Electronic Companion – Reliability-Constrained Distribution Network Expansion Planning with an AC Load Flow Model: A Mixed-Integer Linear Programming Approach

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This appendix presents the piecewise linearization used for the quadratic terms in (24) and (30). As well as the set of restrictions to linearize the products of binary variables within the right-hand side of (37).

A. *Linearization used for the quadratic terms in (24) and (30)*

Such terms are represented by the generic form z^2, where z is a continuous variable. If z denotes a variable that can take both positive and negative values, the linearization is reduced to the positive orthant, i.e., $|z|^2$ is linearized rather than z^2. According to [1], $|z|$ can be equivalently represented by the sum of two auxiliary non-negative variables z^+ and z^- complying with the following set of linear constraints:

$$z^+ - z^- = z \tag{1}$$

$$0 \leq z^+ \leq \overline{z} \tag{2}$$

$$0 \leq z^- \leq \overline{z} \tag{3}$$

where $\overline{z}$ is the upper bound for $|z|$.

The linearization thus comprises two steps:

1) The replacement of z^2 in (24) and (30) with $\sum\limits_{\kappa=1}^{K} \sigma_{z,\kappa}\Delta_{z,\kappa}$, where K is the number of blocks into which $|z|$ is discretized; $\sigma_{z,\kappa}$ is the slope of $|z|^2$ in the κth block, and $\Delta_{z,\kappa}$ is a continuous variable representing the contribution of the κth block to the value of $|z|$. The slopes are defined as $\sigma_{z,\kappa} = \dfrac{1}{\bar{\Delta}_{z,\kappa}}\left[\left(\sum\limits_{\nu=1}^{\kappa}\bar{\Delta}_{z,\nu}\right)^2 - \left(\sum\limits_{\nu=1}^{\kappa-1}\bar{\Delta}_{z,\nu}\right)^2\right], \forall \kappa = 1,\ldots,K,$ where $\bar{\Delta}_{z,\kappa}$ is the width of the κth block.
2) The incorporation of (1)–(3) and the following constraints:

$$z^+ + z^- = \sum_{\kappa=1}^{K} \Delta_{z,\kappa} \tag{4}$$

$$0 \leq \Delta_{z,\kappa} \leq \bar{\Delta}_{z,\kappa}; \forall \kappa = 1,\ldots,K. \tag{5}$$

The relationship between z^+, z^-, and $\Delta_{z,\kappa}$ is modeled in (4) whereas the upper and lower bounds for variables $\Delta_{z,\kappa}$ are set in (5).

Note that if z denotes a non-negative variable, auxiliary variables z^+ and z^- and constraints (1)–(3) are no longer needed and the left-hand side of (4) can be replaced with z.

B. *Set of restrictions to linearize the products of binary variables*

Within right hand side of (37) it presents non-linearities due to the multiplication of binary variables $y^{ES}_{i,t}$ and $y^{R}_{i,c,t}$. To deal with this situation the set of restrictions (6)–(3) is added by each variable product as follows:

$$v \leq w_a \tag{6}$$

$$v \leq w_b \tag{7}$$

$$v \geq w_a + w_b - 1 \tag{8}$$

Where v is a positive auxiliary variable that represents the product of the binary variables w_a,w_b. The number of auxiliary variables v and the associated set of restrictions will depend on the possible combinations of the transformer alternatives that can be added.

References

[1] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Programming*. Belmont, MA, USA: Athena Scientific, 1997.