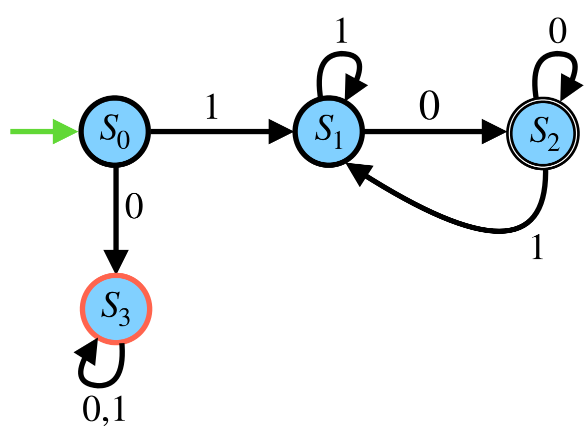
Nick G. Toth
Oct. 12, 2020
CIS 420
Assignment I

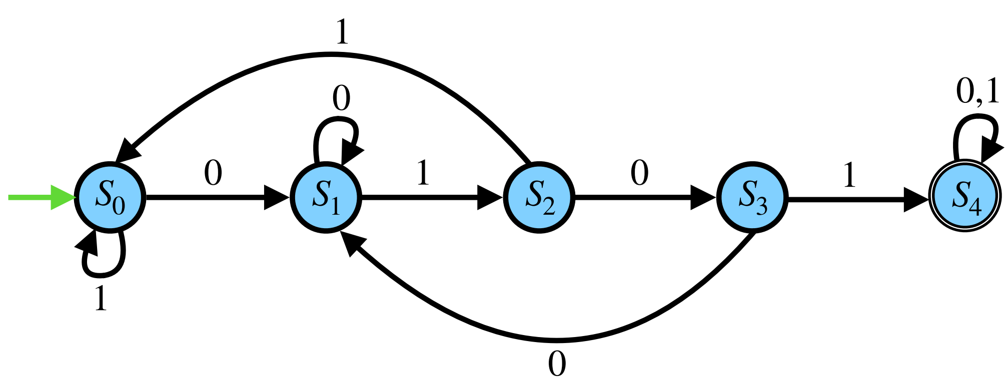
Notes: 1) Rejecting states are indicated by a red border.
2) Final states are indicated by a double border.
3) I did the graduate exercises, however, I am not in the graduate class, so feel free to ignore them. If you do check out those solutions, I'd appreciate any feedback!

Exercise 1.6

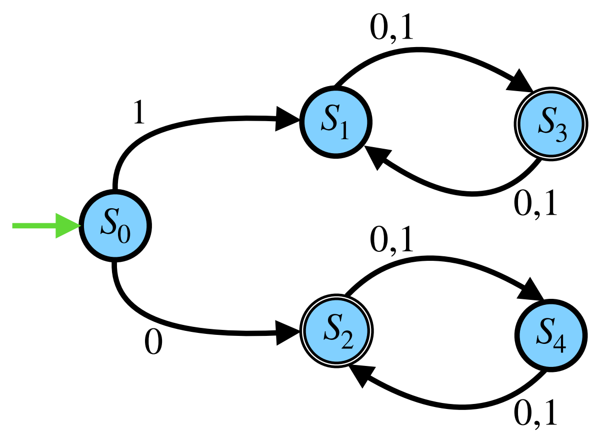
$a)$ **Language:** $\{w \mid w$ **begins with 1 and ends with 0**$\}$.



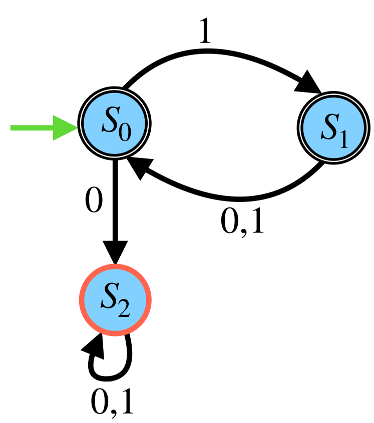
$c)$ **Language:** $\{w \mid w$ **contains the substring 0101**$\}$.



