FPGA Development for the LHCb Vertex Locator Upgrade

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Abstract

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Contents

1 Scrambler

Due to radiation levels inside the detector chamber, the main data processing takes place in a concrete bunker away from the detector. To facilitate this, 20 optical linkes (per modual) are used to transfer the data from the front end VELO to the Data Aquizition FPGA (DAQ). When comunicating data digitaly, the transfering modual (TX) and the recieving modual (RX) must have syncrinised clocks. In these case, the (name form dataflow) is the TX, and the DAQ is teh RX. When achieving syncronised close, there are two main approunches:

I. Transmit the TX clock to the RX modual - used in I^2C and SPI communication.

II. Use bit-changes in the data to continuously synchronise the RX clock.

The former of these options, although the more convienient, is not appropriate for the VELO as it is suseptable to unforseens delays that could cause desyncronisation of the clocks to the data. The latter, while more invarient during delays, requires data with a high density of tranitions to reduce the likelyhood of a desyncronisation event. Becuase delays in the data are possible, the latter option has been selected.

1.1 The Role of Scrambling Data in the VELO

For the reasons described in Section 1, it is nessesary to ensure that the data has large density of transitions before being transmitted from the front-end detector to the DAQ modual. However, as the majority of super pixel hitmaps are empty, the data has a bais towards *'0'*s. This reduces the frequancy of transitions in the data - increasing the probability of a desyncronisation event. It is therefor nesseccary to scramble the data prior to transmition and descramble the data in the DAQ FPGA.

Scrambling and later descrambling the data is not a trivial exercise. The scrambleing (TX) modual and descrambling (RX) modual must use a sycronised *'key'*, that is used in both the scrambling and descrambling processes. In the FPGA, the *'key'* is derived from the previous states of the data. There are two methods when generating this *'key'*:

Additive The *'key'* is generated by evolving the previous *'key'* at each itteration of data using the incoming frame.

Multiplicative The *'key'* is generated from the previos n frames. (Here n is a variable specific to the algorithm).

1.2 Scrambler Options

Three scrambling algorithums have been concidered:

this section is out of date, but I dont have access to the uptoday section untill after I sent this. I have just not included it for now.

1.3 Cross Checks

Ofcourse, the main prioritys when scrambling data is ensuring that the data in recoverable. For all three scramblers, the algorithum was sysnthesised in Quartus[1] and simulated in Modelsim[2]. The aim of sysnthesising and simulating the scramblers in these programs was to ensure that the design was physical in term of on-board logic gates, and to check that the scrambled data was recoverable.

Furthermore, a C++ simulation was created for the three scramblers. This simulated had two purposes: firstly the output of the C++ can checked against the Modelsim simulated cehck consistancy; secondly to simulate the scrambler over a much larger simple of data as Modelsim simulations are less time effecient. In attition to the cross checks, the C++ code allowed for the injection of a desycronisation event. As expected, the additive scarmbler was unable to recover any data post-desycronisation, however the Intermediate and VeloPix scarmblers both recovered the *'key'* after two frames and returned to descrambling data.

1.4 Algorithm Analysis

One assumtion made is that fully scrambled data will be indistinguisable from randomly generated data. For this reason, the three algorithm are not only tested against eachother and the pre-scrambled data but also randomly generated binary. The randomly generated data was created using the Python *'random'* library, selecting a *'0'* or *'1'* with equal probubility. While the Python *'random'* library is only sudo-random, on the scale of this example (i.e. $>$ 100,000 frames), it is by far sufficient.

A more mathematically rigorous approuch, however, is to evaluate the system abstractly in the framework of statistical physics. In this abstraction, the 120 bit frame (with the header and parity removed) is concidered a ensemble; microstates are the particular form of the frames; and macroscopic quantities can be calculated by averaging a large number of frames (i.e. the desync data). For the analysis outlined in section 1.4.1, predictions will be made using these principles and outlined in section 1.4.2.

In the context of the statisical model, it is reasonable to concider the degree of *'scrambledness'* analogous to entropy. This analogy is not disimular to the common interpritation of entropy as a measure od dissorder. Therefor a scrambled system can be assumed to one of maximum entropy; and from Boltzmans law,

$$S \sim ln(\Omega) \tag{1.1}$$

where Ω is the number of microstates assosiated with the macrostate, we learn that this state of maximum entropy is a macrostate with the maximum number of assosiated microstates.

