Applying the Spalart-Allmaras Eddy-Viscosity Model in MATLAB to Benchmark Cases

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This report documents the reproduction of the results of Spalart and Allmaras [1] and Spalart and Garbaruk [2] utilizing a MATLAB [3] code that discretizes the governing equations by applying central differencing as the original paper did. The results have then been compared to plots from the original papers as a means of ensuring the validity of the MATLAB script generated by the author.

# Nomenclature

*cb1* = first empirical constant in the model

*cb2* = second empirical constant in the model

*f* = central differencing sub functions

*i* = time-like index

*j* = space-like index

*Kν* = model tuning parameter

*r* = radial coordinate

*r0* = vortex size parameter

*t* = time

*u* = mean velocity in *x* direction

*uj* = mean velocity components

*uθ* =tangential velocity

*x* = streamwise coordinate

*xj*= Cartesian coordinates

*y* = coordinate normal to stream wise direction

*Γ* = circulation

*νt* = eddy viscosity

*φ* = arbitrary function

*ψ* = arbitrary function

*ω* = vorticity

*σ* = turbulent Prandtl number

# Introduction

THE Spalart-Allmaras (SA) eddy-viscosity model (EVM) was first introduced by Spalart and Allmaras [1] in 1994 as a one-equation EVM to further improve the accuracy of turbulence modeling over algebraic models while being computationally cheaper than the k-ω and k-ε two-equation models which had been developed in the 70s and 80s. Since its original publication, the SA model has grown to become one of, if not the most widely used turbulence models in computation fluid dynamics due to its balance of accuracy and computational cost. As a means of better understanding the SA model this paper attempts to recreate and validate the results of the original paper using MATLAB. Three different one-dimensional flow cases were solved; a planar mixing layer, a planar wake, and a mature vortex from Spalart and Garbaruk [2].

# Governing Equations

To solve these three cases, the high-Reynolds number version of the SA model was utilized which takes the following form

For the two planar cases, equation (1) is can be simplified to

Which can then be combined with the following two equations

Where *cb1* = 0.1355 and *cb2* = 0.622. The axisymmetric case has a similar set of governing equations, but has been modified due to the rotational nature of the problem to the following

While equations (2)-(7) are perfectly adequate, some rearrangements can be made to make the discrete forms more agreeable. The most extensive change is to equations (2) and (6) which can be made into the equivalent forms

with a less major change being made to equation (5)

Armed with equations (3)-(5) and (7)-(10) the three cases now have the underlying physics modeled in a way that a discrete form can be constructed conveniently within MATLAB.

# Discretization

To discretize this problem an explicit scheme was used in time while central differencing was used in space. The form of the explicit scheme is as follows

Where *φ* is an arbitrary function. The central difference scheme for the first derivative is defined as

Where *x* can be any space like term such as *y* or *r*. To calculate the second derivative terms which are constructed as Shu-Osher fluxes a more expansive central difference scheme was used which is as follows

Where *ψ* is also an arbitrary function.

Applying equations (11)-(15) to the governing equations discussed above a MATLAB code can be made which can find results given some initial conditions.

# Results and Discussion

Below outlines the step value, domains, and initial conditions for each of the three benchmark cases and plots the quantities of interest. Plots from the original cases run by Spalart and Allmaras [1] and Spalart and Garbaruk [2] are also included to provide comparison.

#### A. Planar Mixing Layer

The step values for the first case are *Δy* = 0.05 and *Δt* = 1×10-3 with the domain being *y* = [-20, 20] and *t* = [0, 100]. The initial conditions for this case are as follows

The simulation resulted in the following quantities of interest

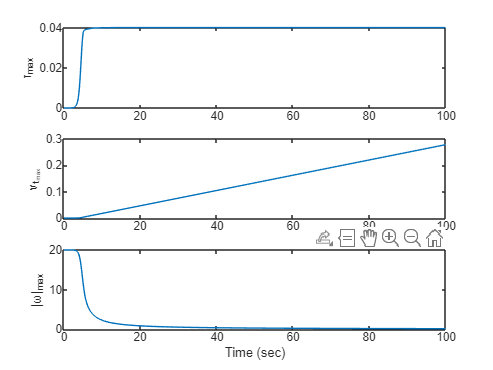


Fig. Maximum shear stress, eddy viscosity, and vorticity for planar mixing layer.

