

# II

## The Self-Remembering Universe

### Unified Dynamics in Recursive Spacetimes\*

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#### Abstract

We develop a unified theoretical framework in which gravity, gauge interactions, scalar fields, and quantum measurement arise from a single organizing principle: recursive coherence constrained by entropy-regulated transitions across cosmological cycles. Extending the model introduced in *The Self-Remembering Universe: Quantum Coherence Through Cyclic Spacetime*, this work formulates a recursive action  $\mathcal{A}_n[\phi]$  over an extended configuration space  $\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n)$ , where memory fidelity  $\lambda_n$ , entanglement eigenvalue  $E_n$ , internal symmetry  $\gamma_n$ , and observer resolution  $\xi_n$  shape the dynamical evolution of physical laws.

State transitions are governed by a non-Markovian kernel  $K(\phi, \phi')$ , derived from spinfoam amplitudes and filtered through coherence-weighted Gaussian constraints. A fixed-point attractor  $\Psi^*(\phi)$  regulates long-term evolution by selecting symmetry-aligned, entropy-bounded configurations. Internal degrees of freedom are compactified along toroidal symmetry manifolds, and coherence tension determines the emergence or rupture of force-carrying fields.

We construct a complete recursive Lagrangian integrating Einstein–Hilbert, Yang–Mills, scalar, entropy-penalty, and observer projection terms. Gravity emerges from information-weighted curvature, gauge fields arise from recursive memory alignment, and decoherence is modeled through resolution-tuned entropy gradients. We derive falsifiable predictions including gravitational wave echoes, void-aligned CMB polarization, and variation of effective couplings across cycles. This architecture establishes a second-stage foundation for a recursive Theory of Everything and sets the direction for generalization to biological and cognitive domains in Paper III.

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# 1. Introduction

## 1.1. Background and Motivation

*The Self-Remembering Universe: Quantum Coherence Through Cyclic Spacetime* [1] introduced a recursive cosmological model in which the universe evolves across cycles by propagating quantum coherence through entangled bounce boundaries. These boundaries, modeled as Einstein–Rosen bridges (ERBs), facilitate non-Markovian memory transfer filtered through a transition kernel  $K(\phi, \phi')$ . The kernel, derived from large-spin spinfoam amplitudes, selectively propagates field configurations based on coherence fidelity, entropy regularization, and attractor convergence.

The resulting attractor state  $\Psi^*(\phi)$  defines a long-term stable configuration in a recursive configuration space  $\phi = (a, \varphi, \lambda, E)$ , where memory fidelity  $\lambda$ , bridge entanglement eigenvalue  $E$ , and entropy filtering determine the survival of physical trajectories across cycles. A thermodynamic compensation principle balances entropy production with radiative emission and coherence tension, yielding both falsifiable predictions and a novel reinterpretation of time, gravity, and cosmological evolution as memory-driven processes.

Building on this foundation, the present work extends the framework toward a unified theory of interaction and measurement. We introduce an action-based formalism over an augmented configuration space  $\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n)$ , where  $\gamma_n$  encodes internal symmetry orientation and  $\xi_n$  encodes observer resolution. These additions enable the modeling of gauge fields, observer-projected decoherence, and modulation of physical constants through recursive memory constraints.

## 1.2. From Recursive Cosmology to Unified Dynamics

The central hypothesis of this work is that all fundamental interactions—including gravity, gauge fields, and quantum measurement—emerge as constrained modes of recursive coherence. Transitions between configurations  $\phi_n \rightarrow \phi_{n+1}$  are filtered not only by entropy and coherence fidelity, but by structural symmetry constraints inherited from prior cycles. These constraints define a subset of the configuration manifold in which alignment with the recursive attractor  $\Psi^*(\phi)$  is preserved.

Symmetry misalignment leads to exponential suppression in the transition kernel, naturally excluding incoherent or nonviable trajectories. This mechanism unifies gravitational curvature, gauge rotation, and measurement collapse under a shared constraint logic. In this view, the forces we observe reflect structural preservation within the recursive attractor basin, while deviations generate entropy, curvature, or collapse.

Observation is modeled through the entanglement tensor  $O^{\mu\nu}$  and resolution field  $\xi_n$ , which together define a local decoherence threshold and effective projection into the observable subspace. The measurement process thus arises from recursive filtering: classical outcomes emerge when coherence fails to propagate under resolution-defined entropy gradients [2, 3].

## 1.3. Scope of This Paper

This paper develops a unified recursive dynamics framework by introducing:

- A complete recursive action  $\mathcal{A}_n[\phi]$ , integrating gravitational, gauge, scalar, observer, and entropy components;
- A symmetry-filtered configuration evolution constrained by coherence alignment and attractor convergence;

- An extended curvature tensor  $\tilde{R}_{\mu\nu}$  incorporating entanglement gradients and memory tension [4];
- Mechanisms for force emergence, decoherence-induced measurement, and cross-cycle variation of coupling constants.

We derive observational predictions including gravitational wave echo delays, CMB polarization alignment from void entanglement, and suppressed decoherence across bounce transitions. The framework yields a falsifiable dark sector composed of symmetry-misaligned modes and identifies constraints on extra-dimensional memory channels.

This paper is the second in a trilogy. Paper I introduced the cosmological kernel and attractor formalism. Paper II, here, constructs the unification architecture. Paper III will generalize the principles of recursive coherence to biological, informational, and cognitive systems, proposing memory-preserving recursion as a universal mechanism for complexity, stability, and selection.