The entropic argument of Equation 1.1 is not only mathematical founded. For a scramble algorithum to hold for all possible data sets, it must also be capable of outputing all possible permutations. As such, assuming all possible output are equally likely, the count of each macroscopic output will be proportional to the number of microstates assosiated.

1.4.1 Messurements of the Algorithms

To compare the effecincy of the three algorithums in section 1.2, the algorithums where run over the same unput data and compared for the following measures:

Number of Transitions Per Frame
This meassure counts the total number of bit transitions (i.e. $bit(n) \neq bit(n-1)$) in a 120 bit frame. The header and parity information was not included as they are not scrambled. This is an important test as one of the roles of the scrambler is to maximise the number of transitions.

Common Bit Chain Length
One of the downfalls of the 'Number of Transitions Per Frame' analysis is that the two hypethetical 20 bit frames,

a) 10101010101111111111,

b) 10011001100110011001,

both with 10 transitions, are concidered equaly. However, (b) is clearly a more suitable output for data transfer as (a) has a large probability of desyncronisated due to the long chains of *'1'*s in the right most bits. It is therefore also nessecary to evaluate the length of common bit chains within the scrambled data as shorter chains are more suitable for data transfer.

Bit Asymetry
Pre-scramble, the data had a large bais towards *'0'*s due to the majority of the hitmaps being empty. Scrambled data, via entropic arguments, *should* show zero bias eitherway. Therefor, by investigating how the inbalance of *'1'*s and *'0'*s evolves over many frames, any bias in the scrambler can be found.

1.4.2 Statistical Predictions

Number of Transitions Per Frame

Consider a particle in a symmetric, descrete time-dependent, two state system,

$$p_0(t) = p_1(t) = 0.5, \quad : \quad \forall\ t \in \mathbb{N} \tag{1.2}$$

at each time itteration,

$$p_{i\to j}(t) = 0.5. \quad : \quad i, j = [0\ 1], \quad \forall\ t \in \mathbb{N} \tag{1.3}$$

However, assuming zero bias and detailed balance, as $p_{1\to 0}(t)$ is equal in both probalility and importance to $p_{0\to 1}(t)$, the probability of a bit change shall herefore be refered to as $p_t(t)$.

Over a n step process, analogous to a n bit frame, the probalility distribution of the number of transitions N_t is given by Binomial statistics,

$$f(N_t) = \frac{n!}{N_t!(n-N_t)!}\ p^{N_t}\ (1-p)^{n-N_t} \tag{1.4}$$

Simplified for the special case $p = p_t = 0.5$,

$$f_t(N_t) = \frac{n!}{N_t!(n-N_t)!}\ (p_t)^n \tag{1.5}$$

For $n = 120$, we can calulate,

$$< N_t >^{Binomial} = \sum_{N_t=0}^{n-1} N_t\ f(N_t) = n\ p_t = 60 \tag{1.6}$$

$$\sigma_{N_t}^{Binomial} = \sqrt{n\ p_t^2} = 5.48 \tag{1.7}$$

Furthermore, when concidering the entropic argument in section 1.4 equation 1.1, the number of microstates corespoding to each macrostate N_t can be related to equation 1.5,

$$\Omega_t = \binom{n}{N_t} = \frac{n!}{N_t!(n-N_t)!} \tag{1.8}$$

$$< N_t >^{Entropic} = MAX[S_t] = MAX[\Omega_t] \tag{1.9}$$

This can be numerically solved,

$$< N_t >^{Entropic} = 60 \tag{1.10}$$

The result of equation 1.10 is consistant with Equation **??**. This is an important as a *'sanity check'* as the descrepincy would indicate an issue in the theory.

