```
clc
clear
close all
```

Functions

```
function res = centralDifference(u,delta_x)
    res = (u(2)-u(1))/(2*delta_x) ;
end

function res = ShuOsher(phi,psi,delta_x)
    i = 2 ;
    f_pos = 1/2*(phi(i)+phi(i+1))*(psi(i+1)-psi(i))/delta_x ;
    f_neg = 1/2*(phi(i)+phi(i-1))*(psi(i)-psi(i-1))/delta_x ;

    res = (f_pos-f_neg)/delta_x ;
end
```

Constants

```
c_b1 = 0.1355 ;
c_b2 = 0.622 ;
sigma = 2/3 ;

Gamma = 1 ;
r_0 = 1 ;
K_v = 0.024 ;
```

Planar Mixing Layer

```
% deltas
delta_y = 0.05 ;
delta_t = 1e-3 ;

% initializing space and time axes
y = -20:delta_y:20 ;
t = 0:delta_t:100 ;
iy = length(y) ;
it = length(t) ;

% initializing arrays
u = zeros(1,iy) ;
nu_t = zeros(1,iy) ;
omega = zeros(1,iy) ;
dudt = zeros(1,iy) ;
dnu_tdt = zeros(1,iy) ;

tau_max = zeros(1,it) ;
```

```
nu_t_max = zeros(1,it) ;
omega_max = zeros(1,it) ;

% initial conditions
nu_t(:) = 1e-8 ;
u(1:find(y==0)) = -1 ;
u(find(y==0):iy) = 1 ;

% solver loop
for i = 1:it
    % progress
    if mod(t(i), 1) == 0
        time = t(i)
    end

    % saving data at t = 50,100
    if i == find(t==50)
        u_50 = u ;
        nu_t_50 = nu_t ;
        yOmega_max_50 = y*omega_max(i-1) ;
        nu_tOmega_max_50 = nu_t*omega_max(i-1) ;
    elseif i == find(t==100)
        u_100 = u ;
        nu_t_100 = nu_t ;
