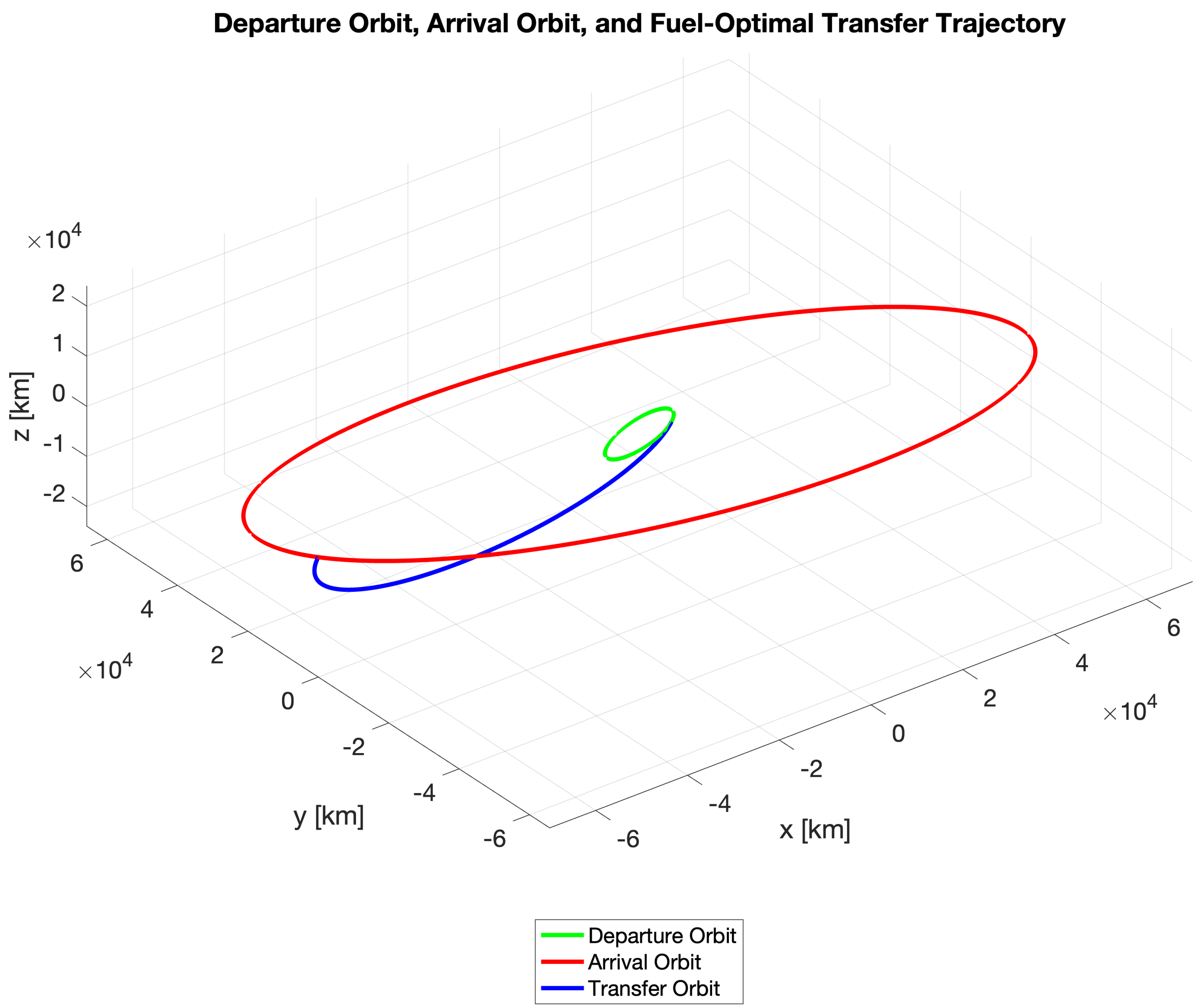
$$(x_P)_1 = a_1 \cos F\,, \qquad (y_P)_1 = a_1 \sin F\,, \qquad (z_P)_1 = 0$$
$$(x_P)_2 = a_2 \cos F\,, \qquad (y_P)_2 = a_2 \sin F\,, \qquad (z_P)_2 = 0$$
$$(x_P)_T = R_T \cos F_T\,, \qquad (y_P)_T = R_T \sin F_T\,, \qquad (z_P)_T = 0$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I_1) & \sin(I_1) \\ 0 & -\sin(I_1) & \cos(I_1) \end{bmatrix} \begin{bmatrix} (x_P)_1 \\ (y_P)_1 \\ (z_P)_1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I_2) & \sin(I_2) \\ 0 & -\sin(I_2) & \cos(I_2) \end{bmatrix} \begin{bmatrix} (x_P)_2 \\ (y_P)_2 \\ (z_P)_2 \end{bmatrix}$$

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I_1 + \alpha_{FO}) & \sin(I_1 + \alpha_{FO}) \\ 0 & -\sin(I_1 + \alpha_{FO}) & \cos(I_1 + \alpha_{FO}) \end{bmatrix} \begin{bmatrix} (x_P)_T \\ (y_P)_T \\ (z_P)_T \end{bmatrix}$$

We then simply plot x_1, y_1, z_1, x_2, y_2, z_2, and x_T, y_T, z_T to obtain the plots we need:



