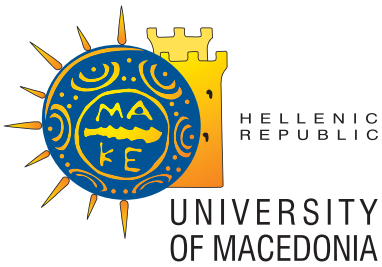

Probabilistic Modeling and Reasoning

Homework — 2

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February 2025

Problem 1

Details provided in the "README.txt" file.

Problem 2

$$p(x, d, e, t, l, b, a, s) = p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(t \mid a)p(a)p(s) \tag{1}$$

$$p(d) = \sum_{x,e,t,l,b,a,s \in \{0,1\}} p(x, d, e, t, l, b, a, s) \tag{2}$$

Part 1

$$p(d) = \sum_{x,e,t,l,b,a,s \in \{0,1\}} p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(t \mid a)p(a)p(s)$$

$$p(d) = \sum_{x,e,t,l,b,s \in \{0,1\}} p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(t \mid a = 0)p(a = 0)p(s) + p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(t \mid a = 1)p(a = 1)p(s)$$

$$p(d) = \sum_{x,e,t,l,b,s \in \{0,1\}} p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(s) \cdot \Big(p(t \mid a = 0)p(a = 0) + p(t \mid a = 1)p(a = 1) \Big)$$

$$p(d) = \sum_{x,e,t,l,b,s \in \{0,1\}} p(d \mid e, b)p(b \mid s)p(x \mid e)p(e \mid t, l)p(l \mid s)p(s) \cdot \Big(p(t \mid a = 0)0.99 + p(t \mid a = 1)0.01 \Big)$$

$$
\begin{aligned}
p(d) = \sum_{x,b,s \in \{0,1\}} & p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(e=1 \mid t=0,l=1)p(l=1 \mid s)p(s) \\
& \cdot \Big(p(t=0 \mid a=0)0.99 + p(t=0 \mid a=1)0.01 \Big) + \\
& p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(e=1 \mid t=1,l=0)p(l=0 \mid s)p(s) \\
& \cdot \Big(p(t=1 \mid a=0)0.99 + p(t=1 \mid a=1)0.01 \Big) + \\
& p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(e=1 \mid t=1,l=1)p(l=1 \mid s)p(s) \\
& \cdot \Big(p(t=1 \mid a=0)0.99 + p(t=1 \mid a=1)0.01 \Big) + \\
& p(d \mid e=0,b)p(b \mid s)p(x \mid e=0)p(e=0 \mid t=0,l=0)p(l=0 \mid s)p(s) \\
& \cdot \Big(p(t=0 \mid a=0)0.99 + p(t=0 \mid a=1)0.01 \Big)
\end{aligned}
$$

$$
\begin{aligned}
p(d) = \sum_{x,b,s \in \{0,1\}} & p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s) \\
& \cdot \Big(p(t=0 \mid a=0) \cdot 0.99 + p(t=0 \mid a=1) \cdot 0.01 \Big) + \\
& p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=0 \mid s)p(s) \\
& \cdot \Big(p(t=1 \mid a=0) \cdot 0.99 + p(t=1 \mid a=1) \cdot 0.01 \Big) + \\
& p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s) \\
& \cdot \Big(p(t=1 \mid a=0) \cdot 0.99 + p(t=1 \mid a=1) \cdot 0.01 \Big) + \\
& p(d \mid e=0,b)p(b \mid s)p(x \mid e=0)p(l=0 \mid s)p(s) \\
& \cdot \Big(p(t=0 \mid a=0) \cdot 0.99 + p(t=0 \mid a=1) \cdot 0.01 \Big)
\end{aligned}
$$

$$p(d) = \sum_{x,b,s \in \{0,1\}} p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s)\Big(0.99 \cdot 0.99 + 0.95 \cdot 0.01\Big) +$$
$$p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=0 \mid s)p(s)\Big(0.01 \cdot 0.99 + 0.05 \cdot 0.01\Big) +$$
$$p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s)\Big(0.01 \cdot 0.99 + 0.05 \cdot 0.01\Big) +$$
$$p(d \mid e=0,b)p(b \mid s)p(x \mid e=0)p(l=0 \mid s)p(s)\Big(0.99 \cdot 0.99 + 0.95 \cdot 0.01\Big)$$