        yOmega_max_100 = y*omega_max(i-1) ;
        nu_tOmega_max_100 = nu_t*omega_max(i-1) ;
    end

    % spatial loop
    for j = 2:iy-1
        omega(j) = -centralDifference(u([j-1 j+1]),delta_y) ;
        dudt(j) = ShuOsher(nu_t(j-1:j+1),u(j-1:j+1),delta_y) ;
        dnu_tdt(j) = c_b1*abs(omega(j))*nu_t(j) + 1/sigma *
((1+c_b2)*ShuOsher(nu_t(j-1:j+1),nu_t(j-1:j+1),delta_y) - c_b2 * nu_t(j) *
ShuOsher([1 1 1],nu_t(j-1:j+1),delta_y)) ;
    end

    % saving data at each time step
    tau_max(i) = max(abs(-nu_t.*omega)) ;
    nu_t_max(i) = max(nu_t) ;
    omega_max(i) = max(abs(omega)) ;

    % update
    u = u+ delta_t*dudt ;
    nu_t = nu_t + delta_t*dnu_tdt ;
end
```

```
time =
0
time =
1
```

```
time =
2
time =
3
time =
4
time =
5
time =
6
time =
7
time =
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time =
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time =
95
time =
96
time =
97
```
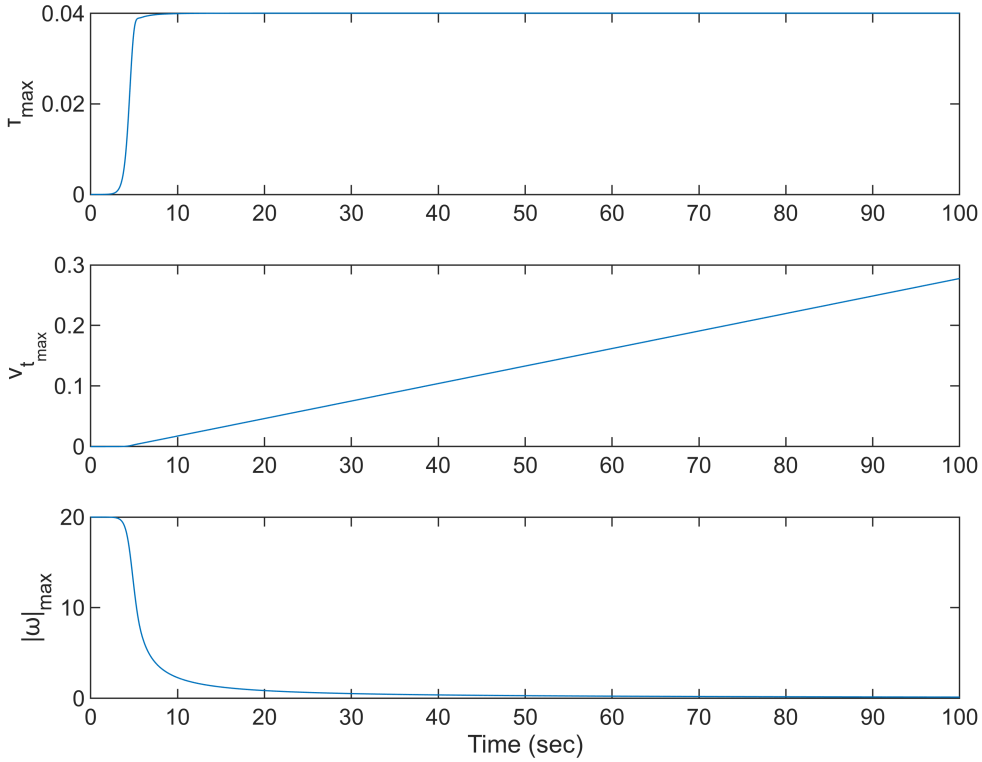
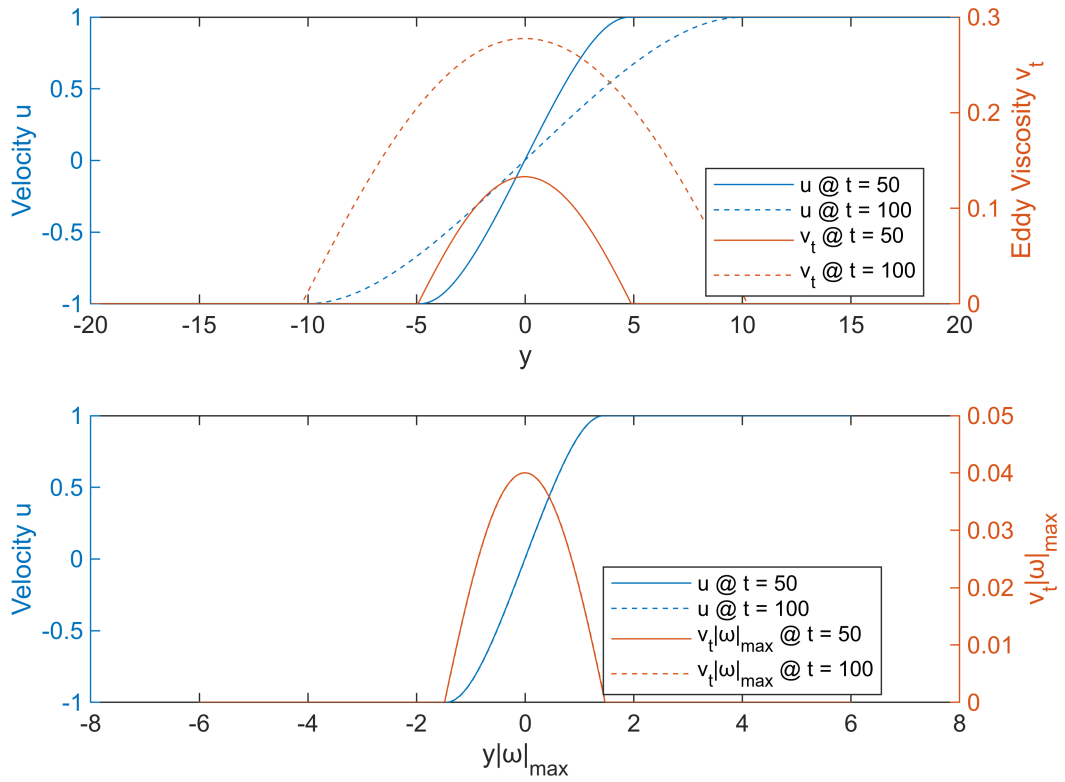

```
time =
98
time =
99
time =
100
```

Planar Mixing Layer Plots

```
% deltas
delta_y = 0.05 ;
delta_t = 1e-3 ;

% initializing space and time axes
y = -20:delta_y:20 ;
t = 0:delta_t:100 ;

figure
subplot(3,1,1)
plot(t,tau_max)
ylabel('τ_m_a_x')
subplot(3,1,2)
plot(t,nu_t_max)
ylabel('v_{t_m_a_x}')