AERSP 450 HW 2 - Melik Demirel - The following is the code for Problem 2 Numerical:

```
% Author: Melik Demirel
% mcd5703
% PSU ID: 952718091
% This code solves HW2 Problem 2
clc; close all; clear;

%% Givens
mu = 3.986*10^5; % km^3*s^-2
% Before
a1 = 7000; % km
e1 = 0;
I1 = 60; % deg
% After
a2 = 70000; % km
e2 = 0;
I2 = 20; % deg

%% Calculations for Analytic Part 1:
disp('--- Analytic Part 1: ---')
% From calculations:
r1 = a1;
r2 = a2;

% aT
aT = (1/2) * (r1+r2);
fprintf('aT = %.3f km.\n', aT);

% k, l, pT
k = 2 * r1 * r2;
l = r1 + r2;
pT = k/l;
fprintf('k = %.3f km^2.\n', k);
fprintf('l = %.3f km.\n', l);
fprintf('pT = %.3f km.\n', pT);

% eT
eT = sqrt(1-pT/aT);
fprintf('eT = %.6f.\n', eT);

%% Calculations for Analytic Part 2:
disp('--- Analytic Part 2: ---')

% Delta
delta = I2 - I1;
fprintf('δ = %.3f deg.\n', delta);

% Alpha:
alpha = -20;
fprintf('α = %.3f deg.\n', alpha);

% Beta
beta = delta - alpha;
fprintf('β = %.3f deg.\n', beta);
```

```
% IT
IT = I1 + alpha;
fprintf('IT = %.3f deg.\n', IT);

%% Calculations for Analytic Part 3:
disp('--- Analytic Part 3: ---')

% Velocity magnitudes
v1 = sqrt(mu*(2/r1-1/a1));
v1T = sqrt(mu*(2/r1-1/aT));
v2T = sqrt(mu*(2/r2-1/aT));
v2 = sqrt(mu*(2/r2-1/a2));
fprintf('v1 = %.3f km/s.\n', v1);
fprintf('v1T = %.3f km/s.\n', v1T);
fprintf('v2T = %.3f km/s.\n', v2T);
fprintf('v2 = %.3f km/s.\n', v2);

% Changes in velocity
Dv1 = sqrt(v1^2 + v1T^2 - 2*v1*v1T*cosd(alpha));
Dv2 = sqrt(v2^2 + v2T^2 - 2*v2*v2T*cosd(beta));
fprintf('Δv1 = %.3f km/s.\n', Dv1);
fprintf('Δv2 = %.3f km/s.\n', Dv2);

%% Calculations for Numeric Part 1:
disp('--- Numeric Part 1: ---')

% Alpha and beta
alpha = -50:3:50;
beta = I2-I1-alpha;

% Vector delta v1 and delta v2:
Dv1 = sqrt(v1^2 + v1T^2 - 2*v1*v1T*cosd(alpha));
Dv2 = sqrt(v2^2 + v2T^2 - 2*v2*v2T*cosd(beta));

% Sum of delta v1 and delta v2:
Dv = Dv1+Dv2;

% Output alpha and Dv
AlphaVSDv = horzcat(alpha', Dv');
AlphaVsDv = array2table(AlphaVSDv, "VariableNames",{'α [°]', 'Δv [km/s]'})

%% Calculations for Numeric Part 2:
disp('--- Numeric Part 2: ---')

% alpha vs. Dv
fig1 = figure(Position=[0,0,1000,1000]);
plot(alpha, Dv, 'r', 'LineWidth', 2);
xlabel('\alpha [°]')
ylabel('\Deltav = \Deltav_1 + \Deltav_2 [km/s]')
title(['Total Change in Velocity (\Deltav) vs. ', ...
    'Transfer-Departure Orbit Angle (\alpha)'])
grid on;
fontsize(fig1,'scale',2)
```

```
% Find fuel optimal (FO) Dv and alpha;
[Dv_FO, Dv_FO_ind] = min(Dv);
alpha_FO = alpha(Dv_FO_ind);
fprintf('Optimal α = %.3f °.\n', alpha_FO);
fprintf('Optimal Δv = %.3f km/s.\n', Dv_FO);

% alpha vs. Dv with min shown on plot
fig2 = figure(Position=[0,0,1000,1000]);
plot(alpha, Dv, 'r', 'LineWidth', 2);
xlabel('\alpha [°]')
ylabel('\Deltav = \Deltav_1 + \Deltav_2 [km/s]')
title(['Total Change in Velocity (\Deltav) vs. ', ...
    'Transfer-Departure Orbit Angle (\alpha)'])
grid on;
hold on;
scatter(alpha_FO,Dv_FO,100,'blue','filled')
hold off;
opText = ['Fuel optimal \Deltav vs. \alpha [', num2str(alpha_FO), ...
            '°, ', num2str(Dv_FO), 'km/s]'];
legend('\Deltav vs. \alpha',opText)
fontsize(fig2,'scale',2)

% Export graphs
FileType = '.png';
SN1 = 'P2np2_1';
ST1 = [SN1, FileType];
FN1 = fullfile(ST1);
exportgraphics(fig1, FN1, 'Resolution', 300, 'BackgroundColor', 'white');
SN2 = 'P2np2_2';
ST2 = [SN2, FileType];
FN2 = fullfile(ST2);
exportgraphics(fig2, FN2, 'Resolution', 300, 'BackgroundColor', 'white');

%% Calculations for Numeric Part 3:
disp('--- Numeric Part 3: ---')

% Independent variables
F = 0:0.1:360;
FT = 0:0.1:180;

% Radius of transfer
RT = (aT*(1-eT^2))./(1+eT*cosd(FT));

% Equatorial plane polar plots:
x1 = a1 * cosd(F);
y1 = a1 * sind(F);
z1 = 0 .* x1; % Just zeros
x2 = a2 * cosd(F);
y2 = a2 * sind(F);
z2 = 0 .* x2; % Just zeros
xT = RT .* cosd(FT);
yT = RT .* sind(FT);
zT = 0 .* xT; % Just zeros

% Function for the DCM R1
```

```matlab
R1 = @(theta) ...
    [1, 0, 0; ...
     0, cosd(theta), sind(theta); ...
     0, -sind(theta), cosd(theta)];

% Application of inclination to departure and arrival
for i = 1:length(F)
    departure = R1(I1) * [x1(i); y1(i); z1(i)];
    arrival = R1(I2) * [x2(i); y2(i); z2(i)];
    x1(i) = departure(1);
    y1(i) = departure(2);
    z1(i) = departure(3);
    x2(i) = arrival(1);
    y2(i) = arrival(2);
    z2(i) = arrival(3);
end

% Application of inclination to transfer
for i = 1:length(FT)
    transfer = R1(I1-alpha_FO) * [xT(i); yT(i); zT(i)];
    xT(i) = transfer(1);
    yT(i) = transfer(2);
    zT(i) = transfer(3);
end

% Plot
fig3 = figure(Position=[0,0,1000,1000]);
plot3(x1,y1,z1,'g', LineWidth=3);
hold on;
plot3(x2,y2,z2,'r', LineWidth=3);
plot3(xT,yT,zT,'b', LineWidth=3);
axis equal
grid on;
xlabel('x [km]');
ylabel('y [km]');
zlabel('z [km]');
title('Departure Orbit, Arrival Orbit, and Fuel-Optimal Transfer
Trajectory');
legend('Departure Orbit', 'Arrival Orbit', 'Transfer Orbit',
Location='south');
fontsize(fig3, 'scale', 1.7)

% Export graph
FileType = '.png';
SN3 = 'P2np3';
ST3 = [SN3, FileType];
FN3 = fullfile(ST3);
exportgraphics(fig3, FN3, 'Resolution', 300, 'BackgroundColor', 'white');
```

Melik Demirel   Subscript definition:
E → Earth, M → Mars, T → transfer

(1)

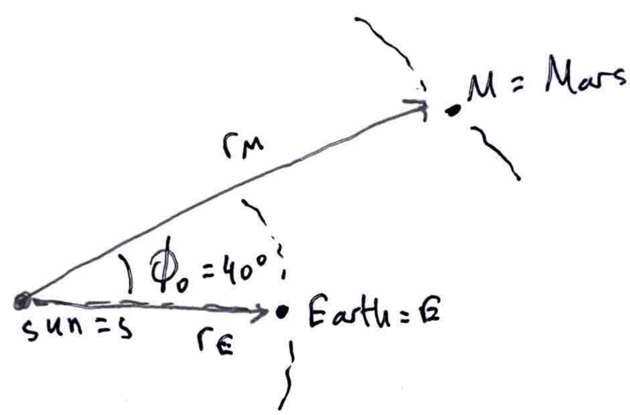
3.1 (a) Given: $\mu_s = 1.327 \times 10^{11}$ km^3/s^2, Coplanar circular orbits