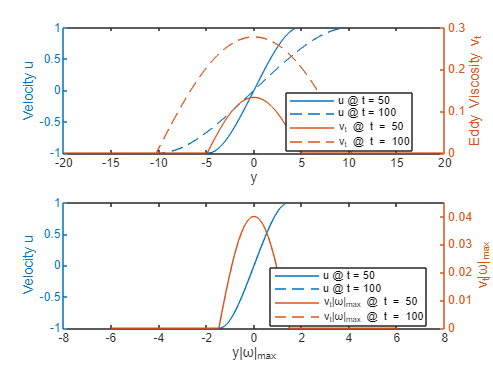


Fig. Velocity and eddy viscosity profiles for planar mixing layer.

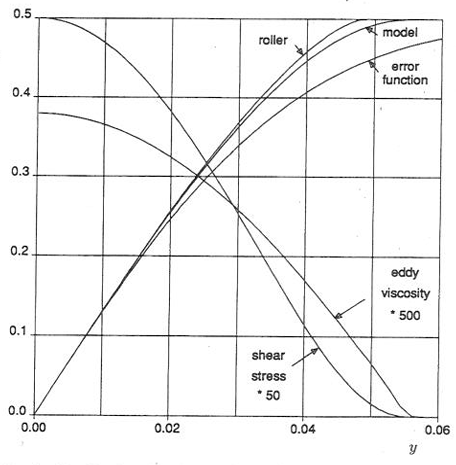


Fig. Velocity and eddy viscosity profiles from Spalart and Allmaras [2].

Investigating weighted results in Fig. 2 which is what Fig. 3 is plotted against as well we find that at *t* = 50 and *t* = 100 that *u*(0.0451) = 0.0526 and *u*(0.0433) = 0.0469 respectively which corresponds to the plot from Fig. 3 but is off by about one order of magnitude. The profiles shapes are qualitatively very which indicates that the scaling of the values being plotted is different and not that the MATLAB model itself is outright wrong.

## B. Planar Wake

The step values for the first case are *Δy* = 0.05 and *Δt* = 5×10-3 with the domain being *y* = [-25, 25] and *t* = [0, 3000]. The initial conditions for this case are as follows

The simulation resulted in the following quantities of interest.

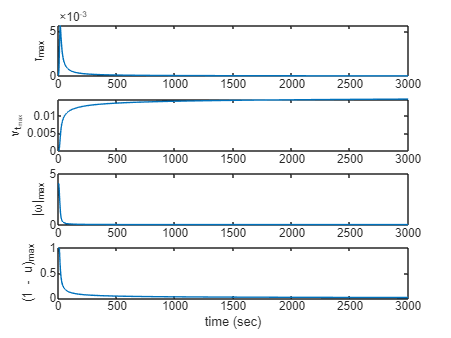


Fig. Maximum shear stress, eddy viscosity, vorticity, and velocity deficit for planar wake.

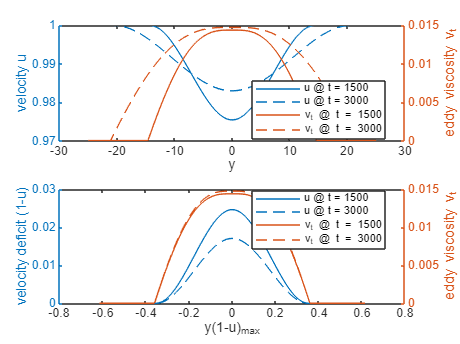


Fig. Velocity and eddy viscosity profiles for planar wake.

The planar wake presents an interest alternative case that utilizes the same solver as the planar mixing layer. The behavior here is what would be expected for a wake. There is an interesting deviation in the plot weighted against maximum velocity deficit in Fig. 5. Unlike the first case where the weighted plot did not evolve over time, here it can be observed that as time progresses the velocity deficit is decaying over time. Intuitively that makes sense as one would expect that a wake would decrease in strength the longer it exists as the wake dissipates its energy away.