subplot(3,1,3)
plot(t,omega_max)
ylabel('|ω|_m_a_x')
xlabel('Time (sec)')
```




```
%sgtitle('τ_m_a_x, v_{t_m_a_x}, and |ω|_m_a_x vs t')

figure
subplot(2,1,1)
yyaxis left
plot(y,u_50,y,u_100)
ylabel('Velocity u')
yyaxis right
plot(y,nu_t_50,y,nu_t_100)
ylabel('Eddy Viscosity v_t')
legend('u @ t = 50','u @ t = 100','v_t @ t = 50','v_t @ t = 100','Location','best')
xlabel('y')
%title('u and v_t vs y')
subplot(2,1,2)
yyaxis left
plot(yOmega_max_50,u_50,yOmega_max_100,u_100)
ylabel('Velocity u')
yyaxis right
plot(yOmega_max_50,nu_tOmega_max_50,yOmega_max_100,nu_tOmega_max_100)
ylabel('v_t|ω|_m_a_x')
legend('u @ t = 50','u @ t = 100','v_t|ω|_m_a_x @ t = 50','v_t|ω|_m_a_x @ t =
100','Location','best')
xlabel('y|ω|_m_a_x')
```




```
%title('u and v_t|ω|_m_a_x vs y|ω|_m_a_x')
```

Planar Wake

```
% deltas
delta_y = 0.05 ;
delta_t = 5e-3 ;

% initializing space and time axes
y = -25:delta_y:25 ;
t = 0:delta_t:3000 ;
iy = length(y) ;
it = length(t) ;

% initializing arrays
nu_t = zeros(1,iy) ;
omega_pw = zeros(1,iy) ;
dudt_pw = zeros(1,iy) ;
dnu_tdt_pw = zeros(1,iy) ;

tau_max_pw = zeros(1,it) ;
nu_t_max_pw = zeros(1,it) ;
omega_max_pw = zeros(1,it) ;
u_deficit_max_pw = zeros(1,it) ;
```

```
% initial conditions
nu_t(:) = 1e-5 ;
u = 1 - exp(-25.*y.^2) ;

% solver loop
for i = 1:it
