

FPGA Development for the LHCb Vertex Locator Upgrade

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Abstract

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1 Introduction

1.1 The Standard Model of Particle Physics

Central to the modern study of particle physics is the standard model,

$$\begin{aligned}
L_{GWL} = & \sum_f (\bar{\Psi}_f (i\gamma^\mu \partial_\mu - m_f) \Psi_f - e Q_f \bar{\Psi}_f \gamma^\mu \Psi_f A_\mu) + \frac{g}{\sqrt{2}} \sum_i (\bar{a}_L^i \gamma^\mu b_L^i W_\mu^+ + \bar{b}_L^i \gamma^\mu a_L^i W_\mu^-) \\
& + \frac{g}{2x_w} \sum_f \bar{\Psi}_f \gamma^\mu (I_f^3 - 2s_w^2 Q_f - I_6 e_f \gamma_5) \Psi_f Z_\mu - \frac{1}{4} |\partial_\mu A_v - \partial_v A_\mu - ie(W_\mu^- W_v^+ - W_\mu^+ W_v^-)|^2 \\
& - \frac{1}{2} |\partial_\mu W_v^+ - \partial_v W_\mu^+ - ie(W_\mu^+ A_v - W_v^+ A_\mu) + ig' c_w (W_\mu^+ Z_v - W_v^+ Z_\mu)|^2 \\
& - \frac{1}{4} |\partial_\mu Z_v - \partial_v Z_\mu + ig' c_w (W_\mu^- W_v^+ - W_\mu^+ W_v^-)|^2 - \frac{1}{2} M_\eta^2 \eta^2 - \frac{g M_\eta^2}{8 M_W} \eta^3 - \frac{g'^2 M_\eta^2}{32 M_W} \eta^4 \\
& + |M_W W_\mu^+ + \frac{g}{2} \eta W_\mu^+|^2 + \frac{1}{2} |\partial_\mu \eta + i M_Z Z_\mu + \frac{ig}{2c_w} \eta Z_\mu|^2 - \sum_f \frac{gm_f}{2M_W} \bar{\Psi}_f \Psi_f \eta. \quad (1.1)
\end{aligned}$$

The standard model, shown in equation 1.1, is a quantum field theory that describes the fundamental particles and how they interact. While this report does require, or attempt, a detailed understanding the intricate detail of the standard model; the aim of many particle physics experiments is to verify, measure and expand the model. Despite being the current best theory to explain particle interactions, the model is not complete. There are many undescribed phenomena, such as the matter domination in the universe, that require physics beyond the standard model in order to be described. To that end, major international efforts, namely in the form of the Large Hadron Collider, aim to gain further knowledge and understanding of the underlying physics of the universe. [1]

1.2 Field Programmable Gate Arrays

1.3 The LHCb Experiment

One experiment at the Large Hadron Collider is Large Hadron Collider beauty (LHCb). Located at intersection point 8, LHCb is designed to study rare particle physics phenomena, such as lepton flavour violation and CP violation. The decays studied in the LHCb are via exotic hadronic decays of Bottom or Charm quarks that form short lived hadrons. These hadrons, commonly B mesons, travel in the order of a few cm in the detector before decaying. As such, B meson decays can be identified by decay products that propagate via a secondary vertex.