## C. Mature Vortex

The step values for the first case are *Δr* = 0.2 and *Δt* = 5×10-3 with the domain being *y* = [0, 100] and *t* = [0, 2000]. The initial conditions for this case are as follows

Where *Kν* = 0.024, *Γ* = 1, and *r0* = 1.

The simulation resulted in the following quantities of interest.

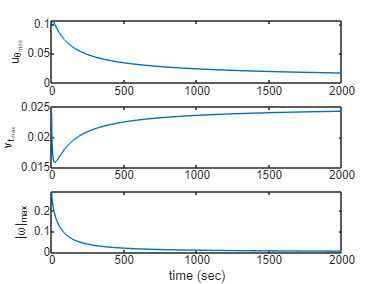


Fig. Maximum velocity, eddy viscosity, and vorticity for mature vortex.

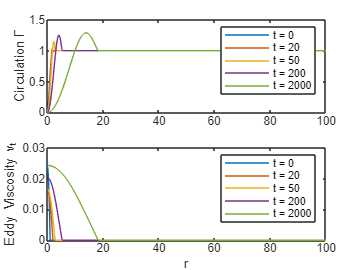


Fig. Circulation and eddy viscosity profile for mature vortex.

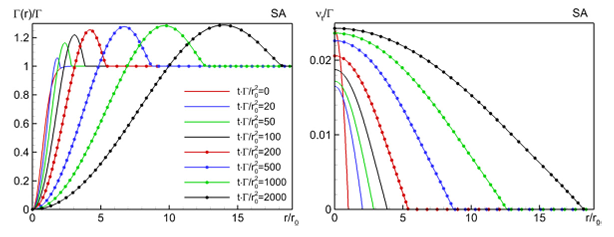


Fig. Profiles from Spalart and Garbaruk [2].

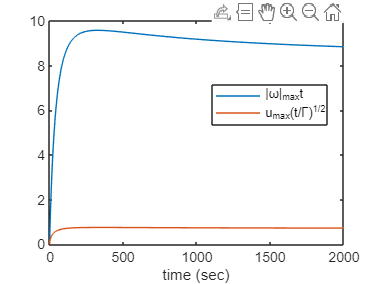


Fig. Self-similarity values for mature vortex.

Qualitatively the profile of the mature vortex seems to align well with Spalart and Garbaruk’s findings. Quantitatively this is best exemplified by the circulation profile for *t* = 2000 where the results shown in Fig. 7 settle to *Γ* ≈ 1 at *t* ≈ 18 which is the same as it is in Fig. 8. Both profiles also peak at a *Γ* ≈ 1.25. The self-similarity values of Fig. 9 also are in agreement with results from [2]. In that paper it was stated that *ωmaxt* = 8.5 and *Vmax(t/Γ)0.5* = 0.71 which the results of the MATLAB code are in agreement with getting *ωmaxt* = 8.84 and *Vmax(t/Γ)0.5* = 0.73 when *t* = 2000. The plots are still trending toward the correct similarity solution but did not get to a self-similar state in the 2000 second run time.

# Conclusion

This report has outlined the creation of benchmark results of the SA EVM model within MATLAB. The results found have been consistent with Spalart and Allmaras [1] as well as Spalart and Garbaruk [2] in all three cases with only minor differences from those original reports which are likely due to small differences in the numeric between the different codes. There is a large difference in the values of planar mixing layer plots but that seems to be a matter of representing the data more than the code itself which worked for the other two cases.

# References

*Reports, Theses, and Individual Papers*

[1] Spalart P. and Allmaras S., “A one-equation turbulence model for aerodynamic flows,” La Recherche Aérospatiale, 1994.

[2] Spalart P. and Garbaruk A.,“The Predictions of Common Turbulence Models in a Mature Vortex,” Flow, Turbulence and Combustion, 2019.

*Computer Software*

[3] MATLAB, Ver. R2024b, MathWorks, Natick, MD, 2024.

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