$$p(d) = \sum_{x,b,s \in \{0,1\}} p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s) \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=0 \mid s)p(s) \cdot 0.0104 +$$
$$p(d \mid e=1,b)p(b \mid s)p(x \mid e=1)p(l=1 \mid s)p(s) \cdot 0.0104 +$$
$$p(d \mid e=0,b)p(b \mid s)p(x \mid e=0)p(l=0 \mid s)p(s) \cdot 0.9896$$

$$p(d) = \sum_{x,b \in \{0,1\}} p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1)p(l=1 \mid s=0)p(s=0) \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1)p(l=0 \mid s=0)p(s=0) \cdot 0.0104 +$$
$$p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1)p(l=1 \mid s=0)p(s=0) \cdot 0.0104 +$$
$$p(d \mid e=0,b)p(b \mid s=0)p(x \mid e=0)p(l=0 \mid s=0)p(s=0) \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1)p(l=1 \mid s=1)p(s=1) \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1)p(l=0 \mid s=1)p(s=1) \cdot 0.0104 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1)p(l=1 \mid s=1)p(s=1) \cdot 0.0104 +$$
$$p(d \mid e=0,b)p(b \mid s=1)p(x \mid e=0)p(l=0 \mid s=1)p(s=1) \cdot 0.9896$$

$$p(d) = \sum_{x,b \in \{0,1\}} p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.01 \cdot 0.5 \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.99 \cdot 0.5 \cdot 0.0104 +$$
$$p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.01 \cdot 0.5 \cdot 0.0104 +$$
$$p(d \mid e=0,b)p(b \mid s=0)p(x \mid e=0) \cdot 0.99 \cdot 0.5 \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.1 \cdot 0.5 \cdot 0.9896 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.9 \cdot 0.5 \cdot 0.0104 +$$
$$p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.1 \cdot 0.5 \cdot 0.0104 +$$
$$p(d \mid e=0,b)p(b \mid s=1)p(x \mid e=0) \cdot 0.9 \cdot 0.5 \cdot 0.9896$$

$$
\begin{aligned}
p(d) = \sum_{x,b \in \{0,1\}} & p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.004948 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.005148 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x \mid e=1) \cdot 0.000052 + \\
& p(d \mid e=0,b)p(b \mid s=0)p(x \mid e=0) \cdot 0.489852 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.04948 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.00468 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x \mid e=1) \cdot 0.00052 + \\
& p(d \mid e=0,b)p(b \mid s=1)p(x \mid e=0) \cdot 0.44532
\end{aligned}
$$

$$
\begin{aligned}
p(d) = \sum_{b \in \{0,1\}} & p(d \mid e=1,b)p(b \mid s=0)p(x=0 \mid e=1) \cdot 0.004948 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x=0 \mid e=1) \cdot 0.005148 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x=0 \mid e=1) \cdot 0.000052 + \\
& p(d \mid e=0,b)p(b \mid s=0)p(x=0 \mid e=0) \cdot 0.489852 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=0 \mid e=1) \cdot 0.04948 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=0 \mid e=1) \cdot 0.00468 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=0 \mid e=1) \cdot 0.00052 + \\
& p(d \mid e=0,b)p(b \mid s=1)p(x=0 \mid e=0) \cdot 0.44532 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x=1 \mid e=1) \cdot 0.004948 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x=1 \mid e=1) \cdot 0.005148 + \\
& p(d \mid e=1,b)p(b \mid s=0)p(x=1 \mid e=1) \cdot 0.000052 + \\
& p(d \mid e=0,b)p(b \mid s=0)p(x=1 \mid e=0) \cdot 0.489852 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=1 \mid e=1) \cdot 0.04948 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=1 \mid e=1) \cdot 0.00468 + \\
& p(d \mid e=1,b)p(b \mid s=1)p(x=1 \mid e=1) \cdot 0.00052 + \\
& p(d \mid e=0,b)p(b \mid s=1)p(x=1 \mid e=0) \cdot 0.44532
\end{aligned}
$$