    % progress
    if mod(t(i), 50) == 0
        time = t(i)
    end

    % saving data at t = 1500,3000
    if i == find(t==1500)
        u_1500 = u ;
        nu_t_1500 = nu_t ;
    elseif i == find(t==3000)
        u_3000 = u ;
        nu_t_3000 = nu_t ;
    end

    % spatial loop
    for j = 2:iy-1
        omega_pw(j) = -centralDifference(u([j-1 j+1]),delta_y) ;
        dudt_pw(j) = ShuOsher(nu_t(j-1:j+1),u(j-1:j+1),delta_y) ;
        dnu_tdt_pw(j) = c_b1*abs(omega_pw(j))*nu_t(j) + 1/sigma * ((1+c_b2) *
ShuOsher(nu_t(j-1:j+1),nu_t(j-1:j+1),delta_y) - c_b2 * nu_t(j) * ShuOsher([1 1
1],nu_t(j-1:j+1),delta_y)) ;
    end

    % saving data at each time step
    tau_max_pw(i) = max(abs(-nu_t.*omega_pw)) ;
    nu_t_max_pw(i) = max(nu_t) ;
    omega_max_pw(i) = max(abs(omega_pw)) ;
    u_deficit_max(i) = max(1-u) ;

    % update
    u = u+ delta_t*dudt_pw ;
    nu_t = nu_t + delta_t*dnu_tdt_pw ;
end
```

```
time =
0
time =
50
time =
100
time =
150
time =
200
time =
250
time =
```

```
300
time =
350
time =
400
time =
450
time =
500
time =
550
time =
600
time =
650
time =
700
time =
750
time =
800
time =
850
time =
900
time =
950
time =
1000
time =
1050
time =
1100
time =
1150
time =
1200
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time =
1500
time =
1550
time =
1600
time =
