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```
% Project 1
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clc; clear;

syms x y mu;

P1 = sqrt((x+mu)^2 + y^2);
P2 = sqrt((x-1+mu)^2 + y^2);

U = 0.5*(x^2 + y^2) + ((1-mu)/P1) + (mu/P2);

% Partial derivatives
U_x = diff(U, x);
U_xx = diff(U, x, 2);
U_xy = diff(U_x, y);
U_yy = diff(U, y, 2);

A = [0 0 1 0;
     0 0 0 1;
     U_xx U_xy 0 2;
     U_xy U_yy -2 0];

% Lists the values of mu for sun-earth, earth-moon, and saturn-titan respectively
Mu = [3.0039*(10^(-7)); 1.2151*(10^(-2)); 2.366*(10^(-4))];

% X and Y value pairs of the lagrange points for various CR3BP systems
L1_Points = [0.995363, 0;
             0.836915, 0;
             0.9575, 0];
L2_Points = [1.004637, 0;
             1.15568, 0;
             1.0425, 0];
L3_Points = [-1.00001, 0;
             -1.00506, 0;
             -1.0001, 0];
L4_Points = [0.5-Mu(1), sqrt(3)/2; % Sun-Earth system (system 1)
             0.5-Mu(2), sqrt(3)/2; % Earth-Moon system (system 2)
             0.5-Mu(3), sqrt(3)/2]; % Saturn-Titan System (system 3)

L5_Points = [0.5-Mu(1), -sqrt(3)/2; % Sun-Earth system
             0.5-Mu(2), -sqrt(3)/2; % Earth-Moon system
             0.5-Mu(3), -sqrt(3)/2]; % Saturn-Titan System
```

Calculating the eigenvalues

Step 1: Plug x & y values into matrix A Step 2: Compute eigenvalues

```
% L1 Points
fprintf("L1 Points: \n");
for i = 1: 3
    A_subbed = subs(A, [x y mu], [L1_Points(i,1),L1_Points(i,2), Mu(i)] );
```

```

eigenvalues = eig(A_subbed);
eigenvaluePairs = [real(eigenvalues), imag(eigenvalues)]; % Separates real and imaginary
fprintf('System %d: %f%+fi\t%f%+fi\t%f%+fi\t%f%+fi\n\n', i, eigenvaluePairs');
end

% L2 Points
fprintf("L2 Points: \n");
for i = 1: 3
    A_subbed = subs(A, [x y mu], [L2_Points(i,1),L2_Points(i,2), Mu(i)] );
    eigenvalues = eig(A_subbed);
    eigenvaluePairs = [real(eigenvalues), imag(eigenvalues)]; % Separates real and imaginary
    fprintf('System %d: %f%+fi\t%f%+fi\t%f%+fi\t%f%+fi\n\n', i, eigenvaluePairs');
end

% L3 Points
fprintf("L3 Points: \n");
for i = 1: 3
    A_subbed = subs(A, [x y mu], [L3_Points(i,1),L3_Points(i,2), Mu(i)] );
    eigenvalues = eig(A_subbed);
    eigenvaluePairs = [real(eigenvalues), imag(eigenvalues)]; % Separates real and imaginary
    fprintf('System %d: %f%+fi\t%f%+fi\t%f%+fi\t%f%+fi\n\n', i, eigenvaluePairs');
end

% L4 Points
fprintf("L4 Points: \n");
for i = 1: 3
    A_subbed = subs(A, [x y mu], [L4_Points(i,1),L4_Points(i,2), Mu(i)] );
    eigenvalues = eig(A_subbed);
    eigenvaluePairs = [real(eigenvalues), imag(eigenvalues)]; % Separates real and imaginary
    fprintf('System %d: %f%+fi\t%f%+fi\t%f%+fi\t%f%+fi\n\n', i, eigenvaluePairs');
end

% L5 Points
fprintf("L5 Points: \n");
for i = 1: 3
    A_subbed = subs(A, [x y mu], [L5_Points(i,1),L5_Points(i,2), Mu(i)] );
    eigenvalues = eig(A_subbed);
    eigenvaluePairs = [real(eigenvalues), imag(eigenvalues)];
    fprintf('System %d: %f%+fi\t%f%+fi\t%f%+fi\t%f%+fi\n\n', i, eigenvaluePairs');
end

```

L1 Points:

| | | | |
|------------------------------|--------------------|--------------------|---------------------|
| System 1: 0.000000+2.078287i | 2.519272+0.000000i | 0.000000-2.078287i | -2.519272+0.000000i |
|------------------------------|--------------------|--------------------|---------------------|

| | | | |
|------------------------------|---------------------|--------------------|--------------------|
| System 2: 0.000000-2.334416i | -2.932103+0.000000i | 0.000000+2.334416i | 2.932103+0.000000i |
|------------------------------|---------------------|--------------------|--------------------|

| | | | |
|------------------------------|--------------------|--------------------|---------------------|
| System 3: 0.000000+2.136985i | 2.615131+0.000000i | 0.000000-2.136985i | -2.615131+0.000000i |
|------------------------------|--------------------|--------------------|---------------------|

L2 Points:

| | | | |
|------------------------------|--------------------|--------------------|---------------------|
| System 1: 0.000000+2.071217i | 2.507668+0.000000i | 0.000000-2.071217i | -2.507668+0.000000i |
|------------------------------|--------------------|--------------------|---------------------|

| | | | |
|------------------------------|--------------------|--------------------|---------------------|
| System 2: 0.000000+1.862693i | 2.158754+0.000000i | 0.000000-1.862693i | -2.158754+0.000000i |
|------------------------------|--------------------|--------------------|---------------------|

| | | | |
|------------------------------|--------------------|--------------------|---------------------|
| System 3: 0.000000+2.050232i | 2.473148+0.000000i | 0.000000-2.050232i | -2.473148+0.000000i |
|------------------------------|--------------------|--------------------|---------------------|

L3 Points:

| | | | |
|------------------------------|--------------------|--------------------|--------------------|
| System 1: 0.000000+0.999971i | 0.000000+0.009385i | 0.000000-0.999971i | 0.000000-0.009385i |
|------------------------------|--------------------|--------------------|--------------------|

| | | | |
|--------------------------------|-----------------------|----------------------|-----------------------|
| System 2: $0.000000+1.010428i$ | $0.177949+0.000000i$ | $0.000000-1.010428i$ | $-0.177949+0.000000i$ |
| System 3: $0.000000-1.000203i$ | $-0.024662+0.000000i$ | $0.000000+1.000203i$ | $0.024662+0.000000i$ |
| L4 Points: | | | |
| System 1: $0.000000+0.999999i$ | $0.000000+0.001424i$ | $0.000000-0.999999i$ | $0.000000-0.001424i$ |
| System 2: $0.000000-0.954499i$ | $0.000000-0.298214i$ | $0.000000+0.954499i$ | $0.000000+0.298214i$ |
| System 3: $0.000000+0.999200i$ | $0.000000+0.039990i$ | $0.000000-0.999200i$ | $0.000000-0.039990i$ |
| L5 Points: | | | |
| System 1: $0.000000+0.999999i$ | $0.000000+0.001424i$ | $0.000000-0.999999i$ | $0.000000-0.001424i$ |
| System 2: $0.000000-0.954499i$ | $0.000000-0.298214i$ | $0.000000+0.954499i$ | $0.000000+0.298214i$ |
| System 3: $0.000000+0.999200i$ | $0.000000+0.039990i$ | $0.000000-0.999200i$ | $0.000000-0.039990i$ |