$r_E = 1$ AU, $r_M = 1.524$ AU, 1 AU = 149.6×10^6 km

TOF = 183 Days, Mars is 40° ahead of Earth = ϕ_0

(b) Find Δf (c) solution process:

Figure 3.1:



Coplanar circular orbits → $\boxed{e_E = e_M = 0}$

$$r_E = \frac{a_E(1-e_E^2)}{1+e_E\cos f_E} \xrightarrow{e_E=0} \boxed{a_E = r_E}$$

$$r_M = \frac{a_M(1-e_M^2)}{1+e_M\cos f_M} \xrightarrow{e_M=0} \boxed{a_M = r_M}$$

for future calculations (parts 2-5)

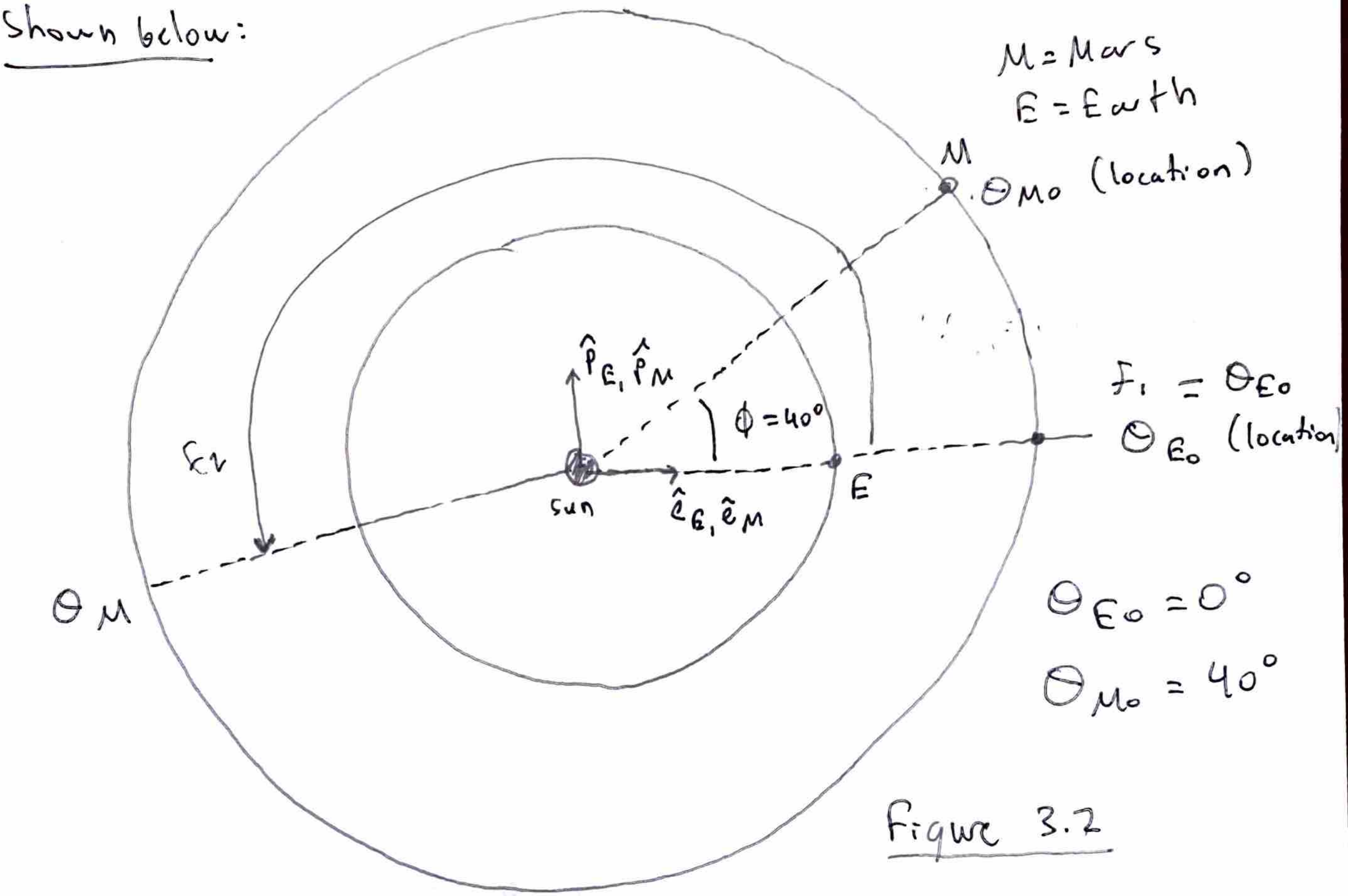
$$\text{Tof} = 183 \text{ days} \cdot \frac{24 \cdot 60 \cdot 60 \text{ s}}{\text{days}} = \boxed{15811200 \text{ s}}$$

$$\boxed{\begin{aligned} r_E &= 149600000 \text{ km} \\ r_M &= 227990400 \text{ km} \end{aligned}}$$

2

Aligned $\hat{e}-\hat{p}$ for E and M means $\underline{\Delta f = \Delta f_T = f_2 - f_1}$

Shown below:



$\theta_{E0} = 0°$

$\theta_{M0} = 40°$

Figure 3.2

Because circular orbits:

$$M = f = E = \theta = n(t - t_0) + \theta_0$$

Earth: $\theta_E = \theta_{E0} + n_E(t - t_0)$, Mars: $\theta_M = \theta_{M0} + n_M(t - t_0)$

we can subtract θ_{E0} from both sides of the Mars equation

$$\underbrace{\theta_M - \theta_{E0}} = \theta_{M0} + n_m(t - t_0) - \theta_{E0}$$

$$f_2 = \theta_{M0} + n_m \Delta t - \cancel{\theta_{E0}}^{\,0} \qquad \boxed{\Delta t = \text{ToF}}$$

$$f_2 = \theta_{M0} + n_m \text{ToF} \qquad n_M = \sqrt{\frac{\mu_S}{a_m^3}}$$

with everything relative to θ_{E0} → $\theta_{M0} = \phi_0 + \theta_{E0}$ → $\theta_{M0} = 40° = \phi_0$

Since we alligned $\hat{e}-\hat{p}$ frame for Mars and Earth as seen in figure 3.2, we realize that $\Delta f = \Delta f_T = f_2 - f_1$. From the geometry and calculations, to summarize, we have:

$$\Delta f = f_2 - f_1 \;,\quad f_1 = 0^\circ \;,\quad f_2 = \phi_0 + \sqrt{\frac{\mu_s}{a_m^3}} \cdot ToF$$

$$\boxed{\Delta f = \sqrt{\frac{\mu_s}{a_m^3}} \cdot ToF + \phi_0}$$

$$\Delta f = \left(\sqrt{\frac{1.327 \cdot 10^{11}}{(227990400)^3}} \cdot 15811200 \right) \underbrace{\frac{180}{\pi}}_{\text{rad to deg}} + 40^\circ = \boxed{135.8624^\circ}$$