$e)$ **Language: $\{w \mid w$ starts with 0 and has odd length or starts with 1 and has even length $\}$.**



$i)$ **Language: $\{w \mid$ Every odd position of w is 1 $\}$.**



Exercise 1.18

$a)$ **Language: $\{w \mid w$ begins with 1 and ends with 0$\}$.**

Regular Expression: $1(0 \cup 1)^*0$.

$c)$ **Language: $\{w \mid w$ contains the substring 0101$\}$.**

Regular Expression: $(0 \cup 1)^*0101(0 \cup 1)^*$.

$e)$ **Language: $\{w \mid w$ starts with 0 and has odd length or starts with 1 and has even length$\}$.**

Regular Expression: $0(00 \cup 01 \cup 10 \cup 11)^* \cup 1(00 \cup 01 \cup 10 \cup 11)^*(0 \cup 1)$.

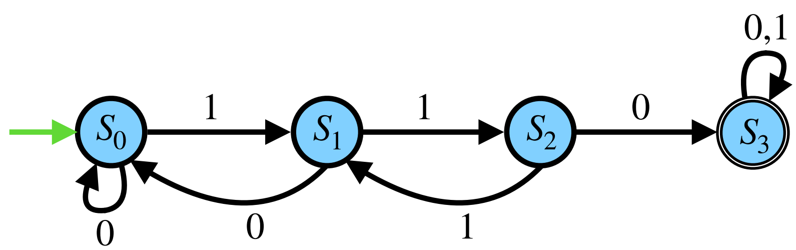
$i)$ **Language: $\{w \mid$ Every odd position of w is 1$\}$.**

Regular Expression: $\epsilon \cup (((1(0 \cup 1))^*) \cup ((1(00 \cup 01 \cup 10 \cup 11))^*) \cup 1)$.

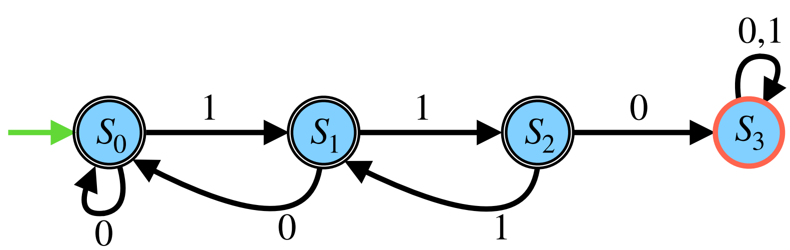
Graduate Exercises (for fun)

$1.6\,f\,)$ **Language:** $L = \{w \,|\, w$ **does not contain the substring 110** $\}$.

We begin by designing a DFA for the language $L^c := \{w \,|\, w$ contains the substring 110 $\}$.