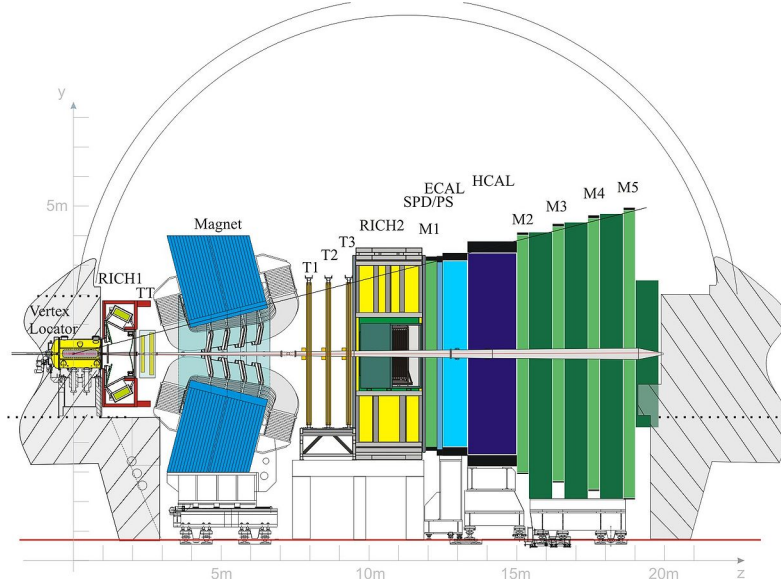


Figure 1.1: The LHCb Detector along the bending plane.

As B mesons are light (in comparison to other particles studied in the LHC), the decay products are produced at a shallow angle relative to the beam pipe; this is the driving factor in the design of the experiment. LHCb is a single arm forward spectrometer. Surrounding the point of collision is the Vertex Locator (VELO), this high precision detector uses silicon strips to detect ionising particles as they propagate from a collision and provides the coordinates of the particle in terms of R^1 and ϕ^2 . By reconstructing the paths of particles back to the intersection point, it can be identified whether or not the particular decay products are a product of the primary vertex³, or a secondary vertex⁴.

The Rich detector, comprised of two subdetectors either side of the magnet, uses Cherenkov radiation to deduce the velocity of the particle. The silicon trackers, labeled TT and T1-3 in Figure 1.1, calculate the angle deflection by the magnet. By combining the velocity and angle of deflection, the mass, momentum and energy of the particles can be deduced from simple relativistic kinematics.

The muon detectors, labeled M1-5 in Figure 1.1, are important to detect muons. This is of particular importance on LHCb as muons can be easily misidentified as charged pions, due to their similar mass.

HCAL and ECAL, shown in Figure 1.1, are hadronic and electromagnetic calorimeters respectively. Both measure the total energy of incoming particles. As the calorimeters are absorbing the particles they detect, any leptonic particle reaching the M2-5 muon detectors can be assumed to be a muon. Electrons and Photons are absorbed by the ECAL and any Tauons would have decayed long before reaching the far muon detectors.

¹Radial distance from the beam pipe.

²Azimuthal angle.

³The position at which the protons collided.

⁴The decay point of a short lived particle. i.e. B Meson.

1.4 LHCb Upgrade

With the advancements in accelerator technology, the detectors must also advance in order to make best use of the accelerators. The LHC is scheduled to increase its luminosity during Long Shutdown 3 (LS3), and as such LHCb will have to cope with this larger luminosity. The front end electronics of LHCb implement a hardware trigger and this is limited to a 1MHz maximum readout speed. Post LS3, LHCb will have to cope with a luminosity of $\mathcal{L} = 2.10^{33} \text{cm}^{-2} \text{s}^{-1}$, this is significantly greater than the current $\mathcal{L} = 4.10^{32} \text{cm}^{-2} \text{s}^{-1}$. A simple luminosity increase will not significantly increase the statistics for some statistical error dominated channels. To achieve this, greater resolution of the VELO and fully software triggers are required. The main goals of the 2019 upgrade are as follows:

- Increase the luminosity to $\mathcal{L} = 2.10^{33} \text{cm}^{-2} \text{s}^{-1}$.
- Read data from the detector at the bunch crossing frequency, 40 Mhz.
- Convert to a fully software based trigger.

1.4.1 VELO Upgrade

The main new feature of the upgraded VELO is the switch from silicon strips to pixels. The upgraded VELO features 12 ASIC chips with 128 x 128 [2].

1.4.2 The Role of FPGA's in the VELO Upgrade

1.4.3 Data Flow and Low Level Interface

2 Scrambler

Due to radiation levels inside the detector chamber, the main data processing takes place in a concrete bunker away from the detector. To facilitate this, 20 optical links (per module) are used to transfer the data from the front end VELO to the Data Acquisition FPGA (DAQ). When communicating data digitally, the transferring module (TX) and the receiving module (RX) must have synchronised clocks. In these cases, the GWT serialiser is the TX, and the DAQ is the RX. When achieving synchronised clock, there are two main approaches:

- Transmit the TX clock with the data to the RX module - used in I²C and SPI communication.
- Use bit-changes in the data to continuously synchronise the RX clock.