$\boxed{3.2}$ Melik Demirel

4

(a) Given Δf, r_E, r_M, everything found in Part 1 / figures, TOF, everything given in part 1

(b) To-find: a_T and e_T of the transfer orbit (Iris probe)

(c) Solution process:

we start by assuming that $a = a_{min}$

$$a_{min} = \frac{1}{4}\left(r_E + r_M + \sqrt{r_E^2 + r_M^2 - 2 r_E r_M \cos \Delta f}\right)$$

$$a_{min} = \frac{1}{4}\Big((149600000) + (227990400) + \big[(149600000)^2 + (227990400)^2 - 2(149600000)(227990400)\cos(135.8624)\big]^{1/2}\Big)$$

$$\boxed{a_{min} = 1.8219 \cdot 10^8 \text{ km}}$$

↳ this will be used later

Then, the process is as follows:

Step 1 given a (arbitrary)

$$p_1 = \frac{2ak\ell - km + k\sqrt{m(8a^2 - 4a\ell + m)}}{2a\ell^2 - 4am}$$

$$p_2 = \frac{2ak\ell - km - k\sqrt{m(8a^2 - 4a\ell + m)}}{2a\ell^2 - 4am}$$

where: $m = r_M r_E(1 + \cos \Delta f)$, $k = r_M r_E(1 - \cos \Delta f)$

$\ell = r_M + r_E$

we now have p_1, p_2

Melik Demirel

step 2)

let $P_1, P_2 = P_i$ for the general calculations...

→ Calculate F & G functions for $i = 1, 2$:

$$F_i = 1 - \frac{r_M}{P_i}(1 - \cos\Delta f)$$

$$\dot{F}_i = \sqrt{\frac{\mu_s}{P_i}}\tan\left(\frac{\Delta f}{2}\right)\left(\frac{1-\cos\Delta f}{P_i} - \frac{1}{r_E} - \frac{1}{r_M}\right)$$

$$G_i = \frac{r_M r_E}{\sqrt{\mu_s P_i}}\sin(\Delta f)$$

$P_1, P_2 \longrightarrow F_i, \dot{F}_i, G_i$ where $i = 1, 2$

---

step 3)

$$\left.\begin{aligned}(\cos\Delta E)_i &= 1 - \frac{(1-F_i)}{a} r_E \\ (\sin\Delta E)_i &= \frac{-r_E r_M \dot{F}_i}{\sqrt{\mu a}}\end{aligned}\right\} \quad \Delta E_i = \underline{\text{atan2}}\left((\sin\Delta E)_i, (\cos\Delta E_i)\right)$$

$F_i, \dot{F}_i$ where $i = 1, 2 \longrightarrow \Delta E_i$ where $i = 1, 2$ corresponding to $P_i = P_1, P_2$

---

step 4)

$$\Delta t_i = G_i + \sqrt{\frac{a^3}{\mu_s}}\left(\Delta E_i - (\sin\Delta E)_i\right)$$

$\therefore$ $\underline{P_i = P_1, P_2 \ (i = 1, 2) \rightarrow \Delta t_i \text{ where } i = 1, 2}$

So $a \rightarrow P_1, P_2 \rightarrow \Delta t_1, \Delta t_2$

Let $f(a,i)$ be a function that inputs a and outputs Δt_1 if $i=1$, Δt_2 if $i=2$, or both if $j=0$. 6

We can construct this function $f(a,j)$ using steps 1-4 outlined previously. (we choose p_1 if $i=1$ and p_2 if $i=2$)

Initially, we test $\underline{f(a_{min}, 0)}$:

$a_{min} = 1.8219 \cdot 10^8$ km

$\rightarrow p_1 = p_2 \approx 166831862$ km (step 1)

$\rightarrow F = -1.347$, $\dot{F} = -0.000000005$, $G = 5048034$. (step 2)

$\rightarrow \Delta E = 2.758$ rad (step 3)

$\rightarrow \Delta t_1 = \Delta t_2 = 244.733$ days (step 4)

$\hookrightarrow$ TOF a_{min}

here, we see that $\Delta t_1 = \Delta t_2$ @ a_{min} is <u>greater than</u> TOF = 183 days

Since $\Delta f < 180°$, $TOF_d < TOF_{amin}$, <u>we use Δt_1 and p_1 from here on out</u> (opposite of page 5 of lecture 16).

Let's test $f(a_{max} = a_{min} \cdot 20, 1)$ ($i=1$ for p_1)

$\rightarrow p_1 = 242962570$ km

$\rightarrow F = -0.612$, $\dot{F} = -0.00000023$, $G = 4183040$ ......

$\rightarrow \Delta E = 0.366$ rad

$\rightarrow \Delta t_1 = 105.063$ days

Then, let's test $L\left(\left(\frac{a_{min} + a_{max}}{2}\right) = a_n, 1\right)$: $a_n =$ 1912977366 km

→ $P_1 = 240795792$ km

→ $f = -0.626$, $\dot{f} = -0.00000023$, $G = 4201819.015$

→ $\Delta E = 0.510$ rad

→ $\Delta t_1 = 106.595$ days

. . . .

Now, we start to see a trend:

$t_{amin} = 244.73$ days | $TOF = 183$ days | $t_{an} = 106.59$ days | $t_{amax} = 105.63$ days

$a_{min} = a_i$ | a_T @ TOF | a_n | $a_{max} = a_f$

→ if $TOF > \Delta t_1$ → set upper a (such as a_{max}) to a_n (Ex: new $a_f = a_n$)

if $TOF < \Delta t_1$ → set lower a (such as a_{min}) to a_n (Ex: new $a_i = a_n$)

→ we recalculate a_n as $\left(\frac{a_i + a_f}{2}\right)$

→ repeat until $|TOF - \Delta t_1| < 0.001$ condition is met...

Table of few iterations

a_n km	p_n km	F	$\dot{F}$	G	ΔE rad	Δt days
1047582843	236938488	-0.653	-0.00000022	4235883	0.701	109.463
614881582	230676593	-0.698	-0.00000021	4292991	0.943	114.545
⋮	⋮	⋮	⋮	⋮	⋮	⋮
195206672 $= a_T$	184404833 $= p_T$	-1.124	0.00000012	4801485	2.249	183 = ToF

$\therefore \boxed{a_T = 1.9521 \cdot 10^8 \text{ km}}, \quad p_T = 1.8440 \cdot 10^8 \text{ km}$

Then, we can calculate e_T:

$$p_T = a_T(1 - e_T^2) \rightarrow e_T = \sqrt{1 - \frac{p_T}{a_T}}$$

$$e_T = \sqrt{1 - \frac{1.8440 \cdot 10^8}{1.9521 \cdot 10^8}} = \boxed{0.2352}$$

a_T = semimajor axis

e_T = eccentricity of iris probe.

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Problem 3

3. Analytic:

(a) GIVEN: $\mu_s\ a_E, r_E, e_E, a_M, r_M, e_M, a_T, e_T, p_T, \Delta f$ (and everything else found in previous parts)

(b) TO-FIND:

The total value of Δv.