Common Bit Chain Length

The probability of a chain of length n is,

$$p_n = p_1(1 - p_t)^{n-1}, \quad : \quad n \in \mathbb{N}, \quad n > 1 \tag{1.11}$$

where p_1 is the number of chains of lenght 1. As $p_1 = N_0(1 - p_t)$, where N_0 is the total number of chains,

$$\frac{N_n}{N_0} = (1 - p_t)^n, \quad : \quad n \in \mathbb{N}, \quad n > 1 \tag{1.12}$$

where N_n in the number of chains of length n. Takeing the log of both sides,

$$\begin{aligned} log\left(\frac{N_n}{N_0}\right) &= n\ log(1 - p_t), \\ log(N_n) &= n\ log(1 - p_t) + log(N_0). \end{aligned} \tag{1.13}$$

Therefor, for a graph of $log(N_n)$ against n for a large sample of data, the gradient would be $log(1 - p_t)$. In this case, as $p_t = 0.5$,

$$log(1 - p_t) = -0.30\ . \tag{1.14}$$

Bit Asymetry

$A_{1,0}$, the measure of assymetry of *'1'*s and *'0'*s is defined as,

$$A_{1,0} = N_1 - N_0, \tag{1.15}$$

where N_1 and N_0 are the number of *'1'*s and *'0'*s respectively. We can concider the evolution of $A_{1,0}$ with frame t of size n as a stockastic itterative map with zero deterministic growth [3],

$$A_{1,0}(nt + n\ \Delta t) = A_{1,0}(nt) + \mathcal{N}(nt) \tag{1.16}$$

Where $\mathcal{N}$ is an independant random variable picked from a gausian distribution. While $A_{1,0}(t) \in \mathbb{Z}$, in the limit of large nt we can approximate that $A_{1,0}$ is continious.

If we concider the moments of $A_{1,0}$,

$$< A_{1,0}(nt = M\ n\ \Delta t) > = \sum_{m=0}^{M-1} \mathcal{N}(m\ n\ \Delta t), \tag{1.17}$$

$$\begin{aligned} < A_{1,0}(nt = M\ n\ \Delta t)^2 > &= \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \mathcal{N}(m\ n\ \Delta t) \mathcal{N}(m'\ n\ \Delta t)\ \delta_{mm'} \\ &= \sum_{m=0}^{M-1} < \mathcal{N}(m\ n\ \Delta t)^2 > . \end{aligned} \tag{1.18}$$

Clearly, in Equation 1.17, $< A >= 0$. In Equation 1.18, we assume the variance is of form $(\Delta t)^\alpha$ [3]. Then,

$$< A_{1,0}(nt = M\ \Delta t)^2 > = M(n\ \Delta t)^\alpha. \tag{1.19}$$

Running the analysis over the frames $t = 0$ to t_f, the number of bits sampled is $M = t_f/n\ \Delta t$. Substituting this into Equation 1.19,

$$< A_{1,0}(nt = M\ n\ \Delta t)^2 > = t_f\ (n\ \Delta t)^{\alpha-1}. \tag{1.20}$$

Conciding the three cases of α in the approximation of continious $n\Delta t$:

- $\boldsymbol{\alpha > 1}$: Here $A_{1,0} \to 0$ as $n\ \Delta t \to 0$.
- $\boldsymbol{\alpha < 1}$: Here $A_{1,0} \to \infty$ as $n\ \Delta t \to 0$.
- $\boldsymbol{\alpha = 1}$: This is the only sensible choice.