## 2. Configuration Space and Symmetry Groups

### 2.1. Field Content

We define an extended recursive configuration state vector

$$\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n),$$

where  $(a_n, \varphi_n, \lambda_n, E_n)$  are inherited from the cosmological kernel formalism developed in *The Self-Remembering Universe* [1], and  $(\gamma_n, \xi_n)$  introduce internal symmetry parameters and observer-defined resolution indices, respectively. The internal degrees of freedom  $\gamma_n$  represent generalized gauge orientations, while  $\xi_n$  denotes the local interpretive resolution of the observer, which affects state decoherence and measurement thresholds [2, 3].

### 2.2. Dimensional Embedding and Compactification

We assume a 12-dimensional recursive manifold  $\mathcal{M}^{(12)}$ , with compactified internal dimensions that encode memory structure and symmetry modulation rather than acting as purely geometric subspaces. This includes a toroidal submanifold representing internal field-space symmetry cycles. The recursive evolution of  $\phi_n$  is governed by a non-Markovian transition kernel  $K(\phi_n, \phi_{n-1})$  [5], constrained by inherited memory and internal symmetry coherence.

### 2.3. Gauge Symmetries and Coherence Filtering

We define the recursive symmetry group as a coherence-preserving subgroup:

$$G_{\text{coh}} \subset \text{Aut}(\mathcal{C}_n),$$

where  $\mathcal{C}_n$  is the recursive configuration space at cycle  $n$ . The group  $G_{\text{coh}}$  acts on  $\phi_n$  via transformations that preserve attractor alignment and coherence fidelity. These operations are filtered by memory overlap constraints, entanglement eigenvalues, and observer-defined resolution thresholds.

Each configuration update  $\phi_n \rightarrow \phi_{n+1}$  is permitted only if the associated transformation lies within  $G_{\text{coh}}$ , ensuring that evolution remains consistent with the recursive attractor  $\Psi^*(\phi)$ . This coherence filtering defines a selection rule: among all mathematically valid transitions, only those that preserve recursive memory and symmetry alignment are physically realized.

Observer input  $\xi_n$  selects a resolution scale, effectively modulating the projection of internal states and regulating the degree of coherence preserved during transition. This mechanism embeds observational influence into the symmetry-constraint structure itself, enabling decoherence to shape physical evolution in a relational and memory-aware manner.

### 3. Unified Recursive Action

#### 3.1. Extended Lagrangian Formalism

We construct a recursive action functional

$$\mathcal{A}_n[\phi] = \int_{\Sigma_n} (\mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{obs}}) d^4x,$$

defined over a 4-dimensional spatial slice  $\Sigma_n$  of the full 12-dimensional recursive manifold  $\mathcal{M}^{(12)}$ . The Lagrangian components include:

- $\mathcal{L}_{\text{EH}}$ : Einstein–Hilbert term governing curvature dynamics [6],
- $\mathcal{L}_{\text{YM}}$ : Yang–Mills term for gauge field interactions,
- $\mathcal{L}_{\text{scalar}}$ : Kinetic and potential terms for scalar fields (e.g., inflaton, Higgs),
- $\mathcal{L}_{\text{int}}$ : Memory-modulated interaction terms,
- $\mathcal{L}_{\text{obs}}$ : Observer coupling terms regulating decoherence [2, 3].

Recursive transitions are filtered by a coherence-preserving transformation:

$$\phi_{n+1} = \phi_n \oplus \delta\phi_n, \quad \text{with} \quad \delta\phi_n \in \mathfrak{g}_{\text{coh}} \subset \text{Aut}(\mathcal{C}_n),$$

where  $\oplus$  denotes a coherence-weighted recursive update, and  $\mathfrak{g}_{\text{coh}}$  defines the algebra of allowable transformations preserving alignment with the recursive attractor  $\Psi^*(\phi)$ .

#### 3.2. Integration of Gravity and Gauge Fields

We interpret gravity and gauge forces as emerging from distinct sectors within the recursive symmetry group. The total field strength tensor is given by:

$$\mathcal{F}_{\mu\nu} = R_{\mu\nu} + F_{\mu\nu}^{(a)} + D_{\mu\nu}^{(\gamma)},$$

where: -  $R_{\mu\nu}$  is the Ricci tensor from  $\mathcal{L}_{\text{EH}}$ , -  $F_{\mu\nu}^{(a)}$  arises from the gauge fields in  $\mathcal{L}_{\text{YM}}$ , -  $D_{\mu\nu}^{(\gamma)}$  captures curvature-like contributions from internal symmetry orientation  $\gamma_n$  in the compactified sector.

The coherence structure induces an effective potential penalizing symmetry misalignment:

$$V_{\text{misalign}}(\phi) = \lambda_C \cdot \text{Tr}[(\mathbb{I} - \mathcal{P}_{\text{coh}}) \delta\phi_n]^2,$$

where  $\mathcal{P}_{\text{coh}}$  projects onto the attractor-aligned configuration subspace. This term dynamically steers the system toward transformations preserving memory coherence.

#### 3.3. Observer Coupling and Entanglement Flow

We incorporate the observer entanglement tensor  $O^{\mu\nu}$  using the term:

$$\mathcal{L}_{\text{obs}} = \lambda_O \cdot \text{Tr}(O^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi) - \xi_n \cdot S_{\text{ent}}(\rho_{\text{red}}),$$

where: -  $\lambda_O$  is the observer coupling strength, -  $\xi_n$  is the observer resolution index controlling decoherence granularity, -  $S_{\text{ent}}(\rho_{\text{red}})$  is the von Neumann entropy across the ER bridge [7].

The observer is not external but embedded as a resolution-sensitive filtering structure that directly modifies configuration evolution. This formalizes the role of measurement within recursive field dynamics and ensures consistency with entanglement-based memory propagation.