(c) SOLUTION PROCESS:

We first use the energy (Vis-viva) equation to obtain the magnitudes of velocity for the departure orbit v_E, the departing transfer orbit velocity v_{ET}, the arriving transfer orbit velocity v_{MT}, and the arrival orbit velocity v_M:

$$\varepsilon = \frac{1}{2}v^2 - \frac{\mu_s}{r} = -\frac{\mu_s}{2a} \rightarrow v = \sqrt{\mu_s\left(\frac{2}{r} - \frac{1}{a}\right)}$$

The radii at the point of departure are the same for the departure orbit and the transfer. This is also true at the point of arrival for the arrival and departure orbits:

$$v_E = \sqrt{\mu_s\left(\frac{2}{r_E} - \frac{1}{a_E}\right)}, \qquad v_{ET} = \sqrt{\mu_s\left(\frac{2}{r_E} - \frac{1}{a_T}\right)}, \qquad v_{MT} = \sqrt{\mu_s\left(\frac{2}{r_M} - \frac{1}{a_T}\right)}, \qquad v_M = \sqrt{\mu_s\left(\frac{2}{r_M} - \frac{1}{a_M}\right)}$$

Plugging in the given values:

$$v_E = \sqrt{1.327 * 10^{11}\left(\frac{2}{149600000} - \frac{1}{149600000}\right)} = 29.783\,\frac{\text{km}}{\text{s}},$$

$$v_{ET} = \sqrt{1.327 * 10^{11}\left(\frac{2}{149600000} - \frac{1}{1.9521 * 10^8}\right)} = 33.080\,\frac{\text{km}}{\text{s}}$$

$$v_{MT} = \sqrt{1.327 * 10^{11}\left(\frac{2}{227990400} - \frac{1}{1.9521 * 10^8}\right)} = 22.007\,\frac{\text{km}}{\text{s}},$$

$$v_M = \sqrt{1.327 * 10^{11}\left(\frac{2}{227990400} - \frac{1}{227990400}\right)} = 24.126\,\frac{\text{km}}{\text{s}}$$

We then use the analytical velocity change formula (cosine law):

$$\Delta v_E = \sqrt{v_{ET}^2 + v_E^2 - 2v_{ET}v_E\cos(\gamma_{ET} - \gamma_E)}$$

$$\Delta v_M = \sqrt{v_M^2 + v_{MT}^2 - 2v_M v_{MT}\cos(\gamma_{MT} - \gamma_M)}$$

To calculate the flight path angle:

$$\tan(\gamma) = \frac{e \cdot \sin f}{1 + e \cdot \cos f} \rightarrow \gamma = \arctan\left(\frac{e \cdot \sin f}{1 + e \cdot \cos f}\right)$$

$$r = \frac{p}{1 + e\cos(f)} \rightarrow f = \arccos\left(\frac{1}{e}\left(\frac{p}{r} - 1\right)\right)$$

The true anomaly equation lends a positive value if the probe is ascending, and a negative value if probe is descending. Since $\Delta f = 135.8624°$ and f_1 was 0° (so f_1 and f_2 are both less than 180°), both impulses are in the ascending zone or positive true anomaly values, which also means both γ values are positive. So, with values from part 1:

$$f_{ET} = \arccos\left(\frac{1}{e_T}\left(\frac{p_T}{r_E} - 1\right)\right) = \arccos\left(\frac{1}{0.2352}\left(\frac{184404833}{149600000} - 1\right)\right) = 8.497°$$

$$f_{MT} = \arccos\left(\frac{1}{e_T}\left(\frac{p_T}{r_M} - 1\right)\right) = \arccos\left(\frac{1}{0.2352}\left(\frac{184404833}{227990400} - 1\right)\right) = 144.360°$$

$$\gamma_E = \arctan\left(\frac{e_E \sin f_E}{1 + e_E \cos f_E}\right) = \arctan(0) = 0°$$

$$\gamma_{ET} = \arctan\left(\frac{e_T \sin f_{ET}}{1 + e_T \cos f_{ET}}\right) = \arctan\left(\frac{0.2352 \sin 8.497°}{1 + 0.2352 \cos 8.497°}\right) = 1.615°$$

$$\gamma_{MT} = \arctan\left(\frac{e_T \sin f_{MT}}{1 + e_T \cos f_{MT}}\right) = \arctan\left(\frac{0.2352 \sin 144.360°}{1 + 0.2352 \cos 144.360°}\right) = 9.618°$$

$$\gamma_M = \arctan\left(\frac{e_M \sin f_M}{1 + e_M \cos f_M}\right) = \arctan(0) = 0°$$

We can then plug values in:

$$\Delta v_E = \sqrt{(33.080)^2 + (29.783)^2 - 2(33.080)(29.783)\cos\big((1.615°) - (0°)\big)} = 3.413\frac{\text{km}}{\text{s}}$$

$$\Delta v_M = \sqrt{(24.126)^2 + (22.007)^2 - 2(24.126)(22.007)\cos\big((0°) - (9.618°)\big)} = 4.406\frac{\text{km}}{\text{s}}$$

Finally, we sum the result:

$$\Delta v = \Delta v_E + \Delta v_M = 3.413 + 4.406 = \mathbf{7.820\frac{km}{s}}$$

4. <u>Numeric:</u>

(a) GIVEN: $\mu_s\ a_E, r_E, e_E, a_M, r_M, e_M, a_T, e_T, p_T, \Delta f$ (and everything else found in previous parts)

(b) TO-FIND:

Plot the Earth, Mars, and Transfer orbits.

(c) SOLUTION PROCESS:

We first define the range for true anomaly F, where $F = [0, 360]°$. Then we can plot the Earth and Mars orbits as polar plots:

$$x_E = r_E \cos F\,, \qquad y_E = r_E \sin F$$
$$x_M = r_M \cos F\,, \qquad y_M = r_M \sin F$$

Likewise, we can then define a true anomaly range for the transfer orbit $F_T = [f_{ET}, f_{MT}] = [8.497, 144.360]°$. With this, we can begin an initial plot of the transfer orbit (x_P and y_P), prior to rotating it with a DCM based on the value of f_1.