The former of these options, although widely used in conventional electronics, requires a finely tuned clock accounting for all possible delays. The latter, while negating cons of the former, requires data with a high density of transitions to reduce the likelihood of a desynchronisation event. Because delays in the data are possible, the latter option has been selected.

As it is necessary to ensure that the data has large density of transitions before being transmitted from the front-end detector to the DAQ module. However, as the majority of super pixel hitmaps are empty, the data has a bias towards '0's. This reduces the frequency of transitions in the data - increasing the probability of a desynchronisation event. It is therefore necessary to scramble the data prior to transmission and descramble the data in the DAQ FPGA.

Scrambling and later descrambling the data is not a trivial exercise. The scrambling (TX) module and descrambling (RX) module must use a synchronised 'key', that is used in both the scrambling and descrambling processes. In the FPGA, the 'key' is derived from the previous states of the data. There are two methods when generating this 'key':

Additive The 'key' is generated by evolving the previous 'key' at each iteration of data using the incoming frame.

Multiplicative The 'key' is generated from the previous n frames. (Here n is a variable specific to the algorithm).

2.1 Scrambler Options

Three scrambling algorithms have been considered:

Additive Scrambler

This scrambler was originally implemented and used two sets of two-input XOR logic gates. As the name implies, this scrambler used additive key generation which is dependent on all previous input frames since the last reset signal.

Intermediate Scrambler

Created by Karol Hennessy, and deriving its name arbitrarily from the order of consideration, this multiplicative scrambler combines the current and previous frames to generate the 'key'. Therefore, in the event of desynchronisation, only two frames are lost before the 'key' is automatically recovered. This feature alone is a significant improvement over the Additive Scrambler.

VeloPix Scrambler

This is the current implemented scramble algorithm in the DAQ and VeloPix code. Like the Intermediate Scrambler, it uses multiplicative 'key' generation. However, the VeloPix scrambler is compatible with further constraints enforced by the ASIC, including the number of combinational logic operations. The Intermediate Scrambler was designed purely for simulation purposes and as such does not meet these constraints.

2.2 Cross Checks

The main priority when scrambling data, is ensuring that the data is recoverable. For all three scramblers, the algorithm was synthesised in Quartus³ and simulated in Modelsim⁴. The aim of synthesising and simulating the scramblers in these programs was to ensure that the design was both physical in term of on-board logic gates, and to check that the scrambled data was recoverable, respectively.

Furthermore, a C++ simulation was created for the three scramblers. This simulation had two main purposes: firstly to cross check the output of the C++ against the Modelsim simulations; secondly to simulate the scrambler over a much larger simple of data as Modelsim simulations are less time efficient. In addition to the cross checks, the C++ code allowed for the injection of a desynchronization event, in which the ‘key’ is lost. As expected, the Additive Scrambler was unable to recover any data post desynchronization, however the intermediate and VeloPix scramblers both recovered the ‘key’ after two frames and continued to recover data.

2.3 Algorithm Analysis

For analytical purposes, it is assumed that fully scrambled data is indistinguishable from randomly generated data. For this reason, the three algorithms are not only tested against each other and the pre-scrambled QWT data but also randomly generated binary. The randomly generated data was created using the Python ‘random’ library, selecting a ‘0’ or ‘1’ with equal probability. While the Python ‘random’ library is only pseudo-random, on the scale of this example (i.e. $\gg 100,000$ frames), it is by far sufficient.

A more mathematically rigorous approach, however, is to evaluate the system abstractly in the framework of statistical physics. In this abstraction, the 120 bit frame (with the header and parity removed) is considered an ensemble; microstates are the particular form of the frames; and macroscopic quantities can be calculated by averaging a large number of frames (i.e. the desync data). For the analysis outlined in section 2.3.1, predictions will be made using these principles and outlined in section 2.3.2.

In the context of the statistical model, it is reasonable to consider the degree of ‘scrambledness’ analogous to entropy. This analogy is not dissimilar to the common interpretation of entropy as a measure of disorder.

$$S \sim \ln(\Omega) \tag{2.1}$$

where Ω is the number of microstates associated with the macrostate, we learn that this state of maximum entropy is a macrostate with the maximum number of associated microstates.