$$
\begin{aligned}
p(d) = \sum_{b \in \{0,1\}} & p(d \mid e=1,b)p(b \mid s=0) \cdot 0.02 \cdot 0.004948 + \\
& p(d \mid e=1,b)p(b \mid s=0) \cdot 0.02 \cdot 0.005148 + \\
& p(d \mid e=1,b)p(b \mid s=0) \cdot 0.02 \cdot 0.000052 + \\
& p(d \mid e=0,b)p(b \mid s=0) \cdot 0.95 \cdot 0.489852 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.02 \cdot 0.04948 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.02 \cdot 0.00468 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.02 \cdot 0.00052 + \\
& p(d \mid e=0,b)p(b \mid s=1) \cdot 0.95 \cdot 0.44532 + \\
& p(d \mid e=1,b)p(b \mid s=0) \cdot 0.98 \cdot 0.004948 + \\
& p(d \mid e=1,b)p(b \mid s=0) \cdot 0.98 \cdot 0.005148 + \\
& p(d \mid e=1,b)p(b \mid s=0) \cdot 0.98 \cdot 0.000052 + \\
& p(d \mid e=0,b)p(b \mid s=0) \cdot 0.05 \cdot 0.489852 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.98 \cdot 0.04948 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.98 \cdot 0.00468 + \\
& p(d \mid e=1,b)p(b \mid s=1) \cdot 0.98 \cdot 0.00052 + \\
& p(d \mid e=0,b)p(b \mid s=1) \cdot 0.05 \cdot 0.44532
\end{aligned}
$$

$$
\begin{aligned}
p(d) = & p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.004948 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.005148 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.000052 + \\
& p(d \mid e=0, b=0)p(b=0 \mid s=0) \cdot 0.95 \cdot 0.489852 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.02 \cdot 0.04948 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.02 \cdot 0.00468 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.02 \cdot 0.00052 + \\
& p(d \mid e=0, b=0)p(b=0 \mid s=1) \cdot 0.95 \cdot 0.44532 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.004948 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.005148 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.000052 + \\
& p(d \mid e=0, b=0)p(b=0 \mid s=0) \cdot 0.05 \cdot 0.489852 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.98 \cdot 0.04948 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.98 \cdot 0.00468 + \\
& p(d \mid e=1, b=0)p(b=0 \mid s=1) \cdot 0.98 \cdot 0.00052 + \\
& p(d \mid e=0, b=0)p(b=0 \mid s=1) \cdot 0.05 \cdot 0.44532 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.004948 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.005148 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.000052 + \\
& p(d \mid e=0, b=1)p(b=1 \mid s=0) \cdot 0.95 \cdot 0.489852 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.02 \cdot 0.04948 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.02 \cdot 0.00468 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.02 \cdot 0.00052 + \\
& p(d \mid e=0, b=1)p(b=1 \mid s=1) \cdot 0.95 \cdot 0.44532 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.004948 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.005148 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.000052 + \\
& p(d \mid e=0, b=1)p(b=1 \mid s=0) \cdot 0.05 \cdot 0.489852 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.98 \cdot 0.04948 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.98 \cdot 0.00468 + \\
& p(d \mid e=1, b=1)p(b=1 \mid s=1) \cdot 0.98 \cdot 0.00052 + \\
& p(d \mid e=0, b=1)p(b=1 \mid s=1) \cdot 0.05 \cdot 0.44532
\end{aligned}
$$