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$$R_T = \frac{a_T(1-e_T^2)}{1+e_T\cos F_T}$$

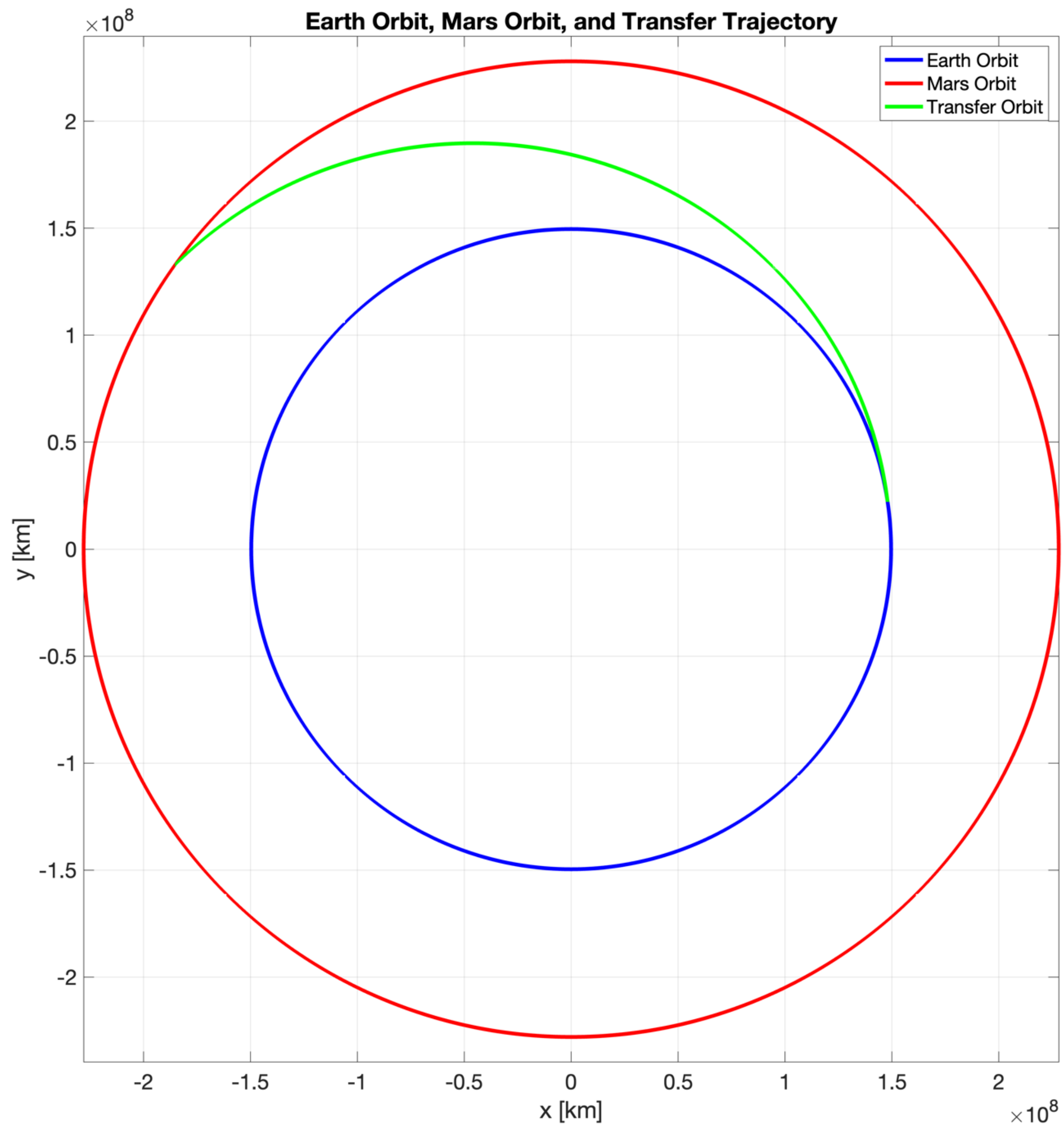
$$(x_P)_T = R_T\cos F_T\,, \qquad (y_P)_T = R_T\sin F_T$$

Since we then recall that f_1 is 0° (recall Figure 1.1 and Problem 1 for such a rotation), there happens to be no need for the DCM, and so we simply have:

$$R_T = \frac{a_T(1-e_T^2)}{1+e_T\cos F_T}$$

$$x_T = R_T\cos F_T\,, \qquad y_T = R_T\sin F_T$$

We then simply plot x_E, y_E, x_M, y_M, and x_T, y_T to obtain the plots we need:



The MATLAB code for this section is posted at the end of part 5, because it also contains the marking system used in part 5.

5. Numeric:

(a) GIVEN: μ_s a_E, r_E, e_E, a_M, r_M, e_M, a_T, e_T, p_T, Δf (and everything else found in previous parts)

(b) TO-FIND:

Label Mars and Earth on the plot.

(c) SOLUTION PROCESS:

@ Departure: (d) → subscript

Earth: moved from $\theta_{E0} = 0$ to $f_{ET} = 8.497 = \theta_{Ed}$

$$\therefore \boxed{x_{Ed} = r_E \cos(f_{ET}),\ y_{Ed} = r_E \sin(f_{ET})}$$

$$\theta_{Ed} = \theta_{E0} + \sqrt{\frac{\mu_s}{a_E^3}}(\Delta t_d) \rightarrow \boxed{\Delta t_d = \frac{f_{ET} - 0}{\sqrt{\frac{\mu_s}{a_E^3}}}}$$

Mars: $\theta_{Md} = \phi_0 + \sqrt{\frac{\mu_s}{a_M^3}}(\Delta t_d)$

$$\therefore \boxed{x_{Md} = r_M \cos(\theta_{Md}),\quad y_{Md} = r_M \sin(\theta_{Md})}$$

@ Arrival (a) → subscript

$$\text{Mars: } \theta_{Ma} = \phi_0 + \left(\sqrt{\frac{\mu_s}{a_M^3}}\ \overbrace{TOF}^{\boxed{15811200\ s}}\right)\frac{180}{\pi} + \theta_{Ed}$$

