Nick G. Toth
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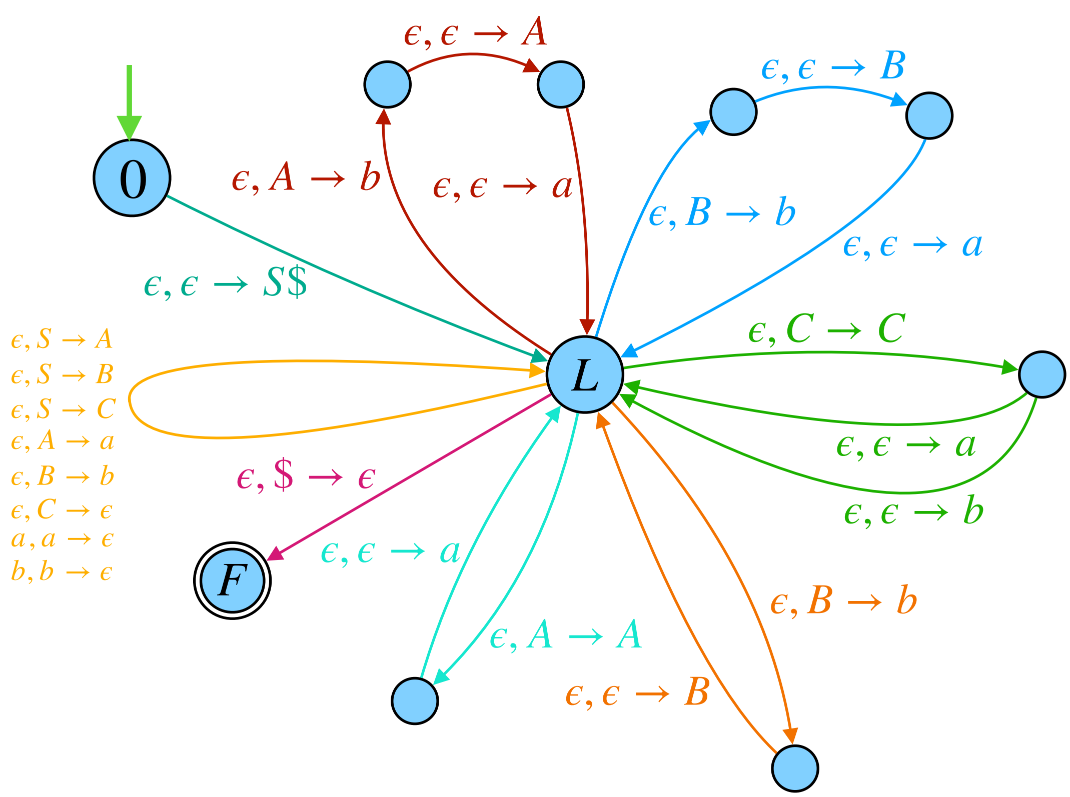
CIS 420
Assignment IV

Exercise 1

Draw a PDA diagram for the complement of the language $L := \{a^n b^n : n \geq 0\}$.

Let $R = \begin{cases} S \to A \,|\, B \,|\, C \\ A \to aAb \,|\, aA \,|\, a \\ B \to aBb \,|\, Bb \,|\, b \\ C \to \epsilon \,|\, aC \,|\, bC \end{cases}$.

