Probabilistic Modeling and Reasoning

Homework — 5

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Problem 1

After renaming the variables as following, the probability we want to calculate is:

Old variable name	New variable name
fuse assembly malfunction	x_0
drum unit	x_1
toner out	x_2
poor paper quality	x_3
worn roller	x_4
burning smell	x_5
poor print quality	x_6
wrinkled pages	x_7
multiple pages fed	x_8
paper jam	x_9

Table 1: Mapping of original variable names to new variable names.

$$p(x_1 = 1 \mid x_7 = 0, x_5 = 0, x_3 = 1) = 0.5075$$

It was calculated in python utilizing the "pyAgrum" library in the Python script named *Problem1_EM.py*

Problem 2

The Python script is named *Gaussian_Mixture.py* and generates the output file named *em_output.txt*

Problem 3

The marginal likelihood is determined by summing over x_2:

$$p(x_1 = i) = \sum_{j=1}^{2} \theta_{ij}.$$

1. For the first matrix:

$$\theta^{(1)} = \begin{pmatrix} 0.3 & 0.3 \\ 0.2 & 0.2 \end{pmatrix}$$

we compute:

$$p(x_1 = 1) = \theta_{11} + \theta_{12} = 0.3 + 0.3 = 0.6, \quad p(x_1 = 2) = \theta_{21} + \theta_{22} = 0.2 + 0.2 = 0.4.$$

2. For the second matrix:

$$\theta^{(2)} = \begin{pmatrix} 0.2 & 0.4 \\ 0.4 & 0 \end{pmatrix}$$

we compute:

$$p(x_1 = 1) = \theta_{11} + \theta_{12} = 0.2 + 0.4 = 0.6, \quad p(x_1 = 2) = \theta_{21} + \theta_{22} = 0.4 + 0 = 0.4.$$

Since the marginal probabilities $p(x_1 = i)$ are identical for both matrices ($p(x_1 = 1) = 0.6$ and $p(x_1 = 2) = 0.4$), both $\theta^{(1)}$ and $\theta^{(2)}$ yield the same marginal likelihood score.

Problem 4

Output of the "EM" function (implemented in *EM.m*) for input EM(1.9,0.2), where 1.9 is the starting value for θ and 0.2 is the starting value for $q(h = 2)$.

