Step P and b

In[ ]:= `g[zbarb_, u0_] = 22/100 Log[4 zbarb] (1 - 2 Exp[-zbarb (u0 - 1) / 15])`

Out[ ]= $$\frac{11}{50}\left(1-2\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\right)\mathrm{Log}[4\,zbarb]$$

In[ ]:= `expArg = Exp[-zbarb (u0 - 1) / 15]`

Out[ ]= $$e^{-\frac{1}{15}(-1+u0)\,zbarb}$$

In[ ]:= `D[g[zbarb, u0], zbarb]`

Out[ ]= $$\frac{11\left(1-2\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\right)}{50\,zbarb}-\frac{11}{375}\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\,(1-u0)\,\mathrm{Log}[4\,zbarb]$$

In[ ]:= $$\frac{11\left(1-2\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\right)}{50\,zbarb}\Big/ g[zbarb, u0]$$

Out[ ]= $$\frac{1}{zbarb\,\mathrm{Log}[4\,zbarb]}$$

In[ ]:= $$\mathrm{Simplify}\left[D[g[zbarb, u0], zbarb] == \frac{g[zbarb, u0]}{zbarb\,\mathrm{Log}[4\,zbarb]}+\frac{11}{375}\,expArg\,(-1+u0)\,\mathrm{Log}[4\,zbarb]\right]$$

Out[ ]= True

In[ ]:= `N[11 / 375]`

Out[ ]= 0.0293333

In[ ]:= `D[g[zbarb, u0], u0]`

Out[ ]= $$\frac{11}{375}\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\,zbarb\,\mathrm{Log}[4\,zbarb]$$

In[ ]:= $$\frac{11}{375}\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\,zbarb\,\mathrm{Log}[4\,zbarb]$$

Out[ ]= $$\frac{11}{375}\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\,zbarb\,\mathrm{Log}[4\,zbarb]$$

In[ ]:= $$\mathrm{Simplify}\left[D[g[zbarb, u0], u0] == \frac{11}{375}\,expArg\,zbarb\,\mathrm{Log}[4\,zbarb]\right]$$

Out[ ]= True

In[ ]:= $$h[zbarz\_, u0\_] = 1 - 10\,(1 - 1 / (1 + u0 / 10)) \Big/ zbarb^2$$

Out[ ]= $$1-\frac{10\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^2}$$

In[•]:= `D[h[zbarb, u0], zbarb]`

Out[•]= $$\frac{20\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^3}$$

In[•]:= `Simplify[D[h[zbarb, U0], zbarb] == 20 (1 - 1 / (1 + U0 / 10)) / zbarb^3]`

Out[•]= True

In[•]:= `D[h[zbarb, u0], u0]`

Out[•]= $$-\frac{1}{\left(1+\frac{u0}{10}\right)^2 zbarb^2}$$

In[•]:= `Simplify[D[h[zbarb, u0], u0] == -Power[((1 + u0 / 10) zbarb), -2]]`

Out[•]= True

In[•]:= `b[rbar_, phi0_, f_] = Sqrt[2] (1 + Sqrt[1 - rbar phi0 / f]) / rbar`

Out[•]= $$\frac{\sqrt{2}\left(1+\sqrt{1-\frac{phi0\ rbar}{f}}\right)}{rbar}$$

In[•]:= `D[b[rbar, phi0, f], rbar]`

Out[•]= $$-\frac{phi0}{\sqrt{2}\ f\ rbar\sqrt{1-\frac{phi0\ rbar}{f}}}-\frac{\sqrt{2}\left(1+\sqrt{1-\frac{phi0\ rbar}{f}}\right)}{rbar^2}$$

In[•]:= `Simplify[D[b[rbar, phi0, f], rbar] ==`
`-b[rbar, phi0, f] / rbar - phi0 / ( f rbar Sqrt[2 (1 - phi0 rbar / f)])]`

Out[•]= True

In[•]:= `Simplify[D[b[rbar, phi0, f], phi0]]`

Out[•]= $$-\frac{1}{f\sqrt{2-\frac{2\ phi0\ rbar}{f}}}$$

In[•]:= `Simplify[D[b[rbar, phi0, f], phi0] == -1 / (f Sqrt[2 (1 - phi0 rbar / f)])]`

Out[•]= True

In[•]:= `Simplify[D[b[rbar, phi0, f], f]]`

Out[•]= $$\frac{phi0}{f^2\sqrt{2-\frac{2\ phi0\ rbar}{f}}}$$

In[•]:= `Simplify[D[b[rbar, phi0, f], f] == phi0 / (Power[f, 2] Sqrt[2 (1 - phi0 rbar / f)])]`

Out[•]= True

In[•]:= `D[g[zbarb, u0] h[zbarb, u0]^4, u0]`

Out[•]=
$$-\frac{22\left(1-2\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\right)\left(1-\frac{10\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^2}\right)^3 Log[4\,zbarb]}{25\left(1+\frac{u0}{10}\right)^2 zbarb^2}+\frac{11}{375}\,e^{-\frac{1}{15}(-1+u0)\,zbarb}\left(1-\frac{10\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^2}\right)^4 zbarb\,Log[4\,zbarb]$$