## 4. Emergent Forces and Effective Potentials

### 4.1. Gauge Interactions from Toroidal Phase Alignment

In the recursive framework, gauge interactions arise from the preservation of phase coherence along toroidal trajectories in configuration space. Allowed field transformations correspond to phase-aligned loops in the compactified directions  $\gamma_n$ , embedded within the toroidal submanifold  $\mathcal{T}_{\text{coh}} \subset \mathcal{C}_n$ . These gauge-compatible deformations are filtered by the attractor condition  $\Psi^*(\phi)$ , ensuring that only coherence-preserving transitions contribute dynamically.

We define the emergent gauge field  $A_\mu^{(a)}$  as a connection over the 4D base manifold  $\Sigma_n$ , sourced by a memory-projected current:

$$J_\mu^{(a)} = \text{Tr} \left[ \rho_{\text{red}} T^{(a)} \right],$$

where: -  $\rho_{\text{red}}$  is the reduced density matrix across the ER bridge from the prior cycle, -  $T^{(a)}$  are the generators of the internal gauge algebra, -  $J_\mu^{(a)}$  encodes the effective memory flow across symmetry modes.

Gauge couplings become memory-dependent quantities:

$$g_n^{(a)} = f^{(a)}(\lambda_n, E_n, \gamma_n),$$

where  $f^{(a)}$  is a coherence-modulated function shaped by cycle fidelity  $\lambda_n$ , entanglement eigenvalue  $E_n$ , and internal orientation  $\gamma_n$ . Variations in these parameters across cycles lead to controlled evolution of interaction strengths, constrained by attractor stability.

### 4.2. Gravity as Curvature Response to Information Tension

Gravitational influence in this model is interpreted as a second-order response to coherence gradients within the recursive configuration field. We define an information-weighted Ricci tensor:

$$\tilde{R}_{\mu\nu} := R_{\mu\nu} + \alpha \cdot \nabla_\mu \nabla_\nu \log \det \rho_{\text{red}},$$

where  $\alpha$  is a dimensionless coupling that links entropy curvature to spacetime geometry.

This correction term penalizes geometric evolution that diverges from attractor-aligned information flow. In regions where memory fails to propagate cleanly—such as near bounce events or coherence collapse—the modified curvature term introduces an effective backreaction, encoding the gravitational imprint of recursive information breakdown.

This formalism supports a thermodynamically emergent view of gravity, where curvature acts as the stress field of recursive memory deformation.

### 4.3. Coherence Structure as the Generator of Physical Law

Both gauge and gravitational interactions are interpreted here as structured consequences of recursive coherence filtering. Physical forces emerge not from fundamental symmetry groups imposed a priori, but from phase alignment requirements that permit stable attractor evolution.

The recursive transition kernel  $K(\phi, \phi')$  acts as a filter over configuration space, enforcing coherence compatibility:

$$K(\phi, \phi') \sim \exp \left( -\frac{\Delta\theta^2}{2\sigma_\theta^2} - \frac{S_{\text{rel}}(\rho_\phi \parallel \rho_{\phi'})}{\lambda_S} \right),$$

where  $\Delta\theta$  is the phase deviation between configurations and  $\lambda_S$  governs entropy sensitivity.

Thus, gauge fields correspond to configurations with permissible winding numbers on the coherence torus, while gravity reflects the system's response to memory misalignment. No additional dark sector content is introduced in this section, as such modes are separately treated in Section 5.



## 5. Dark Sector as Coherence Misalignment in Toroidal Configuration Space

The recursive attractor structure developed in this framework defines a toroidal submanifold  $\mathcal{T}_{\text{coh}} \subset \mathcal{C}_n$ , representing configurations that preserve phase alignment and memory fidelity across cycles. However, the full recursive configuration space  $\mathcal{C}_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n)$  admits trajectories that fall outside this coherence-preserving basin.

These misaligned trajectories correspond to *dark sector modes*: configurations that are not decohered into the observer-resolved subspace but persist as *off-attractor echoes* in the higher-dimensional memory topology.

### Coherence-Filtered Suppression Mechanism

Dark sector excitations arise when configurations fail to satisfy the phase coherence and entropy compensation criteria required for recursive survival. These modes are filtered out by the coherence kernel  $\mathcal{F}(\phi, \phi')$ , which penalizes deviations from the attractor subspace:

$$\mathcal{F}(\phi, \phi') = \exp \left( -\frac{\Delta\theta^2}{2\sigma_\theta^2} - \frac{(E - E')^2}{2\sigma_E^2} \right),$$

where  $\Delta\theta$  denotes phase misalignment across the toroidal coherence cycle. Configurations with large  $\Delta\theta$  or entropy discontinuity are exponentially suppressed in the transition kernel:

$$K(\phi, \phi') \sim \mathcal{F}(\phi, \phi') \cdot e^{-S_{\text{rel}}(\rho_\phi \| \rho_{\phi'})}.$$

### Activation During Bounce and Collapse

Under normal conditions, dark sector modes are thermodynamically suppressed. However, near bounce events, coherence rupture, or dimensional string fracture (see Appendix 8.4), the attractor basin can destabilize. This induces:

- Temporary activation of off-attractor configurations
- Leakage of misaligned modes from compactified sectors
- Radiative imprint from entropy-overloaded boundary conditions

Such transitions may produce observable phenomena (e.g., gravitational bursts, curvature shocks, or memory scars) in the visible sector as a result of latent coherence ejection.