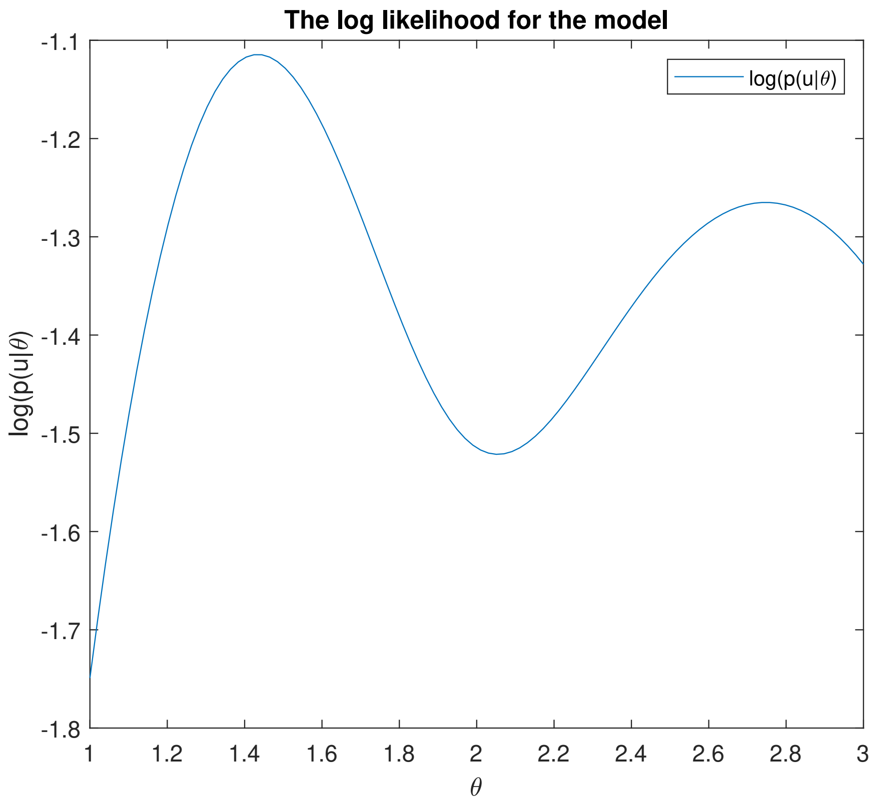

Figure 1: The log likelihood

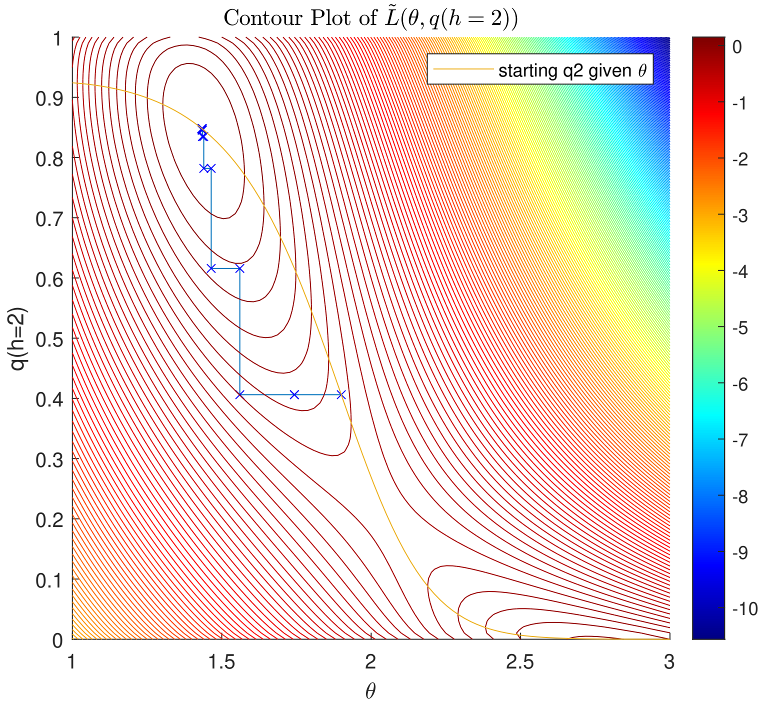

Figure 2: Convergence using 1.9 for the starting value for θ

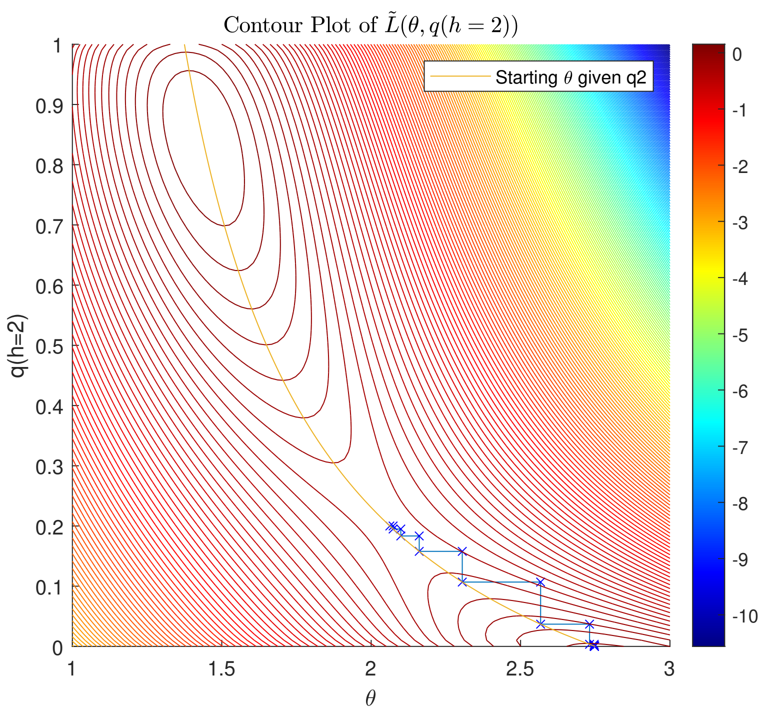

Figure 3: Convergence using 0.2 for the starting value for $q(h=2)$