2x2x2 Planner Problem

The economy consists of 2 individuals, A and B, and two firms, X and Y. Each individual takes utility from a combination of consumption, k, and leisure, ℓ. Each individual is endowed with one unit of time which can be used toward leisure (either own or that of the other individual) and labor supply to the firms. Specifically, if individual A takes leisure ℓ_A, then the time $1 - \ell_A$ is available to be allocated to either the firms or to B's leisure. The utility functions are given by,

$$u_A(k, \ell) = \alpha k + \ell$$

$$u_B(k, \ell) = \ln(k) + \ln(\ell),$$

where α reflects A's marginal rate of substiution between consumption and leisure.

Firms X and Y take labor from the individuals and use it to produce the consumption good. The production technology is represented by the production functions,

$$F_X(h) = \sqrt{\gamma h}$$

$$F_Y(h) = \sqrt{h},$$

where h is labor input. γ reflects a productivity differential between firms X and Y.

A social planner chooses an *allocation* $(k_A, \ell_A, k_B, \ell_B, h_X, h_Y)$ so as to maximize the welfare criterion,

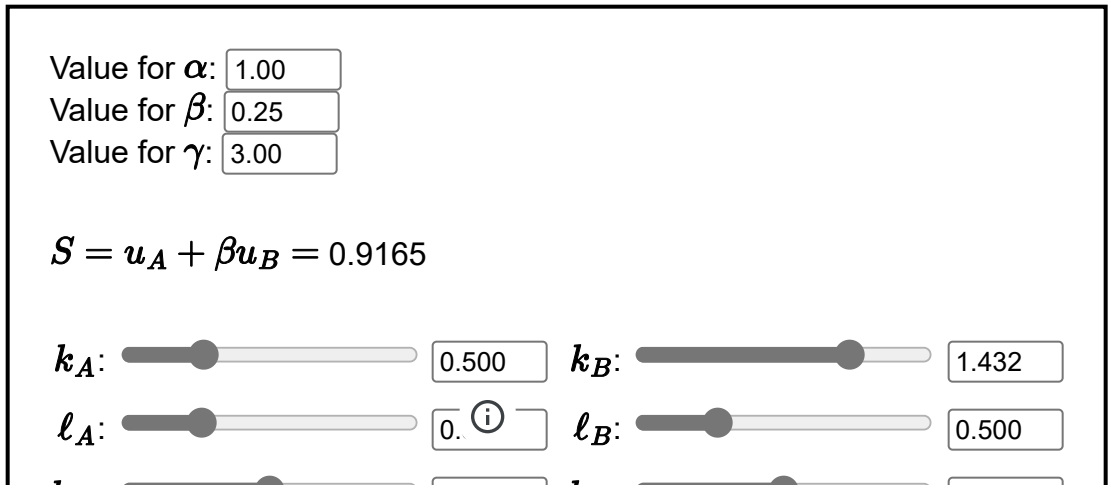
$$S(k_A, \ell_A, k_B, \ell_B, h_X, h_Y) = u_A(k_A, \ell_A) + \beta u_B(k_B, \ell_B),$$

subject to the *feasibility* constraints,

$$k_A + k_B = F_X(h_X) + F_Y(h_Y)$$

$$h_X + h_Y = 2 - \ell_A - \ell_B.$$

β is the weight the social planner places on individual B's utility relative to indvidual A.



h_X: 0.500 h_Y: 0.500

$u_A(k_A, \ell_A) =$ 1.000
$u_B(k_B, \ell_B) =$ -0.334
$h = 2 - \ell_A - \ell_B =$ 1.000
$k = F_X(h_X) + F_Y(h_Y) =$ 1.932

Last updated: 12/18/2023

Last edited November, 2023