$$
\begin{aligned}
p(d) = & 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.004948 + \\
& 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.005148 + \\
& 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.000052 + \\
& 0.1 \cdot 0.7 \cdot 0.95 \cdot 0.489852 + \\
& 0.7 \cdot 0.4 \cdot 0.02 \cdot 0.04948 + \\
& 0.7 \cdot 0.4 \cdot 0.02 \cdot 0.00468 + \\
& 0.7 \cdot 0.4 \cdot 0.02 \cdot 0.00052 + \\
& 0.1 \cdot 0.4 \cdot 0.95 \cdot 0.44532 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.004948 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.005148 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.000052 + \\
& 0.1 \cdot 0.7 \cdot 0.05 \cdot 0.489852 + \\
& 0.7 \cdot 0.4 \cdot 0.98 \cdot 0.04948 + \\
& 0.7 \cdot 0.4 \cdot 0.98 \cdot 0.00468 + \\
& 0.7 \cdot 0.4 \cdot 0.98 \cdot 0.00052 + \\
& 0.1 \cdot 0.4 \cdot 0.05 \cdot 0.44532 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.004948 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.005148 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.000052 + \\
& 0.8 \cdot 0.3 \cdot 0.95 \cdot 0.489852 + \\
& 0.9 \cdot 0.6 \cdot 0.02 \cdot 0.04948 + \\
& 0.9 \cdot 0.6 \cdot 0.02 \cdot 0.00468 + \\
& 0.9 \cdot 0.6 \cdot 0.02 \cdot 0.00052 + \\
& 0.8 \cdot 0.6 \cdot 0.95 \cdot 0.44532 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.004948 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.005148 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.000052 + \\
& 0.8 \cdot 0.3 \cdot 0.05 \cdot 0.489852 + \\
& 0.9 \cdot 0.6 \cdot 0.98 \cdot 0.04948 + \\
& 0.9 \cdot 0.6 \cdot 0.98 \cdot 0.00468 + \\
& 0.9 \cdot 0.6 \cdot 0.98 \cdot 0.00052 + \\
& 0.8 \cdot 0.6 \cdot 0.05 \cdot 0.44532
\end{aligned}
$$

$$p(d) = 0.4359706$$

Part 2

$$p(d = 1 \mid s = 1) = \frac{p(d = 1, s = 1)}{p(s = 1)} \tag{3}$$

$$p(d = 1, s = 1) = \sum_{x,e,t,l,b,a \in \{0,1\}} p(x, d = 1, e, t, l, b, a, s = 1) \tag{4}$$

$$\begin{aligned}
p(d = 1, s = 1) = & p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.02 \cdot 0.04948 + \\
& p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.02 \cdot 0.00468 + \\
& p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.02 \cdot 0.00052 + \\
& p(d = 1 \mid e = 0, b = 0)p(b = 0 \mid s = 1) \cdot 0.95 \cdot 0.44532 + \\
& p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.98 \cdot 0.04948 + \\
& p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.98 \cdot 0.00468 + \\
& p(d = 1 \mid e = 1, b = 0)p(b = 0 \mid s = 1) \cdot 0.98 \cdot 0.00052 + \\
& p(d = 1 \mid e = 0, b = 0)p(b = 0 \mid s = 1) \cdot 0.05 \cdot 0.44532 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.02 \cdot 0.04948 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.02 \cdot 0.00468 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.02 \cdot 0.00052 + \\
& p(d = 1 \mid e = 0, b = 1)p(b = 1 \mid s = 1) \cdot 0.95 \cdot 0.44532 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.98 \cdot 0.04948 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.98 \cdot 0.00468 + \\
& p(d = 1 \mid e = 1, b = 1)p(b = 1 \mid s = 1) \cdot 0.98 \cdot 0.00052 + \\
& p(d = 1 \mid e = 0, b = 1)p(b = 1 \mid s = 1) \cdot 0.05 \cdot 0.44532
\end{aligned}$$

$$
\begin{aligned}
p(d=1,s=1) = & 0.70.4 \cdot 0.02 \cdot 0.04948 + \\
& 0.70.4 \cdot 0.02 \cdot 0.00468 + \\
& 0.70.4 \cdot 0.02 \cdot 0.00052 + \\
& 0.10.4 \cdot 0.95 \cdot 0.44532 + \\
& 0.70.4 \cdot 0.98 \cdot 0.04948 + \\
& 0.70.4 \cdot 0.98 \cdot 0.00468 + \\
& 0.70.4 \cdot 0.98 \cdot 0.00052 + \\
& 0.10.4 \cdot 0.05 \cdot 0.44532 + \\
& 0.90.6 \cdot 0.02 \cdot 0.04948 + \\
& 0.90.6 \cdot 0.02 \cdot 0.00468 + \\
& 0.90.6 \cdot 0.02 \cdot 0.00052 + \\
& 0.80.6 \cdot 0.95 \cdot 0.44532 + \\
& 0.90.6 \cdot 0.98 \cdot 0.04948 + \\
& 0.90.6 \cdot 0.98 \cdot 0.00468 + \\
& 0.90.6 \cdot 0.98 \cdot 0.00052 + \\
& 0.80.6 \cdot 0.05 \cdot 0.44532
\end{aligned}
$$