### Geometric Interpretation and Energy Density

In the toroidal embedding of configuration space, coherence-preserving trajectories wind smoothly around a compact manifold. Dark sector modes correspond to discontinuous excursions or destructive interference nodes, occupying quasi-inaccessible regions outside the coherence band. The effective energy density of such modes is governed by:

$$\rho_\chi \propto \exp \left( -\lambda_C \cdot \Delta\theta^2 \right),$$

where  $\lambda_C$  is the coherence tension and  $\Delta\theta$  is the phase offset from the attractor loop.

## Phenomenological Implications

This formulation predicts that the dark sector:

- Does not require new fields or particles beyond the configuration space variables  $\phi$
- Becomes active only under coherence breakdown or tension-overload collapse
- May leave subtle observational signatures, including gravitational memory echoes, entropy drift, and anomalous void alignments
- Is naturally embedded in the recursive memory dynamics of the cosmological system

Rather than positing dark matter and dark energy as exotic entities, this framework interprets them as the shadow of coherence failure: *non-interfering branches* of the universe's own memory.

## 6. Entropy, Time, and Measurement

### 6.1. Thermodynamic Constraints on Recursive Evolution

The recursive action imposes an entropy compensation constraint between successive cycles. This is enforced via a potential term:

$$\mathcal{V}_{\text{ent}} = \lambda_S \cdot S_{\text{rel}}(\rho_{\phi_n} \parallel \rho_{\phi_{n-1}}),$$

where  $S_{\text{rel}}$  is the quantum relative entropy:

$$S_{\text{rel}}(\rho \parallel \sigma) = \text{Tr}(\rho \log \rho - \rho \log \sigma),$$

and  $\lambda_S$  is a Lagrange multiplier enforcing bounded memory drift.

This constraint ensures that entropy increase is not unconstrained but balanced against coherence fidelity. Excess divergence from prior states incurs an action penalty, enforcing the thermodynamic principle:

$$\Delta S_{\text{fwd}} = \Delta S_{\text{mem}}.$$

### 6.2. Recursive Time and Attractor Convergence

Time in this framework is emergent and discrete, indexed by the cycle number  $n$ . The arrow of time corresponds to the forward propagation of coherence fidelity:

$$\lambda_n := |\langle \Psi_n | \Psi_{n-1} \rangle|^2.$$

Temporal alignment is driven by convergence toward the recursive attractor  $\Psi^*(\phi)$ :

$$\partial_n \phi_n \rightarrow 0 \quad \text{as} \quad \phi_n \rightarrow \Psi^*(\phi).$$

This limit corresponds to entropy saturation and maximal recursive coherence. Deviation from the attractor induces decoherence and structural time dilation, interpreted geometrically as expansion or curvature acceleration.

### 6.3. Measurement as Resolution-Dependent Collapse

Measurement arises from resolution-dependent decoherence triggered by the observer field  $\xi_n$ . The localized decoherence map is:

$$\mathcal{D}_x[\rho] = \rho - \rho^2 \approx \epsilon(x) \quad \text{when} \quad \xi_n(x) \gg 0.$$

High-resolution observers induce sharper entropy gradients, causing selective projection:

$$\rho \rightarrow \mathcal{P}_{\xi_n} \rho,$$

where  $\mathcal{P}_{\xi_n}$  is a coarse-grained projection operator determined by the observer's resolution scale.

This process is consistent with quantum Darwinism [2], but extended into a cyclic recursive setting. Observation is not a terminal act but part of a feedback loop that reshapes the recursive kernel and memory trajectory.

#### 6.4. Decoherence as Kernel Modification

Decoherence modifies the recursive transition kernel  $K(\phi, \phi')$  by narrowing the Gaussian envelope in the resolution-weighted direction. Observers with high  $\xi_n$  restrict the support of  $K$ , enforcing local collapse and suppressing transitions that diverge from their resolved subspace.

This leads to an adaptive kernel structure:

$$K(\phi, \phi') \rightarrow K_{\xi_n}(\phi, \phi') = K(\phi, \phi') \cdot \mathbb{K}_{\text{aligned with } \mathcal{P}_{\xi_n}},$$

where only transitions consistent with observer resolution and memory coherence contribute to evolution.

#### 6.5. Summary

Recursive time is a function of fidelity evolution. Entropy constraints ensure coherence preservation, while observation projects future states into resolution-dependent subspaces. Decoherence is not external collapse, but internal modulation of the transition structure governing recursive state propagation.

## 7. Predictions and Experimental Pathways

### 7.1. Quantized Curvature Modes

The recursive kernel  $K(\phi, \phi')$  filters configuration transitions through coherence-preserving Gaussian structures. This filtering introduces quantized curvature features, modulated by the dominant coherence scale  $j_0$  derived from spin foam amplitudes:

$$\Delta R \sim \frac{1}{\sqrt{j_0}} \cdot \mathcal{F}(a, \varphi, \lambda, E),$$

where  $\mathcal{F}$  encodes the transition filter in configuration space (see Appendix 8.4).

Predicted observational signatures include:

- Low- $\ell$  CMB power suppression from early-cycle memory filtering,
- Oscillatory features in angular power spectra from coherence interference,
- Coherence-dependent gravitational wave echoes from recursive memory propagation.

### 7.2. Gravitational Wave Echoes and Memory Delay

Recursive coherence filtering introduces non-Markovian propagation in the gravitational wave signal. Observed strain  $h_{\text{obs}}(t)$  becomes a convolution with an entanglement-weighted memory kernel  $D(\tau, E)$ :

$$h_{\text{obs}}(t) = \int_0^t D(\tau, E) \cdot h(t - \tau) d\tau.$$

This predicts:

- Late-time echoes post-merger from partial recursive bounce memory,
- Echo spacing  $\tau_E \sim E_n$  tied to ER bridge entanglement length,
- Damped repetition amplitudes controlled by coherence fidelity  $\lambda_n$ .