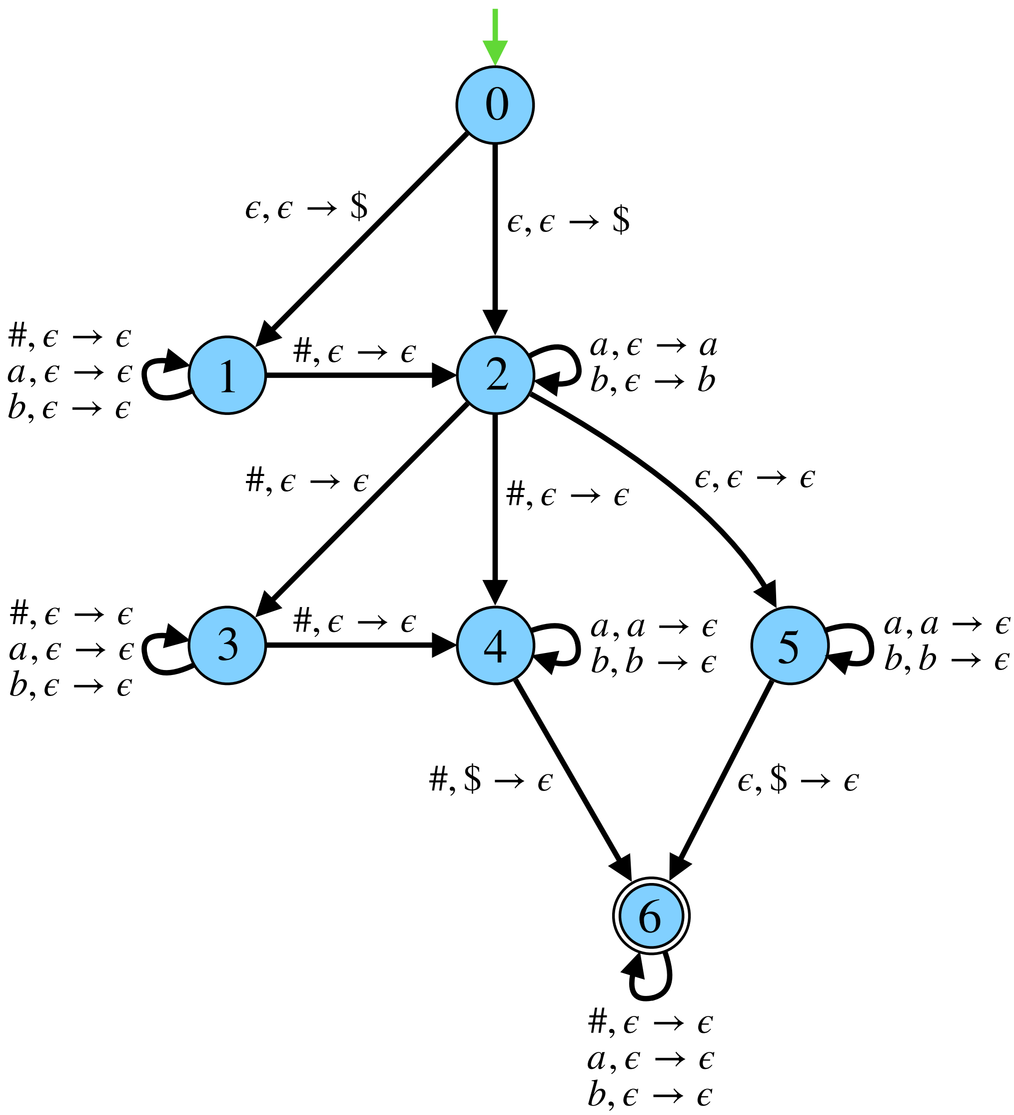
R is a set of rules for the grammar of L, so we can apply the algorithm given in theorem 2.12/20 to R to obtain a PDA for L.



