

M, J, Z<sub>b</sub>

In[88]:= **M[C0\_, C1\_, C2\_] = C0 (Z0 / A0) + C1 (Z1 / A1) + C2 (Z2 / A2)**

$$\text{Out[88]} = \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2}$$

In[89]:= **D[M[C0, C1, C2], C1]**

$$\text{Out[89]} = \frac{Z1}{A1}$$

In[130]:= **Simplify[D[M[C0, C1, C2], C1] == Z1 / A1]**

Out[130]= True

In[90]:= **D[M[C0, C1, C2], A1]**

$$\text{Out[90]} = -\frac{C1 Z1}{A1^2}$$

In[131]:= **Simplify[D[M[C0, C1, C2], A1] == -\frac{C1 Z1}{A1^2}]**

Out[131]= True

Do this relative to log[J]

In[91]:= **1J[C0\_, C1\_, C2\_] = (C0 Z0 Log[J0] / A0 + C1 Z1 Log[J1] / A1 + C2 Z2 Log[J2] / A2) / M[C0, C1, C2]**

$$\text{Out[91]} = \frac{\frac{C0 Z0 \text{Log}[J0]}{A0} + \frac{C1 Z1 \text{Log}[J1]}{A1} + \frac{C2 Z2 \text{Log}[J2]}{A2}}{\frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2}}$$

In[110]:= **D[1J[C0, C1, C2], C1]**

$$\text{Out[110]} = \frac{Z1 \text{Log}[J1]}{A1 \left( \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2} \right)} - \frac{Z1 \left( \frac{C0 Z0 \text{Log}[J0]}{A0} + \frac{C1 Z1 \text{Log}[J1]}{A1} + \frac{C2 Z2 \text{Log}[J2]}{A2} \right)}{A1 \left( \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2} \right)^2}$$

In[115]:= **Simplify[ \frac{Z1 \text{Log}[J1]}{A1 \left( \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2} \right)} M[C0, C1, C2] ]**

$$\text{Out[115]} = \frac{Z1 \text{Log}[J1]}{A1}$$

In[117]:= **\frac{Z1 \left( \frac{C0 Z0 \text{Log}[J0]}{A0} + \frac{C1 Z1 \text{Log}[J1]}{A1} + \frac{C2 Z2 \text{Log}[J2]}{A2} \right)}{A1 \left( \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2} \right)^2} M[C0, C1, C2] / 1J[C0, C1, C2]**

$$\text{Out[117]} = \frac{Z1}{A1}$$

$$\text{In[132]:= Simplify}\left[D[J[C0, C1, C2], C1] == \left(\frac{Z1}{A1}\right) \left(\frac{1}{M[C0, C1, C2]}\right) (\text{Log}[J1] - J[C0, C1, C2])\right]$$

Out[132]= True

Now account for the Log[J]

$$\text{In[129]:= D[Exp[F[X]], X]$$

$$\text{Out[129]= } e^{F[X]} F'[X]$$

$$\text{In[128]:= Simplify}\left[D[\text{Exp}[J[C0, C1, C2]], C1] == \left(\frac{J[C0, C1, C2]}{M[C0, C1, C2]}\right) \left(\frac{Z1}{A1}\right) (\text{Log}[J1] - J[C0, C1, C2])\right]$$

Out[128]= True

$$\text{In[133]:= Simplify}[D[J[C0, C1, C2], C0] == (J[C0, C1, C2] / M[C0, C1, C2]) (Z0 / A0) (\text{Log}[J0] - J[C0, C1, C2])]$$

Out[133]= True

$$\text{In[98]:= Simplify}[D[J[C0, C1, C2], C1] == (J[C0, C1, C2] / M[C0, C1, C2]) (Z1 / A1) (\text{Log}[J1] - J[C0, C1, C2])]$$

Out[98]= True

$$\text{In[101]:= Simplify}[D[J[C0, C1, C2], C2] == (J[C0, C1, C2] / M[C0, C1, C2]) (Z2 / A2) (\text{Log}[J2] - J[C0, C1, C2])]$$

Out[101]= True

Now consider partial of J relative to  $J_i$

$$\text{In[99]:= D[J[C0, C1, C2], J1] M[C0, C1, C2] / J[C0, C1, C2]$$

$$\text{Out[99]= } \frac{C1 Z1}{A1 J1}$$

$$\text{In[126]:= Simplify}[D[J[C0, C1, C2], J1] == (J[C0, C1, C2] / M[C0, C1, C2]) (Z1 / A1) (C1 / J1)]$$

Out[126]= True

Now consider partial of J relative to  $A_i$

$$\text{In[102]:= D[J[C0, C1, C2], A1] / J[C0, C1, C2]$$

$$\text{Out[102]= } -\frac{C1 Z1 \text{Log}[J1]}{A1^2 \left(\frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2}\right)} + \frac{C1 Z1 \left(\frac{C0 Z0 \text{Log}[J0]}{A0} + \frac{C1 Z1 \text{Log}[J1]}{A1} + \frac{C2 Z2 \text{Log}[J2]}{A2}\right)}{A1^2 \left(\frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2}\right)^2}$$

$$\text{In[103]:= } \frac{C1 Z1 \text{Log}[J1]}{A1^2 \left(\frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2}\right)} M[C0, C1, C2]$$

$$\text{Out[103]= } \frac{C1 Z1 \text{Log}[J1]}{A1^2}$$

$$\text{In[104]:= } \frac{C1 Z1 \left( \frac{C0 Z0 \text{Log}[J0]}{A0} + \frac{C1 Z1 \text{Log}[J1]}{A1} + \frac{C2 Z2 \text{Log}[J2]}{A2} \right)}{A1^2 \left( \frac{C0 Z0}{A0} + \frac{C1 Z1}{A1} + \frac{C2 Z2}{A2} \right)^2} M[C0, C1, C2] / LJ[C0, C1, C2]$$

$$\text{Out[104]:= } \frac{C1 Z1}{A1^2}$$

The result

$$\text{In[124]:= } \text{Simplify}[D[J[C0, C1, C2], A1] == (J[C0, C1, C2] / M[C0, C1, C2]) (Z1 / A1) (C1 / A1) (LJ[C0, C1, C2] - \text{Log}[J1])]$$

$$\text{Out[124]:= } \text{True}$$

## Consider $Z_b$

$$\text{In[106]:= } Zbarb[C0_, C1_, C2_] = (C0 Z0^{1/2} + C1 Z1^{1/2} + C2 Z2^{1/2})^2$$

$$\text{Out[106]:= } (C0 \sqrt{Z0} + C1 \sqrt{Z1} + C2 \sqrt{Z2})^2$$

$$\text{In[107]:= } \text{Simplify}[D[Zbarb[C0, C1, C2], C1], \{C0 > 0, C1 > 0, C2 > 0, Z0 > 0, Z1 > 0, Z2 > 0\}]$$

$$\text{Out[107]:= } 2 (C1 Z1 + C0 \sqrt{Z0 Z1} + C2 \sqrt{Z1 Z2})$$

$$\text{In[139]:= } \text{Simplify}[D[Zbarb[C0, C1, C2], C1] == 2 \text{Sqrt}[Z1 Zbarb[C0, C1, C2]], \{C0 > 0, C1 > 0, C2 > 0, Z0 > 0, Z1 > 0, Z2 > 0\}]$$

$$\text{Out[139]:= } \text{True}$$