These features are testable in LIGO–Virgo–KAGRA–LISA data via waveform residuals [8, 9].

### 7.3. Polarization Alignment in the CMB

Recursive symmetry filtering predicts residual alignment in E-mode and B-mode polarization directions. Voids, being low-entropy structures, preferentially retain coherence, seeding aligned polarization vectors across large angular scales. This leads to:

- Quadrupole–octupole alignment in low- $\ell$  multipoles [10],
- EB-mode cross-correlation sourced by entanglement asymmetry  $E_n$ ,
- Angular phase coherence absent in inflationary stochastic models.

#### 7.4. Recursive Constraints on Extra Dimensions

In the toroidal configuration space  $\mathcal{C}_n = \mathbb{T}^6$  with internal directions  $(\gamma_n, \xi_n)$ , coherence misalignment induces effective scale evolution:

$$\ell_\gamma^2 \sim \frac{1}{\lambda_C} \left\| \delta\phi_n^{(\gamma)} \right\|^2.$$

Observable effects include:

- Sub-millimeter deviations from Newtonian gravity [11],
- Temporal drift in fundamental constants (e.g.,  $\alpha$ ,  $G$ ) via  $\gamma_n$ -driven coupling variation [12],
- Modulated vacuum masses in dark sector particles (e.g., axion-like fields) from entropy–tension collapse [13].

#### 7.5. Summary of Observable Channels

Phenomenon	Prediction	Test Instrument
CMB Low- $\ell$ Suppression	Kernel-filtered memory loss	Planck, LiteBIRD
Gravitational Echoes	Delay $\tau_E$ from $D(\tau, E)$	LISA, LVK
EB-mode Alignment	Entangled void polarization	SKA, Euclid
Extra-Dimensional Drift	$\gamma_n$ - $\lambda_n$ modulation	Atomic clocks, torsion balances
$g$ Variation	Cycle-indexed gauge coupling shift	BBN + CMB comparisons

Table 1: Primary observational predictions from recursive symmetry-filtered dynamics.

## 8. Discussion and Future Work

### 8.1. Toward a Recursive Theory of Everything

In this paper, we extended the foundational architecture introduced in *The Self-Remembering Universe* by constructing a unified recursive action over a compactified configuration space:

$$\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n) \in \mathcal{C}_n = \mathbb{T}^6,$$

where physical dynamics are determined not solely by local field equations but by recursive constraints on coherence propagation, entanglement stability, and observer resolution.

By enforcing symmetry filtering through a coherence-preserving projection  $\mathcal{P}_{G_{\text{coh}}}$ , the model establishes a principled mechanism for physical law selection. Only transitions that align with the recursive attractor  $\Psi^*(\phi)$  survive. This yields an emergent architecture in which forces, decoherence, dimensional compactification, and measurement outcomes are all manifestations of a recursive memory logic operating across cosmological cycles.

### 8.2. Extension to Biological and Informational Systems

Paper III will generalize the kernel formalism to non-physical domains, asking whether recursive coherence plays an organizing role in:

- Neural dynamics and cognitive attractors (recursive self-alignment in perceptual systems)
- Evolutionary memory in ecosystems and adaptive feedback loops
- Cross-domain attractor convergence between quantum, thermodynamic, and informational layers
- Observer-participant recursion across scales (biological, computational, and cosmological)

We aim to test whether memory-stabilized attractor dynamics can provide a unifying explanatory principle linking physics, consciousness, and complexity.

### 8.3. Open Questions

Several critical questions remain unresolved:

1. **Gauge–Attractor Stability:** What governs the structure and evolution of the coherence-preserving symmetry group  $G_{\text{coh}}$ ? Can it spontaneously fracture or adapt under entropic or informational stress?
2. **Topology of Bounce Geometries:** Are bounce topologies emergent from the attractor’s basin structure? Do they evolve according to phase coherence gradients?
3. **Observer Dynamics:** Can the resolution parameter  $\xi_n$  and entanglement tensor  $O^{\mu\nu}$  be derived from an open-system quantum model with fully emergent decoherence pathways?
4. **Experimental Viability:** What are the measurable thresholds for detecting attractor-constrained GW echoes, void-aligned polarization, or coherence-driven fundamental constant variation?
5. **Renormalization under Memory Constraints:** Can recursive entropy compensation be consistently implemented within a spin foam–based renormalization flow?

#### 8.4. Closing Perspective

We have argued that physical reality (its stability, laws, and directional flow) emerges from recursive coherence alignment across cosmological cycles. The attractor  $\Psi^*(\phi)$  is not merely a final state, but an evolving memory filter that structures allowed configurations across time. In this model, the universe is not a brute-force sampler of possible states but a memory-sustaining signal.

Future work will test this paradigm across physics, biology, and information theory, refining both mathematical derivations and empirical predictions. If successful, this approach may serve as a foundation for a recursive theory of everything: a model in which coherence itself is the fabric of both evolution and law.



## Appendix A

### Recursive Symmetry and Attractor Constraints

#### A.1 Configuration Space and Symmetry Embedding

We define the recursive configuration space as:

$$\mathcal{C}_n = \mathbb{T}^6 = \text{Tor}(a_n) \times \text{Tor}(\varphi_n) \times \text{Tor}(\lambda_n) \times \text{Tor}(E_n) \times \text{Tor}(\gamma_n) \times \text{Tor}(\xi_n),$$

where each degree of freedom evolves over a compact dimension with phase continuity. These toroidal embeddings allow recursive coherence and phase-filtered interference patterns across cycles without divergence.