Exercise 2

Draw a PDA diagram for the language L defined by

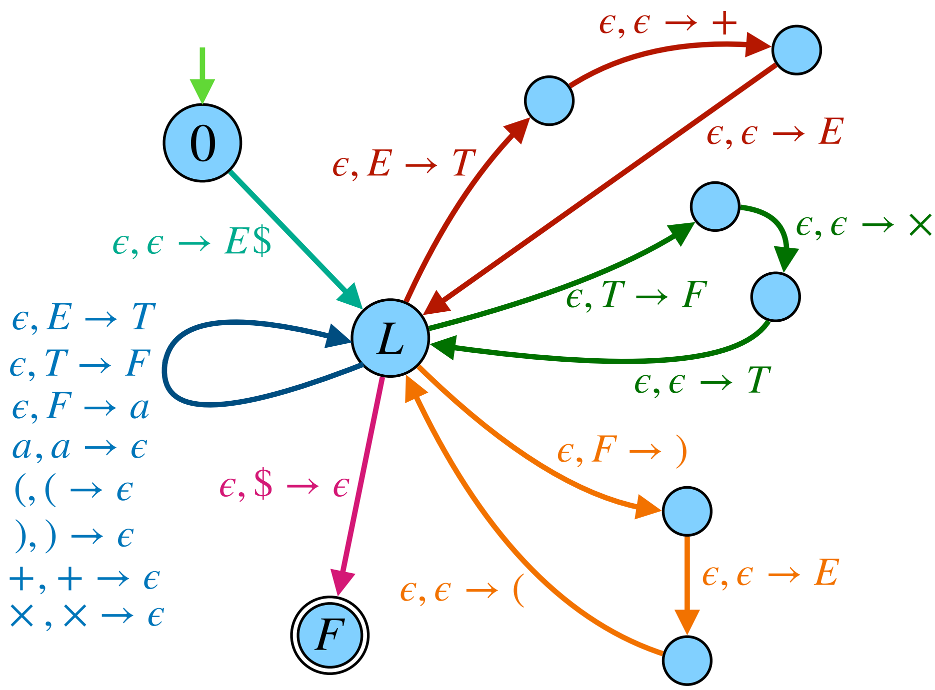
$$\{x_1\#x_2\#\ldots\#x_k : k \geq 1,\ x_i \in \{a,b\}^*,\ \text{and } x_i = x_j^R \text{ for some } i \text{ and } j\,.\}.$$



Exercise 3 (Sipser 2.11)

Convert grammar G_4 from exercise 2.1 into a PDA using the procedure from theorem 2.12/20.

$$
\begin{aligned}
&E \rightarrow E + T \mid T \\
&T \rightarrow T \times F \mid F \\
&F \rightarrow (E) \mid a
\end{aligned}
$$



Exercise 4 (Sipser 2.14)

Convert the following CFG into an equivalent CFG in Chomsky Normal Form using the procedure given in theorems 2.6 and 2.9.

$$
G_0 = \begin{cases} A \rightarrow BAB \mid B \mid \epsilon \\ B \rightarrow 00 \mid \epsilon \end{cases}
$$

Step 1 - Add a new start variable S_0 to G_0.

$$G_1 = \begin{cases} S_0 \to A \\ A \to BAB \mid B \mid \epsilon \\ B \to 00 \mid \epsilon \end{cases}$$

Step 2 - Eliminate $B \to \epsilon$ from G_1.

$$G_2 = \begin{cases} S_0 \to A \\ A \to BA \mid AB \mid BAB \mid B \mid \epsilon \\ B \to 00 \end{cases}$$

Step 3 - Eliminate $A \to \epsilon$ from G_2.

$$G_3 = \begin{cases} S_0 \to A \mid \epsilon \\ A \to BA \mid AB \mid BAB \mid B \\ B \to 00 \end{cases}$$

Step 4 - Eliminate $A \to B$ from G_3.

$$G_4 = \begin{cases} S_0 \to A \mid \epsilon \\ A \to BA \mid AB \mid BAB \mid 00 \\ B \to 00 \end{cases}$$

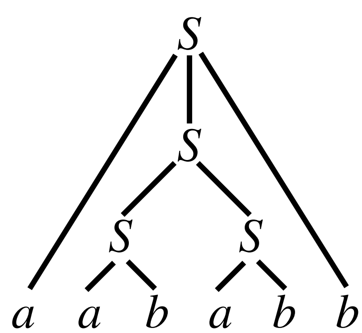
Step 5 - Eliminate $S \to BAB$ from G_4.

$$G_5 = \begin{cases} S_0 \to BA \mid AB \mid BC \mid DD \mid \epsilon \\ A \to BA \mid AB \mid BC \mid DD \\ B \to DD \\ C \to AB \\ D \to 0 \end{cases}$$

Exercise 5 (Sipser 2.15)

Give a counter-example to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generate by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \to SS$ and call the resulting grammar G'. This grammar is supposed to generate $A*$.

Consider the language $\left\{a^n b^n : n \geq 0\right\}$ which is generated by the grammar G whose rules are $R := S \to aSb \,|\, \epsilon$. Now consider the grammar G' obtained by adding the rule $S \to SS$ to R in G. Observe that the string $aababb$ is an element of $L(G')$



But $aababb \notin L(G')*$, so clearly $L(G') \neq L(G)*$, and thus the construction fails to prove that the class of context free languages is closed under star.