ME751-Assignment 7

Problem 1. [SimEngine3D: MATLAB/Python/C] This problem builds on the problem in the previous assignment. The schematic of the mechanism is shown in Figure 1. The rigid body is subjected to a motion specified as $q(t) = \frac{\pi}{4}\cos(2t)$. Recycle as much as possible of the code you already generated.

Perform an Inverse Dynamics Analysis to compute the amount of torque that you would have to apply to the pendulum to make it move as indicated by the specified motion. Assume that $L = 2$, and the cross-section of the bar is a square of width 0.05. The density of the material is $\rho = 7,800$. All units are SI. For this problem, please provide a plot that displays the value of the torque as a function of time for $t \in [0, 10]$.

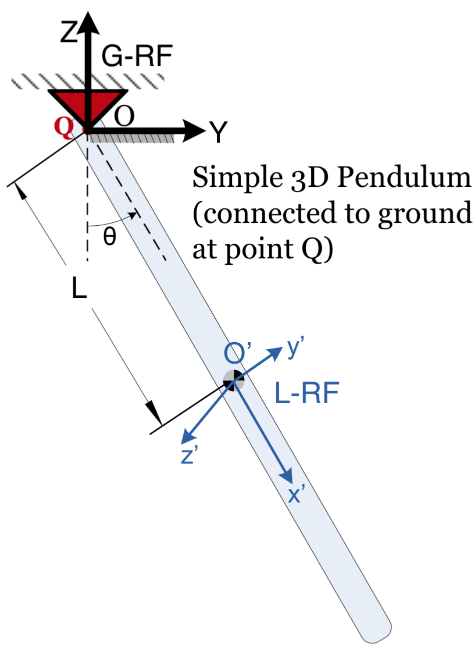

Figure 1: Pendulum with revolute joint.

Problem 2. Consider the following IVP (discussed in class, see also handout example on the computation of the Jacobian):

$$\begin{cases} \dot{x} = -x - \dfrac{4xy}{1+x^2} \\ \dot{y} = x(1 - \dfrac{y}{1+x^2}) \end{cases},$$

where

$$\begin{cases} x(0) = 0,\ y(0) = 2 \\ t \in [0, 20] \end{cases}.$$

Apply Backward Euler to find an approximation of the exact solution of this IVP. Generate plots of x and y, respectively, that you include as part of your HW.

Problem 3. For this problem, you will have to perform a convergence analysis, a concept explained at the end of the problem. Consider the following IVP:

$$\dot{y} = -y^2 - \frac{1}{t^4} \; ,$$

where

$$\begin{cases} y(1) = 1 \\ t \in [1, 10] \end{cases} .$$

a) Prove that the exact solution of this IVP is $y(t) = \dfrac{1}{t} + \dfrac{1}{t^2}\tan(\dfrac{1}{t} + \pi - 1)$ by showing that it satisfies both the scalar ODE above and the IC specified.

b) Generate the Backward Euler convergence plot for the above IVP

c) Generate the BDF convergence plot for the above IVP. Note:

   i) Display the convergence plot in the same figure you used for the Backward Euler analysis

   ii) Use the 4th order BDF formula in this exercise

   iii) Use the exact solution above to generate the required starting points for the BDF formula

d) Measure the slope of the two plots and comment whether the two values come in line with your expectations

Convergence analysis: The idea in a convergence analysis is to understand how the quality of the solution improves as you reduce the step size h. If the error gets cut in half when h is halved, you are dealing with a first order method. The method will converge with second order if when you decrease h by a factor of two, the error is reduced by a factor of four. Likewise, a third order method would reduce the solution by a factor of 1000, if you cut the step size by a factor of 10. And so on. Two important aspects: (i) how do you compute the error $|y(t_n) - y_n|$? and, (ii) where do you compute the error? For (i), in general, one would generate a "ground truth" solution by taking a tiny step-size h. Luckily, for our problem we have an analytical solution, so computing the error will be easy. For (ii), the error is often times evaluated at the end of the solution interval. For us, this would be $t = 10\,\mathrm{s}$.