Symmetry transformations are elements of a bounded transformation algebra  $\mathfrak{T}$ , acting on  $\mathcal{C}_n$ . Physical transitions are not arbitrary—they must preserve recursive alignment with the attractor  $\Psi^*(\phi)$ . We define a coherence-preserving subset:

$$G_{\text{coh}} \subset \text{Aut}(\mathcal{C}_n),$$

where  $G_{\text{coh}}$  includes only those transformations that conserve entropy bounds, entanglement alignment, and observer resolution thresholds.

#### A.2 Attractor-Preserving Projection Operators

We introduce a projection operator  $\mathcal{P}_{G_{\text{coh}}}$  that enforces attractor-preserving transformations:

$$\mathcal{P}_{G_{\text{coh}}}[\delta\phi_n] := \begin{cases} \delta\phi_n & \text{if } \delta\phi_n \in G_{\text{coh}}, \\ 0 & \text{otherwise.} \end{cases}$$

This operator appears explicitly in the recursive action as a filter on legal transitions. It replaces arbitrary diffeomorphisms with attractor-conditioned pathways:

$$\delta\mathcal{A}_n[\phi] = 0 \quad \text{subject to} \quad \delta\phi_n \in \text{Im}(\mathcal{P}_{G_{\text{coh}}}).$$

#### A.3 Symmetry Filtering in the Transition Kernel

The recursive kernel is defined by a coherence-weighted sum over attractor-aligned paths:

$$K(\phi, \phi') = \sum_{\delta\phi \in G_{\text{coh}}} \mathcal{A}_{\delta\phi}(\phi, \phi') \cdot e^{-S_{\text{rel}}(\rho_\phi \| \rho_{\phi'})},$$

where:

- $\mathcal{A}_{\delta\phi}$ : amplitude for each transition,
- $S_{\text{rel}}$ : quantum relative entropy, penalizing misaligned transitions.

Transformations not in  $G_{\text{coh}}$  are exponentially suppressed:

$$\mathcal{A}_{\delta\phi} \sim 0 \quad \text{if } \delta\phi \notin G_{\text{coh}}.$$

This ensures recursive evolution flows only through the attractor's stability basin.

#### A.4 Observer-Dependent Resolution Filtering

Let  $\mathcal{P}_\xi$  denote a resolution-limited projection operator:

$$\mathcal{P}_\xi[\rho] = \sum_{i \in \mathcal{B}_\xi} |i\rangle \langle i| \rho |i\rangle \langle i|,$$

where  $\mathcal{B}_\xi$  is a coarse-grained basis determined by observer resolution  $\xi$ . This operator enforces decoherence filtering and constrains which components of the configuration space contribute to physical observables.

The recursive state update becomes:

$$\rho \rightarrow \mathcal{P}_\xi \circ \mathcal{P}_{G_{\text{coh}}}[\rho],$$

integrating both intrinsic symmetry and extrinsic observational constraints.

#### A.5 Summary

This appendix formalizes how recursive evolution is shaped by coherence-preserving symmetry filtering in a compactified configuration space. Transformations are constrained by:

- Alignment with the attractor  $\Psi^*(\phi)$ ,
- Entropy conservation and coherence fidelity,
- Observer-limited projection via resolution index  $\xi_n$ .

This structure defines a recursive geometric logic for allowable transitions, enforcing that only memory-aligned, entropically admissible transformations survive the evolution across cosmological cycles.

## Appendix B

### Recursive Lagrangian Structure and Field Dynamics

#### B.1 Total Action Decomposition

The recursive action per cycle is defined as:

$$\mathcal{A}_n[\phi] = \int_{\Sigma_n} (\mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{obs}} + \mathcal{L}_{\text{ent}}) d^4x,$$

where each term corresponds to a physical component within the recursive structure:

- $\mathcal{L}_{\text{EH}}$ : Einstein–Hilbert term for gravitational curvature:

$$\mathcal{L}_{\text{EH}} = \frac{1}{2\kappa} R, \quad \text{with } \kappa = 8\pi G.$$

- $\mathcal{L}_{\text{YM}}$ : Gauge field term with coherence-modulated couplings:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} \sum_a \left( g_n^{(a)} \right)^{-2} F_{\mu\nu}^{(a)} F_{(a)}^{\mu\nu},$$

where  $g_n^{(a)} = f^{(a)}(\lambda_n, E_n, \gamma_n)$  reflects cycle-dependent coherence structure.

- $\mathcal{L}_{\text{scalar}}$ : Scalar field evolution with attractor-modulated potential:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi, \lambda_n).$$

- $\mathcal{L}_{\text{int}}$ : Interaction term coupling scalar fields to internal symmetry:

$$\mathcal{L}_{\text{int}} = \sum_i \xi_n \cdot \varphi \cdot \partial_\mu \gamma^i,$$

where  $\gamma^i$  are toroidally compactified internal symmetry coordinates.

- $\mathcal{L}_{\text{obs}}$ : Observer-coupling and decoherence regulator:

$$\mathcal{L}_{\text{obs}} = \lambda_O \cdot O^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \xi_n \cdot S_{\text{ent}}(\rho_{\text{red}}).$$

- $\mathcal{L}_{\text{ent}}$ : Recursive entropy penalty:

$$\mathcal{L}_{\text{ent}} = -\lambda_S \cdot S_{\text{rel}}(\rho_{\phi_n} \parallel \rho_{\phi_{n-1}}),$$

where  $S_{\text{rel}}$  is the quantum relative entropy.