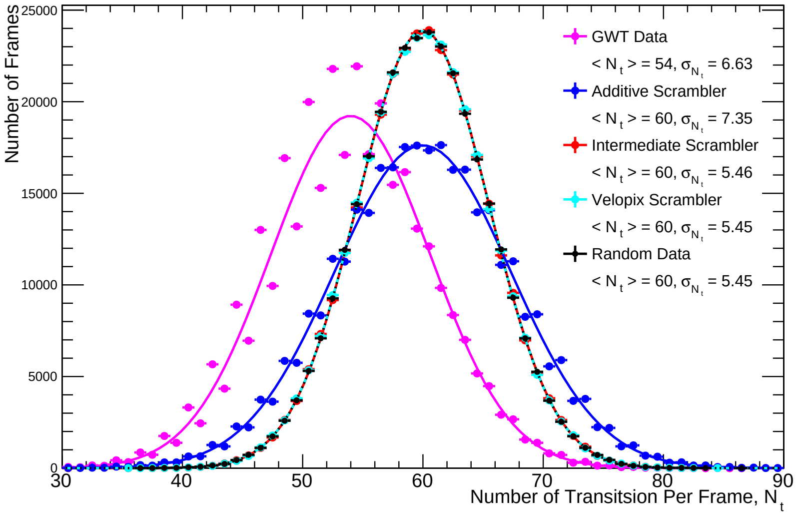
With $\alpha = 1$,

$$< A_{1,0}(nt = M\ n\ \Delta t)^2 >= M(n\ \Delta t). \tag{1.21}$$

And thus,

$$\sigma_{A_{1,0}} = \sqrt{< A_{1,0}^2 > - < A_{1,0} >^2} = \sqrt{< A_{1,0}^2 >} = \sqrt{n\ \Delta t}. \tag{1.22}$$

1.4.3 Results of Analysis



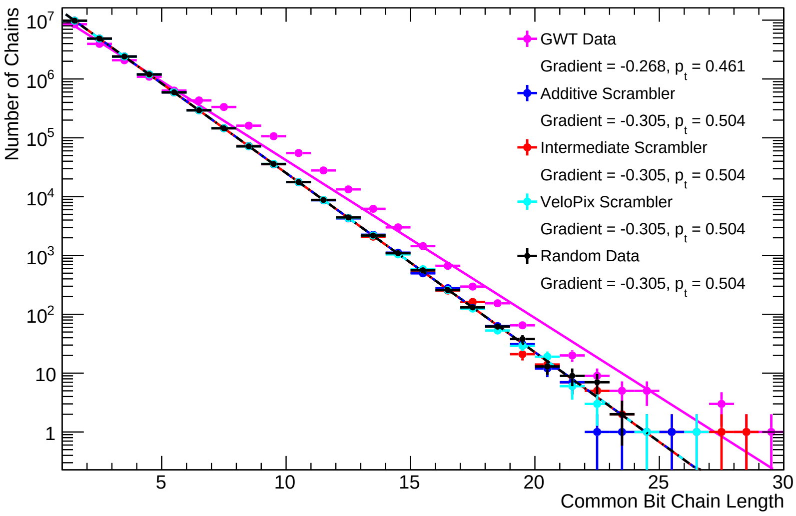


Figure 1.1: Results of the *'Number of Transitions Per Frame'* analysis (Top) and the *'Common Bit Chain Length'* analysis (Bottom). The results for the Random Data, Intermediate Scrambler and VeloPix Scrambler overlap for the *'Number of Transitions Per Frame'* analysis. The results for the Random Data, Additive Scrambler, Intermediate Scrambler and VeloPix Scrambler approximatly overlap for the *'Common Bit Chain Length'* analysis.

The results from the *'Number of Transitions Per Frame'* analysis, shown in Figure 1.1, show a strong corelation between the Intermediate and VeloPix Scramblers with the randomly generated data. These results are withing 1% agreement with the theoretical predictions for $< N_t >= 60$ and $\sigma_{N_t} = 5.48$, made in Section 1.4.2. The remarkable consistancy between the theoretical predictions and the randomly gernerated data provides confidance in both the theory, and the scrambled nature of the Intermediate and VeloPix scrambler outputs.

All three scramblers, the random data, and the theoretical predictions are all consistant to within 1%. Comparing the two results for the Additive Scrambler, its shown that

