

Computing A and B from f, p, a and b. We convert  $\epsilon$  into  $(a-b)/b$ .

Take the expression for B

$$\begin{aligned} \text{In[ ]:= } & \mathbf{B[b\_ , f\_ , eps\_ , p\_ , phi0\_]} = \text{Simplify} \left[ \left( b^2 f (1 + \text{eps}) - p - \text{phi0 } b (2 + \text{eps}) \right) / \text{eps} \right] \\ \text{Out[ ]:= } & - \frac{-b^2 (1 + \text{eps}) f + p + b (2 + \text{eps}) \text{phi0}}{\text{eps}} \end{aligned}$$

Substitute  $\epsilon = (a - b)/b$

$$\begin{aligned} \text{In[ ]:= } & \mathbf{Bp[b\_ , f\_ , a\_ , p\_ , phi0\_]} = \text{Simplify}[\mathbf{B[b, f, (a - b) / b, p, phi0]}] \\ \text{Out[ ]:= } & \frac{b (a b f - p - a \text{phi0} - b \text{phi0})}{a - b} \end{aligned}$$

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{Bp[b, f, a, p, phi0]} == \frac{b (a b f - p - \text{phi0} (a + b))}{a - b} \right]$$

Out[ ]:= True

Take the particle wrt a

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{D[Bp[b, f, a, p, phi0], a]} == \frac{b (p + 2 b \text{phi0} - b^2 f)}{(a - b)^2} \right]$$

Out[ ]:= True

Take the particle wrt b

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{D[Bp[b, f, a, p, phi0], b]} == \frac{a^2 (2 b f - \text{phi0}) + b^2 \text{phi0} - a (b (b f + 2 \text{phi0}) + p)}{(a - b)^2} \right]$$

Out[ ]:= True

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{D[Bp[b, f, a, p, phi0], phi0]} == - \frac{b (a + b)}{a - b} \right]$$

Out[ ]:= True

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{D[Bp[b, f, a, p, phi0], f]} == \frac{a b^2}{a - b} \right]$$

Out[ ]:= True

$$\text{In[ ]:= } \mathbf{Simplify} \left[ \mathbf{D[Bp[b, f, a, p, phi0], p]} == - \frac{b}{a - b} \right]$$

Out[ ]:= True

$$\text{In[ ]:= } \mathbf{A[B\_ , b\_ , phi0\_ , f\_ , eps\_]} = (B / b + \text{phi0} - b f) (1 + \text{eps}) / \text{eps}$$

$$\text{Out[ ]:= } \frac{(1 + \text{eps}) \left( \frac{B}{b} - b f + \text{phi0} \right)}{\text{eps}}$$

In[<sup>⊗</sup>]:= **Ap[b\_, phi0\_, f\_, a\_, p\_] = Simplify[A[Bp[b, f, a, p, phi0], b, phi0, f, (a - b) / b]]**

Out[<sup>⊗</sup>]= 
$$\frac{a (b^2 f - p - 2 b \text{phi0})}{(a - b)^2}$$

In[<sup>⊗</sup>]:= **Simplify[Ap[b, phi0, f, a, p] ==  $\frac{a (b^2 f - p - 2 b \text{phi0})}{(a - b)^2}$ ]**

Out[<sup>⊗</sup>]= True

In[<sup>⊗</sup>]:= **Simplify[D[Ap[b, phi0, f, a, p], b] ==  $\frac{2 a (a b f - p - \text{phi0} (a + b))}{(a - b)^3}$ ]**

Out[<sup>⊗</sup>]= True

In[<sup>⊗</sup>]:= **Simplify[D[Ap[b, phi0, f, a, p], a] ==  $\frac{(a + b) (-b^2 f + p + 2 b \text{phi0})}{(a - b)^3}$ ]**

Out[<sup>⊗</sup>]= True

In[<sup>⊗</sup>]:= **Simplify[D[Ap[b, phi0, f, a, p], phi0] ==  $-\frac{2 a b}{(a - b)^2}$ ]**

Out[<sup>⊗</sup>]= True

In[<sup>⊗</sup>]:= **Simplify[D[Ap[b, phi0, f, a, p], f] ==  $\frac{a b^2}{(a - b)^2}$ ]**

Out[<sup>⊗</sup>]= True

In[<sup>⊗</sup>]:= **Simplify[D[Ap[b, phi0, f, a, p], p] ==  $-\frac{a}{(a - b)^2}$ ]**

Out[<sup>⊗</sup>]= True