

## Step F ( $\chi$ )

In[77]:= **Fchi**[A\_, B\_, b\_, phi0\_, chi\_, eps\_, dz\_] =  
**Exp**[-dz chi] (phi0 + B / (b + chi) - A b eps / (b (1 + eps) + chi)) / (b + chi)

$$\text{Out[77]} = \frac{e^{-\chi dz} \left( \frac{B}{b+\chi} - \frac{A b \text{eps}}{\chi + b (1+\text{eps})} + \text{phi0} \right)}{b + \chi}$$

In[78]:= **Fchi**[A, B, b, phi0, chi, eps, dz]

$$\text{Out[78]} = \frac{e^{-\chi dz} \left( \frac{B}{b+\chi} - \frac{A b \text{eps}}{\chi + b (1+\text{eps})} + \text{phi0} \right)}{b + \chi}$$

In[79]:= **D**[Fchi[A, B, b, phi0, chi, eps, dz], A]

$$\text{Out[79]} = - \frac{b e^{-\chi dz} \text{eps}}{(b + \chi) (\chi + b (1 + \text{eps}))}$$

In[80]:= **Simplify**[D[Fchi[A, B, b, phi0, chi, eps, dz], A] ==  
-Exp[-dz chi] (b eps) / ((b + chi) (chi + b (1 + eps)))]

Out[80]= True

In[81]:= **D**[Fchi[A, B, b, phi0, chi, eps, dz], B]

$$\text{Out[81]} = \frac{e^{-\chi dz}}{(b + \chi)^2}$$

In[83]:= **D**[Fchi[A, B, b, phi0, chi, eps, dz], B]

$$\text{Out[83]} = \frac{e^{-\chi dz}}{(b + \chi)^2}$$

In[84]:= **Simplify**[D[Fchi[A, B, b, phi0, chi, eps, dz], B] == Exp[-dz chi] (b + chi)<sup>-2</sup>]

Out[84]= True

In[85]:= **D**[Fchi[A, B, b, phi0, chi, eps, dz], b]

$$\text{Out[85]} = \frac{e^{-\chi dz} \left( -\frac{B}{(b+\chi)^2} + \frac{A b \text{eps} (1+\text{eps})}{(\chi + b (1+\text{eps}))^2} - \frac{A \text{eps}}{\chi + b (1+\text{eps})} \right)}{b + \chi} - \frac{e^{-\chi dz} \left( \frac{B}{b+\chi} - \frac{A b \text{eps}}{\chi + b (1+\text{eps})} + \text{phi0} \right)}{(b + \chi)^2}$$

In[86]:= **Simplify**[  
 $e^{-dz \chi} \left( \frac{B}{b + \chi} - \frac{A b \text{eps}}{\chi + b (1 + \text{eps})} + \text{phi0} \right) == \text{Fchi}[A, B, b, \text{phi0}, \chi, \text{eps}, dz] (b + \chi)$ ]

Out[86]= True

$$\text{In[97]:= Simplify}\left[\frac{A b \text{eps} (1 + \text{eps})}{(\text{chi} + b (1 + \text{eps}))^2} - \frac{A \text{eps}}{\text{chi} + b (1 + \text{eps})}\right]$$

$$\text{Out[97]= } -\frac{A \text{chi} \text{eps}}{(b + \text{chi} + b \text{eps})^2}$$

$$\text{In[115]:= Simplify}\left[D[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], b] == \left(-\text{Exp}[-dz \text{chi}] \left(\frac{B}{(b + \text{chi})^2} + \frac{A \text{chi} \text{eps}}{(b + \text{chi} + b \text{eps})^2}\right) - F\text{chi}[A, B, b, \text{phi0}, \text{chi}, dz]\right) / (b + \text{chi})\right]$$

Out[115]= True

$$\text{In[88]:= D}[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], \text{phi0}]$$

$$\text{Out[88]= } \frac{e^{-\text{chi} dz}}{b + \text{chi}}$$

$$\text{In[89]:= Simplify}[D[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], \text{phi0}] == \text{Exp}[-dz \text{chi}] / (b + \text{chi})]$$

Out[89]= True

$$\text{In[102]:= D}[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], \text{chi}]$$

$$\text{Out[102]= } \frac{e^{-\text{chi} dz} \left(-\frac{B}{(b + \text{chi})^2} + \frac{A b \text{eps}}{(\text{chi} + b (1 + \text{eps}))^2}\right)}{b + \text{chi}} - \frac{e^{-\text{chi} dz} \left(\frac{B}{b + \text{chi}} - \frac{A b \text{eps}}{\text{chi} + b (1 + \text{eps})} + \text{phi0}\right) dz e^{-\text{chi} dz} \left(\frac{B}{b + \text{chi}} - \frac{A b \text{eps}}{\text{chi} + b (1 + \text{eps})} + \text{phi0}\right)}{(b + \text{chi})^2}$$

$$\text{In[109]:= Simplify}\left[\left(-\frac{e^{-\text{chi} dz} \left(\frac{B}{b + \text{chi}} - \frac{A b \text{eps}}{\text{chi} + b (1 + \text{eps})} + \text{phi0}\right)}{(b + \text{chi})^2} - \frac{dz e^{-\text{chi} dz} \left(\frac{B}{b + \text{chi}} - \frac{A b \text{eps}}{\text{chi} + b (1 + \text{eps})} + \text{phi0}\right)}{b + \text{chi}}\right) / F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz]\right]$$

$$\text{Out[109]= } -\frac{1 + b dz + \text{chi} dz}{b + \text{chi}}$$

$$\text{In[114]:= Simplify}\left[D[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], \text{chi}] == \left(e^{-\text{chi} dz} \left(\frac{A b \text{eps}}{(\text{chi} + b (1 + \text{eps}))^2} - \frac{B}{(b + \text{chi})^2}\right) - (1 + (b + \text{chi}) dz) F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz]\right) / (b + \text{chi})\right]$$

Out[114]= True

$$\text{In[95]:= Simplify}[D[F\text{chi}[A, B, b, \text{phi0}, \text{chi}, \text{eps}, dz], \text{eps}]]$$

$$\text{Out[95]= } -\frac{A b e^{-\text{chi} dz}}{(b + \text{chi} + b \text{eps})^2}$$

In[96]:= **Simplify**[**D**[**Fchi**[**A**, **B**, **b**, **phi0**, **chi**, **eps**, **dz**], **eps**] == - (**A b**  $e^{-\chi dz}$ ) / (**b + chi + b eps**)<sup>2</sup>]

Out[96]= **True**

In[92]:= **D**[**Fchi**[**A**, **B**, **b**, **phi0**, **chi**, **eps**, **dz**], **dz**]

Out[92]= 
$$-\frac{\chi e^{-\chi dz} \left( \frac{B}{b+\chi} - \frac{A b \text{eps}}{\chi+b(1+\text{eps})} + \text{phi0} \right)}{b+\chi}$$

In[93]:= **Simplify**[**D**[**Fchi**[**A**, **B**, **b**, **phi0**, **chi**, **eps**, **dz**], **dz**] == - **chi** **Fchi**[**A**, **B**, **b**, **phi0**, **chi**, **dz**]

Out[93]= **True**