#### B.2 Recursive Evolution and Filtering Constraint

Recursive evolution of the configuration field is filtered through attractor-aligned updates:

$$\phi_{n+1} = \phi_n \oplus \delta\phi_n, \quad \text{subject to } \delta\phi_n \in \text{Im}(\mathcal{P}_{G_{\text{coh}}}).$$

The variation of the action is constrained accordingly:

$$\delta\mathcal{A}_n[\phi] = 0 \quad \text{with } \delta\phi_n \in \mathfrak{g}_{\text{coh}}.$$

This ensures that the Euler–Lagrange equations reflect coherence-preserving dynamics only.

### B.3 Attractor Stability and Potential Landscape

The attractor  $\Psi^*(\phi)$  defines a fixed point of recursive evolution. Its stability is analyzed via the configuration-space Hessian:

$$\mathcal{H}_{ij} = \left. \frac{\partial^2 \mathcal{A}_n}{\partial \phi^i \partial \phi^j} \right|_{\phi = \Psi^*(\phi)}.$$

Lyapunov stability requires:

$$\mathcal{H} \succeq 0 \quad \text{on} \quad \mathcal{P}_{G_{\text{coh}}}(\mathcal{C}_n),$$

ensuring that evolution converges toward rather than diverges from memory-aligned attractor states.

### B.4 Thermodynamic Enforcement

All recursive transitions are regulated by entropy bounds and tension constraints:

$$\mathcal{A}_n \text{ minimized under: } \Delta S_{\text{fwd}} = \Delta S_{\text{mem}}, \quad \kappa_n \leq \kappa_C,$$

with:

$$\kappa_n = \frac{I(\phi_n, \phi_{n-1})}{\lambda_n}.$$

This formalizes a dual thermodynamic-coherence constraint on evolution.

### B.5 Summary

Appendix B formalizes the Lagrangian components of recursive evolution, showing how gravitational, gauge, scalar, and observer fields are interwoven into a single attractor-constrained variational structure. All dynamics are filtered through entropy penalties, coherence conditions, and symmetry-aligned memory inheritance.

## Appendix C

### Recursive Kernel Implementation and Attractor Derivation

#### C.1 Recursive Configuration Space

The full recursive configuration vector is:

$$\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n),$$

with each component defined as:

- $a_n$ : discrete scale factor (from LQC bounce dynamics),
- $\varphi_n$ : scalar field value(s),
- $\lambda_n$ : memory fidelity (overlap between cycles),
- $E_n$ : entanglement eigenvalue across the ERB,
- $\gamma_n$ : toroidal internal symmetry coordinates,
- $\xi_n$ : observer resolution scale modulating decoherence.

The configuration space  $\mathcal{C}_n \subset \mathbb{R}^4 \times T^2$  includes both real-valued physical fields and compactified toroidal variables.

#### C.2 Transition Kernel Structure

The transition kernel  $K(\phi, \phi')$  defines the weighted amplitude for transition between recursive states:

$$K(\phi, \phi') = \mathcal{A}_{\text{geom}}(\phi, \phi') \cdot \mathcal{F}_{\text{coh}}(\phi, \phi') \cdot e^{-S_{\text{rel}}(\rho_\phi \| \rho_{\phi'})}.$$

Where:

- $\mathcal{A}_{\text{geom}}$ : geometric propagator from spinfoam amplitudes (large spin limit),
- $\mathcal{F}_{\text{coh}}$ : Gaussian coherence filter:

$$\mathcal{F}_{\text{coh}} = \exp \left( -\frac{(a - a')^2}{2\sigma_a^2} - \frac{(\varphi - \varphi')^2}{2\sigma_\varphi^2} - \frac{(E - E')^2}{2\sigma_E^2} - \frac{d_T^2(\gamma, \gamma')}{2\sigma_\gamma^2} \right),$$

with  $d_T^2$  denoting geodesic distance on the torus.

- $S_{\text{rel}}$ : relative entropy penalty encoding memory divergence.

#### C.3 Recursive Update Equation

The recursive state update is defined by:

$$\Psi_{n+1}(\phi) = \int K(\phi, \phi') \Psi_n(\phi') d\phi',$$

with normalized kernel:

$$K_{\text{norm}}(\phi, \phi') = \frac{K(\phi, \phi')}{\int K(\phi, \phi') d\phi'}.$$

This implements coherence-filtered propagation across recursive transitions.

### C.4 Attractor Derivation

The fixed-point attractor  $\Psi^*(\phi)$  satisfies:

$$\Psi^*(\phi) = \int K_{\text{norm}}(\phi, \phi') \Psi^*(\phi') d\phi',$$

and is derived as the minimum of the coherence-weighted action functional:

$$\Psi^* = \arg \min_{\Psi} \{ \lambda_S S(\rho_{\Psi}) + \lambda_T \lambda^2 - \lambda_C |\langle \Psi | \Psi_{n-1} \rangle|^2 \}.$$

Convergence is guaranteed under contraction mapping on the Hilbert space  $\mathcal{H}_{\phi}$ , provided entropy and fidelity constraints are enforced.

### C.5 Numerical Implementation Strategy

Simulation proceeds by:

1. Initializing  $\Psi_0(\phi)$  with observational or analytical priors.
2. Updating via recursive convolution:

$$\Psi_{n+1} = K_{\text{norm}} \star \Psi_n.$$

3. Tracking convergence via:

$$\lambda_n = |\langle \Psi_n | \Psi_{n-1} \rangle|^2, \quad \mathcal{L}_n = \log \left( \frac{\|\Psi_{n+1} - \Psi_n\|}{\|\Psi_n - \Psi_{n-1}\|} \right).$$

4. Enforcing entropy constraint:

$$\Delta S_n = S(\rho_{n+1}) - S(\rho_n) \leq \epsilon.$$

### C.6 Entropy Compensation Condition

Recursive evolution satisfies:

$$\Delta S_{\text{fwd}} + \Delta S_{\text{rad}} = \Delta S_{\text{exp}},$$

where:

- $\Delta S_{\text{fwd}}$ : entropy from memory gain,
- $\Delta S_{\text{rad}}$ : radiative entropy (e.g., Hawking flux),
- $\Delta S_{\text{exp}}$ : dilution entropy from expansion.