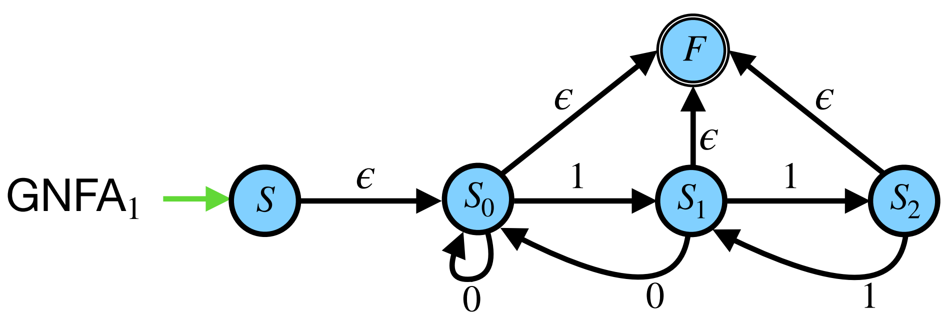


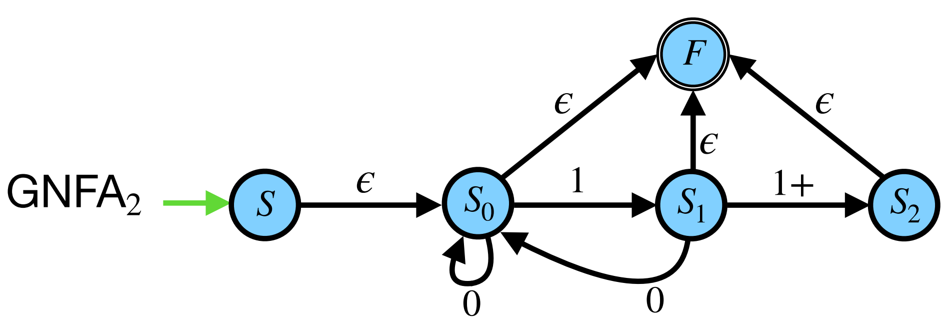
Now we can swap the accepting and non-accepting states of the DFA for L^c to obtain a DFA for the complementary language, namely $(L^c)^c = L$.



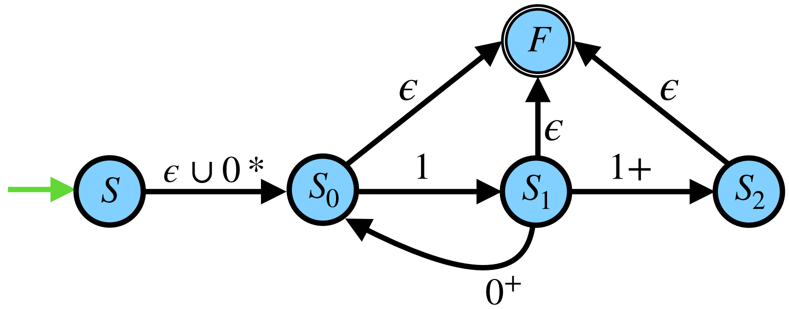
$1.18\,f\,)$ **Language:** $\{w \,|\, w$ **does not contain the substring 110** $\}$.

We can find a regular expression for this language by constructing a GNFA from the DFA produced in $1.6\,f$, and reducing it.

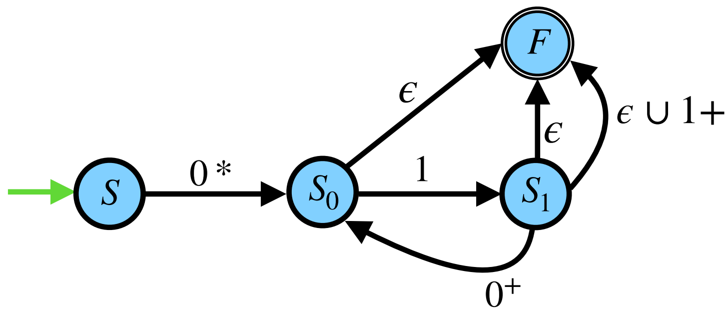




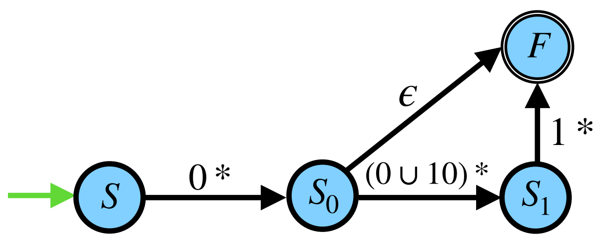
GNFA$_3$



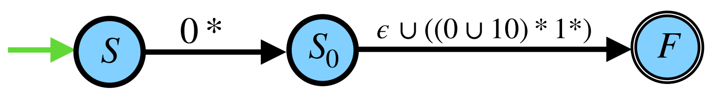
GNFA$_4$



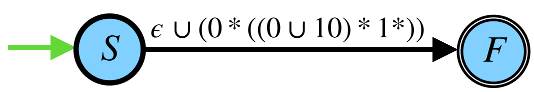
GNFA$_5$



GNFA$_6$



GNFA$_7$



Assuming I didn't make any mistakes, we can recognize the given language with the regular expression $\epsilon \cup (0*((0 \cup 10)*1*))$. Reduced further: $(0 \cup 10)*1*$.