The entropic argument of Equation 2.1 is not only mathematically founded. For a scramble algorithm to hold for all possible data sets, it must also be capable of outputting all

possible permutations. As such, assuming all possible output are equally likely, the count of each macroscopic output will be proportional to the number of microstates associated.

2.3.1 Measurements of the Algorithms

To compare the effecincy of the three algorithms in section 2.1, the algorithms where run over the same unput data and compared for the following measures:

Number of Transitions Per Frame

This measure counts the total number of bit transitions (i.e. $bit(n) \neq bit(n-1)$) in a 120 bit frame. The header and parity information was not included as they are not scrambled. This is an important test as one of the roles of the scrambler is to maximise the number of transitions.

Common Bit Chain Length

One of the downfalls of the ‘Number of Transitions Per Frame’ analysis is that the two hypethetical 20 bit frames,

a) 10101010101111111111,

b) 10011001100110011001,

both with 10 transitions, are considered equaly. However, (b) is clearly a more suitable output for data transfer as (a) has a large probability of desynchronised due to the long chains of ‘1’s in the right most bits. It is therefore also nessecary to evaluate the length of common bit chains within the scrambled data as shorter chains are more suitable for data transfer.

Bit Asymetry

Pre-scramble, the data had a large bais towards ‘0’s due to the majority of the hitmaps being empty. Scrambled data, via entropic arguments, *should* show zero bias eitherway. Therefor, by investigating how the number of ‘1’s - ‘0’s evolves over many frames, any bias in the scrambler can be found.

2.3.2 Statistical Predictions

Number of Transitions Per Frame

Consider a particle in a symmetric, descrete time-dependent, two state system,

$$p_0(t) = p_1(t) = 0.5 \quad : \quad \forall t \in \mathbb{N}, \quad (2.2)$$

At each time itteration,

$$p_{i \rightarrow j}(t) = 0.5 \quad : \quad i, j = [0 \ 1], \quad \forall t \in \mathbb{N}. \quad (2.3)$$

However, assuming zero bias and detailed balance, as $p_{1 \rightarrow 0}(t)$ is equal in both probability and importance to $p_{0 \rightarrow 1}(t)$, the probability of a bit change shall herefore be referred to as $p_t(t)$.

Over a n step process, analogous to a n bit frame, the probability distribution of the number of transitions N_t is given by Binomial statistics,

$$f(N_t) = \frac{n!}{N_t!(n - N_t)!} p^{N_t} (1 - p)^{n - N_t} \quad (2.4)$$

Simplified for the special case $p = p_t = 0.5$,

$$f_t(N_t) = \frac{n!}{N_t!(n - N_t)!} (p_t)^n \quad (2.5)$$

For $n = 120$, we can calculate,

$$\langle N_t \rangle^{Binomial} = \sum_{N_t=0}^{n-1} N_t f(N_t) = n p_t = 60 \quad (2.6)$$

$$\sigma_{N_t}^{Binomial} = \sqrt{n p_t^2} = 5.48 \quad (2.7)$$

Furthermore, when considering the entropic argument in section 2.3 equation 2.1, the number of microstates corresponding to each macrostate N_t can be related to equation 2.5,

$$\Omega_t \sim \frac{n!}{N_t!(n - N_t)!} \quad (2.8)$$

$$\langle N_t \rangle^{Entropic} = MAX[S_t] = MAX[\Omega_t] \quad (2.9)$$

This can be numerically solved,

$$\langle N_t \rangle^{Entropic} = 60 \quad (2.10)$$

While the result of equation 2.10 does not contribute anything new, it is important as a ‘*sanity check*’. Because the system can be described as in section 2.3, it would indicate a problem in the theoretical framework if the result did not match.

