Appendix – Reliability-Based Expansion Planning of AC Distribution Networks Using a Mixed-Integer Linear Programming Model

Alejandra Tabares, *Member, IEEE,* Gregorio Muñoz-Delgado, *Member, IEEE,* John F. Franco, *Senior Member, IEEE,* José M. Arroyo, *Fellow, IEEE,* and Javier Contreras, *Fellow, IEEE*

This appendix presents the piecewise linear approximation of the quadratic terms in (29) and (35) and the exact linearization scheme used for the products of binary variables in (42).

A. Linearization of the Quadratic Terms in (29) and (35)

For expository purposes, the quadratic terms in (29) and (35) are represented by the generic form z^2, where z is a continuous variable. If z denotes a variable that can take both positive and negative values, the linearization is reduced to the positive orthant, i.e., $|z|^2$ is linearized rather than z^2. According to [EC.1], $|z|$ can be equivalently represented by the sum of two auxiliary non-negative variables z^+ and z^- complying with the following set of linear constraints:

$$z^+ - z^- = z \tag{EC.1}$$

$$0 \leq z^+ \leq \overline{z} \tag{EC.2}$$

$$0 \leq z^- \leq \overline{z} \tag{EC.3}$$

where $\overline{z}$ is the upper bound for $|z|$.

The piecewise linearization thus comprises two steps:

1) The replacement of z^2 in (29) and (35) with $\sum\limits_{\kappa=1}^{K} \sigma_{z,\kappa}\Delta_{z,\kappa}$, where K is the number of blocks into which $|z|$ is discretized, $\sigma_{z,\kappa}$ is the slope of $|z|^2$ in the κth block, and $\Delta_{z,\kappa}$ is a continuous variable representing the contribution of the κth block to the value of $|z|$. The slopes are defined as $\sigma_{z,\kappa} = \dfrac{1}{\bar{\Delta}_{z,\kappa}}\left[\left(\sum\limits_{\nu=1}^{\kappa}\bar{\Delta}_{z,\nu}\right)^2 - \left(\sum\limits_{\nu=1}^{\kappa-1}\bar{\Delta}_{z,\nu}\right)^2\right], \kappa = 1,\ldots,K,$ where $\bar{\Delta}_{z,\kappa}$ is the width of the κth block.
2) The incorporation of (EC.1)–(EC.3) and the following constraints:

$$z^+ + z^- = \sum_{\kappa=1}^{K} \Delta_{z,\kappa} \tag{EC.4}$$

$$0 \leq \Delta_{z,\kappa} \leq \bar{\Delta}_{z,\kappa};\ \kappa = 1,\ldots,K. \tag{EC.5}$$

The relationship between z^+, z^-, and $\Delta_{z,\kappa}$ is modeled in (EC.4) whereas the upper and lower bounds for variables $\Delta_{z,\kappa}$ are set in (EC.5).
Note that if z denotes a non-negative variable, auxiliary variables z^+ and z^- and constraints (EC.1)–(EC.3) are no longer needed and the left-hand side of (EC.4) can be replaced with z.

B. Linearization of the Products of Two Binary Variables in (42)

A linear equivalent for the product of two binary variables $x \in \{0,1\}$ and $y \in \{0,1\}$ is obtained as follows [EC.2]:

1) Replace the product xy with a new binary variable z.
2) Introduce the following new expressions:

$$z \in \{0,1\} \tag{EC.6}$$

$$z \leq x \tag{EC.7}$$

$$z \leq y \tag{EC.8}$$

$$z \geq x + y - 1. \tag{EC.9}$$

Expression (EC.6) imposes the integrality of the newly added binary variable z. Expressions (EC.6)–(EC.8) make sure that if either x or y are equal to 0, the new binary variable z is also equal to 0. Analogously, expressions (EC.7)–(EC.9) ensure that z is equal to 1 if both binary variables, x and y, are equal to 1.

References

[EC.1] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Programming*. Belmont, MA, USA: Athena Scientific, 1997.

[EC.2] F. Glover and E. Woolsey, "Converting the 0-1 polynomial programming problem to a 0-1 linear program," *Oper. Res.*, vol. 22, no. 1, pp. 180–182, Jan.-Feb. 1974.