1650
time =
1700
time =
1750
time =
1800
time =
1850
time =
```

```
1900
time =
1950
time =
2000
time =
2050
time =
2100
time =
2150
time =
2200
time =
2250
time =
2300
time =
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time =
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time =
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2500
time =
2550
time =
2600
time =
2650
time =
2700
time =
2750
time =
2800
time =
2850
time =
2900
time =
2950
time =
3000
```
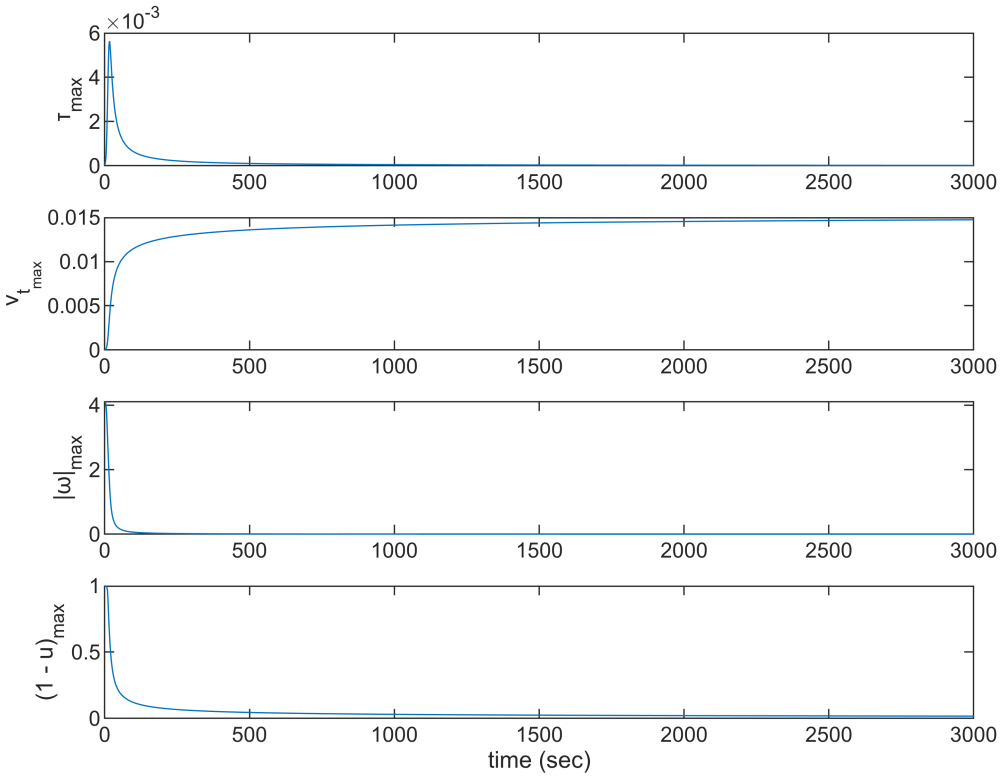

Planar Wake Plots

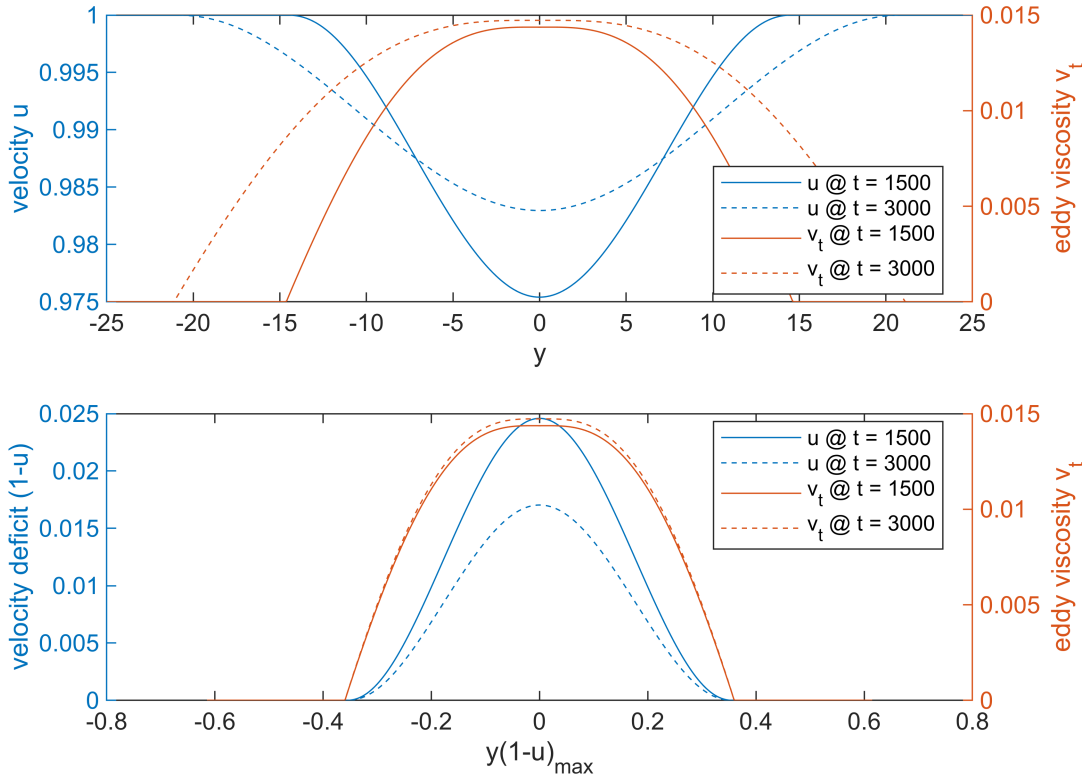
```
figure
subplot(4,1,1)
plot(t,tau_max_pw)
ylabel('τ_m_a_x')
subplot(4,1,2)
plot(t,nu_t_max_pw)
ylabel('v_{t_m_a_x}')
subplot(4,1,3)
plot(t,omega_max_pw)
ylabel('|ω|_m_a_x')
subplot(4,1,4)
```

```
plot(t,u_deficit_max)
ylabel('(1 - u)_m_a_x')
xlabel('time (sec)')
```




```
%sgtitle('τ_m_a_x, v_{t_m_a_x},|ω|_m_a_x, and (1 - u)_m_a_x vs t')

figure
subplot(2,1,1)
yyaxis left
plot(y,u_1500,y,u_3000)
ylabel('velocity u')
yyaxis right
plot(y,nu_t_1500,y,nu_t_3000)
ylabel('eddy viscosity v_t')
legend('u @ t = 1500','u @ t = 3000','v_t @ t = 1500','v_t @ t =
3000','Location','best')
xlabel('y')
%title('u and v_t vs y')
subplot(2,1,2)
yyaxis left
plot(y*u_deficit_max(t==1500),1-u_1500,y*u_deficit_max(t==3000),1-u_3000)