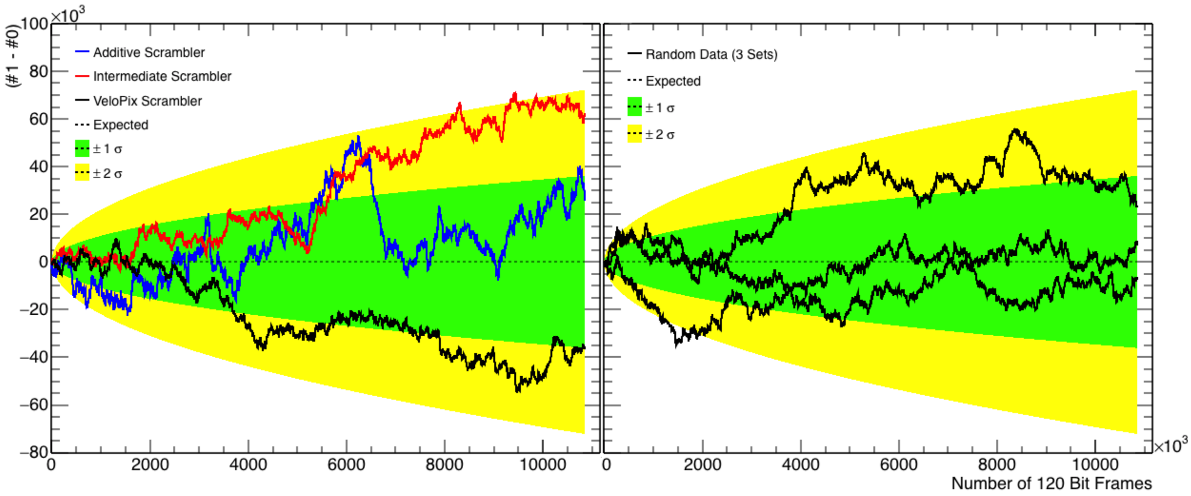


Figure 1.2: The results of the *'Bit Asymetry'* analysis.

while the frequancy of longer chains is consistant with random data; but as the variance of transitions is larger than predicted, the long and short trains are more localy clustered.

The *'Bit Asymetry'* of each scrambler, shown in Figure 1.2, is consistant with the theoretical prediction. The deviation of $A_{1,0}$ for the predicted mean of 0 is fully consistant with stockastic noise. The random data also shows consistancy. This gives confidance in the assumtpions made in Section 1.4.2.

One notible feature of Figure1.2 is the steap grandient of the additive scrambler a $t \sim 6.10^6$. However, as the data stays within the theoretical limits and the *'drop'* is of approximatly $\Delta A_{1,0} \sim 60.10^3$ over the range $n\ \Delta t \sim 1.2.10^8$ it would be difficult to construnct any argument claiming that this feature is of statistically significance

(I am tempted to run χ^2 analysis for a fit of y=0 so show that the data the data is consistant with the model, but am nut sure this will actually add to the argument?)

1.5 Conclusion

The consistancy of random data and the theoretical predictions justifies the assumptions and approximations made in Section 1.4 and Section 1.4.2. Furthermore, the conformation of the statistical model allows for accurate comparisons to be made form predicted values and their measured counterparts.

The Additive Scrambler, while consistant with the *'Chain Length'* and *'Bit Asymetry'* analysis, has a variance in the transition frequancy that leads the concultion that long and short chains are locally clusted. This is not ideal for data transfer. Many sequenchal long chains increase the probability of TX-RX clock desycronisation. Furthermore, the additive scrambler will not recover from this loss of syncronisation, as the *'key'* will never be recovered without a common reset signal.

	$< N_t >$	σ_{N_t}	Gradient	p_t
GQT data	54	6.63	-0.268	0.460
Additive Scrambler	60	7.35	-0.305	0.504
Intermediate Scrambler	60	5.45	-0.305	0.504
Velopix Scrambler	60	5.46	-0.305	0.504
Random Data	60	5.45	-0.305	0.504
Theoretical Prediction	60	5.48	-0.3	0.5

Table 1.1: The combined results of the algorithum analysis.

The Intermediate Scrambler produced an output consistant with random data. This makes the algorithm suitable of data transfer. As already mentioned[1], however, the scrambler is designed for computer simulated. As such, it is not suitable for implementation as it does not meet the additions requirments of the ASIC.

The VeloPix Scrambler, like the Intermediate Scrambler, produces a statistically scrambled output. Furthermore, the algorithum in inline with the additional requirments of the ASIC. As such, it ideal for implementation, and hense is currently the choice algorithm for use in the 2019 VELO upgrade.

[1]Note to Marco: this is in the scrabler options section

References

[1] Altera. *Quartus Prime Software*. 2015. URL: https://www.altera.com/products/design-software/fpga-design/quartus-prime/overview.html (visited on 12/2015).

[2] Mentor Graphics. *ModelSim - Leading Simulation and Debugging*. 2015. URL: https://www.mentor.com/products/fpga/model/ (visited on 12/2015).

[3] Kurt Jacobs. *Stochastic Processes for Physicists - Understanding Noisy Systems*. Cambridge University Press, 2010. ISBN: 9780521765428.