$$p(d=1,s=1) = 0.276404$$

Therefore, by substituting in equation (3):

$$p(d=1 \mid s=1) = \frac{0.276404}{0.5} = 0.552808$$

Part 3

$$p(d=1 \mid s=0) = \frac{p(d=1,s=0)}{p(s=0)} \tag{5}$$

$$p(d=1,s=0) = \sum_{x,e,t,l,b,a \in \{0,1\}} p(x,d=1,e,t,l,b,a,s=0) \tag{6}$$

$$
\begin{aligned}
p(d=1,s=0) = & p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.004948 + \\
& p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.005148 + \\
& p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.02 \cdot 0.000052 + \\
& p(d=1 \mid e=0,b=0)p(b=0 \mid s=0) \cdot 0.95 \cdot 0.489852 + \\
& p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.004948 + \\
& p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.005148 + \\
& p(d=1 \mid e=1,b=0)p(b=0 \mid s=0) \cdot 0.98 \cdot 0.000052 + \\
& p(d=1 \mid e=0,b=0)p(b=0 \mid s=0) \cdot 0.05 \cdot 0.489852 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.004948 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.005148 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.02 \cdot 0.000052 + \\
& p(d=1 \mid e=0,b=1)p(b=1 \mid s=0) \cdot 0.95 \cdot 0.489852 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.004948 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.005148 + \\
& p(d=1 \mid e=1,b=1)p(b=1 \mid s=0) \cdot 0.98 \cdot 0.000052 + \\
& p(d=1 \mid e=0,b=1)p(b=1 \mid s=0) \cdot 0.05 \cdot 0.489852 +
\end{aligned}
$$

$$
\begin{aligned}
p(d=1,s=0) = & 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.004948 + \\
& 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.005148 + \\
& 0.7 \cdot 0.7 \cdot 0.02 \cdot 0.000052 + \\
& 0.1 \cdot 0.7 \cdot 0.95 \cdot 0.489852 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.004948 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.005148 + \\
& 0.7 \cdot 0.7 \cdot 0.98 \cdot 0.000052 + \\
& 0.1 \cdot 0.7 \cdot 0.05 \cdot 0.489852 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.004948 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.005148 + \\
& 0.9 \cdot 0.3 \cdot 0.02 \cdot 0.000052 + \\
& 0.8 \cdot 0.3 \cdot 0.95 \cdot 0.489852 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.004948 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.005148 + \\
& 0.9 \cdot 0.3 \cdot 0.98 \cdot 0.000052 + \\
& 0.8 \cdot 0.3 \cdot 0.05 \cdot 0.489852 +
\end{aligned}
$$

$$p(d=1,s=0) = 0.1595666$$

Therefore, by substituting in equation (5):

$$p(d = 1 \mid s = 0) = \frac{0.1595666}{0.5} = 0.3191332$$

Problem 3

Examine the following independence relations from the Chest Clinic Bayesian network.

a) $d \perp\!\!\!\perp s$ False

Independence of Dyspnea (d) and Smoking (s) $d \perp\!\!\!\perp s$.