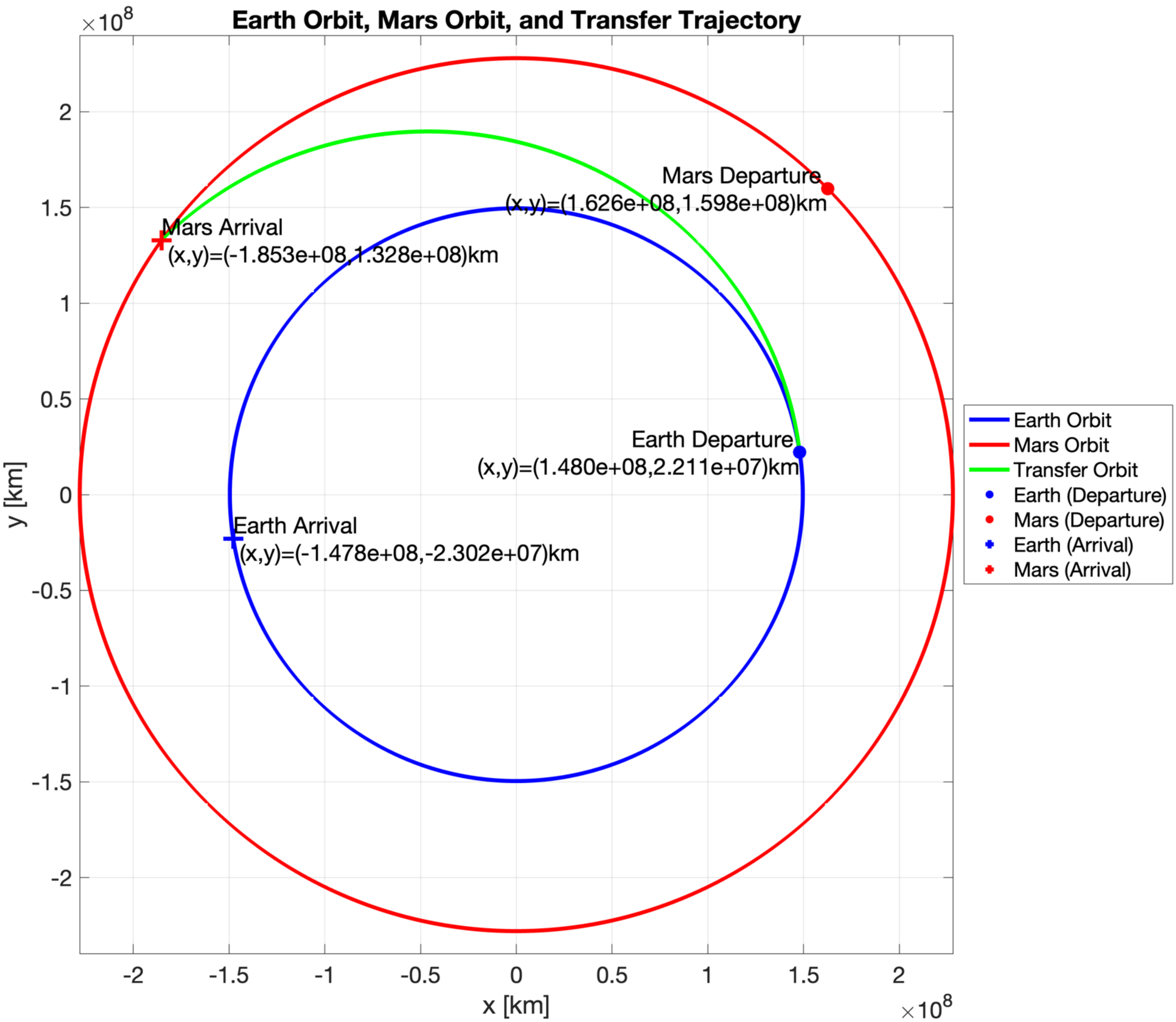
amount Earth moved before launch

$$\boxed{\therefore x_{Ma} = r_M \cos(\theta_{Ma}),\quad y_{Ma} = r_M \sin(\theta_{Ma})}$$

$$\text{Earth: } \theta_{Ea} = \underbrace{\theta_{Ed}}_{\text{start}} + \underbrace{\left(\sqrt{\frac{\mu_s}{a_E^3}}\ TOF\right)\frac{180}{\pi}}_{\text{change in orbit}}$$

$$\therefore \boxed{x_{Ea} = r_E \cos(\theta_{Ea}),\quad y_{Ea} = r_E \sin(\theta_{Ea})}$$

Calculated values are plotted in the graph…


Earth Orbit, Mars Orbit, and Transfer Trajectory
×10^8
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
y [km]
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x [km]
×10^8
Mars Departure
(x,y)=(1.626e+08,1.598e+08)km
Mars Arrival
(x,y)=(-1.853e+08,1.328e+08)km
Earth Departure
(x,y)=(1.480e+08,2.211e+07)km
Earth Arrival
(x,y)=(-1.478e+08,-2.302e+07)km
Earth Orbit
Mars Orbit
Transfer Orbit
Earth (Departure)
Mars (Departure)
Earth (Arrival)
Mars (Arrival)