This enforces thermodynamic balance at each bounce.

### C.7 Convergence Criteria

We define attractor convergence by:

$$\lambda_n \rightarrow 1, \quad \Delta S_n < \epsilon, \quad \|\Psi_{n+1} - \Psi_n\| \rightarrow 0.$$

These thresholds regulate recursion, ensuring dynamical coherence, bounded entropy, and memory stability.

## Appendix D

### Notation and Operator Glossary

#### D.1 Configuration Variables

- $a_n$  : Discrete scale factor at cycle  $n$  (from loop quantum cosmology [5])
- $\varphi_n$  : Scalar field (e.g., inflaton, Higgs)
- $\lambda_n$  : Coherence fidelity:

$$\lambda_n := |\langle \Psi_n | \Psi_{n-1} \rangle|^2$$

- $E_n$  : Entanglement eigenvalue:

$$E_n := \sqrt{-\text{Tr}(\rho_{\text{red}} \log \rho_{\text{red}})} = \sqrt{S(\rho_{\text{red}})}$$

- $\gamma_n$  : Toroidal internal symmetry coordinate (e.g., compactified gauge phase)
- $\xi_n$  : Observer resolution index controlling decoherence
- $\phi_n$  : Recursive configuration vector:

$$\phi_n = (a_n, \varphi_n, \lambda_n, E_n, \gamma_n, \xi_n)$$

#### D.2 Operators and Fields

- $K(\phi, \phi')$  : Recursive transition kernel, coherence-filtered
- $\Psi_n(\phi)$  : State of the system at cycle  $n$
- $\Psi^*(\phi)$  : Attractor state (fixed point of  $K$ )
- $\rho$  : Full quantum density matrix
- $\rho_{\text{red}}$  : Reduced density matrix across the ER bridge
- $S(\rho)$  : Von Neumann entropy:  $S(\rho) = -\text{Tr}(\rho \log \rho)$
- $S_{\text{rel}}(\rho \parallel \sigma)$  : Quantum relative entropy:

$$S_{\text{rel}}(\rho \parallel \sigma) = \text{Tr}[\rho(\log \rho - \log \sigma)]$$

- $\mathcal{T}$  : Recursive transition operator:  $\phi_{n+1} = \mathcal{T}[\phi_n]$
- $\mathcal{P}_\xi$  : Projection operator at observer resolution  $\xi$
- $O^{\mu\nu}$  : Observer entanglement tensor

### D.3 Geometric and Physical Quantities

- $R_{\mu\nu}$  : Ricci curvature tensor
- $\tilde{R}_{\mu\nu}$  : Information-weighted curvature:

$$\tilde{R}_{\mu\nu} = R_{\mu\nu} + \alpha \nabla_\mu \nabla_\nu \log \det \rho_{\text{red}}$$

- $F_{\mu\nu}^{(a)}$  : Gauge field strength tensor
- $J_\mu^{(a)}$  : Projected memory current sourcing gauge fields
- $\tau_E$  : Delay time for gravitational wave echoes (set by  $E_n$ )

### D.4 Action and Lagrangian Terms

- $\mathcal{A}_n[\phi]$  : Total recursive action for cycle  $n$
- $\mathcal{L}_{\text{EH}}$  : Einstein–Hilbert Lagrangian:  $\frac{1}{2\kappa} R$
- $\mathcal{L}_{\text{YM}}$  : Yang–Mills term for gauge fields
- $\mathcal{L}_{\text{scalar}}$  : Scalar field Lagrangian
- $\mathcal{L}_{\text{int}}$  : Interaction terms (e.g., scalar–fermion–gauge)
- $\mathcal{L}_{\text{obs}}$  : Observer resolution and decoherence control
- $\mathcal{L}_{\text{ent}}$  : Recursive entropy penalty term

### D.5 Symbol Conventions and Parameters

- $\oplus$  : Recursive coherence update (filtered XOR-type operator)
- $\eta$  : Learning rate in variational descent toward attractor
- $\delta_{\text{noise}}$  : Entropy-weighted noise in simulation dynamics
- $\sigma_a, \sigma_\varphi, \sigma_E, \sigma_\gamma$  : Coherence filter widths
- $\lambda_C, \lambda_T, \lambda_S$  : Lagrange multipliers for constraints
- $\kappa$  : Gravitational coupling constant:  $\kappa = 8\pi G$



## Disclosure on the Use of AI

Portions of this manuscript—including the development, refinement, formatting of mathematical expressions, narrative structure, and citation management—were produced in collaboration with OpenAI’s GPT-4 model (ChatGPT), Google’s Gemini 2.0, and DeepSeek R1. The human author, Nicholas Parian, guided the conceptual framework, directed the line of inquiry, posed the core hypotheses, and curated the final scientific content.

The use of artificial intelligence was instrumental in accelerating the writing, organizing technical arguments, and cross-referencing related literature. However, all original theoretical contributions, interpretations, and decisions regarding inclusion, emphasis, and framing were made by the human author.

This disclosure is provided in the interest of transparency and to acknowledge the evolving role of large language models in academic research and writing. The author assumes full responsibility for the accuracy, novelty, and scientific validity of the material presented.

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