Common Bit Chain Length

The probability of a chain of length n is,

$$p_n = p_1(1 - p_t)^{n-1}, \quad : \quad n \in \mathbb{N}, \quad n > 1 \quad (2.11)$$

where p_1 is the number of chains of length 1. As $p_1 = N_0(1 - p_t)$, where N_0 is the total number of chains,

$$\frac{N_n}{N_0} = (1 - p_t)^n, \quad : \quad n \in \mathbb{N}, \quad n > 1 \quad (2.12)$$

where N_n is the number of chains of length n . Taking the log of both sides,

$$\begin{aligned} \log \left(\frac{N_n}{N_0} \right) &= n \log(1 - p_t), \\ \log(N_n) &= n \log(1 - p_t) + \log(N_0). \end{aligned} \quad (2.13)$$

236 Therefor, for a graph of $\log(N_n)$ against n for a large sample of data, the gradient would be $\log(1 - p_t)$. In this case, as $p_t = 0.5$,

$$\log(1 - p_t) = -0.30. \quad (2.14)$$

238 Bit Asymetry

240 $A_{1,0}$, the assymetry of '1's and '0's is defined as,

$$A_{1,0} = N_1 - N_0, \quad (2.15)$$

242 where N_1 and N_0 are the number of '1's and '0's respectively. We can consider the evolution of $A_{1,0}$ with frame t of size n as a stockastic iterative map with zero deterministic growth [5],

$$A_{1,0}(nt + n \Delta t) = A_{1,0}(nt) + \mathcal{N}(nt) \quad (2.16)$$

244 Where \mathcal{N} is an independant random variable picked from a gaussian distribution. While $A_{1,0}(t) \in \mathbb{Z}$, in the limit of large nt we can approximate that $A_{1,0}$ is continious.

246 If we consider the moments of $A_{1,0}$,

$$\langle A_{1,0}(nt = M n \Delta t) \rangle = \sum_{m=0}^{M-1} \mathcal{N}(m n \Delta t), \quad (2.17)$$

$$\begin{aligned} \langle A_{1,0}(nt = M n \Delta t)^2 \rangle &= \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \mathcal{N}(m n \Delta t) \mathcal{N}(m' n \Delta t) \delta_{mm'} \\ &= \sum_{m=0}^{M-1} \langle \mathcal{N}(m n \Delta t)^2 \rangle. \end{aligned} \quad (2.18)$$

248 Clearly, in Equation 2.17, $\langle A_{1,0} \rangle = 0$. In Equation 2.18, we assume the variance is of form $(n \Delta t)^\alpha$ [5]. Then,

$$\langle A_{1,0}(nt = M n \Delta t)^2 \rangle = M(n \Delta t)^\alpha. \quad (2.19)$$

250

Running the analysis over the frames $t = 0$ to t_f , the number of bits sampled is $M = t_f/n \Delta t$. Substituting this into Equation 2.19,

$$\langle A_{1,0}(nt = M n \Delta t)^2 \rangle = t_f (n \Delta t)^{\alpha-1}. \quad (2.20)$$

Concidering the three cases of α in the approximation of continious $n\Delta t$:

252

• $\alpha > 1$: Here $A_{1,0} \rightarrow 0$ as $\Delta t \rightarrow 0$.

• $\alpha < 1$: Here $A_{1,0} \rightarrow \infty$ as $\Delta t \rightarrow 0$.

254

• $\alpha = 1$: This is the only sensible choice.

With $\alpha = 1$,

$$\langle A_{1,0}(nt = M n \Delta t)^2 \rangle = M(n \Delta t). \quad (2.21)$$

256

And thus,

$$\sigma_{A_{1,0}} = \sqrt{\langle A_{1,0}^2 \rangle - \langle A_{1,0} \rangle^2} = \sqrt{\langle A_{1,0}^2 \rangle} = \sqrt{n \Delta t}. \quad (2.22)$$