Typical case

In[•]:= `P[g_, h_, F_, rbar_] = g h^4 F / rbar`

Out[•]=
$$\frac{F\,g\,h^4}{rbar}$$

In[•]:= `D[P[g[zbar, u0], h[zbar, u0], F, rbar], zbar]`

Out[•]=
$$\frac{11\left(1-2\,e^{-\frac{1}{15}(-1+u0)\,zbar}\right)F\left(1-\frac{10\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^2}\right)^4}{50\,rbar\,zbar}-\frac{11\,e^{-\frac{1}{15}(-1+u0)\,zbar}\,F\,(1-u0)\left(1-\frac{10\left(1-\frac{1}{1+\frac{u0}{10}}\right)}{zbarb^2}\right)^4 Log[4\,zbar]}{375\,rbar}$$

In[•]:= `Simplify[D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], zbar] ==`
`D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], gg[zbar, u0]] D[gg[zbar, u0], zbar] +`
`D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], hh[zbar, u0]] D[hh[zbar, u0], zbar]]`

Out[•]= True

In[•]:= `Simplify[D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], gg[zbar, u0]] ==`
`P[gg[zbar, u0], hh[zbar, u0], F, rbar] / gg[zbar, u0]]`

Out[•]= True

In[•]:= `Simplify[D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], hh[zbar, u0]] ==`
`4 P[gg[zbar, u0], hh[zbar, u0], F, rbar] / hh[zbar, u0]]`

Out[•]= True

In[•]:=
```
Simplify[
 D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], zbar] == P[gg[zbar, u0], hh[zbar, u0], F, rbar]
   (1 / gg[zbar, u0] D[gg[zbar, u0], zbar] + 4 / hh[zbar, u0] D[hh[zbar, u0], zbar])]
```

Out[•]= True

In[•]:=
```
Simplify[
 D[P[gg[zbar, u0], hh[zbar, u0], F, rbar], u0] == P[gg[zbar, u0], hh[zbar, u0], F, rbar]
   (1 / gg[zbar, u0] D[gg[zbar, u0], u0] + 4 / hh[zbar, u0] D[hh[zbar, u0], u0])]
```

Out[•]= True

Special case : Limit $g\,h^4$

In[•]:= `gh4[b_, rbar_, phi0_, f_] = 0.9 b rbar^2 (b - 2 phi0 / f)`

Out[•]= $0.9\, b \left(b - \frac{2\, \text{phi0}}{f}\right) \text{rbar}^2$

In[•]:= `P[bb_, rbar_, phi0_, ff_] = gh4[bb, rbar, phi0, ff] ff / rbar^2`

Out[•]= $0.9\, \text{bb}\, \text{ff} \left(\text{bb} - \frac{2\, \text{phi0}}{\text{ff}}\right)$

In[•]:= `Simplify[D[P[b, rbar, phi0, f], b]]`

Out[•]= $1.8\, b\, f - 1.8\, \text{phi0}$

In[•]:= `Simplify[D[P[b, rbar, phi0, f], phi0]]`

Out[•]= $-1.8\, b$

In[•]:= `Simplify[D[P[bb, rbar, phi0, f], f]]`

Out[•]= $0.9\, \text{bb}^2$

In[•]:= `D[Expand[P[bb, rbar, phi0, f]], rbar]`

Out[•]= 0

In[•]:= `P[bb_, rbar_, phi0_, ff_] = gh4[bb, rbar, phi0, ff] ff / rbar^2`

Out[•]= $0.9\, \text{bb}\, \text{ff} \left(\text{bb} - \frac{2\, \text{phi0}}{\text{ff}}\right)$

In[•]:= `D[P[bb[F], rbar, phi0, F], F]`

Out[•]= $0.9\, \text{bb}[F] \left(-\frac{2\, \text{phi0}}{F} + \text{bb}[F]\right) + 0.9\, F \left(-\frac{2\, \text{phi0}}{F} + \text{bb}[F]\right) \text{bb}'[F] + 0.9\, F\, \text{bb}[F] \left(\frac{2\, \text{phi0}}{F^2} + \text{bb}'[F]\right)$

In[•]:=
$$\text{Simplify}\left[D[P[bb[F], rbar, phi0, F], F] == \frac{P[bb[F], rbar, phi0, F]}{F} + \frac{P[bb[F], rbar, phi0, F]}{bb[F]} D[bb[F], F] + 0.9\, F\, bb[F] \left(\frac{2\, phi0}{F^2} + D[bb[F], F]\right)\right]$$

Out[•]= True

In[•]:=
$$\text{Simplify}\left[D[P[bb[F], rbar, phi0, F], F] == D[P[bb[F], rbar, phi0, F], bb[F]]\, D[bb[F], F] + 0.9\, bb[F]^2\right]$$

Out[•]= True

In[•]:= D[P[bb[F], rbar, phi0, F], rbar]

Out[•]= 0

In[•]:= D[P[bb[F], rbar, phi0, F], phi0]

Out[•]= $-1.8\, bb[F]$

In[•]:=