MODiCuM: Mechanisms for Outsourcing via a Decentralized Computation Market

Theorem 1

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Our goal is to characterize when neither actors have an incentive to act dishonest. We capture this using the concept of Nash equilibrium. Formally, strategy profile (Cj, Cr) is a Nash equilibrium if neither actor can increase its own expected utility by unilaterally changing its strategy choice.

Theorem 1. *Both RP and JC acting honestly is a Nash equilibrium if and only if*

$$p \leq Q\frac{P_m \cdot (2f + M + r) - r - f}{C_j}$$

and

$$p \geq \frac{(1-Q)}{P_j} + \frac{C - C_d}{P_j \cdot (r + f + M)}.$$

Proof. First, JC will act honestly if and only if

$$\begin{aligned} \mathcal{U}_{JC}(C_j, C_r) &\geq \mathcal{U}_{JC}(D_j, C_r) \\ Q \cdot (B - r) + (1-Q) \cdot f - p \cdot C_j &\geq Q \cdot \big[B + (1 - P_m) \cdot f - P_m \cdot (f + M + r)\big] + (1-Q) \cdot f. \end{aligned}$$

Equivalently,

$$\begin{aligned} -p \cdot C_j &\geq Q \cdot \big[r + (1 - P_m) \cdot f - P_m \cdot (f + M + r)\big] \\ p &\leq Q\frac{P_m \cdot (f + M + r) - r - (1 - P_m) \cdot f}{C_j} \\ p &\leq Q\frac{P_m \cdot (2f + M + r) - r - f}{C_j}. \end{aligned}$$

Second, the RP will act honestly if and only if

$$\begin{aligned} \mathcal{U}_{RP}(C_j, C_r) &\geq \mathcal{U}_{RP}(C_j, D_r) \\ Q \cdot r - (1-Q) \cdot (f + M) - C &\geq (1 - p \cdot P_j) \cdot r - p \cdot P_j \cdot (f + M) - C_d. \end{aligned}$$

Equivalently,

$$\begin{aligned} p \cdot P_j \cdot (r + f + M) &\geq (1-Q) \cdot (f + M + r) + C - C_d \\ p &\geq \frac{(1-Q) \cdot (f + M + r) + C - C_d}{P_j \cdot (r + f + M)} \\ p &\geq \frac{(1-Q)}{P_j} + \frac{C - C_d}{P_j \cdot (r + f + M)}. \end{aligned}$$

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