ylabel('velocity deficit (1-u)')
yyaxis right
plot(y*u_deficit_max(t==1500),nu_t_1500,y*u_deficit_max(t==3000),nu_t_3000)
ylabel('eddy viscosity v_t')
legend('u @ t = 1500','u @ t = 3000','v_t @ t = 1500','v_t @ t =
3000','Location','best')
```

```
xlabel('y(1-u)_m_a_x')
```




```
%title('1-u and v_t vs y(1 - u)_m_a_x')
```

Mature Vortex

```
% deltas
delta_r = 0.2 ;
delta_t = 5e-3 ;

% initializing space and time axes
r = 0:delta_r:100 ;
t = 0:delta_t:2000 ;
ir = length(r) ;
it = length(t) ;

% initializing arrays
omega_mv = zeros(1,ir) ;
dudt_mv = zeros(1,ir) ;
dnu_tdt_mv = zeros(1,ir) ;
dnu_tdr = zeros(1,ir) ;

u_max = zeros(1,it) ;
nu_t_max_mv = zeros(1,it) ;
omega_max_mv = zeros(1,it) ;
```

```
% initial conditions
nu_t = max(K_v*Gamma*(1-r.^2./r_0^2),1e-8);
u = Gamma./(2*pi*r).*(1 - exp(-r.^2/r_0^2)) ;

% solver loop
for i = 1:it
    % progress
    if mod(t(i), 20) == 0
        time = t(i)
    end

    % saving data at t = 0,20,50,200,2000
    if i == find(t==0)
        Gamma_0 = 2*pi*u.*r ;
        nu_t_0 = nu_t ;
    elseif i == find(t==20)
        Gamma_20 = 2*pi*u.*r ;
        nu_t_20 = nu_t ;
    elseif i == find(t==50)
        Gamma_50 = 2*pi*u.*r ;
        nu_t_50 = nu_t ;
    elseif i == find(t==200)
        Gamma_200 = 2*pi*u.*r ;
        nu_t_200 = nu_t ;