2.3.3 Results of Analysis

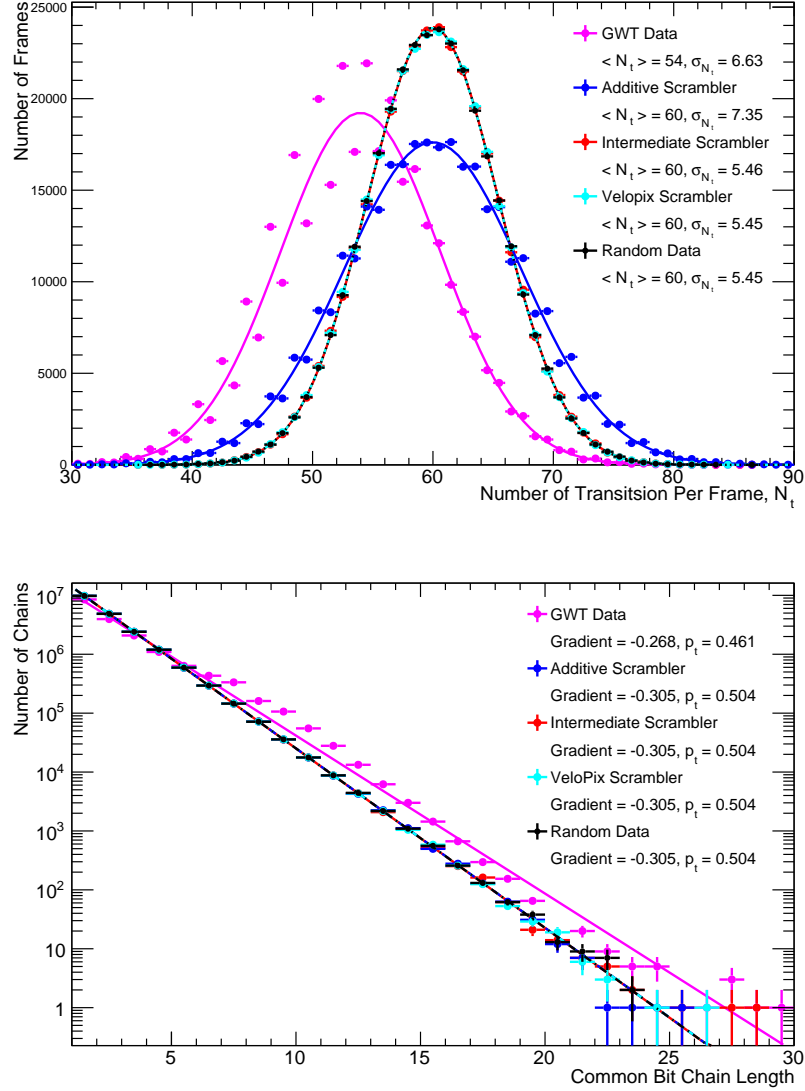


Figure 2.1: Results of the ‘*Number of Transitions Per Frame*’ analysis (Top) and the ‘*Common Bit Chain Length*’ analysis (Bottom). The results for the Random Data, Intermediate Scrambler and VeloPix Scrambler overlap for the ‘*Number of Transitions Per Frame*’ analysis. The results for the Random Data, Additive Scrambler, Intermediate Scrambler and VeloPix Scrambler approximately overlap for the ‘*Common Bit Chain Length*’ analysis.

The results from the ‘*Number of Transitions Per Frame*’ analysis, shown in Figure 2.1, show a strong correlation between the Intermediate and VeloPix Scramblers with the randomly generated data. These results are within 1% agreement with the theoretical predictions for $\langle N_t \rangle = 60$ and $\sigma_{N_t} = 5.48$, made in Section 2.3.2. The remarkable consistency between the theoretical predictions and the randomly generated data provides confidence in both the theory, and the scrambled nature of the Intermediate and VeloPix scrambler outputs.

All three scramblers, the random data, and the theoretical predictions are all consistent to within 1%. Comparing the two results for the Additive Scrambler, it is shown that while the frequency of longer chains is consistent with random data; but as the variance of transitions is larger than predicted, the long and short trains are more locally clustered.

The ‘*Bit Asymmetry*’ of each scrambler, shown in Figure 2.2, is consistent with the theoretical prediction. The deviation of $A_{1,0}$ for the predicted mean of 0 is fully consistent with stochastic noise. The random data also shows consistency. This gives confidence in the assumptions made in Section 2.3.2.

One notable feature of Figure 2.2 is the steep gradient of the additive scrambler at $t \sim 6 \cdot 10^6$. However, as the data stays within the theoretical limits and the ‘*drop*’ is of approximately $\Delta A_{1,0} \sim 60 \cdot 10^3$ over the range $n \Delta t \sim 1.2 \cdot 10^8$ it would be difficult to construct any argument claiming that this feature is of statistical significance.