- **Path 1: s - b - d**
  This path does not contain any colliders, therefore it is not blocked and $d \not\perp\!\!\!\perp s$
- **Path 2: s - l - e - d**
  This path also does not contain any colliders, therefore it is not blocked and $d \not\perp\!\!\!\perp s$

b) $s \perp\!\!\!\perp a$ True

Independence of Smoking (s) and Visiting Asia (a)

- **Path: s - l - e - t - a**
  This path is blocked by e because it is a collider, neither e nor any of its descendants is in the conditioning set. Therefore $d \perp\!\!\!\perp a$

c) $s \perp\!\!\!\perp a \mid e$ False

Conditional independence of Smoking (s) and Visiting Asia (a) given Either Tuberculosis or Lung Cancer (e)

- **Path: s - l - e - t - a**
  This path is not blocked by e because it is a collider that is in the conditioning set. Therefore $d \not\perp\!\!\!\perp a \mid e$

d) $s \perp\!\!\!\perp x \mid d$ False

Conditional Independence of Smoking (s) and Positive X-ray (x) Given Dyspnea (d)

- **Path 1: s - l - e - x**
  This path does not contain any colliders, but it is it is blocked by d, which is a descendant of e. This path cannot induce dependence.
- **Path 2: s - b - d - e - x**
  In this path d is a collider, but because it is in the conditioning set it does not block the path. Therefore $d \not\perp\!\!\!\perp x \mid d$

Problem 4

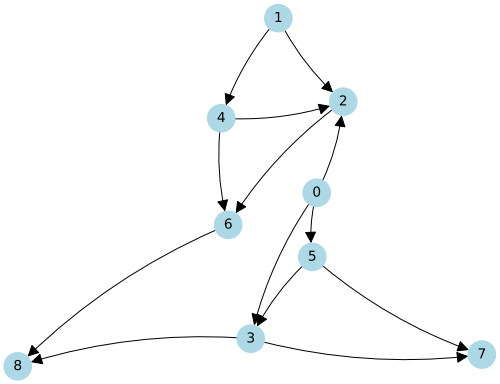

Figure 1: Belief network A

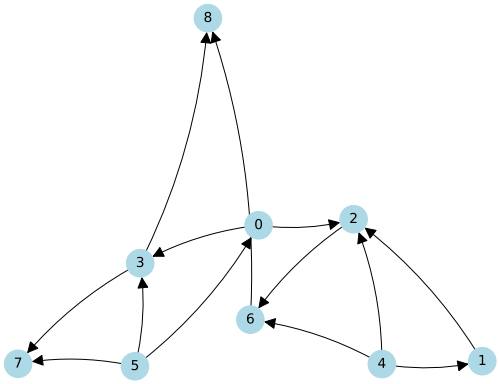

Figure 2: Belief network B

The two belief networks are Markov equivalent.

Problem 5

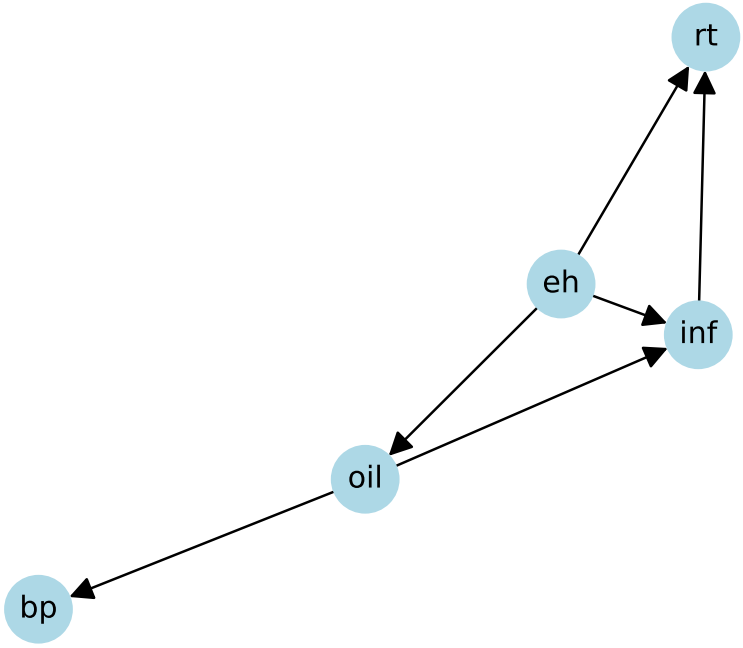

Figure 3: Belief network that models the relation between the variables oil, inf, eh, bp, rt.