    elseif i == find(t==2000)
        Gamma_2000 = 2*pi*u.*r ;
        nu_t_2000 = nu_t ;
    end

    u(1) = 0 ;
    dnu_tdr(1) = 0 ;
    % spatial loop
    for j = 2:ir-1
        dnu_tdr(j) = centralDifference(nu_t([j-1 j+1]),delta_r) ;
        omega_mv(j) = 1/r(j)*centralDifference(r([j-1 j+1]).*u([j-1
j+1]),delta_r) ;
        dudt_mv(j) = 1/r(j)^2 *
(ShuOsher(r(j-1:j+1).^2.*nu_t(j-1:j+1),u(j-1:j+1),delta_r) -
centralDifference(r([j-1 j+1]).*nu_t([j-1 j+1]).*u([j-1 j+1]),delta_r)) ;
        dnu_tdt_mv(j) = c_b1*abs(omega_mv(j))*nu_t(j) + 1/sigma * (nu_t(j)/
r(j)*dnu_tdr(j) + (1+c_b2)*ShuOsher(nu_t(j-1:j+1),nu_t(j-1:j+1),delta_r) -
c_b2*nu_t(j)*ShuOsher([1 1 1],nu_t(j-1:j+1),delta_r)) ;
    end

    % saving data at each time step
    u_max(i) = max(u) ;
    nu_t_max_mv(i) = max(nu_t) ;
    omega_max_mv(i) = max(abs(omega_mv)) ;

    % update
```

```
    u = u + delta_t*dudt_mv ;
    nu_t = nu_t + delta_t*dnu_tdt_mv ;
    nu_t(1) = nu_t(2) ;
end
```

```
time =
0
time =
20
time =
40
time =
60
time =
80
time =
100
time =
120
time =
140
time =
160
time =
180
time =
200
time =
220
time =
240
time =
260
time =
280
time =
300
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560
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1060
time =
1080
time =
1100