AERSP 450 HW 2 - Melik Demirel - The following is the code for Problem 3:

```
% Author: Melik Demirel
% mcd5703
% PSU ID: 952718091
% This code solves HW2 Problem 3
clc; close all; clear;

%% Givens
% Mu of the sun
mu_s = 1.327*10^11; % km^3*s^-2
AU = 149.6*10^6;    % km
% Radii from the sun
rE = 1*AU;          % Earth
rM = 1.524 * AU;    % Mars
% Mars phase angle
phi0 = 40; % deg
% TOF
TOF = 183; % Days
TOF = TOF * 24 * 60 * 60; % s

%% Initial calculations

% Orbit semimajor axes
aE = rE;
aM = rM;
Df = 180/pi * sqrt(mu_s/aM^3) * TOF + phi0;
amin = (1/4) * (rE + rM + sqrt(rE^2 + rM^2 - 2*rE*rM*cosd(Df)));

%% Lambert calculations

a = amin;
output = lambert(a, rM, rE, Df, mu_s, 0);
% Sanity check:
if round(output(1),-3) ~= round(output(2),-3)
    error('For some reason, t1 ≠ t2 at a_min')
end
% Δt at a_min
t_amin = output(1);
% Check which to use
if Df < 180
    if TOF < t_amin
        i = 1;
    else
        i = 2;
    end
else
    if TOF < t_amin
        i = 2;
    else
        i = 1;
    end
end
% In our analysis from here on out we found that we shall use i = 1:
```

```
% Upper bound
disp('Test a_max:')
amax = 20*amin; % a max
t_amax = lambert(amax, rM, rE, Df, mu_s, 1);

% Initial conditions
a_i = amin; % Lower a bound
a_f = amax; % Upper a bound
Dt = t_amax; % Starting t
while abs(Dt - TOF) > 0.001
    % Half value of a
    a_h = (a_i + a_f) / 2;
    disp('For middle a:')
    disp(['a_i = ', num2str(a_i), 'km'])
    disp(['a_f = ', num2str(a_f), 'km'])
    disp(['New a_h = ', num2str(a_h), 'km'])
    [Dt,p] = lambert(a_h, rM, rE, Df, mu_s, 1);
    % Our case was t_amax > t_amin, so we use the following:
    if TOF > Dt
        a_f = a_h;
    else
        a_i = a_h;
    end
end

% Final values of Iris probe
aT = a_h;
pT = p;
eT = sqrt(1-pT/aT);
fprintf('aT = %.3f km.\n', aT);
fprintf('pT = %.3f km.\n', pT);
fprintf('eT = %.4f .\n', eT);

%% Velocity Change:
disp(' ')

% Velocity magnitudes
vE = sqrt(mu_s*(2/rE-1/aE));
vET = sqrt(mu_s*(2/rE-1/aT));
vMT = sqrt(mu_s*(2/rM-1/aT));
vM = sqrt(mu_s*(2/rM-1/aM));
fprintf('vE = %.3f km/s.\n', vE);
fprintf('vET = %.3f km/s.\n', vET);
fprintf('vMT = %.3f km/s.\n', vMT);
fprintf('vM = %.3f km/s.\n', vM);

% Flight path angle
fET = acosd(1/eT*(pT/rE-1));
fMT = acosd(1/eT*(pT/rM-1));
yET = atand(eT*sind(fET)/(1+eT*cosd(fET)));
yMT = atand(eT*sind(fMT)/(1+eT*cosd(fMT)));
DyE = yET - 0;
DyM = 0 - yMT;
fprintf('fET = %.3f °.\n', fET);
fprintf('fMT = %.3f °.\n', fMT);
```

```matlab
fprintf('yET = %.3f °.\n', yET);
fprintf('yMT = %.3f °.\n', yMT);

% Changes in velocity
DvE = sqrt(vE^2 + vET^2 - 2*vE*vET*cosd(DyE));
DvM = sqrt(vM^2 + vMT^2 - 2*vM*vMT*cosd(DyM));
fprintf('ΔvE = %.3f km/s.\n', DvE);
fprintf('ΔvM = %.3f km/s.\n', DvM);

Dv = DvE + DvM;
fprintf('Δv = %.3f km/s.\n', Dv);

%% Orbit Plot:

% Independent variables
F = 0:0.1:360;
FT = fET:0.01:fMT;

% Radius of transfer
RT = (aT*(1-eT^2))./(1+eT*cosd(FT));

% Equatorial plane polar plots:
xE = rE * cosd(F);
yE = rE * sind(F);
xM = rM * cosd(F);
yM = rM * sind(F);
xT = RT .* cosd(FT);
yT = RT .* sind(FT);

% Plot
fig1 = figure(Position=[0,0,1000,1000]);
plot(xE,yE,'b', LineWidth=3);
hold on;
plot(xM,yM,'r', LineWidth=3);
plot(xT,yT,'g', LineWidth=3);
axis equal
grid on;
xlabel('x [km]');
ylabel('y [km]');
title('Earth Orbit, Mars Orbit, and Transfer Trajectory');
legend('Earth Orbit', 'Mars Orbit', 'Transfer Orbit');
fontsize(fig1, 'scale', 1.7)

% Export graph
FileType = '.png';
SN1 = 'P3_1';
ST1 = [SN1, FileType];
FN1 = fullfile(ST1);
exportgraphics(fig1, FN1, 'Resolution', 300, 'BackgroundColor', 'white');

%% Marked orbit plot

% Find the locations of Earth and Mars at departure and arrival:
xED = rE*cosd(fET);
yED = rE*sind(fET);
```

```matlab
Dtd = fET/sqrt(mu_s/aE^3);
oMD = phi0 + sqrt(mu_s/aM^3) * Dtd;
xMD = rM*cosd(oMD);
yMD = rM*sind(oMD);
oMA = fET+phi0+rad2deg(TOF * sqrt(mu_s/aM^3));
xMA = rM*cosd(oMA);
yMA = rM*sind(oMA);
oEA = fET+rad2deg(TOF * sqrt(mu_s/aE^3));
xEA = rE*cosd(oEA);
yEA = rE*sind(oEA);

% Plot
fig2 = figure(Position=[0,0,1000,1000]);
plot(xE,yE,'b', LineWidth=3);
hold on;
plot(xM,yM,'r', LineWidth=3);
plot(xT,yT,'g', LineWidth=3);
scatter(xED, yED, 100, 'blue', 'filled');
scatter(xMD, yMD, 100, 'red', 'filled');
scatter(xEA, yEA, 200, 'blue', '+', 'LineWidth',3);
scatter(xMA, yMA, 200, 'red', '+', 'LineWidth',3);
text(xED, yED, ...
    sprintf('Earth Departure \n (x,y)=(%.3e,%.3e)km',xED,yED),...
    'HorizontalAlignment','right');
text(xMD, yMD, ...
    sprintf('Mars Departure \n (x,y)=(%.3e,%.3e)km',xMD,yMD), ...
    'HorizontalAlignment','right');
text(xEA, yEA, ...
    sprintf('Earth Arrival \n (x,y)=(%.3e,%.3e)km',xEA,yEA),...
    'HorizontalAlignment','left');
text(xMA,yMA, ...
    sprintf('Mars Arrival \n (x,y)=(%.3e,%.3e)km',xMA,yMA),...
    'HorizontalAlignment','left');
axis equal
grid on;
xlabel('x [km]');
ylabel('y [km]');
title('Earth Orbit, Mars Orbit, and Transfer Trajectory');
legend('Earth Orbit', 'Mars Orbit', 'Transfer Orbit', ...
    'Earth (Departure)', 'Mars (Departure)', ...
    'Earth (Arrival)', 'Mars (Arrival)', Location='eastoutside');
fontsize(fig2, 'scale', 1.7)

% Export graph
FileType = '.png';
SN2 = 'P3_2';
ST2 = [SN2, FileType];
FN2 = fullfile(ST2);
exportgraphics(fig2, FN2, 'Resolution', 300, 'BackgroundColor', 'white');

%% Functions

function [output,p] = lambert(a, rM, rE, Df, mu_s, i)
    if i == 1 || i == 0
        % ----- t1 -----
```

```
        % Step 1
        k = rM * rE * (1-cosd(Df));
        m = rM * rE * (1+cosd(Df));
        l = rE + rM;
        p = (2*a*k*l - k*m + k*sqrt(m*(8*a^2 - 4*a*l + m))) / (2*a*l^2 -
4*a*m);

        % Step 2
        F = 1 - (rM / p) * (1-cosd(Df));
        Fdot = sqrt(mu_s/p) * tand(Df/2) * ((1-cosd(Df))/p - (1/rE) -
(1/rM));
        G = (rM*rE) / sqrt(mu_s*p) * sind(Df);

        % Step 3
        CE = 1-((1-F)/a)*rE;
        SE = -(rE*rM*Fdot) / sqrt(mu_s*a);
        DE = atan2(real(SE),real(CE));

        % Step 4
        t1 = G + sqrt(a^3/mu_s) * (DE - SE);
        fprintf('p1 = %.3f km.\n', p);
        fprintf('\t 1: F = %.3f .\n', F);
        fprintf('\t 1: Fdot = %.8f .\n', Fdot);
        fprintf('\t 1: G = %.3f .\n', G);
        fprintf('\t 1: ΔE = %.3f rad .\n', DE);
        fprintf('\t Δt1 = %.3f days .\n', t1/(24*60*60));
    end
    if i == 2 || i == 0
        % ----- t2 -----
        % Step 1
        k = rM * rE * (1-cosd(Df));
        m = rM * rE * (1+cosd(Df));
        l = rE + rM;
        p = (2*a*k*l - k*m - k*sqrt(m*(8*a^2 - 4*a*l + m))) / (2*a*l^2 -
4*a*m);

        % Step 2
        F = 1 - (rM / p) * (1-cosd(Df));
        Fdot = sqrt(mu_s/p) * tand(Df/2) * ((1-cosd(Df)) / p - (1/rE) -
(1/rM));
        G = (rM*rE) / sqrt(mu_s*p) * sind(Df);

        % Step 3
        CE = 1-((1-F)/a)*rE;
        SE = -(rE*rM*Fdot) / sqrt(mu_s*a);
        DE = atan2(real(SE),real(CE));

        % Step 4
        t2 = G + sqrt(a^3/mu_s) * (DE - SE);
        fprintf('p2 = %.3f km.\n', p);
        fprintf('\t 2: F = %.3f .\n', F);
        fprintf('\t 2: Fdot = %.8f .\n', Fdot);
        fprintf('\t 2: G = %.3f .\n', G);
        fprintf('\t 2: ΔE = %.3f rad .\n', DE);
        fprintf('\t Δt2 = %.3f days .\n', t2/(24*60*60));
```

```
    end
    switch i
        case 0
            output = [t1,t2];
        case 1
            output = t1;
        case 2
            output = t2;
    end
    disp(' ')
end
```