(I am tempted to run χ^2 analysis for a fit of $y=0$ so show that the data the data is consistent with the model, but am not sure this will actually add to the argument?)

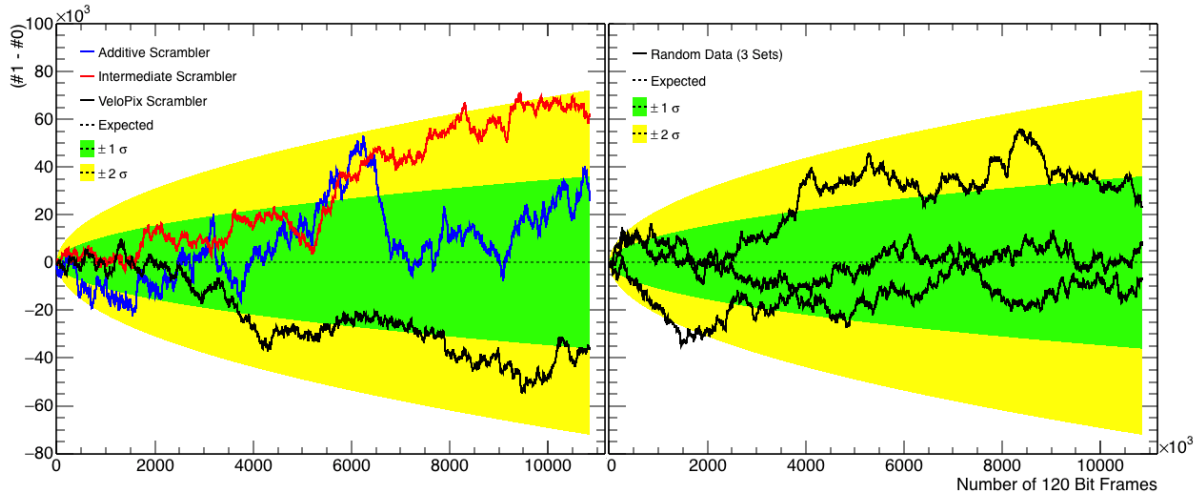


Figure 2.2: The results of the ‘*Bit Asymetry*’ analysis.

2.4 Conclusion

	$\langle N_t \rangle$	σ_{N_t}	Gradient	p_t
GQT data	54	6.63	-0.268	0.460
Additive Scrambler	60	7.35	-0.305	0.504
Intermediate Scrambler	60	5.45	-0.305	0.504
Velopix Scrambler	60	5.46	-0.305	0.504
Random Data	60	5.45	-0.305	0.504
Theoretical Prediction	60	5.48	-0.3	0.5

Table 2.1: The combined results of the algorithm analysis.

The consistency of random data and the theoretical predictions justifies the assumptions and approximations made in Section 2.3 and Section 2.3.2. Furthermore, the conformation of the statistical model allows for accurate comparisons to be made from predicted values and their measured counterparts.

The Additive Scrambler, while consistent with the ‘*Chain Length*’ and ‘*Bit Asymetry*’ analysis, has a variance in the transition frequency that leads to the conclusion that long and short chains are locally clustered. This is not ideal for data transfer. Many sequential long chains increase the probability of TX-RX clock desynchronisation. Furthermore, the additive scrambler will not recover from this loss of synchronisation, as the ‘*key*’ will never be recovered without a common reset signal.

The Intermediate Scrambler produced an output consistent with random data. This

292 makes the algorithm suitable of data transfer. As already mentioned⁵, however, the
scrambler is designed for computer simulated. As such, it is not suitable for implemen-
294 tation as it does not meet the additions requirments of the ASIC.

The VeloPix Scrambler, like the Intermediate Scrambler, produces a statistically scram-
296 bled output. Furthermore, the algorithm in inline with the additional requirments of the
ASIC. As such, it ideal for implementation, and hense is currently the choice algorithm
298 for use in the 2019 VELO upgrade.

⁵Note to Marco: this is in the scrabler options section

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