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time =
1200
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1840
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time =
1860
time =
1880
time =
1900
time =
1920
time =
1940
time =
1960
time =
1980
time =
2000
```
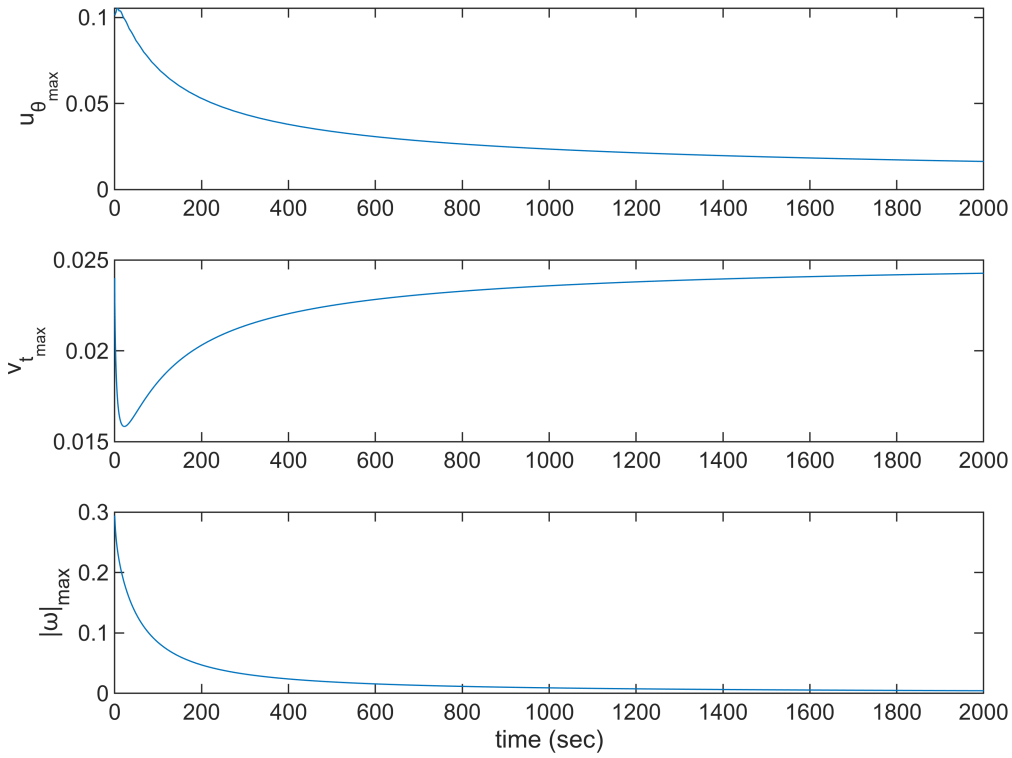
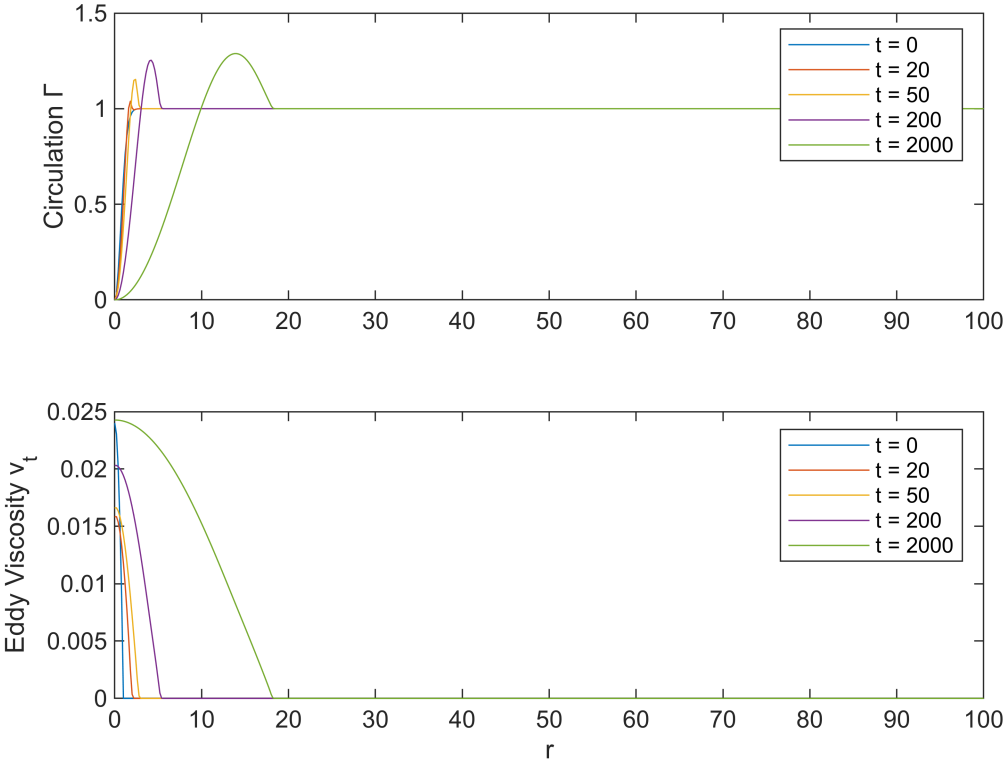

Mature Vortex Plots

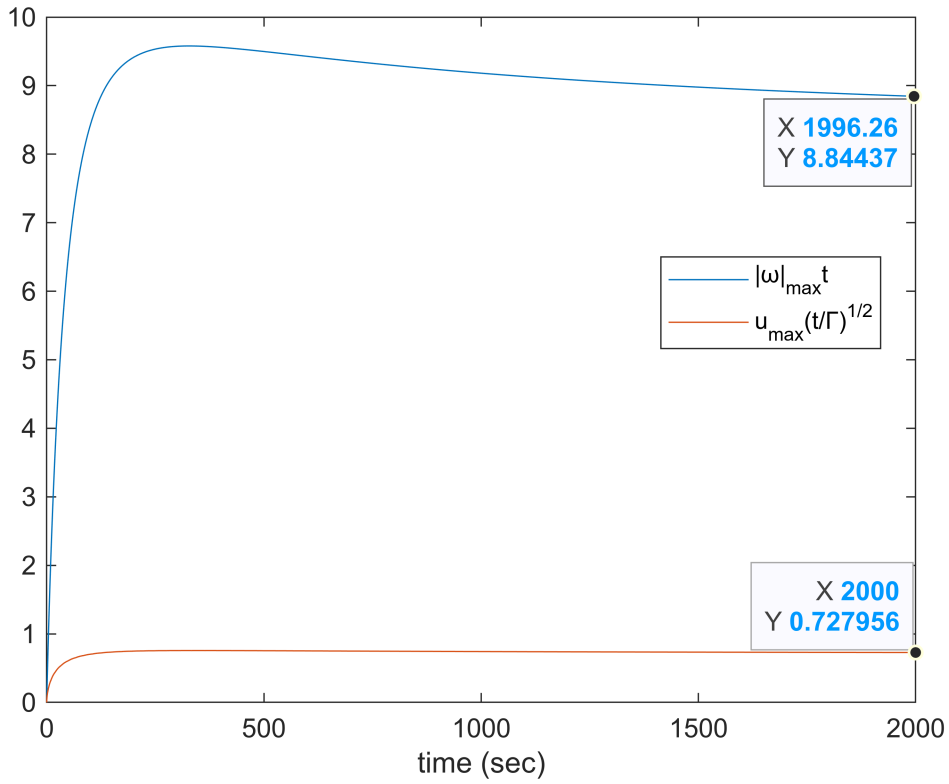
```
figure
subplot(3,1,1)
plot(t,u_max)
ylabel('u_{θ_m_a_x}')
subplot(3,1,2)
plot(t,nu_t_max_mv)
ylabel('v_{t_m_a_x}')
subplot(3,1,3)
plot(t,omega_max_mv)
ylabel('|ω|_m_a_x')
xlabel('time (sec)')
```




```
%sgtitle('τ_m_a_x, v_{t_m_a_x}, and |ω|_m_a_x vs t')

figure
subplot(2,1,1)
plot(r,Gamma_0,r,Gamma_20,r,Gamma_50,r,Gamma_200,r,Gamma_2000)
ylabel('Circulation Γ')
legend('t = 0','t = 20','t = 50','t = 200','t = 2000')
subplot(2,1,2)
plot(r,nu_t_0,r,nu_t_20,r,nu_t_50,r,nu_t_200,r,nu_t_2000)
ylabel('Eddy Viscosity v_t')
legend('t = 0','t = 20','t = 50','t = 200','t = 2000')
xlabel('r')
```




```
%sgtitle('Γ and v_t vs r')

figure
plot(t,omega_max_mv.*t,t,u_max.*sqrt(t/Gamma))
xlabel('time (sec)')
legend('|ω|_m_a_xt','u_m_a_x(t/Γ)^1^/^2','Location','best')
```


10
9
8
7
6
5
4
3
2
1
0
0
500
1000
1500
2000
time (sec)
X 1996.26
Y 8.84437
|ω|max t
umax(t/Γ)1/2
X 2000
Y 0.727956