



MODULE ONE PROBLEM SET

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

PROBLEM 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

- $P(x)$: x was given the placebo
- $D(x)$: x was given the medication
- $M(x)$: x had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.

- $\exists x (P(x) \wedge D(x))$
- *Negation:* $\neg \exists x (P(x) \wedge D(x))$
- *Applying De Morgan's law:* $\forall x (\neg P(x) \vee \neg D(x))$
- *English:* Every patient was either not given the placebo or not given the medication (or both).

- (a) Every patient was given the medication or the placebo or both.
- (b) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \vee q$.)
- (c) There is a patient who had migraines and was given the placebo.

PROBLEM 2

Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences:

$$(a) \neg \forall x (P(x) \wedge \neg Q(x)) \equiv \exists x (\neg P(x) \vee Q(x))$$

$$(b) \neg \forall x (\neg P(x) \rightarrow Q(x)) \equiv \exists x (\neg P(x) \wedge \neg Q(x))$$

$$(c) \neg \exists x (\neg P(x) \vee (Q(x) \wedge \neg R(x))) \equiv \forall x (P(x) \wedge (\neg Q(x) \vee R(x)))$$

PROBLEM 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x, y)$ indicates whether x has sent an email to y , so $M(2, 3)$ is read “Person 2 has sent an email to person 3.” The table below shows the value of the predicate $M(x, y)$ for each (x, y) pair. The truth value in row x and column y gives the truth value for $M(x, y)$.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

Determine if the quantified statement is true or false. Justify your answer.

(a) $\forall x \forall y (x \neq y) \rightarrow M(x, y)$

(b) $\forall x \exists y \neg M(x, y)$

(c) $\exists x \forall y M(x, y)$

PROBLEM 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

- (a) The reciprocal of every positive number less than one is greater than one.
- (b) There is no smallest number.
- (c) Every number other than 0 has a multiplicative inverse.

PROBLEM 5

The sets A , B , and C are defined as follows:

$$A = \text{tall, grande, venti}$$

$$B = \text{foam, no - foam}$$

$$C = \text{non - fat, whole}$$

Use the definitions for A , B , and C to answer the questions. Express the elements using n -tuple notation, not string notation.

(a) Write an element from the set $A \times B \times C$.

(b) Write an element from the set $B \times A \times C$.

(c) Write the set $B \times C$ using roster notation.