Verification of the Spalart-Allmaras Eddy-Viscosity Model in MATLAB

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This report documents the reproduction of the results of Spalart and Allmaras [1] and Spalart and Garbaruk [2] utilizing a MATLAB [3] code that discretizes the governing equations with by applying a central differencing technique…

# Nomenclature

*cb1* = first empirical constant in the model

*cb2* = second empirical constant in the model

*i* = time-like index

*j* = space-like index

*r* = radial coordinate

*t* = time

*u* = mean velocity in *x* direction

*uj* = mean velocity components

*uθ* =tangential velocity

*x* = streamwise coordinate

*xj*= Cartesian coordinates

*y* = coordinate normal to streamwise direction

*νt* = eddy viscosity

*ω* = vorticity

*σ* = turbulent Prandtl number

# Introduction

THE Spalart-Allmaras (SA) eddy-viscosity model (EVM) was first introduced by Spalart and Allmaras [1] in 1994 as a one-equation EVM to further improve the accuracy of turbulence modeling over algebraic models while being computationally cheaper than the k-ω and k-ε two-equation models which had been developed in the 70s and 80s. Since its original publication, the SA model has grown to become one of, if not the most widely used turbulence models in computation fluid dynamics due to its balance of accuracy and computational cost. As a means of better understanding the SA model this paper attempts to recreate and validate the results of the original paper using MATLAB. Three different one-dimensional flow cases were solved; a planar mixing layer, a planar wake, and a mature vortex from Spalart and Garbaruk [2].

# Governing Equations

To solve these three cases, the high-Reynolds number version of the SA model was utilized which takes the following form

For the two planar cases, equation (1) is can be simplified to

which can then be combined with the following two equations

to fully define the dynamics of this problem. The axisymmetric case has a similar set of governing equations, but has been modified due to the rotational nature of the problem to the following

While equations (2)-(7) are perfectly adequate, some rearrangements can be made to make the discrete forms more agreeable. The most extensive change is to equations (2) and (6) which can be made into the equivalent forms

with a less major change being made to equation (5)

Armed with equations (3)-(5) and (7)-(10) the three cases now have the underlying physics modeled in a way that a discrete form can be constructed conveniently within MATLAB.

# Discretization

To discretize this problem an explicit scheme was used in time while central differencing was used in space. The form of the explicit scheme is as follows

where φ is an arbitrary function. The central difference scheme for the first derivative is defined as

where x can be any space like term such as y or r.

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|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 1 Transitions selected for thermometry | | | | | | |
|  | Transition | | |  |  | |
| Line | n² |  | *J*² | Frequency, cm-1 | *FJ*, cm-1 | *G*n, cm-1 |
| a | 0 | P12 | 2.5 | 44069.416 | 73.58 | 948.66 |
| b | 1 | R2 | 2.5 | 42229.348 | 73.41 | 2824.76 |
| c | 2 | R21 | 805 | 40562.179 | 71.37 | 4672.68 |
| d | 0 | R2 | 23.5 | 42516.527 | 1045.85 | 948.76 |

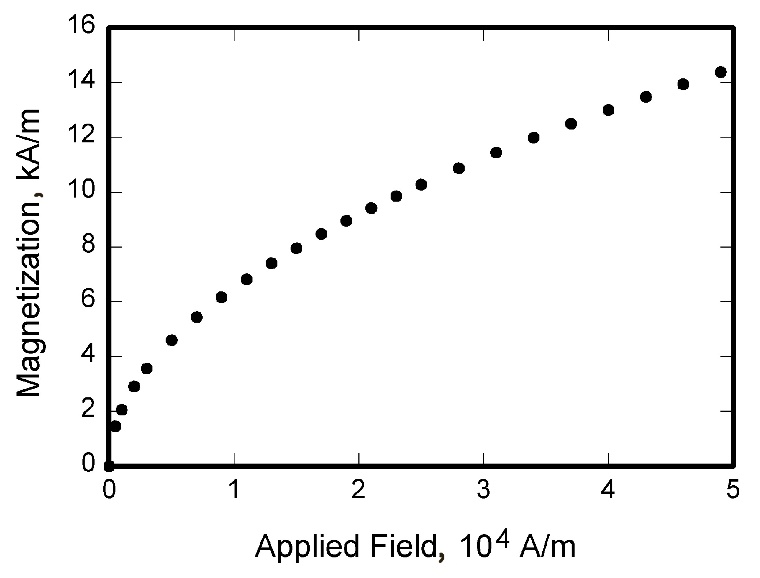


Fig. 1 Magnetization as a function of applied fields.

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# Appendix

An Appendix, if needed, appears **before** research funding information and other acknowledgments.

# References

*Reports, Theses, and Individual Papers*

[11] Chapman, G. T., and Tobak, M., “Nonlinear Problems in Flight Dynamics,” NASA TM-85940, 1984.

[12] Brandis, A. M., Johnston, C. O., and Cruden, B. A., “Nonequilibrium Radiation for Earth Entry,” AIAA Paper 2016-3690, June 2016.

[13] Steger, J. L., Jr., Nietubicz, C. J., and Heavey, J. E., “A General Curvilinear Grid Generation Program for Projectile Configurations,” U.S. Army Ballistic Research Lab., Rept. ARBRL-MR03142, Aberdeen Proving Ground, MD, Oct. 1981.

[14] Tseng, K., “Nonlinear Green’s Function Method for Transonic Potential Flow,” Ph.D. Dissertation, Aeronautics and Astronautics Dept., Boston Univ., Cambridge, MA, 1983.

Government agency reports do not require locations. For reports such as NASA TM-85940, neither insert nor delete dashes; leave them as provided. Place of publication *should* be given, although it is not mandatory, for military and company reports. Always include a city and state for universities. Papers need only the name of the sponsor; neither the sponsor’s location nor the conference name and location is required. *Do not confuse proceedings references with conference papers*.

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[15] Atkins, C. P., and Scantelbury, J. D., “The Activity Coefficient of Sodium Chloride in a Simulated Pore Solution Environment,” *Journal of Corrosion Science and Engineering* [online journal], Vol. 1, No. 1, Paper 2, URL: <http://www.cp/umist.ac.uk/JCSE/vol1/vol1.html> [retrieved 13 April 1998].

[16] Vickers, A., “10-110 mm/hr Hypodermic Gravity Design A,” *Rainfall Simulation Database* [online database], URL: <http://www.geog.le.ac.uk/bgrg/lab.htm> [retrieved 15 March 2006].

Break website addresses after punctuation, and do not hyphenate at line breaks.

*Computer Software*

[17] TAPP, Thermochemical and Physical Properties, Software Package, Ver. 1.0, E. S. Microware, Hamilton, OH, 1992.

Include a version number and the company name and location of software packages.

*Patents*

Patents appear infrequently. Be sure to include the patent number and date.

[18] Scherrer, R., Overholster, D., and Watson, K., Lockheed Corp., Burbank, CA, U.S. Patent Application for a “Vehicle,” Docket No. P-01-1532, filed 11 Feb. 1979.

1. Graduate Research Assistant, Aerospace Engineering, AIAA Student Member [↑](#footnote-ref-1)