Electronic Companion – Reliability-Constrained Distribution Network Expansion Planning with an AC Load Flow Model: A Mixed-Integer Linear Programming Approach

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This appendix presents the piecewise linearization used for the quadratic terms in (24) and (30). Such terms are represented by the generic form z^2, where z is a continuous variable. If z denotes a variable that can take both positive and negative values, the linearization is reduced to the positive orthant, i.e., $|z|^2$ is linearized rather than z^2. According to [1], $|z|$ can be equivalently represented by the sum of two auxiliary non-negative variables z^+ and z^- complying with the following set of linear constraints:

$$z^+ - z^- = z \tag{1}$$

$$0 \leq z^+ \leq \overline{z} \tag{2}$$

$$0 \leq z^- \leq \overline{z} \tag{3}$$

where $\overline{z}$ is the upper bound for $|z|$.

The linearization thus comprises two steps:

1) The replacement of z^2 in (24) and (30) with $\sum_{\kappa=1}^{K} \sigma_{z,\kappa}\Delta_{z,\kappa}$, where K is the number of blocks into which $|z|$ is discretized; $\sigma_{z,\kappa}$ is the slope of $|z|^2$ in the κth block, and $\Delta_{z,\kappa}$ is a continuous variable representing the contribution of the κth block to the value of $|z|$. The slopes are defined as $\sigma_{z,\kappa} = \frac{1}{\bar{\Delta}_{z,\kappa}}\left[\left(\sum_{\nu=1}^{\kappa}\bar{\Delta}_{z,\nu}\right)^2 - \left(\sum_{\nu=1}^{\kappa-1}\bar{\Delta}_{z,\nu}\right)^2\right], \forall\kappa = 1,\ldots,K,$ where $\bar{\Delta}_{z,\kappa}$ is the width of the κth block.
2) The incorporation of (1)–(3) and the following constraints:

$$z^+ + z^- = \sum_{\kappa=1}^{K} \Delta_{z,\kappa} \tag{4}$$

$$0 \leq \Delta_{z,\kappa} \leq \bar{\Delta}_{z,\kappa}; \forall\kappa = 1,\ldots,K. \tag{5}$$

The relationship between z^+, z^-, and $\Delta_{z,\kappa}$ is modeled in (4) whereas the upper and lower bounds for variables $\Delta_{z,\kappa}$ are set in (5).

Note that if z denotes a non-negative variable, auxiliary variables z^+ and z^- and constraints (1)–(3) are no longer needed and the left-hand side of (4) can be replaced with z.

REFERENCES

[1] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Programming*. Belmont, MA, USA: Athena Scientific, 1997.